ECE 210/211 HWs HW 7

Student NAX6 SKC7

TOTAL POINTS

50.5 / 54

QUESTION 1

1 0 / 0

✓ **- 0 pts** Correct

QUESTION 2

2 8.5 / 10

✓ **- 1.5 pts** $$H(2)$$ term has minor error

QUESTION 3

14 pts

3.1 2 / 2

✓ **- 0 pts** Correct

3.2 2 / 2

✓ **- 0 pts** Correct

3.3 2 / 2

✓ **- 0 pts** Correct

3.4 2 / 2

✓ **- 0 pts** Correct

3.5 4 / 4

✓ **- 0 pts** Correct

3.6 2 / 2

✓ **- 0 pts** Correct: Lower resistance leads to narrower passbands. Higher resistance correspons to wider passband

QUESTION 4

4 10 / 10

✓ **- 0 pts** Correct

QUESTION 5

5 10 / 10

✓ **- 0 pts** Correct

QUESTION 6

6 8 / 10

✓ **- 1 pts** Incorrect Magnitude

✓ **- 1 pts** Extra terms

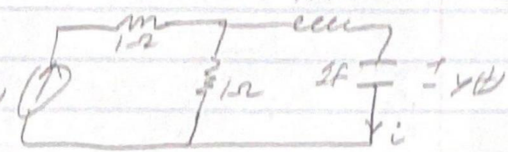
655479542

varenya3

03/05/2022 ECE 210 HW 7 Varenya Jain 1

1. Varenya Jain

2. $f(t) = 1 + 2\cos(t) + \cos(2t)$



$Z_L = j\omega L = j\omega$

$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{\omega}$

$H(\omega) = Y/F$

$F = 1 + 2 + 1 = 4$ Amps

$\hookrightarrow Z_{LC} = j\omega - \frac{j}{\omega} = \frac{j\omega^2 - j}{\omega}$, $I_C = \frac{Y}{(-j/\omega)} = \frac{-\omega Y}{j}$

Current Divider: $I_C = F \cdot \frac{1\Omega}{1\Omega + \left(\frac{j\omega^2 - j}{\omega}\right)}$ } $F \cdot \frac{1}{\left(\frac{\omega + j(\omega^2 - 1)}{\omega}\right)} = \frac{-\omega Y}{j}$

$\hookrightarrow \frac{F\omega}{\omega + j(\omega^2 - 1)} = \frac{-\omega Y}{j} \to \frac{Y}{F} = H(\omega) = \frac{-j}{\omega + j(\omega^2 - 1)}$

$\hookrightarrow y(t) = |H(\omega)| \cdot f(t + \measuredangle H(\omega))$

$2\cos(t) \to H(\omega) = \frac{-j}{1+0} = 0 - j \to |H(1)| = 1 \to \measuredangle H(1) = \tan^{-1}\left(\frac{-1}{0}\right) = -\pi/2$

$\cos(2t) \to H(\omega) = \frac{-j}{2+j3} \cdot \frac{j}{j} = \frac{1}{-3+2j} \to +\pi \to \frac{-j}{2+j3} \cdot \left(\frac{2-3j}{2-3j}\right)$

$\hookrightarrow H(\omega) = \frac{-2j-3}{13} = \frac{-3}{13} - \frac{2}{13}j \to |H(2)| = \sqrt{\frac{9+4}{13^2}} = \frac{\sqrt{13}}{13}$

$\measuredangle H(2) = -\arctan(-2/-3) = -\arctan(2/3)$

$$\boxed{\hookrightarrow y(t) = 2 + 2\cos\left(t - \frac{\pi}{2}\right) + \frac{\sqrt{13}}{13}\cos\left(2t + \pi - \arctan(2/3)\right)}$$

1 **0 / 0**

✓ **- 0 pts** Correct

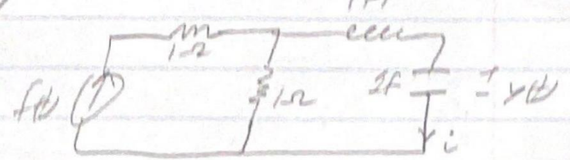
655479542

verenya3

03/05/2022 ECE 210 HW 7 Varenya Jain 1

1. Varenya Jain

2. $f(t) = 1 + 2\cos(t) + \cos(2t)$



$Z_L = j\omega L = j\omega$

$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{\omega}$

$H(\omega) = Y/F$

$F = 1 + 2 + 1 = 4$ Amps

$\hookrightarrow Z_{LC} = j\omega - \frac{j}{\omega} = \frac{j\omega^2 - j}{\omega}$, $I_C = \frac{Y}{(-j/\omega)} = \frac{-\omega Y}{j}$

