

# ECE 210/211 HWs HW 10

Student XRU7 MZG2

TOTAL POINTS

**63.5 / 70**

QUESTION 1

**1 0 / 0**

✓ - **0 pts** Correct

QUESTION 2

30 pts

**2.1 10 / 10**

✓ - **0 pts** Correct

**2.2 10 / 10**

✓ - **0 pts** Correct

**2.3 8.5 / 10**

✓ - **1.5 pts** Graph Incorrect, but Exists

QUESTION 3

20 pts

**3.1 5 / 5**

✓ - **0 pts** Correct

**3.2 10 / 10**

✓ - **0 pts** Correct

**3.3 5 / 5**

✓ - **0 pts** Correct

QUESTION 4

20 pts

**4.1 10 / 10**

✓ - **0 pts** Correct

**4.2 5 / 10**

✓ - **5 pts** Incorrect

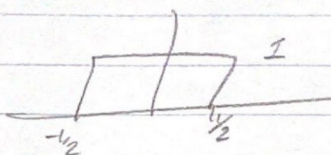
Varenya Jain, varenya3, 655479542

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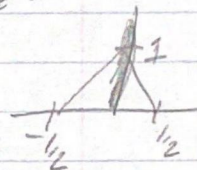


1. Varenya Jain

2a) Unit rect:



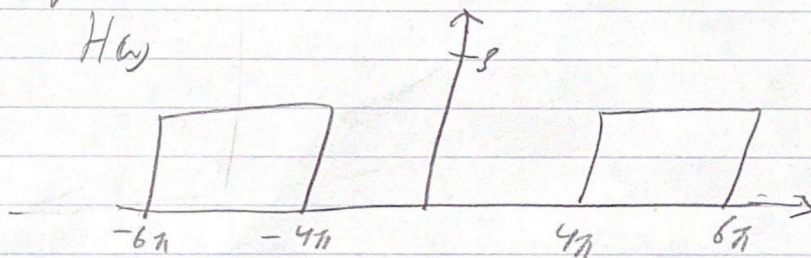
Unit triangle:



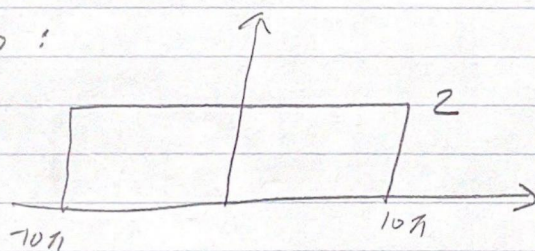
$$H_{\omega} = 3 \text{rect} \left( \frac{\omega - 5\pi}{2\pi} \right) + 3 \text{rect} \left( \frac{\omega + 5\pi}{2\pi} \right)$$

↓

$H_{\omega}$



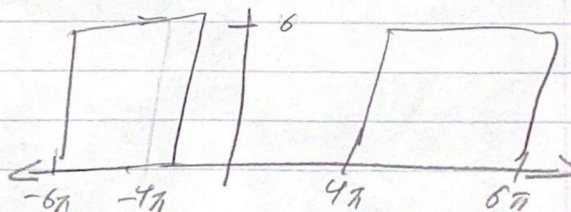
$F_{\omega}$ :



$$F_{\omega} = 2 \text{rect} \left( \frac{\omega}{20\pi} \right)$$

$$Y_{\omega} = F_{\omega} \cdot H_{\omega} :$$

↓  
 $Y_{\omega} \rightarrow y, \omega$



$$\hookrightarrow 6 \text{rect} \left( \frac{\omega - 5\pi}{2\pi} \right) + 6 \text{rect} \left( \frac{\omega + 5\pi}{2\pi} \right) = Y_{\omega} \rightarrow$$

