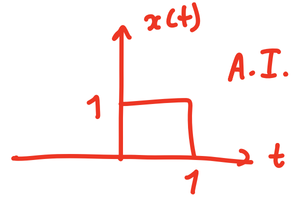
- Laplace transform - Example # 4
  - Determine the Laplace transform of $x(t) = rect(t - \frac{1}{2})$



A.I.V

$$X(s) = \int_{0^-}^{\infty} x(t)\, e^{-st}\, dt = \int_{0}^{1} (1)\, e^{-st}\, dt =$$

$$= \left. \frac{e^{-st}}{-s} \right|_{0}^{1} = \frac{e^{-s} - 1}{-s} = \frac{1 - e^{-s}}{s}$$

pole @ s=0 ? No!

ROC is the entire s-plane

$$\lim_{\substack{s \to 0 \\ -\infty}} \frac{1-e^{-s}}{s} = \lim_{\substack{s \to 0 \\ -\infty}} \frac{\frac{d}{ds}(1-e^{-s})}{\frac{d}{ds}(s)} = \lim_{\substack{s \to 0 \\ -\infty}} \frac{e^{-s}}{1} = \begin{matrix} 1 \\ \infty \end{matrix}$$

$\Rightarrow$ pole @ $s = -\infty$

poles @ $\pm\infty$: hidden poles

• Laplace transform - cont

transfer function

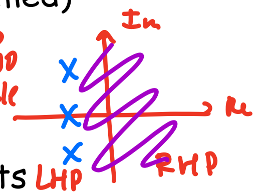
- An LTIC system with impulse response h(t) is BIBO stable if and only if $\hat{H}(s)$ has all of its poles on the left-half plane, where $\hat{H}(s)$ is a rational function $\frac{N(s)}{D(s)}$ in minimal form (polynomial in s divided by polynomial in s, where all possible cancellations of terms between the numerator and denominator has been performed)

- Recall that

$$\hat{H}(s) = \frac{s-2}{s^2-4} = \frac{\cancel{s-2}}{(\cancel{s-2})(s+2)} = \frac{1}{s+2}$$

pole @ $s = -2$

LHP ⇒ BIBO stable



BIBO $\longleftrightarrow$ A.I. $\longrightarrow$ $H(\omega)$ exists

not BIBO $\longleftrightarrow$ not A.I. $\longrightarrow$ $H(\omega)$ might not exist, but $\hat{H}(s)$ might exist

- $\hat{H}(s = j\omega)$ is well-defined if the ROC includes the jω-axis

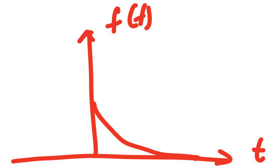
$\Rightarrow$ *all the poles of* $\hat{H}(s)$ must be in the left-half plane.

Note: a pole at $s = \infty$ is NO exception for this rule!

- Laplace transform - cont

  - Recall that

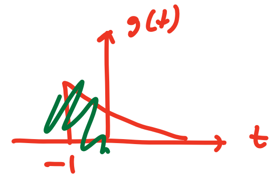
$$f(t) = e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \hat{F}(s) = \frac{1}{s+1}$$



different funct. have same LT!

  - Also

$$g(t) = e^{-t}u(t+1) \rightarrow \hat{G}(s) = \frac{1}{s+1}$$



  - Why?

LT ignores everything before $t=0$

$$\hat{G}(s) = \int_{0^-}^{\infty} g(t)\, e^{-st}\, dt$$

F.T.

$f(t) \leftrightarrow F(\omega)$

unique pairs

L.T:

unique pairs only for causal signals

- Laplace transform - Properties Table 11.2

  - Time-shift:

If $f(t) \overset{\mathcal{L}}{\longleftrightarrow} \hat{F}(s)$ $\mathcal{L}$ is only for causal signals $\longrightarrow$ - all signals

f(t) causal

and $t_0 \geq 0$, then

shift right

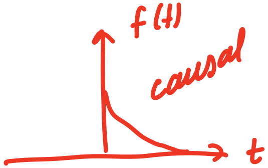
$$g(t) = f(t - t_0) \overset{\mathcal{L}}{\longleftrightarrow} \hat{G}(s) = \hat{F}(s)e^{-st_0}$$

• Laplace transform - Example # 5

$f(t)$ causal ✓

shift right ✓

- If

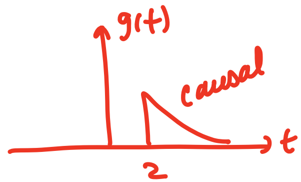


$$f(t) = e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \hat{F}(s) = \frac{1}{s+1}$$

and

$$g(t) = e^{-(t-2)}u(t-2) = f(t-\overset{t_0}{2})$$

- Determine $\hat{G}(s)$



$$\hat{G}(s) = \hat{F}(s)\, e^{-s(t_0)} =$$

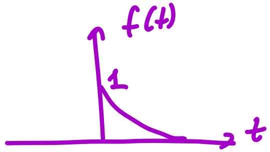
$$= \frac{1}{s+1} e^{-2s}$$

• Laplace transform - Example # 6

f(t) causal ✓

shift right ✗

- If



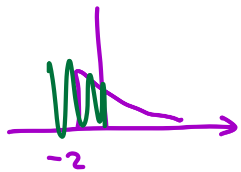
$$f(t) = e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \hat{F}(s) = \frac{1}{s+1}$$

and

$$h(t) = e^{-(t+2)}u(t+2) = \underset{t-(-2)}{f(t+2)}$$

↙ to the left

- Determine $\hat{H}(s)$



Let's pretend the property applies:

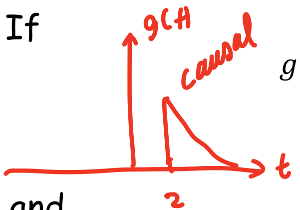
$$\hat{H}(s) = \hat{F}(s)\, e^{2s} = \frac{1}{s+1} e^{2s}$$

$$\hat{H}(s) = \mathcal{L}\{e^{-(t+2)}u(t+2)\} = \mathcal{L}\{e^{-(t+2)}u(t)\} = e^{-2}\mathcal{L}\{e^{-t}u(t)\} = e^{-2}\hat{F}(s) = \frac{e^{-2}}{s+1}$$

different!

- Laplace transform - Example # 7

  - If



$g(t) = e^{-(t-2)}u(t-2) \overset{\mathcal{L}}{\longleftrightarrow} \hat{G}(s) = \dfrac{e^{-2s}}{s+1}$

  and

left shift

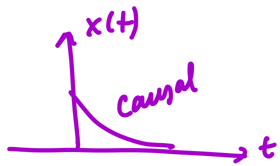
$x(t) = g(t+2) = e^{-t}u(t)$

$\updownarrow \mathcal{L}$

$\hat{X}(s) = \dfrac{1}{s+1}$

  - Determine $\hat{X}(s)$

$$\hat{X}(s) = \hat{G}(s)e^{2s} = \frac{e^{-2s}\cdot e^{2s}}{s+1} = \frac{1}{s+1}$$



g(t) causal ✓

shift right ✗

⇓

property doesn't apply

(or does it? :))

It does work!

Because shifted signal is still causal

- Laplace transform - Properties - cont

  - Time-derivative:

If $f(t) \rightarrow \hat{F}(s)$ works for any $f(t)$

and

$$g(t) = \frac{d}{dt} f(t) \xrightarrow{\mathcal{L}} \hat{G}(s) = s\hat{F}(s) - f(0^-)$$

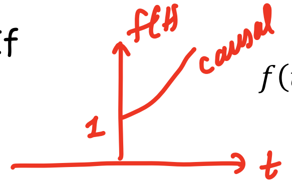
If $f(t)$ is causal:

$$\frac{d}{dt} f(t) \xleftrightarrow{\mathcal{L}} s\hat{F}(s)$$

• Laplace transform - Example # 8

Table 11.1

- If



$$f(t) = e^{3t}u(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad \hat{F}(s) = \frac{1}{s-3}$$

and

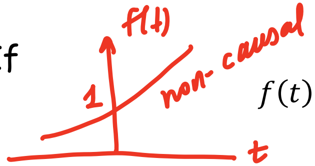
$$g(t) = \frac{d}{dt}f(t)$$

- Determine $\hat{G}(s)$

$$\hat{G}(s) = s\hat{F}(s) = \frac{s}{s-3}$$

• Laplace transform - Example # 9

- If



$$f(t) = e^{3t} \xrightarrow{\mathcal{L}} \hat{F}(s) = \frac{1}{s-3}$$

and

$$h(t) = \frac{d}{dt} f(t)$$

- Determine $\hat{H}(s)$

$$\hat{H}(s) = s\hat{F}(s) - \overbrace{f(0^-)}^{e^{3(0)} = 1} = s\hat{F}(s) - 1 = \frac{s}{s-3} - 1 =$$

$$= \frac{3}{s-3}$$