Current Divider: $I_C = F \cdot \dfrac{1\Omega}{1\Omega + \left(\frac{j\omega^2 - j}{\omega}\right)} \Bigg\} \; \dfrac{F \cdot 1}{\left(\frac{\omega + j(\omega^2 - 1)}{\omega}\right)} = \dfrac{-\omega Y}{j}$

$\hookrightarrow \dfrac{F\omega}{\omega + j(\omega^2 - 1)} = \dfrac{-\omega Y}{j} \rightarrow \dfrac{Y}{F} = H(\omega) = \dfrac{-j}{\omega + j(\omega^2 - 1)}$

$\hookrightarrow y(t) = |H(\omega)| \cdot f(t + \measuredangle H(\omega))$

$2\cos(t) \rightarrow H(\omega) = \frac{-j}{1+0} = 0 - j \rightarrow |H(1)| = 1 \rightarrow \measuredangle H(1) = \tan^{-1}\left(\frac{-1}{0}\right) = -\pi/2$

$\cos(2t) \rightarrow H(\omega) = \frac{-j}{2+j3} \cdot \frac{j}{j} = \frac{1}{-3+2j} \rightarrow +\pi \rightarrow \frac{-j}{2+j3} \cdot \left(\frac{2-3j}{2-3j}\right)$

$\hookrightarrow H(\omega) = \frac{-2j-3}{13} = \frac{-3}{13} - \frac{2}{13}j \rightarrow |H(2)| = \sqrt{\frac{9+4}{13^2}} = \frac{\sqrt{13}}{13}$

$\measuredangle H(2) = -\arctan\left(\frac{-2}{-3}\right) = -\arctan\left(\frac{2}{3}\right)$

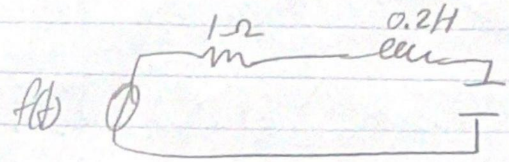
$$\boxed{\hookrightarrow y(t) = 2 + 2\cos\left(t - \frac{\pi}{2}\right) + \frac{\sqrt{13}}{13}\cos\left(2t + \pi - \arctan\left(\frac{2}{3}\right)\right)}$$

2 8.5 / 10

✓ - 1.5 pts $$H(2)$$ term has minor error

2

3.



input f(t)
output i(t)

a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{5} \cdot \frac{1}{20}}} = \frac{1}{\sqrt{1/100}} = \frac{1}{(1/10)} = 10 \text{ rad/s}$

b) $H(\omega) = \frac{Y}{F} = \frac{I}{V_s} \rightarrow I = \frac{V_s}{R + j\omega L + 1/j\omega C}$

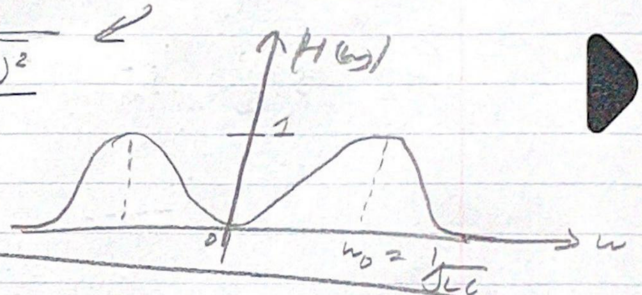
$\hookrightarrow I = \frac{j\omega C}{j\omega RC - \omega^2 LC + 1} \cdot V_s \rightarrow H(\omega)$