1 0 / 0

✓ - 0 pts Correct

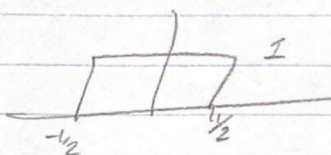
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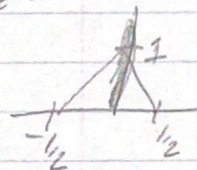


1. Varenya Jain

2a) Unit rect:



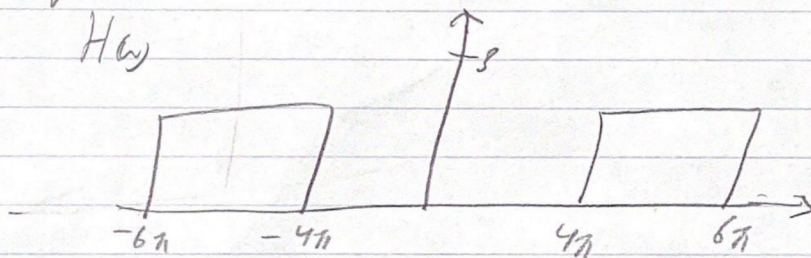
Unit triangle:



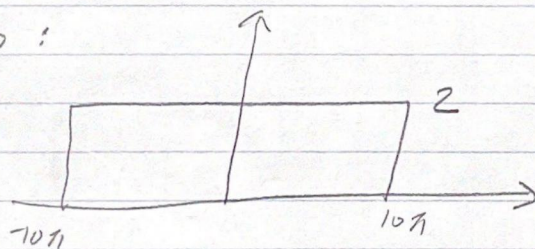
$$H_{\omega} = 3 \text{rect}\left(\frac{\omega - 5\pi}{2\pi}\right) + 3 \text{rect}\left(\frac{\omega + 5\pi}{2\pi}\right)$$

↓

$H_{\omega}$



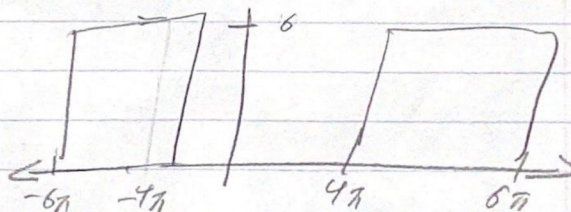
$F_{\omega}$ :



↙  $F_{\omega} = 2 \text{rect}\left(\frac{\omega}{20\pi}\right)$

$$Y_{\omega} = F_{\omega} \cdot H_{\omega} :$$

↓  
 $Y_{\omega} \rightarrow y, \omega$



$$\hookrightarrow 6 \text{rect}\left(\frac{\omega - 5\pi}{2\pi}\right) + 6 \text{rect}\left(\frac{\omega + 5\pi}{2\pi}\right) = Y_{\omega} \rightarrow$$

2

Table 3, #9. → Frequency Shift

$$f(t) \leftrightarrow f(\omega), \quad f(t)e^{j\omega_0 t} \leftrightarrow f(\omega - \omega_0)$$

↓

$$b \operatorname{rect}\left(\frac{\omega - 5\pi}{2\pi}\right) + b \operatorname{rect}\left(\frac{\omega + 5\pi}{2\pi}\right) \rightarrow b f(t) e^{-j5\pi t} + b f(t) e^{j5\pi t}$$

Table 4, #8:  $\operatorname{sinc}(Wt) \leftrightarrow \frac{\pi}{W} \operatorname{rect}\left(\frac{\omega}{2W}\right)$

$$W = \pi \rightarrow f_1(t) = \operatorname{sinc}(\pi t) \quad * \text{we accounted for the } \pm 5\pi \text{ shift}$$

