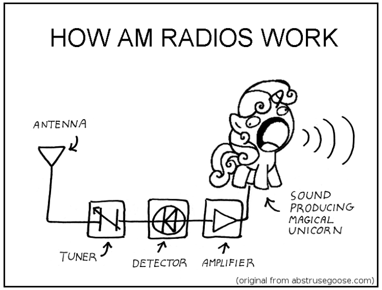
ECE 210 (AL2)

Chapter 8

Modulation and AM Radio



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https://abstrusegoose.com/

Chapter objectives

- Understand modulation
- Understand coherent demodulation of AM signals
- Understand envelope detection of AM signals
- Understand how a superheterodyne receiver with envelope detection works

- Properties of Fourier transform
  - Frequency shift

Recall time shift:

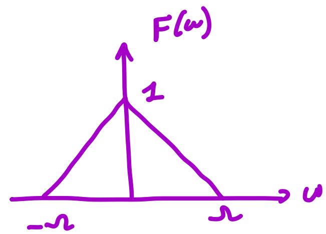
$f(t) \Leftrightarrow F(\omega)$ same sign!

$f(t-t_0) \Leftrightarrow F(\omega)\, e^{j\omega(-t_0)}$

$$f(t) \leftrightarrow F(\omega)$$ opposite sign!

$$g(t) = f(t)e^{j\omega_c t} \leftrightarrow G(\omega) = ?\ F(\omega - \omega_c)$$

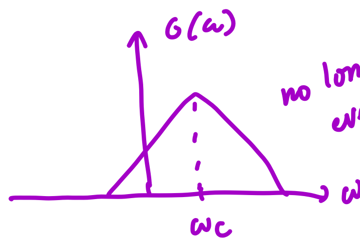
time-varying, specific freq.



real $f(t)$

$F^*(\omega) = F(-\omega)$

$|F(\omega)|$ even



no longer even if $g(t)$ is not real

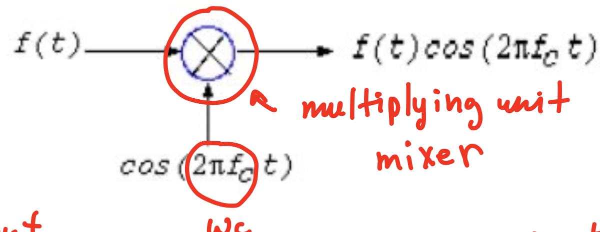
$g(t) = f(t)e^{j\omega_c t}$

- Properties of Fourier transform-cont
  - Modulation

$\omega = 2\pi f$

$$f(t) \leftrightarrow F(\omega)$$

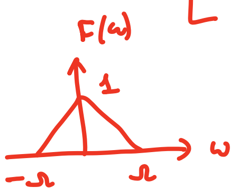
$$x(t) = f(t)\cos(\omega_c t) \leftrightarrow X(\omega) = ? \quad \frac{1}{2}F(\omega-\omega_c) + \frac{1}{2}F(\omega+\omega_c)$$

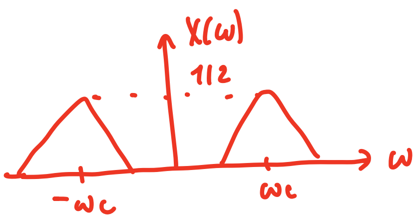


$$f(t)\cdot\left[\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\right] = \frac{1}{2}f(t)\,e^{j\omega_c t} + \frac{1}{2}f(t)\,e^{-j\omega_c t}$$

$$\downarrow \mathcal{F}$$

$$\frac{1}{2}F(\omega-\omega_c) + \frac{1}{2}F(\omega+\omega_c)$$





Multiplication by $\cos(\omega_c t)$ is called modulation.

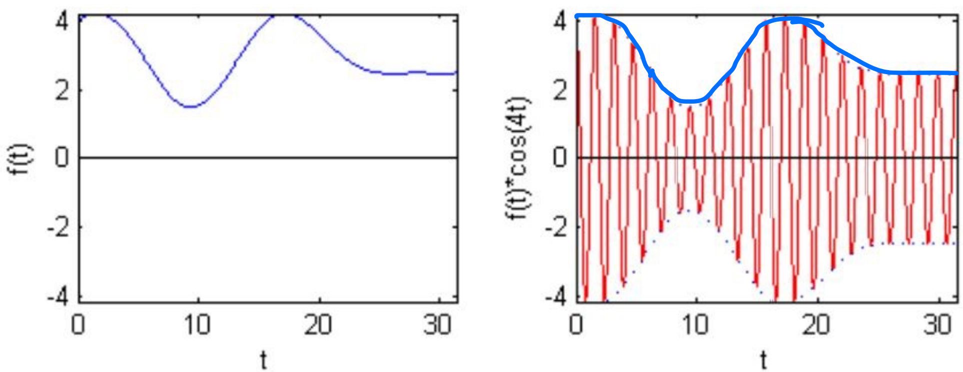
- Properties of Fourier transform-cont
  - Modulation

It's carrying f(t) via its amplitude

$$f(t) \leftrightarrow F(\omega)$$

carrier

$$x(t) = f(t)\cos(\omega_c t) \leftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$



f(t) is modulating the amplitude of $\cos(\omega_c t)$ → amplitude modulation (AM)

- Modulation
  - Why modulate?