$\downarrow$

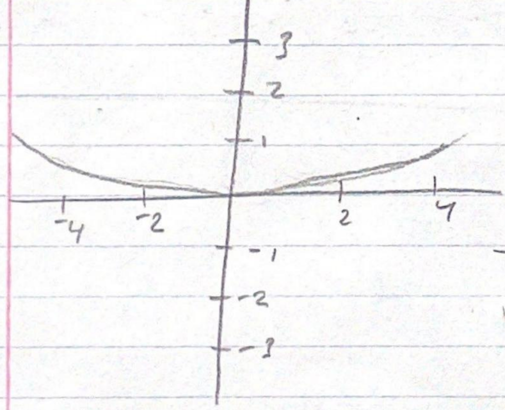
$\boxed{\text{Band-Pass filter}}$

$\hookrightarrow H(\omega) = \frac{|\omega| \cdot C}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$

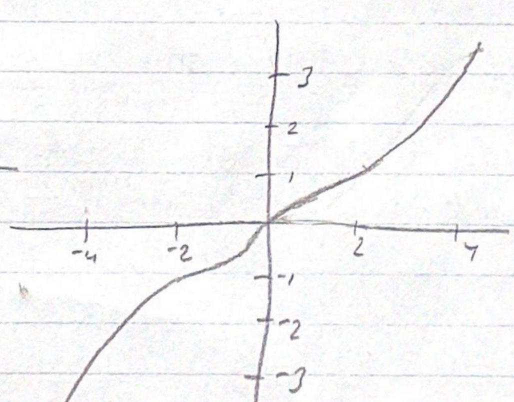
$|H(10)| = 1 \rightarrow$



c) Re { H(ω) }



Im { H(ω) }



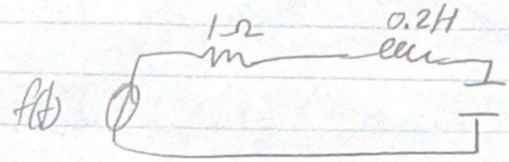
*Approximate drawings, not precise representations

3.1 2 / 2

✓ **- 0 pts** Correct

2

3.



input $f(t)$
output $i(t)$

a) $w_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{\frac{1}{5}\cdot\frac{1}{20}}} = \dfrac{1}{\sqrt{\frac{1}{100}}} = \dfrac{1}{(\frac{1}{10})} = 10$ rad/s

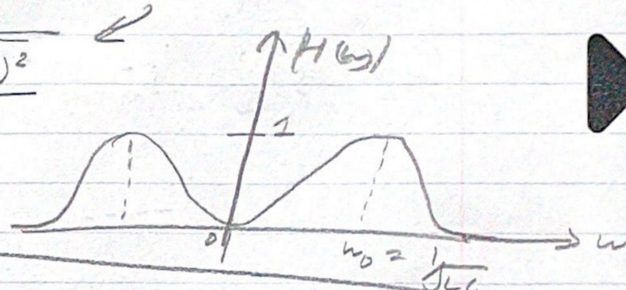
b) $H(\omega) = \dfrac{Y}{F} = \dfrac{I}{V_s} \rightarrow I = \dfrac{V_s}{R + j\omega L + \frac{1}{j\omega C}}$

$\hookrightarrow I = \dfrac{j\omega C}{j\omega RC - \omega^2 LC + 1} \cdot V_s \rightarrow H(\omega)$

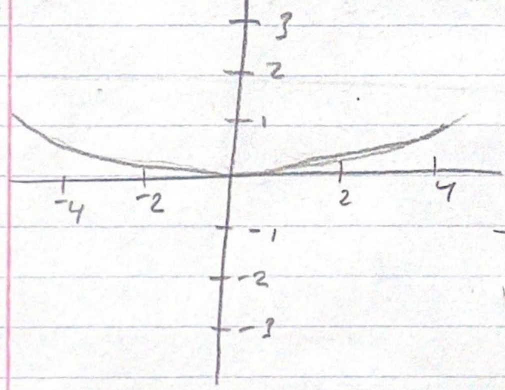
$\boxed{\text{Band-Pass filter}}$

$\hookrightarrow H(\omega) = \dfrac{|\omega| \cdot C}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$

