- Average power and Parseval's theorem
  - Recall the average power for periodic functions

$$P = \frac{1}{T}\int_T |f(t)|^2 dt$$

  - Parseval's theorem says that

$$P = \frac{1}{T}\int_T |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |F_n|^2 = \frac{c_0{}^2}{4} + \sum_{n=1}^{\infty} \frac{c_n{}^2}{2}$$

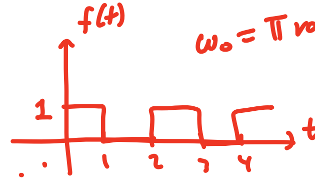
average signal power

" $\left(\frac{c_0}{2}\right)^2$

- Average power - Example # 8

  - Recall the periodic function f(t) with period T = 2s

$$f(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & t \in [1,2) \end{cases} = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{1}{jn\pi} e^{jn\pi t}$$

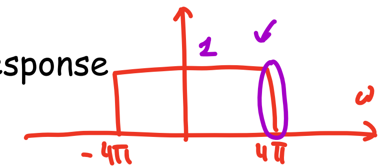


$\omega_0 = \pi$ rad/s

  - Let f(t) be the input to the ideal low-pass filter with frequency response

$$H(\omega) = rect(\frac{\omega}{8\pi})$$

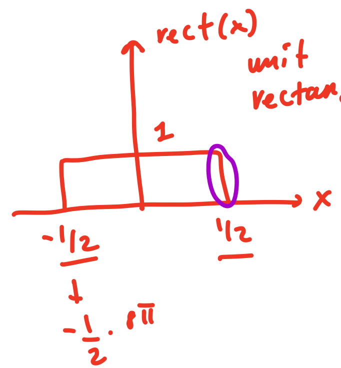
$rect(\frac{4\pi}{8\pi}) = H(\omega) = rect(\frac{1}{2})$



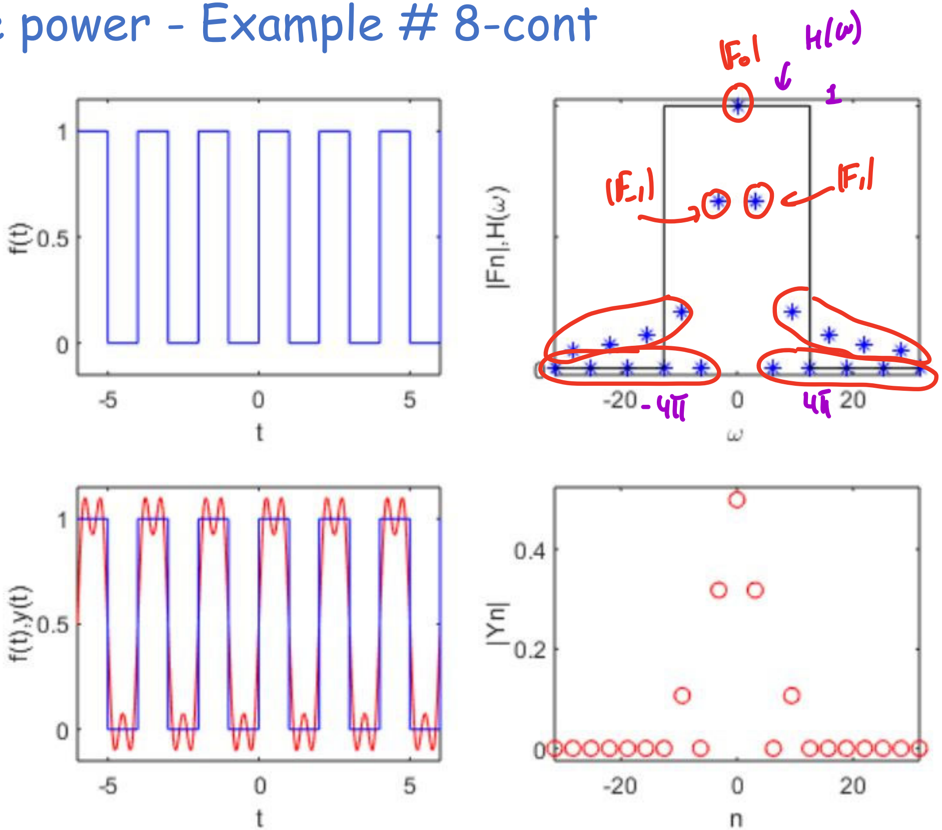
  - Determine the average power of f(t) and of the steady state output, y(t)

$$P_f = \frac{1}{T}\int_T |f(t)|^2 dt = \frac{1}{2}\int_0^1 (1)^2 dt = \frac{1}{2}$$

$$P_f = \sum_{n=-\infty}^{\infty} |F_n|^2 = \left(\frac{1}{2}\right)^2 + \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{1}{n^2\pi^2}$$



- Average power - