• s-domain analysis of LTIC systems - Example # 17

- Consider an LTIC system described by the following ODE:

$$\frac{d^2}{dt^2}y + 5\frac{d}{dt}y + 4y = 2f(t)$$

where

- f(t) = u(t)
- y (0^-) = 1
- y' (0^-) = 0

√ • Determine the characteristic poles and characteristic modes of the system

√ • Determine $\widehat{H}(s)$ and h(t)

√ • Determine if the system is BIBO stable

√ • Determine $y_{ZI}(t), y_{ZS}(t)$ and y(t)

- s-domain analysis of LTIC systems - Example # 17-cont
  - Consider an LTIC system described by the following ODE:

$$\frac{d^2}{dt^2}y + 5\frac{d}{dt}y + 4y = 2f(t) \rightarrow s^2+5s+4$$

  - Determine the characteristic poles and characteristic modes of the system

$$\downarrow \mathcal{L}$$

$$\left(s^2\hat{Y} - s^{2-1}y(0^-) - s^0 y'(0^-)\right) + 5\left(s\hat{Y} - y(0^-)\right) + 4\hat{Y} = 2\hat{F}$$

$$\hat{Y}\underbrace{\left(s^2+5s+4\right)}_{\text{charact. polynomial}} = 2\hat{F} + y(0^-)(s+5) + y'(0^-)$$

$$= (s+4)(s+1)$$

charact. poles: $p_1 = -4$; $p_2 = -1$

charact. modes: e^{-4t}, e^{-t}

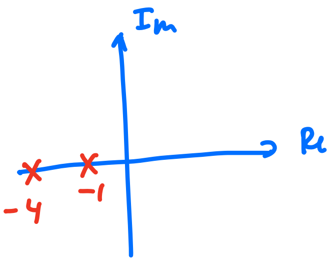
- s-domain analysis of LTIC systems - Example # 17-cont

$$\hat{Y}(s) = \frac{2\hat{F} + \cancel{y(0^-)(s+5) + \dot{y}(0^-)}}{(s+4)(s+1)} \quad = 0 \text{ for z-s.}$$

- Determine $\hat{H}(s)$ and h(t)
- Determine if the system is BIBO stable

$$\hat{Y}_{zs} = \hat{F}\cdot\hat{H} \Rightarrow \frac{2\hat{F}}{(s+4)(s+1)}$$

$\hat{H}(s)$



↑ BIBO stable

$$h(t) = \mathcal{L}^{-1}\{\hat{H}(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{(s+4)(s+1)}\right\} =$$

$$= \mathcal{L}^{-1}\left\{\frac{-2/3}{s+4} + \frac{2/3}{s+1}\right\} = -\frac{2}{3}e^{-4t}u(t) + \frac{2}{3}e^{-t}u(t)$$

- s-domain analysis of LTIC systems - Example # 17-cont

$$\hat{Y}_{zs}(s) = \frac{2\hat{F}}{(s+4)(s+1)} = \frac{2}{(s+4)(s+1)} \cdot \frac{1}{s}$$

↑ pole @ $s=0$, not a char. pole. It comes from the input, not from the system.

- Recall that $f(t) = u(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$
- Determine $y_{Zs}(t)$

$$\hat{Y}_{zs} = \frac{\frac{1}{6}}{s+4} + \frac{-2/3}{s+1} + \frac{1/2}{s}$$

$$\downarrow \mathcal{L}^{-1}$$

$$y_{zs}(t) = \frac{1}{6}e^{-4t}u(t) - \frac{2}{3}e^{-t}u(t) + \frac{1}{2}u(t)$$

- s-domain analysis of LTIC systems - Example # 17-cont

$$\widehat{Y_{ZI}}(s) = \frac{y(0^-)[s+5] + y'(0^-)}{(s+4)(s+1)}$$

- Recall that $y(0^-) = 1$ and $y'(0^-) = 0$
- Determine $y_{ZI}(t)$ $\hat{F} = 0$ (no input)

$$\hat{Y}_{ZI} = \frac{s+5}{(s+4)(s+1)} = \frac{-1/3}{s+4} + \frac{4/3}{s+1}$$

$$\downarrow \mathcal{L}^{-1}$$

$$y_{ZI}(t) = -\frac{1}{3} e^{-4t} u(t) + \frac{4}{3} e^{-t} u(t)$$

ZI response is a linear combination of char. modes

- s-domain analysis of LTIC systems - Example # 17-cont

$$y_{zs}(t) = \frac{1}{2}u(t) + \frac{1}{6}e^{-4t}u(t) - \frac{2}{3}e^{-t}u(t)$$

$$y_{zI}(t) = -\frac{1}{3}e^{-4t}u(t) + \frac{4}{3}e^{-t}u(t)$$

- Determine y(t)

$$y(t) = y_{zs}(t) + y_{zI}(t) = \frac{1}{2}u(t) - \frac{1}{6}e^{-4t}u(t) + \frac{2}{3}e^{-t}u(t)$$

- s-domain analysis of LTIC systems - Example # 18
  - Consider an LTIC system described by the following ODE

$$\frac{d^2}{dt^2}y - \frac{d}{dt}y - 2y = \frac{d}{dt}f - 2f(t)$$

where

- $f(t) = u(t)$
- $y(0^-) = 1$
- $y'(0^-) = 1$

- Determine the characteristic poles and characteristic modes of the system.
- Determine $\widehat{H}(s)$ and h(t)
- Determine if the system is BIBO stable.
- Determine $y_{ZI}(t)$, $y_{zs}(t)$ and y(t)