↓

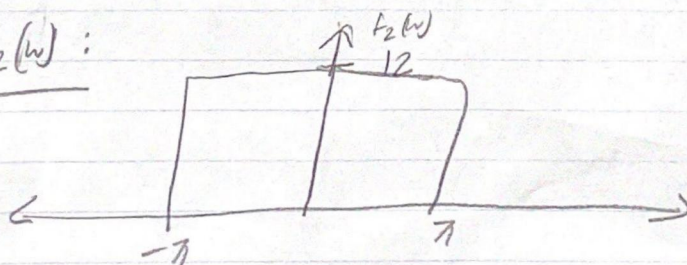
$$\begin{aligned} y_1(t) &= b \operatorname{sinc}(\pi t) e^{-j5\pi t} + b \operatorname{sinc}(\pi t) e^{j5\pi t} \\ &= b \operatorname{sinc}(\pi t) (e^{-j5\pi t} + e^{j5\pi t}) \\ &= b \operatorname{sinc}(\pi t) \cdot 2 \cos(5\pi t) \end{aligned}$$

$$y_1(t) = 12 \operatorname{sinc}(\pi t) \cdot \cos(5\pi t)$$

$$b) \quad y_2(t) = f_2(t) \cos(\omega_0 t) = y_1(t) \rightarrow \boxed{\omega_0 = 5\pi}$$

$$f_2(t) = 12 \operatorname{sinc}(\pi t) \leftrightarrow \underline{12 \operatorname{rect}\left(\frac{\omega}{2\pi}\right) = F_2(\omega)}$$

$F_2(\omega)$ :



with rect  
freq. rate by  $2\pi$

2.1 10 / 10

✓ - 0 pts Correct

2

Table 3, #9. → Frequency Shift

$$f(t) \leftrightarrow f(\omega), \quad f(t)e^{j\omega_0 t} \leftrightarrow f(\omega - \omega_0)$$

↓

$$b \operatorname{rect}\left(\frac{\omega - 5\pi}{2\pi}\right) + b \operatorname{rect}\left(\frac{\omega + 5\pi}{2\pi}\right) \rightarrow b f(t) e^{-j5\pi t} + b f(t) e^{j5\pi t}$$

Table 4, #8:  $\operatorname{sinc}(Wt) \leftrightarrow \frac{\pi}{W} \operatorname{rect}\left(\frac{\omega}{2W}\right)$

$W = \pi \rightarrow f_1(t) = \operatorname{sinc}(\pi t)$  \* we accounted for the  $\pm 5\pi$  shift

↓

$$y_1(t) = b \operatorname{sinc}(\pi t) e^{-j5\pi t} + b \operatorname{sinc}(\pi t) e^{j5\pi t}$$

$$= b \operatorname{sinc}(\pi t) (e^{-j5\pi t} + e^{j5\pi t})$$

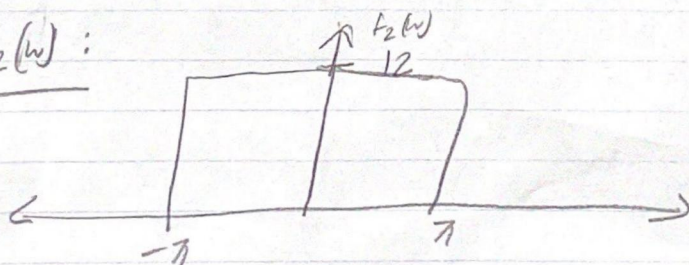
$$= b \operatorname{sinc}(\pi t) \cdot 2 \cos(5\pi t)$$

$$y_1(t) = 12 \operatorname{sinc}(\pi t) \cdot \cos(5\pi t)$$

b)  $y_2(t) = f_2(t) \cos(\omega_0 t) = y_1(t) \rightarrow \boxed{\omega_0 = 5\pi}$

$$f_2(t) = 12 \operatorname{sinc}(\pi t) \leftrightarrow \underline{12 \operatorname{rect}\left(\frac{\omega}{2\pi}\right)} = F_2(\omega)$$