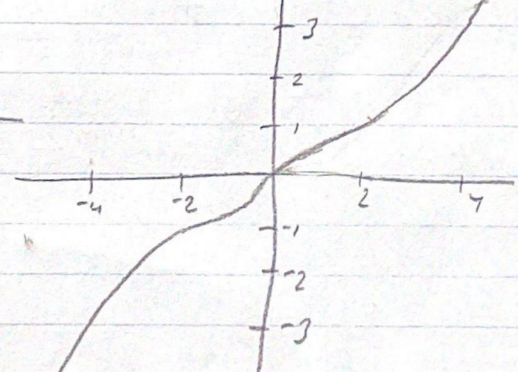
$|H(10)| = 1 \rightarrow$



c) Re { $H(\omega)$ }



Im { $H(\omega)$ }

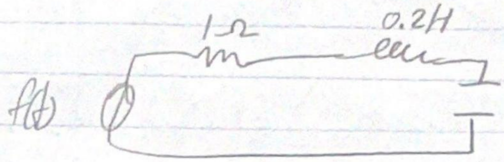


*Approximate drawings, not precise representations

3.2 **2 / 2**

✓ **- 0 pts** Correct

2

3\. 

input f(t)
output i(t)

a) $w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{5}\cdot\frac{1}{20}}} = \frac{1}{\sqrt{\frac{1}{100}}} = \frac{1}{\left(\frac{1}{10}\right)} = 10$ rad/s

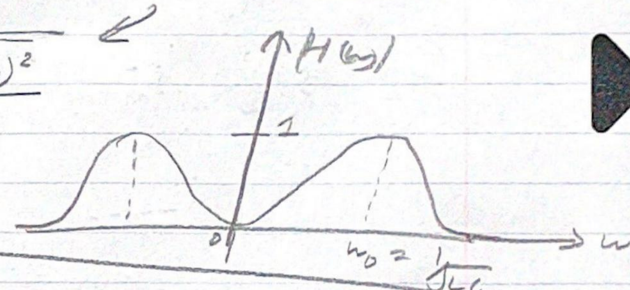
b) $H(w) = \frac{Y}{F} = \frac{I}{V_s} \rightarrow I = \frac{V_s}{R + jwL + \frac{1}{jwC}}$

$\hookrightarrow I = \frac{jwC}{jwRC - w^2LC + 1} \cdot V_s \rightarrow H(w)$

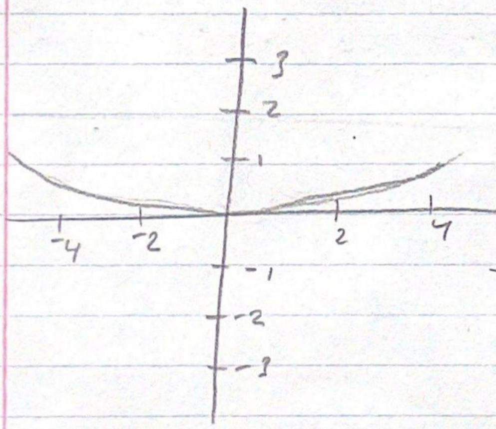
$\boxed{\text{Band-Pass filter}}$

$\hookrightarrow H(w) = \frac{|w| \cdot C}{\sqrt{(1 - w^2LC)^2 + (wRC)^2}}$

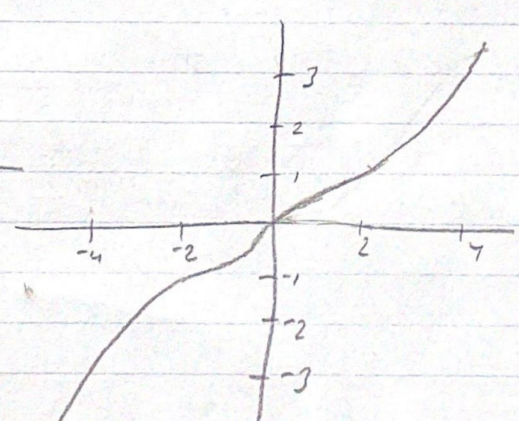
$|H(10)| = 1 \rightarrow$



c) Re { H(w) }



Im { H(w) }



*Approximate drawings, not precise representations

3.3 2 / 2

✓ **- 0 pts** Correct

3

3. d) This circuit might be called a "bandpass" filter because it only allows certain frequencies to pass through and attenuates/rejects all other signals

