- Modulation - Example # 17
  - Determine the Fourier transform of $\underbrace{\mathrm{f(t)}\overbrace{\cos(\omega_c t)}^{g(t)}}$

Freq. convolution property:

$$f(t)g(t) \leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega)$$

$$g(t) = \cos(\omega_c t)$$

$$G(\omega) = \pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)$$

$$f(t)\cos(\omega_c t) \leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega) = \frac{1}{2\pi} F(\omega) * \left(\pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)\right) =$$

$$= \frac{1}{2} F(\omega-\omega_c) + \frac{1}{2} F(\omega+\omega_c)$$

- Periodic signals - Example # 18
  - Determine the Fourier transform of the periodic signal *f(t)*

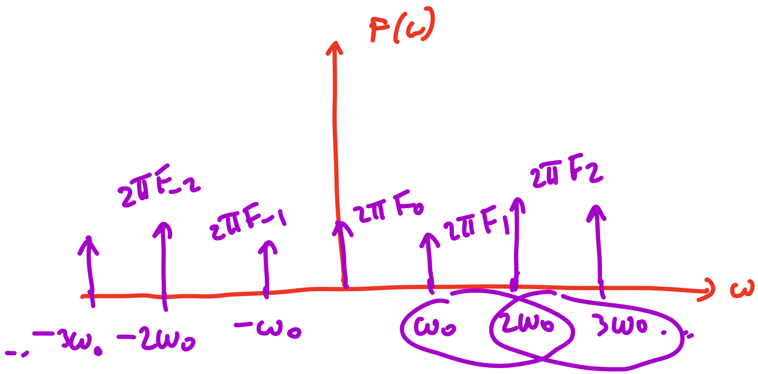
$$\mathcal{F}\left\{ f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \right\}$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \mathcal{F}\left\{ F_n e^{jn\omega_0 t} \right\} = \sum_{n=-\infty}^{\infty} F_n 2\pi \delta(\omega - n\omega_0)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$F_n \leftrightarrow F_n 2\pi\delta(\omega)$$

$$F_n e^{j\omega_0 t} \leftrightarrow F_n 2\pi\delta(\omega - \omega_0)$$



- Impulse train - Example # 19
  - Determine the Fourier transform of the impulse train → periodic signal with period T

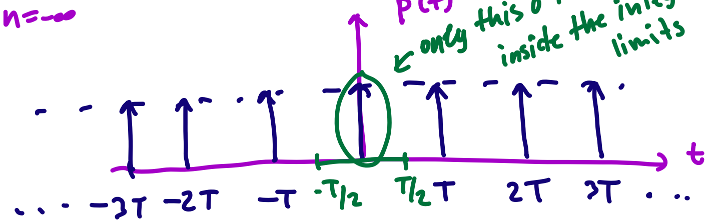
$\omega_0 = \frac{2\pi}{T}$

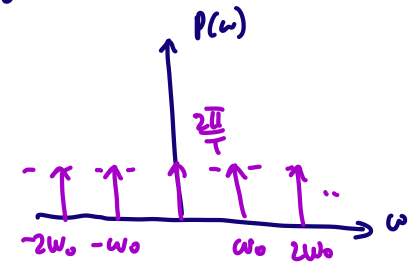
Table 7.2, 24

$$\sum_{n=-\infty}^{\infty} \delta(t-nT) \Leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{T}\right)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

only this δ is inside the integration limits





$$P(\omega) = \sum_{n=-\infty}^{\infty} P_n\, 2\pi\,\delta(\omega - n\omega_0) = \sum_{n=-\infty}^{\infty} \frac{2\pi}{T}\,\delta(\omega - n\omega_0)$$

F.S. coeff.

$$P_n = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} \left(\sum_{m=-\infty}^{\infty} \delta(t-mT)\right) e^{-jn\omega_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jn\omega_0 t}\, dt =$$

sifting

$$= \frac{1}{T}\, e^{-jn\omega_0 (0)} = \frac{1}{T}$$

- Sampling
  - Suppose that the signal *f(t)* is mixed with the impulse train *p(t)c*
  - How can we recover the signal?

$$f(t) \rightarrow \otimes \rightarrow s(t) = f(t)p(t) = \left(\sum_{n=-\infty}^{\infty} \delta(t-nT)\right) f(t) =$$

$$\uparrow$$

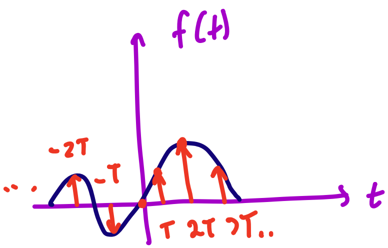
$$p(t) = \sum_{n=\infty}^{\infty} \delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} f(nT)\,\delta(t-nT)$$

$$\mathcal{F} \downarrow$$

$$S(\omega) = \frac{1}{2\pi} F(\omega) * P(\omega) = \frac{1}{2\pi} F(\omega) * \left(\sum_{n=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - n\omega_0)\right) =$$

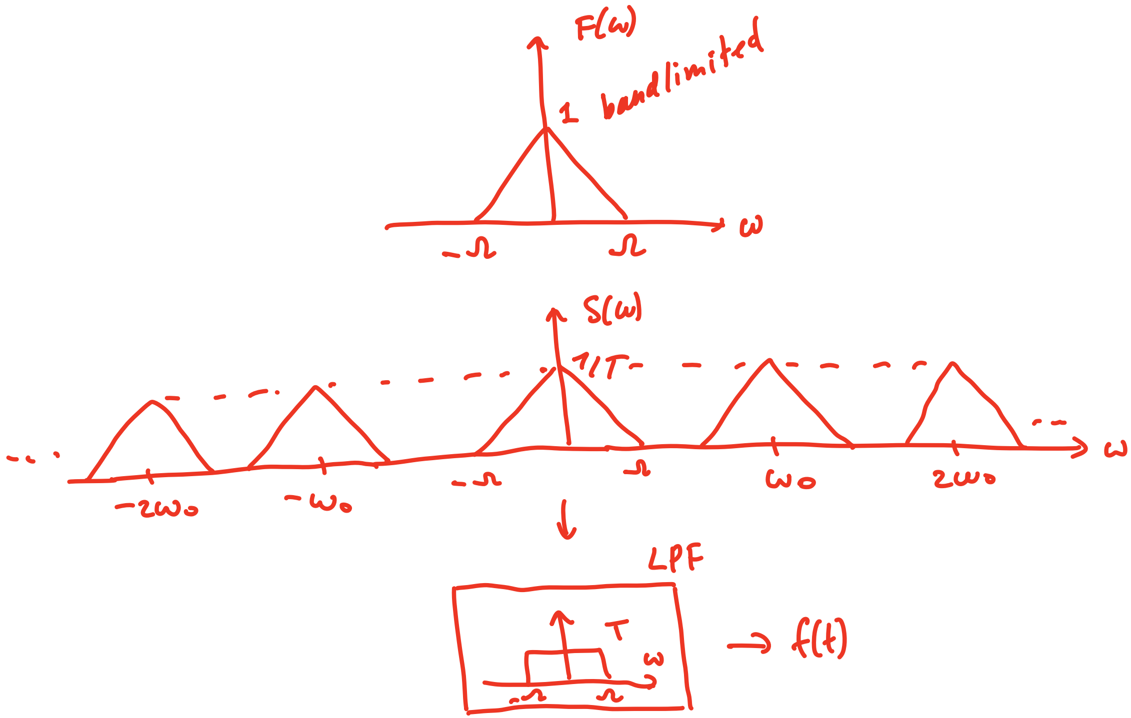
$$= \frac{1}{T}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$



- Sampling - cont

$$S(\omega) = \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\omega_0)$$

- How can we recover the signal?



- Sampling - cont
  - What is happening in the time domain?

$$y(t) = \left(\sum_{n=-\infty}^{\infty} f(nT)\delta(t - nT) \right) * sinc\left(\frac{\pi t}{T}\right) = ?$$

- Sampling - cont
    - Reconstruction formula

$$y(t) = \sum_{n=-\infty}^{\infty} f(nT)sinc\left(\frac{\pi(t-nT)}{T}\right)$$

when does this work?

if $\omega_0 < 2\Omega \Rightarrow$ undersampling

↓

aliasing

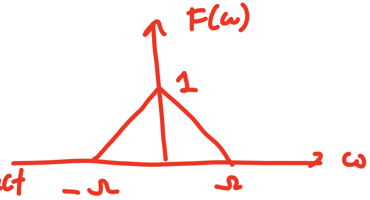
↓

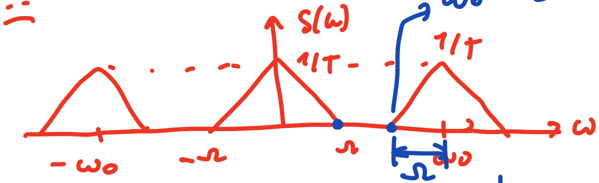
can't reconstruct :(

if $\omega_0 = 2\Omega \Rightarrow$ sampling at Nyquist sampl. freq

⇓

can reconstruct :)





$\omega_0 - \Omega > \Omega \Rightarrow \boxed{\omega_0 \geq 2\Omega}$

Nyquist criterion

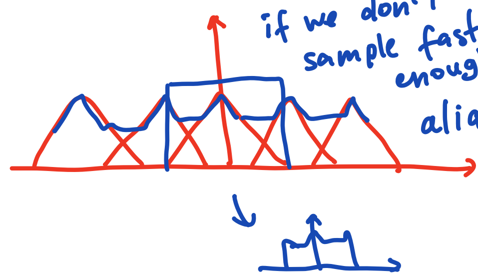
sampling freq.

Nyquist sampling frequency

if $\omega_0 > 2\Omega \Rightarrow$ oversampling

↓

can reconstruct :)



if we don't sample fast enough

aliasing

Chapter objectives

- Understand what convolution represents
- Understand how to convolve two signals
- Understand and be able to apply properties of convolution
- Understand what an impulse represents
- Understand and be able to apply properties of the impulse
- Understand what the impulse response of an LTI system represents
- Understand Fourier Transforms of power signals
- Understand sampling and reconstruction
- Understand Nyquist sampling frequency and aliasing
- Understand the difference between sampling bandwidth and energy bandwidth