$F_2(\omega)$ :



with rect  
freq. rate by  $2\pi$

2.2 10 / 10

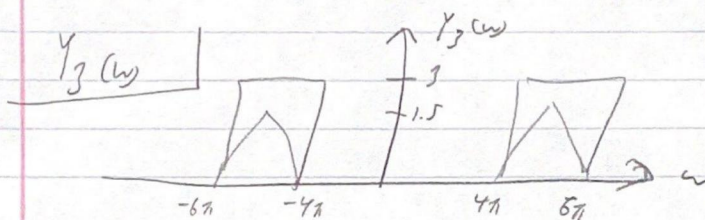
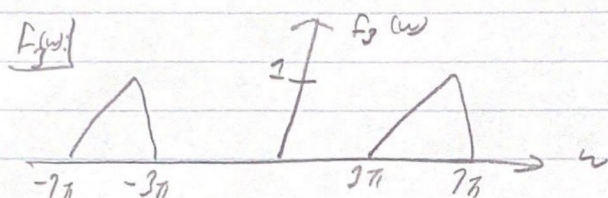
✓ - 0 pts Correct

3

$$2c \quad H(\omega) = 3 \left[ \text{rect} \left( \frac{\omega - 5\pi}{2\pi} \right) + \text{rect} \left( \frac{\omega + 5\pi}{2\pi} \right) \right]$$

$$f_2(\omega) = \Delta \left( \frac{\omega - 5\pi}{4\pi} \right) + \Delta \left( \frac{\omega + 5\pi}{4\pi} \right)$$

$$Y_2(\omega) = H(\omega) \cdot f_2(\omega) \rightarrow \mathcal{F}^{-1} \rightarrow y_2(t) = ?$$



$$\begin{aligned} Y_2(\omega) &= 3 \text{rect} \left( \frac{\omega - 5\pi}{2\pi} \right) + 3 \text{rect} \left( \frac{\omega + 5\pi}{2\pi} \right) + 1.5 \Delta \left( \frac{\omega - 5\pi}{2\pi} \right) + 1.5 \Delta \left( \frac{\omega + 5\pi}{2\pi} \right) \\ &= 3 \cdot f(t) \cdot (e^{j5\pi t} + e^{-j5\pi t}) + 1.5 \cdot f(t) \cdot (e^{j5\pi t} + e^{-j5\pi t}) \\ &= 3 \text{sinc} \left( \frac{\pi t}{2} \right) (e^{j5\pi t} + e^{-j5\pi t}) = \frac{3}{2} \left( \frac{1}{2} \right) \text{sinc}^2 \left( \frac{\pi t}{2} \right) e^{j5\pi t} + \frac{3}{2} \left( \frac{1}{2} \right) \text{sinc}^2 \left( \frac{\pi t}{2} \right) e^{-j5\pi t} \\ y_2(t) &= 3 \text{sinc}(\pi t) \cdot 2 \cos(5\pi t) + \frac{3}{2} \text{sinc}^2 \left( \frac{\pi t}{2} \right) \cdot \cos(5\pi t) = 3 \cos(5\pi t) \cdot (\text{sinc}(\pi t) + \frac{1}{2} \cos(5\pi t)) \\ y_2(t) &= 3 \cos(5\pi t) \cdot (\text{sinc}(\pi t) + \frac{1}{2} \cos(5\pi t)) \end{aligned}$$

2.3 8.5 / 10

✓ - 1.5 pts Graph Incorrect, but Exists

4

3. a)  $f(t) = \text{sinc}(10^5 t)$ , A.M. w/  $\cos(\omega_c t)$ ,  $\omega_c = 10^6 \text{ rad/s}$

$$x(t) = f(t) \cdot \cos(\omega_c t) \Leftrightarrow X(\omega) = \frac{1}{2} (F(\omega - \omega_c) + F(\omega + \omega_c))$$

$$f(t) \cdot \left( \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \right) \Leftrightarrow X(\omega), \quad \boxed{\text{Table \#3, \#10}}$$