1. Antenna length

   Signal wavelength: $\lambda = \frac{c}{f_c}$

   Antenna length for efficient transmission: $L > \frac{\lambda}{4} = \frac{c}{4f_c}$

   Audio bandwidth: ≈ 15KHz ⇒ L > 5Km
   AM radio: (WILL) 580KHz ⇒ L > 130m
   FM radio: 100MHz ⇒ L > 75cm
   Satellite: 10GHz ⇒ L > 7.5mm

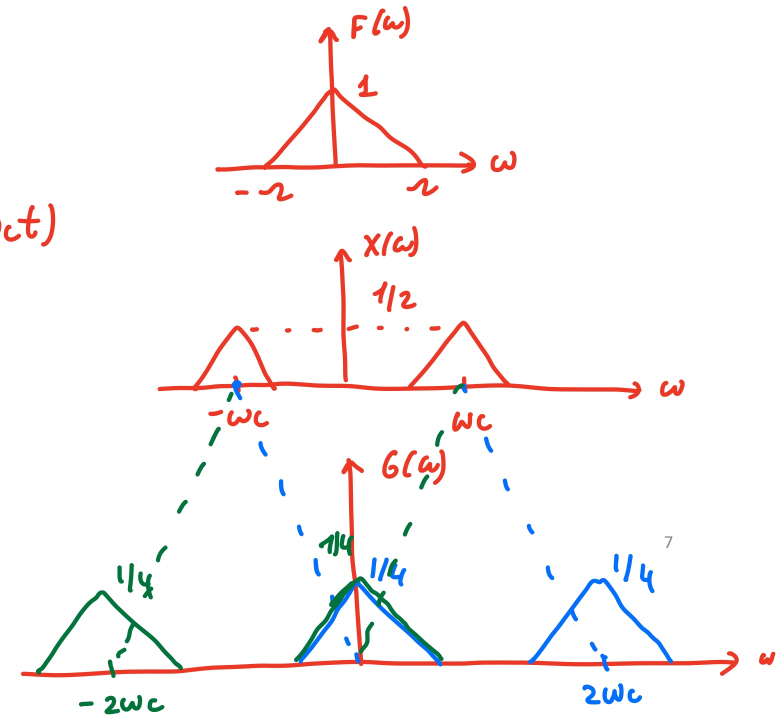
2. Available bandwidth
   Can't all transmit at baseband
   Transmitters are assigned frequency bands

- Coherent demodulation of AM signals
  - How to demodulate?

$$x(t) = \underbrace{f(t)\cos(\omega_c t)}_{x(t)} \longleftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$

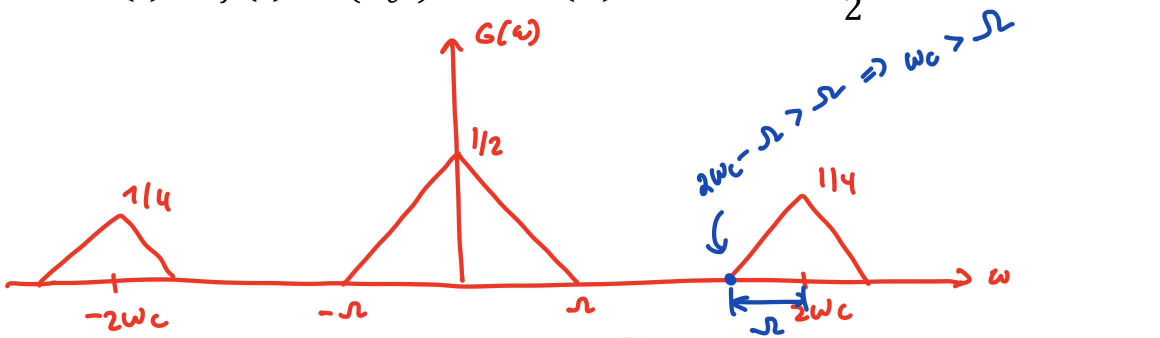
$$\cos(\omega_c t) \longrightarrow \otimes$$

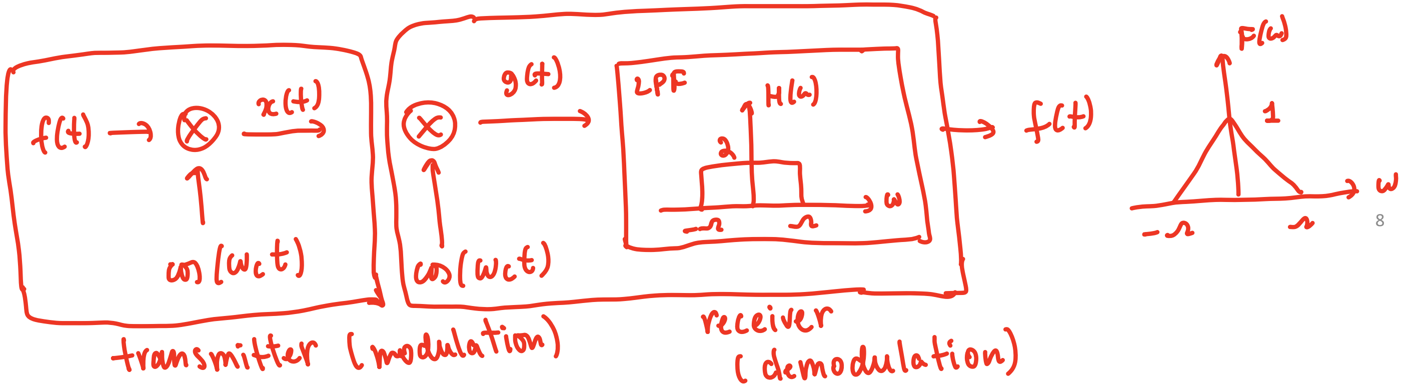
$$g(t) = x(t)\cos(\omega_c t)$$



- Coherent demodulation of AM signals-cont
  - How to demodulate?

$$x(t) = f(t)\cos(\omega_c t) \Leftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$





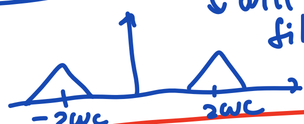
- Coherent demodulation of AM signals-cont
  - What if the channel is not ideal, e.g. there is a time delay?

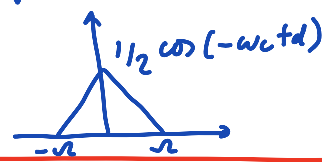
$$\cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$$

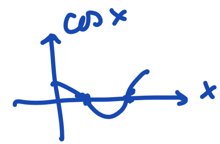
$$g(t) = f(t-t_d)\cos(w_c(t-t_d))\cos(w_c t) =$$

$$= f(t-t_d)\left[\frac{\cos(2w_c t - w_c t_d) + \cos(-w_c t_d)}{2}\right] =$$

$$= \underbrace{\frac{1}{2} f(t-t_d)\cos(2w_c t - w_c t_d)}_{\downarrow \text{ will be filtered by low-pass filter}} + \underbrace{\frac{1}{2} f(t-t_d)\overbrace{\cos(-w_c t_d)}}$$







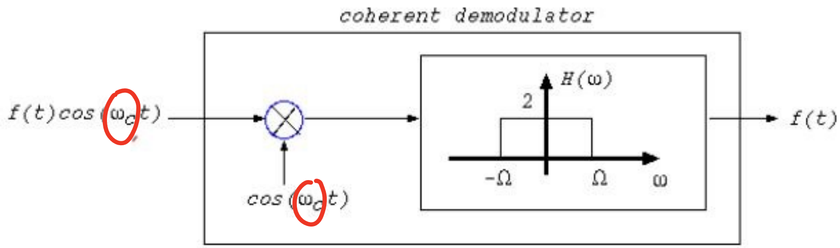
$$g(t) = f(t)\cos(w_c t)\cdot\cos(w_c t) = f(t)\cos^2(w_c t) = f(t)\left[\frac{1+\cos(2w_c t)}{2}\right] =$$

$$= \frac{1}{2}f(t) + \overbrace{\frac{1}{2}f(t)\cos(2w_c t)}$$

$$\downarrow \mathcal{F}$$

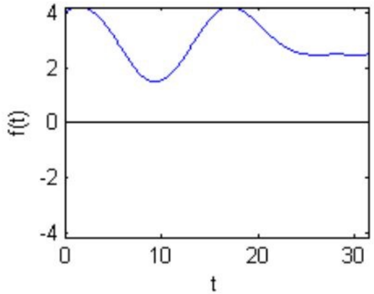
$$\frac{1}{2}F(\omega) + \frac{1}{2}\left[\frac{F(\omega - 2w_c) + F(\omega + 2w_c)}{2}\right]$$

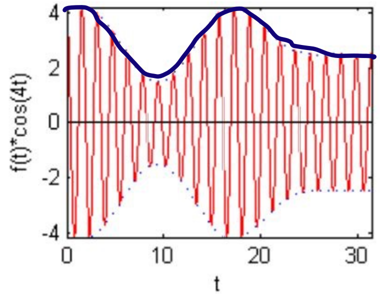
- Coherent demodulation of AM signals-cont



- Needs same phase at modulator and at demodulator.