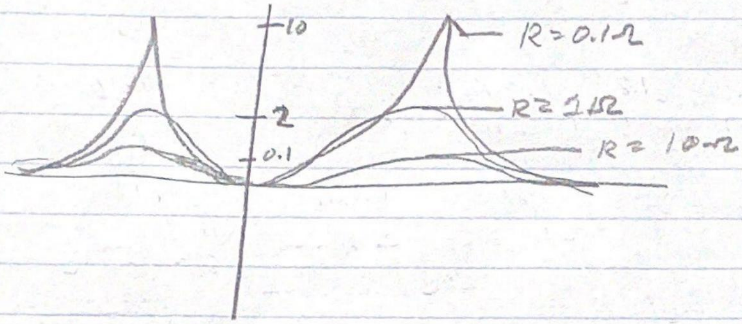
e) $w_0 = \frac{1}{\sqrt{LC}} = 10 \text{ rad/s}$

$H(\omega) = \frac{j\omega C}{j\omega RC - \omega^2 LC + 1}$

$|H(\omega)| = \frac{|\omega| C}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$

$\rightarrow R = 10\,\Omega \rightarrow |H(10)| = 0.1$

$R = 0.1\,\Omega \rightarrow |H(10)| = 10$



f) Decreasing R results in narrower peaks with higher amplitudes, so as R decreases so does the width of the filter's passband.

3.4 2 / 2

✓ **- 0 pts** Correct

3

3. d) This circuit might be called a "bandpass" filter because it only allows certain frequencies to pass through and attenuates/rejects all other signals

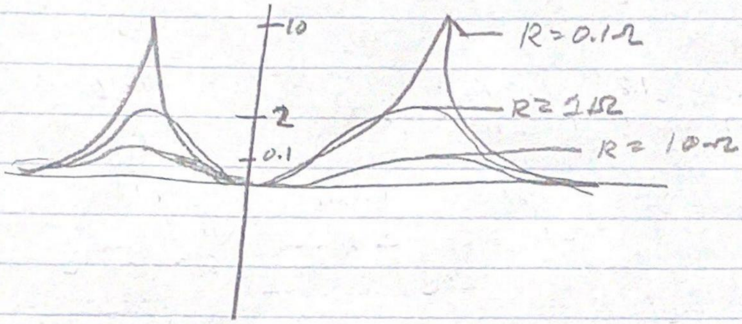
e) $w_0 = \frac{1}{\sqrt{LC}} = 10$ rad/s

$H(\omega) = \frac{j\omega C}{j\omega RC - \omega^2 LC + 1}$

$|H(\omega)| = \frac{|\omega| C}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$

$\rightarrow R = 10\,\Omega \rightarrow |H(10)| = 0.1$

$R = 0.1\,\Omega \rightarrow |H(10)| = 10$



f) Decreasing R results in narrower peaks with higher amplitudes, so as R decreases so does the width of the filter's pass band.

3.5 4 / 4

✓ **- 0 pts** Correct

3.) d) This circuit might be called a "bandpass" filter because it only allows certain frequencies to pass through and attenuates/rejects all other signals

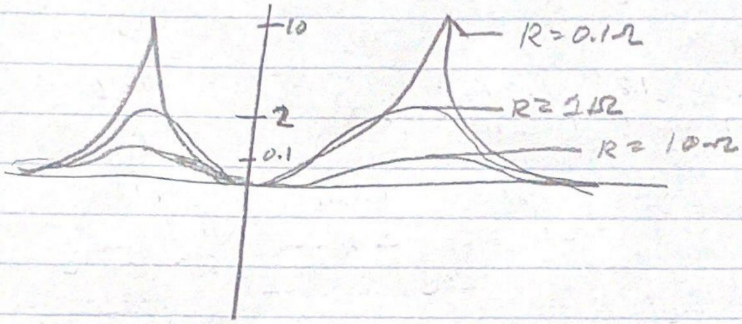
e) $\omega_0 = \frac{1}{\sqrt{LC}} = 10 \text{ rad/s}$

$H(\omega) = \frac{j\omega L}{j\omega RC - \omega^2 LC + 1}$

$|H(\omega)| = \frac{|\omega| L}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$

$\rightarrow R = 10\,\Omega \rightarrow |H(10)| = 0.1$

$R = 0.1\,\Omega \rightarrow |H(10)| = 10$

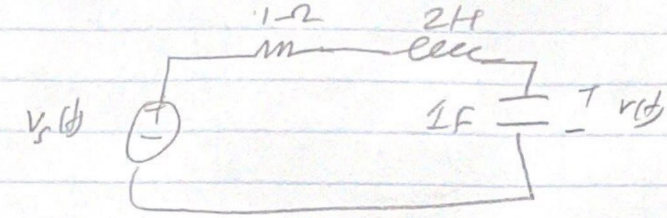


f) Decreasing R results in narrower peaks with higher amplitudes, so as R decreases so does the width of the filter's pass band.