- s-domain analysis of LTIC systems - Example # 18-cont
  - Consider an LTIC system described by the following ODE

$$\frac{d^2}{dt^2}y - \frac{d}{dt}y - 2y = \frac{d}{dt}f - 2f(t)$$

  - Determine the characteristic poles and characteristic modes of the system.

$$\downarrow \mathcal{L}$$

$$s^2\hat{Y} - sy(0^-) - y'(0^-) - \left(s\hat{Y} - y(0^-)\right) - 2\hat{Y} = s\hat{F} - \underbrace{f(0^-)}_{=0} - 2\hat{F}$$

since $f(t) = u(t)$

$$\hat{Y} = \frac{(s-2)\hat{F} + y(0^-)(s-1) + y'(0^-)(1)}{\underbrace{s^2 - s - 2}_{\text{charac. polynomial}}}$$

$$= (s-2)(s+1) \Rightarrow \text{char. poles: } 2, -1$$

char. modes: e^{2t}, e^{-t}

• s-domain analysis of LTIC systems - Example # 18-cont

$$\hat{Y}(s) = \frac{(s-2)\hat{F} \cancel{+ f(0^-) + y(0^-)(s+1) + y'(0^-)}}{\underbrace{(s-2)(s+1)}} \quad = 0 \text{ for zs}$$

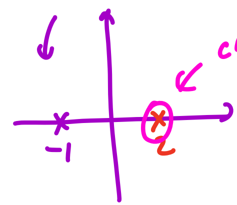
- Determine $\hat{H}(s)$ and h(t)
- Determine if the system is BIBO stable.

$$\hat{Y}_{zs} = \hat{F} \cdot \hat{H} = \frac{\cancel{(s-2)}\hat{F}}{\cancel{(s-2)}(s+1)} \Rightarrow \hat{H} = \frac{1}{s+1}$$

$$h(t) = \mathcal{L}^{-1}\{\hat{H}\} = e^{-t}u(t)$$

LHP ⇒ BIBO stable

cancelled



- s-domain analysis of LTIC systems - Example # 18-cont

$$\widehat{Y_{zs}}(s) = \frac{(s \cancel{-2})\hat{F}}{(s \cancel{-2})(s+1)}$$

- Recall that $f(t) = u(t) \xrightarrow{\mathcal{L}} \hat{F} = \frac{1}{s}$
- Determine $y_{zs}(t)$

$$\hat{Y}_{zs} = \frac{1}{(s+1)} \cdot \frac{1}{s} = \frac{1}{s} + \frac{-1}{s+1}$$

$$\downarrow \mathcal{L}^{-1}$$

$$y_{zs}(t) = u(t) - e^{-t} u(t)$$

- s-domain analysis of LTIC systems - Example # 18-cont

$$\widehat{Y_{ZI}}(s) = \frac{y(0^-)(s \cancel{+} - 1) + y'(0^-)}{(s-2)(s+1)}$$

- Recall that $y(0^-) = 1$ and $y'(0^-) = 1$
- Determine $y_{ZI}(t)$ $\Rightarrow$ $\hat{F} = 0$ (no input)

$$\hat{Y}_{ZI} = \frac{s-1+1}{(s-2)(s+1)} = \frac{s}{(s-2)(s+1)} = \frac{2/3}{s-2} + \frac{1/3}{s+1}$$

$$\downarrow \mathcal{L}^{-1}$$

$$y_{ZI}(t) = \frac{2}{3} e^{2t} u(t) + \frac{1}{3} e^{-t} u(t)$$

$\xrightarrow{t \to \infty} \infty$

Blows up even if no input!

(from the state)

- s-domain analysis of LTIC systems - Example # 18-cont
  - Recall that

$$y_{zs}(t) = u(t) - e^{-t}u(t)$$

$$y_{ZI}(t) = \frac{2}{3}e^{2t}u(t) + \frac{1}{3}e^{-t}u(t)$$

  - Determine y(t)

$$y(t) = y_{zs}(t) + y_{zI}(t) =$$

- Asymptotic~~al~~ stability
  - A system with transient zero-input response is called *asymptotically stable* (referred to earlier as dissipative) if

$$\underbrace{\lim_{t\to\infty} y_{ZI}(t) = 0}$$

BIBO stable → poles of $\hat{H}(s)$ (after we did all cancell.) are on LHP

  - An LTIC system with a rational transfer function, $\hat{H}(s)$, is asymptotically stable if and only if all of its characteristic poles (before pole-zero cancellation) are on the left-half plane.

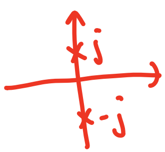
char. poles on RHP or imag. axis ⇒ not assymp. stable

  - A system is *marginally stable* if it has a bounded non-transient zero-input response:

↳ if and only if it has non-repeated char. poles on imag. axis and no ch. poles on RHP

$$\lim_{t\to\infty} y_{ZI}(t) \neq 0 \; but \; |y_{ZI}(t)| \leq C \; for \; some \; C$$

$y_{ZI}(t) = \cos(t)$



$\hat{Y}_{ZI} = \frac{1}{s^2+1}$