- Envelope detection of AM signals





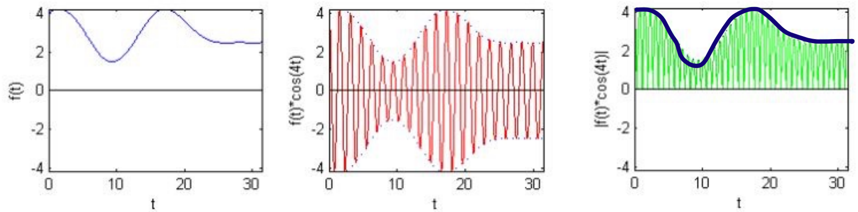
envelope

  - How to recover the envelope?

full-wave rectifier

$$x(t) = f(t)\cos(\omega_c t) \longrightarrow \boxed{|\cdot|} \longrightarrow |x(t)|$$

- Envelope detection of AM signals-cont



$|a+b| \neq |a|+|b|$

not linear!

  - How to recover the envelope?

$$|x(\overset{t-t_d}{t})| = |\underbrace{f(\overset{t-t_d}{t})}_{>0} \cos(\omega_c \overset{t-t_d}{t})| = f(\overset{t-t_d}{t})\,|\cos(\omega_c \overset{t-t_d}{t})| =$$

>0 assume for now

$$= f(\overset{t-t_d}{t})\left[\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(2\omega_c n \overset{t-t_d}{t} + \theta_n)\right] =$$

$$= \underbrace{\frac{c_0}{2} f(\overset{t-t_d}{t})} + \cancel{\sum_{n=1}^{\infty} c_n f(t) \cos(2\omega_c n t + \theta_n)}$$

will be filtered out

$$\downarrow \mathcal{F}$$

$$\frac{c_0}{2} F(\omega) + \sum_{n=1}^{\infty} c_n \left[\frac{F(\omega - 2n\omega_c)\, e^{j\theta_n} + F(\omega + 2n\omega_c)\, e^{-j\theta_n}}{2}\right]$$

$\cos(\omega_c t)$

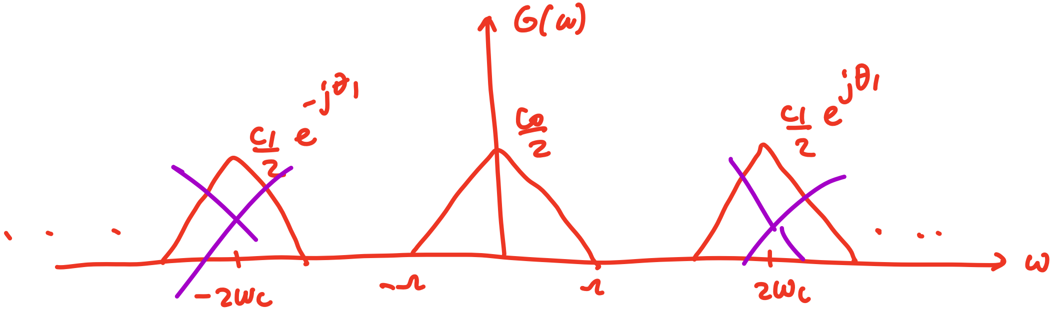
$|\cos(\omega_c t)|$

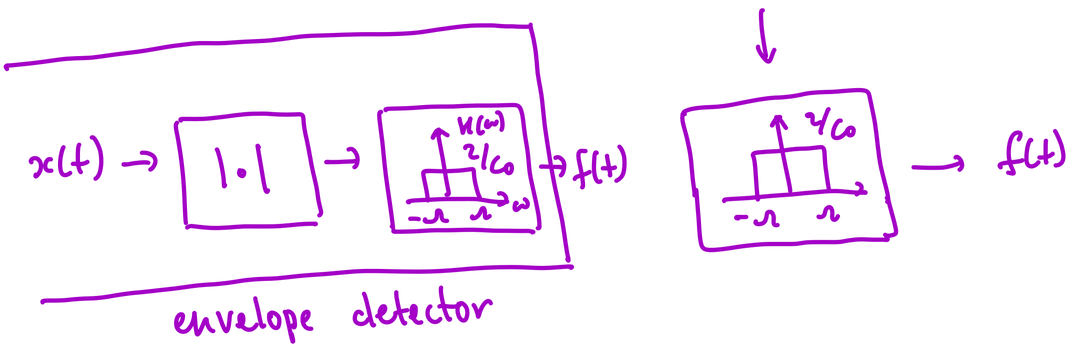
periodic

$\omega_0 = 2\omega_c$

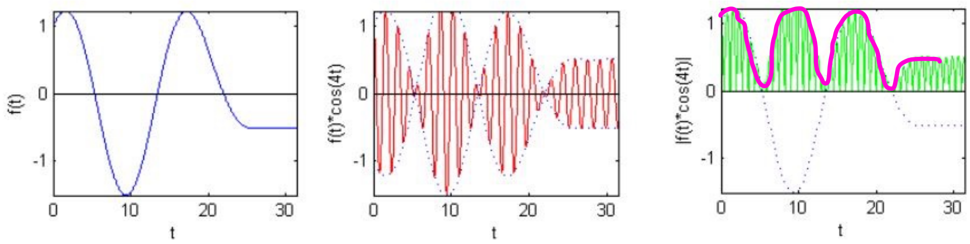
- Envelope detection of AM signals-cont
  - How to recover the envelope?

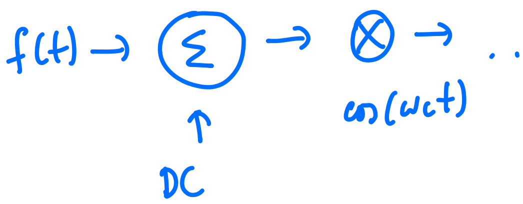
$$|x(t)| = g(t) = f(t)|\cos(\omega_c t)| = \frac{c_0}{2} f(t) + \sum_{n=1}^{\infty} c_n f(t) \cos(n2\omega_c t + \theta_n)$$





- Envelope detection of AM signals-cont
  - What if *f(t)* ≱ 0?

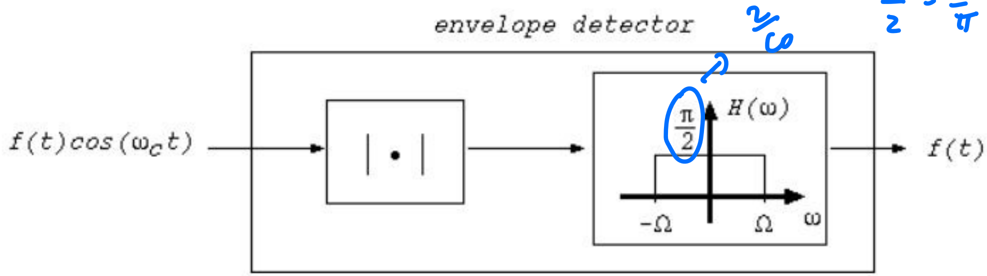




- Envelope detection of AM signals-cont
  - What if the channel is not ideal, e.g. time delay?

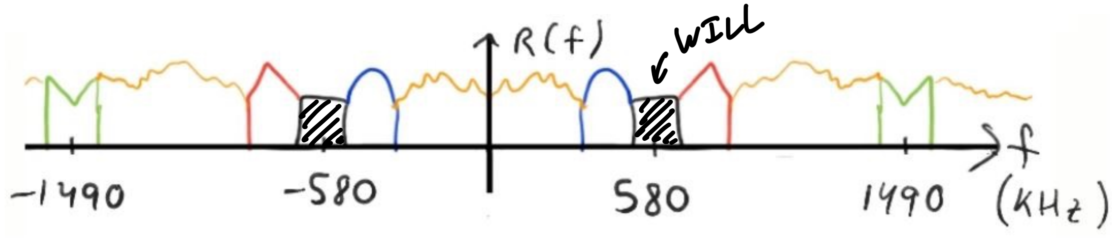
- Envelope detection of AM signals-cont

$\frac{\omega}{2} = \frac{2}{\pi}$



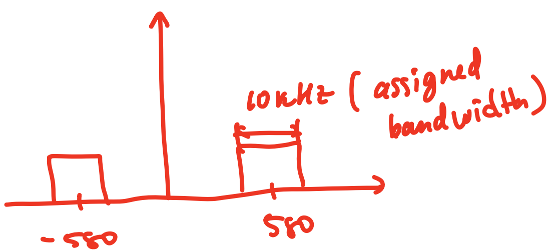
- It is not linear!

- Superheterodyne AM receiver with envelope detection
  - Real wireless transmission has signals across a broad frequency spectrum:



  - For envelope detection to work, need to isolate signal, how?

Need to get our signal by itself, so that envelope detection works.



A BPF would achieve this, but needs to be selective and tunable => expensive!

We will do it in 3 cheaper steps :)

- Superheterodyne AM receiver with envelope detection-cont
  - Heterodyne: demodulate to a lower carrier frequency
    - Typically, f_{IF} = 455KHz (commercial AM in U.S.)
    - Problem?

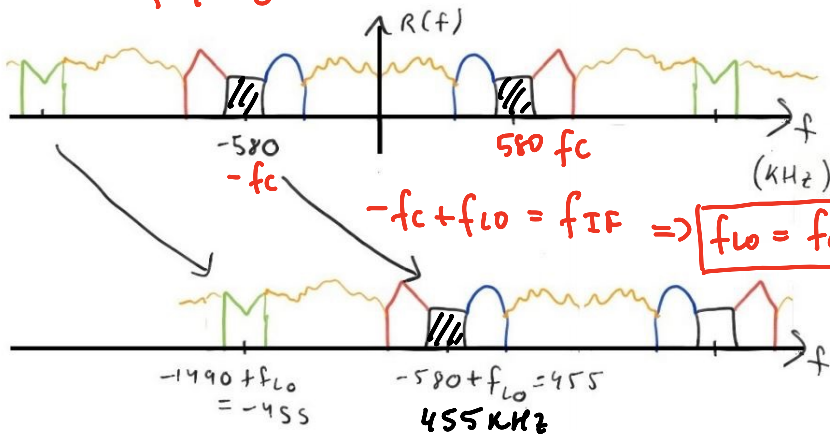
← intermediate frequency

$r(t) \rightarrow \otimes$

$\uparrow$

$\cos(\omega_{LO} t)$

↑ local oscillator

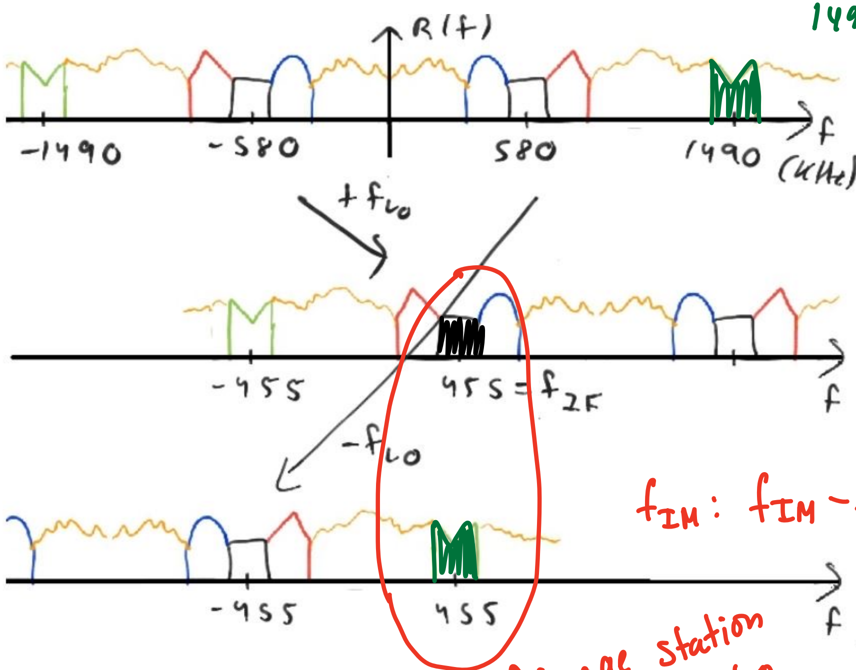


$-f_c + f_{LO} = f_{IF} \Rightarrow \boxed{f_{LO} = f_c + f_{IF}} = 580 + 455 =$

$= 1035$ kHz

- Superheterodyne AM receiver with envelope detection-cont

$1490 - 1035 = 455\,\text{kHz}$



$f_{LO} = f_{IF} + f_c$

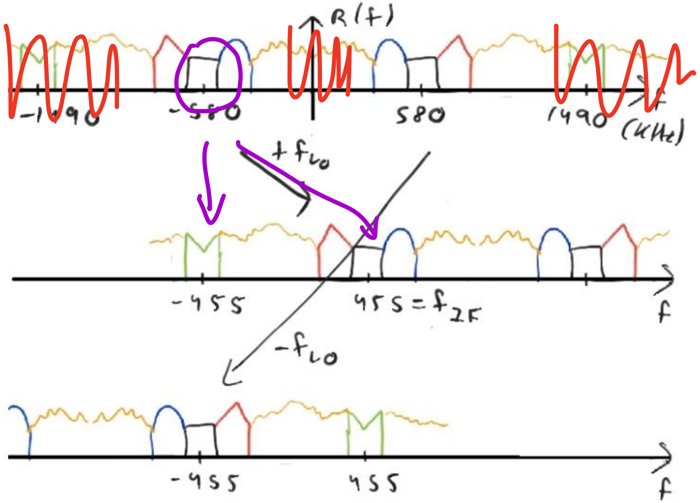
f_{IM}: $f_{IM} - f_{LO} = f_{IF} \Rightarrow$

$f_{IM} = f_{IF} + f_{LO} =$

$= \boxed{2f_{IF} + f_c = f_{IM}}$

image station (lands on top of our station)

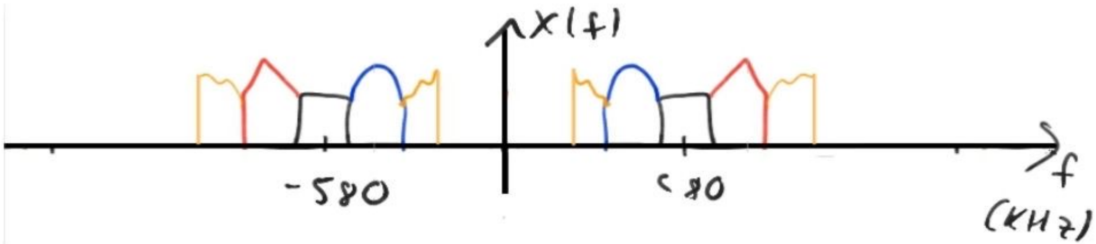
- Superheterodyne AM receiver with envelope detection-cont



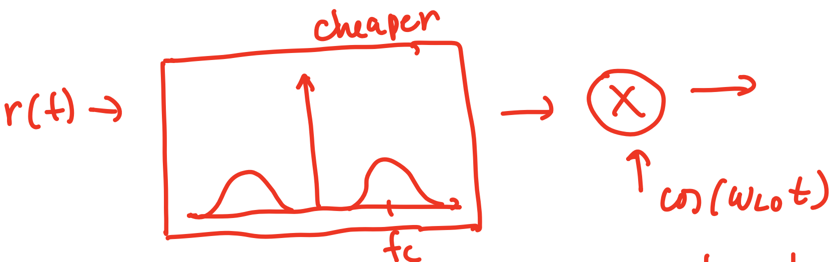
- Image station problem

  How to fix it? Preselector BPF

- Superheterodyne AM receiver with envelope detection-cont
  - Real wireless transmission has signals across a broad frequency spectrum:

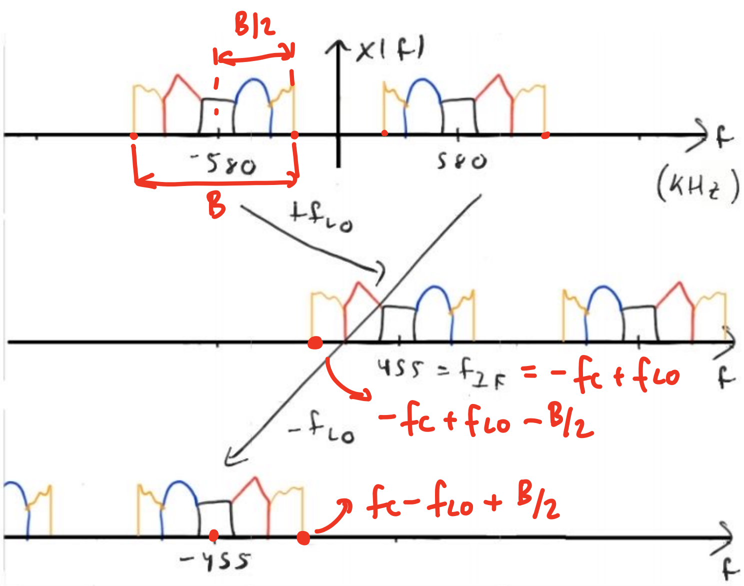


  - Bandwidth of filter?



Preselector BPF : tunable , not selective (so wide)

- Superheterodyne AM receiver with envelope detection-cont



$f_c - f_{LO}$

$$f_c - f_{LO} + \frac{B}{2} < -f_c + f_{LO} - \frac{B}{2} \Rightarrow B < 2\underbrace{(f_{LO} - f_c)}_{f_{IF}} = 2f_{IF}$$

$$B \approx 1\ \text{MHz} \gg 10\ \text{kHz}$$

- Superheterodyne AM receiver with envelope detection-cont

high-LO standard

  - Should we use $f_{LO} = f_c + f_{IF}$ or $f_{LO} = f_c - f_{IF}$?

cheaper!

    - If we use $f_{LO} = f_c + f_{IF}$

      $\underbrace{f_{LO} \in [995, 2155] \text{ KHz}}$

      $\frac{f_{LO,max}}{f_{LO,min}} \simeq 2$

    - If we use $f_{LO} = f_c - f_{IF}$

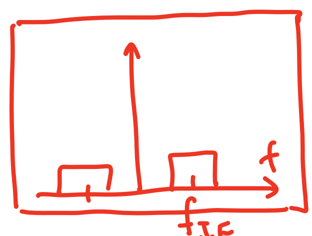
      $\underbrace{f_{LO} \in [85, 1245] \text{ KHz}}$

      $\frac{f_{LO,max}}{f_{LO,min}} \simeq 15$ more demanding design

Now, signal is at 455 KHz

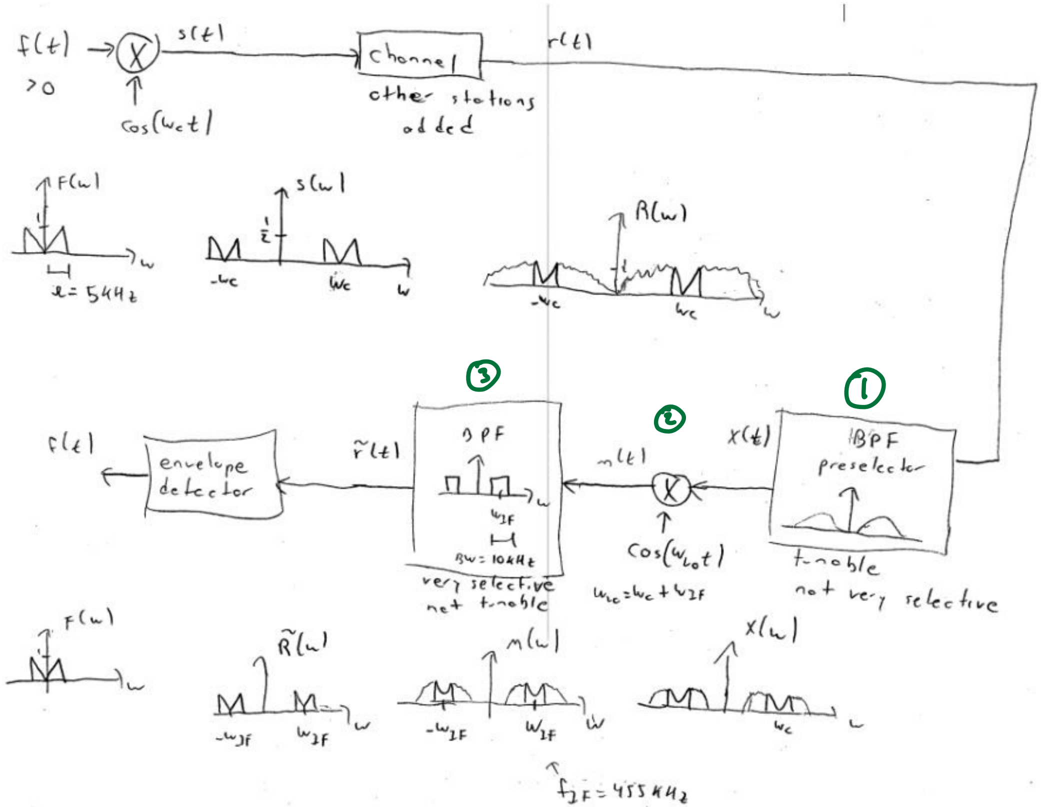
IF filter

$\rightarrow$



$\rightarrow$ ready for the envelope detector ¿

BPF: not tunable, but very selective


f(t)
>0
s(t)
cos(wct)
channel
other stations added
r(t)
F(w)
1
Ω = 5kHz
S(w)
1/2
-wc
wc
w
R(w)
1
①
②
③
BPF
preselector
tunable
not very selective
x(t)
m(t)
cos(wLOt)
wLO=wc+wIF
BPF
wIF
BW=10kHz
very selective
not tunable
r̃(t)
envelope detector
f(t)
F(w)
R̃(w)
-wIF
wIF
m(w)
x(w)
fIF=455kHz

Chapter objectives

- Understand modulation in time and frequency
- Understand coherent demodulation of AM signals
- Understand envelope detection of AM signals
- Understand how a superheterodyne receiver with envelope detection works