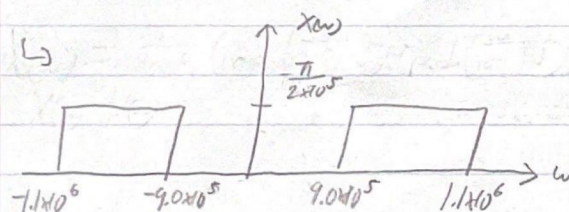
$$H(\omega) = 2 \text{rect} \left( \frac{\omega + 1.05 \omega_c}{10^5} \right) + 2 \text{rect} \left( \frac{\omega - 1.05 \omega_c}{10^5} \right)$$

$$S(\omega) = X(\omega) \cdot H(\omega)$$

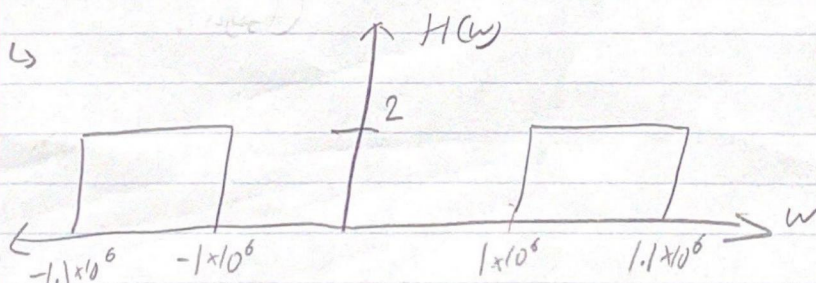
Table 4, #8:  $\text{sinc}(Wt) \Leftrightarrow \frac{\pi}{W} \text{rect} \left( \frac{\omega}{2W} \right)$

$$W = 10^5 \rightarrow \frac{\pi}{10^5} \text{rect} \left( \frac{\omega}{2 \cdot 10^5} \right) = F(\omega)$$

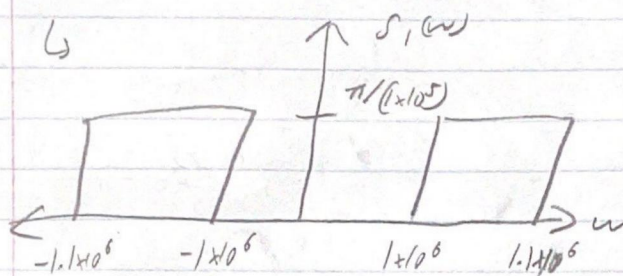
$$X(\omega) = \frac{1}{2} [F(\omega - \omega_c) + F(\omega + \omega_c)] = \frac{\pi}{2 \times 10^5} \left( \text{rect} \left( \frac{\omega - \omega_c}{2 \times 10^5} \right) + \text{rect} \left( \frac{\omega + \omega_c}{2 \times 10^5} \right) \right)$$



$$H(\omega) = 2 \text{rect} \left( \frac{\omega - 1.05 \omega_c}{10^5} \right) + 2 \text{rect} \left( \frac{\omega + 1.05 \omega_c}{10^5} \right)$$



2b)  $S_1(\omega) = X(\omega) \cdot H(\omega) = \text{overlap of graphs}$



$$\hookrightarrow S_1(\omega) = \frac{\pi}{10^5} \text{rect}\left(\frac{\omega - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{\omega + (1.05 \times 10^6)}{10^5}\right)$$

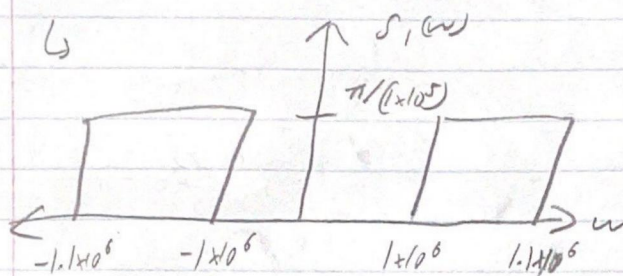
3b) To recover  $f(t)$  from  $S_1(t)$ , we need to mix  $S_1(t)$  with a delayed  $\cos(\omega_c t)$ , pass this mix through a low-pass filter, feed this through an amplifier with a gain of 2 in order to retain the original signal's amplitude. This results in the system returning the original  $f(t)$ .