3.6 2 / 2

✓ **- 0 pts** **Correct: Lower resistance leads to narrower passbands. Higher resistance correspons to wider passband**

4



$$H(\omega) = \frac{Y}{F} = \frac{V_c}{V_s}$$

$$\hookrightarrow H = \frac{V_c}{V_s} = \frac{I}{I} \cdot \frac{Z_c}{Z_T} = \frac{Z_c}{R + j\omega L + 1/j\omega C}$$

$$\hookrightarrow H = \frac{j\omega C}{j\omega RC - \omega^2 RC + 1} \cdot \frac{1}{j\omega C} = \boxed{\frac{1}{j\omega - 2\omega^2 + 1} = H(\omega)}$$

5. input $f(t)$, output $y(t)$ $\Big\rangle$ $\frac{d^2y}{dt^2} + \frac{4dy}{dt} + 2y(t) = \frac{df}{dt} + \frac{d^2f}{dt^2}$

$$H(\omega) = \frac{Y}{F} \hookrightarrow (j\omega)^2 Y + 4j\omega Y + 2Y = j\omega F + (j\omega)^2 F$$

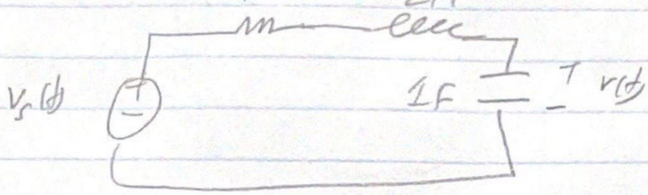
$$\hookrightarrow Y(-\omega^2 + 4j\omega + 2) = F(j\omega - \omega^2)$$

$$\boxed{\hookrightarrow \frac{Y}{F} = \frac{j\omega - \omega^2}{2 + 4j\omega - \omega^2} = H(\omega)}$$

4 10 / 10

✓ **- 0 pts** Correct

4



$$H(\omega) = \frac{Y}{F} = \frac{V_c}{V_s}$$

$$\hookrightarrow H = \frac{V_c}{V_s} = \frac{I}{I} \cdot \frac{Z_c}{Z_T} = \frac{Z_c}{R + j\omega L + 1/j\omega C}$$

$$\hookrightarrow H = \frac{j\omega C}{j\omega RC - \omega^2 RC + 1} \cdot \frac{1}{j\omega C} = \boxed{\frac{1}{j\omega - 2\omega^2 + 1} = H(\omega)}$$

5. input $f(t)$, output $y(t)$ $>$ $\frac{d^2y}{dt^2} + \frac{4dy}{dt} + 2y(t) = \frac{df}{dt} + \frac{d^2f}{dt^2}$

$$H(\omega) = \frac{Y}{F} \quad \hookrightarrow (j\omega)^2 Y + 4j\omega Y + 2Y = j\omega F + (j\omega)^2 F$$

$$\hookrightarrow Y(-\omega^2 + 4j\omega + 2) = F(j\omega - \omega^2)$$

$$\boxed{\hookrightarrow \frac{Y}{F} = \frac{j\omega - \omega^2}{2 + 4j\omega - \omega^2} = H(\omega)}$$

5 10 / 10

✓ **- 0 pts** Correct

6. input: $f(t) = \underbrace{2e^{-2jt}}_{(1)} + \underbrace{(2+j2)e^{-jt}}_{(2)} + \underbrace{(2-j2)e^{jt}}_{(3)} + \underbrace{2e^{2jt}}_{(4)}$

$$H(\omega) = \frac{1+j\omega}{2+j\omega} \to \frac{\sqrt{1+\omega^2}}{\sqrt{4+\omega^2}} \cdot \frac{e^{j\tan^{-1}(\omega/1)}}{e^{j\tan^{-1}(\omega/2)}} = \sqrt{\frac{1+\omega^2}{4+\omega^2}} \cdot e^{j(\tan^{-1}(\omega)-\tan^{-1}(\omega/2))} = H(\omega)$$

$$f = f_1+f_2+f_3+f_4 \to y = H(\omega_1)\cdot f_1 + H(\omega_2)\cdot f_2 + H(\omega_3)\cdot f_3 + H(\omega_4)\cdot f_4$$

↳ $\omega_1 = -2,\ \omega_2 = -1,\ \omega_3 = 1,\ \omega_4 = 2$

$\boxed{2e^{-j2t}}$ $\quad \omega = -2 \to H(\omega) = \sqrt{\frac{1+4}{4+4}} \cdot e^{j\underbrace{(\tan^{-1}(-2)-\tan^{-1}(-1))}_{\angle H(\omega)}}$

↳ $H(\omega) = \frac{\sqrt{5}}{2\sqrt{2}} e^{j(\tan^{-1}(-2)+\pi/4)} \to |H(\omega)| = \frac{\sqrt{5}}{2\sqrt{2}}$

$$y_1 = 2\cdot\frac{\sqrt{5}}{2\sqrt{2}}\cos(-2 + \tan^{-1}(2) + \pi/4)$$

$\boxed{(2+j2)e^{-jt}}$ $\quad \omega = -1 \to H(\omega) = \sqrt{\frac{1+1}{4+1}}\, e^{j(\tan^{-1}(-1)-\tan^{-1}(-1/2))}$

↳ $H(\omega) = \frac{\sqrt{2}}{\sqrt{5}} e^{j(-\pi/4 + \arctan(1/2))} \to |H(\omega)| = \frac{\sqrt{2}}{\sqrt{5}}$

↳ $y_2 = \sqrt{\frac{2}{5}} \cdot (2+j2)\cos(-t - \pi/4 + \arctan(1/2))$

6

$\boxed{(2-j2)e^{jt}}$ $w=1 \rightarrow H(w) = \sqrt{\frac{1+1}{4+1}}\, e^{j(\tan^{-1}(1) - \tan^{-1}(\frac{1}{2}))}$

$\hookrightarrow H(w) = \sqrt{\frac{2}{5}}\, e^{j(\pi/4 - \tan^{-1}(\frac{1}{2}))} \rightarrow \underline{|H(w)| = \sqrt{\frac{2}{5}}}$

$\hookrightarrow \underline{y_3 = \sqrt{\frac{2}{5}} \cdot (2-j2) \cdot \cos(t + \pi/4 - \tan^{-1}(\frac{1}{2}))}$

$\boxed{2e^{2jt}}$ $w=2 \rightarrow H(w) = \sqrt{\frac{1+(2)^2}{4+(2)^2}}\, e^{j(\tan^{-1}(2) - \tan^{-1}(\frac{2}{2}))}$

$\hookrightarrow H(w) = \frac{\sqrt{5}}{2\sqrt{2}}\, e^{j(\tan^{-1}(2) - \pi/4)} \rightarrow \underline{|H(w)| = \frac{5}{2\sqrt{2}}}$

$\hookrightarrow \underline{y_4 = \frac{2 \cdot \sqrt{5}}{2\sqrt{2}} \cos(2t + \tan^{-1}(2) - \pi/4)}$

$\hookrightarrow y(t) = y_1 + y_2 + y_3 + y_4$

$$\boxed{\begin{aligned} \hookrightarrow y(t) &= \sqrt{\frac{5}{2}}\left[\cos(-2t + \tan^{-1}(2) + \pi/4) + \cos(2t + \tan^{-1}(2) - \pi/4)\right] \\ &+ \sqrt{\frac{2}{5}}\left[(2+j2)\cos(-t - \pi/4 + \tan^{-1}(\tfrac{1}{2})) + (2-j2)\cos t + \pi/4 - \tan^{-1}(\tfrac{1}{2})\right] \end{aligned}}$$

6 8 / 10

- ✓ **- 1 pts** Incorrect Magnitude
- ✓ **- 1 pts** Extra terms