3c) A Fourier transform is an even function + a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single Sideband Modulation, aptly named because one sideband (the 2 side bands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.1 5 / 5

✓ - 0 pts Correct

2b)  $S_1(\omega) = X(\omega) \cdot H(\omega) = \text{overlap of graphs}$



$$\hookrightarrow S_1(\omega) = \frac{\pi}{10^5} \text{rect}\left(\frac{\omega - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{\omega + (1.05 \times 10^6)}{10^5}\right)$$

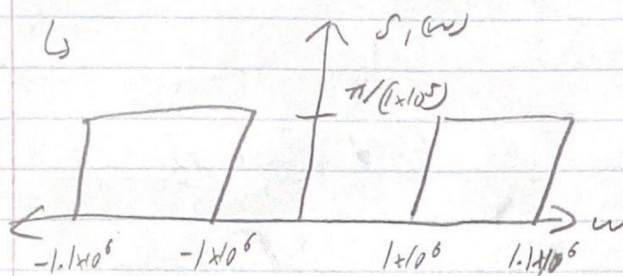
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3c) A Fourier transform is an even function + a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single Sideband Modulation, aptly named because one sideband (the 2 side bands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.2 10 / 10

✓ - 0 pts Correct

2b)  $S_1(\omega) = X(\omega) \cdot H(\omega) = \text{overlap of graphs}$



$$\hookrightarrow S_1(\omega) = \frac{\pi}{10^5} \text{rect}\left(\frac{\omega - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{\omega + (1.05 \times 10^6)}{10^5}\right)$$

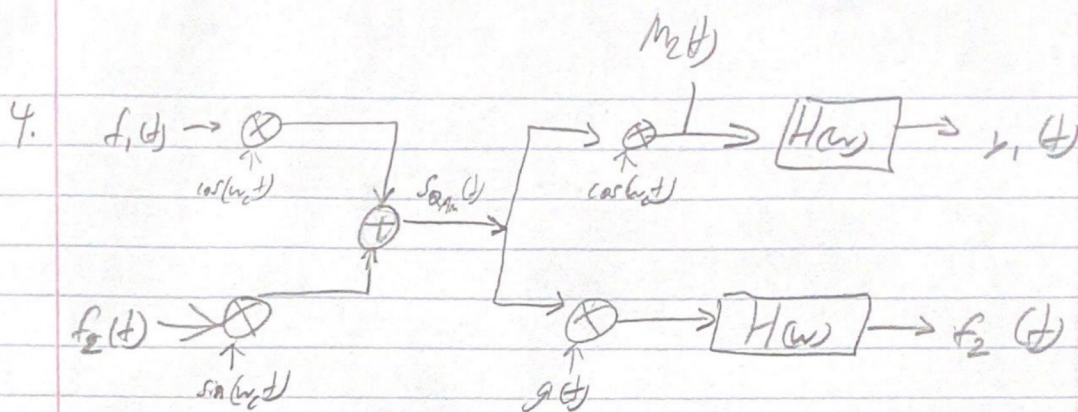
3b) To recover  $f(t)$  from  $S_1(t)$ , we need to mix  $S_1(t)$  with a delayed  $\cos(\omega_c t)$ , pass this mix through a low-pass filter, feed this through an amplifier with a gain of 2 in order to retain the original signal's amplitude. This results in the system returning the original  $f(t)$ .

3c) A Fourier transform is an even function + a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single Sideband Modulation, aptly named because one sideband (the 2 side bands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.3 5 / 5

✓ - 0 pts Correct

6



$f_1(t)$  and  $f_2(t)$  have 100% bandwidth  $\rightarrow B_{100\%} = 40\pi \times 10^3 \text{ rad/s}$

$\hookrightarrow \omega_c = 1 \times 10^6 \text{ rad/s}$ ,  $H(\omega) = 2 \text{rect}\left(\frac{\omega}{20\pi \times 10^3}\right)$

$$M_1(t) = f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)$$

$$\begin{aligned} M_2(t) &= \cos(\omega_c t) \cdot (f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)) \\ &= f_1(t) \cdot \cos^2(\omega_c t) + f_2(t) \cdot \cos(\omega_c t) \sin(\omega_c t) \\ &= f_1(t) \cdot \left(\frac{1}{2}\right) \cdot (1 + 2\cos(2\omega_c t)) + f_2(t) \cdot \left(\frac{1}{2}\right) \cdot (\sin(2\omega_c t)) \end{aligned}$$

$\hookrightarrow 2\omega = 2 \times 10^6 > B_{100\%}$ , Amplitude of  $H(\omega) = 2$

$$f_1(t) \cdot \left(\frac{1}{2}\right) \cdot 2 = f_1(t) \rightarrow \boxed{f_1(t) = f_1(t)}$$

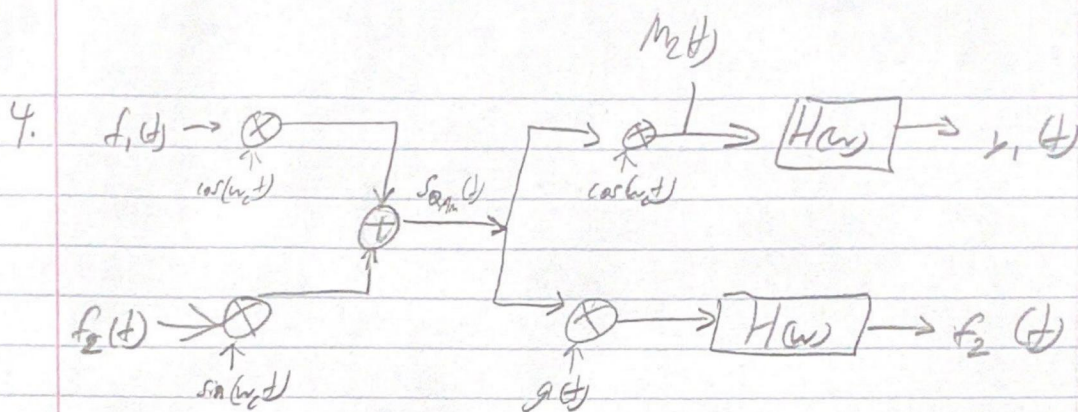
$$M_1(t) = f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)$$

$$\hookrightarrow M_2(t) = [f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)] \cdot \cos(\omega_c t)$$

4.1 10 / 10

✓ - 0 pts Correct

6



$f_1(t)$  and  $f_2(t)$  have 100% bandwidth  $\rightarrow \omega_{100\%} = 40\pi \times 10^3 \text{ rad/s}$

$\Rightarrow \omega_c = 1 \times 10^6 \text{ rad/s}$ ,  $H(\omega) = 2 \text{rect}\left(\frac{\omega}{20\pi \times 10^3}\right)$

$$M_1(t) = f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)$$

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$\Rightarrow 2\omega = 2 \times 10^6 > \omega_{100\%}$ , Amplitude of  $H(\omega) = 2$

$$f_1(t) \cdot \left(\frac{1}{2}\right) \cdot 2 = f_1(t) \rightarrow \boxed{f_1(t) = f_1(t)}$$

$$M_1(t) = f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)$$

$$\Rightarrow M_2(t) = [f_1(t) \cdot \cos(\omega_c t) + f_2(t) \cdot \sin(\omega_c t)] \cdot \cos(\omega_c t)$$

4.2 5 / 10

✓ - 5 pts Incorrect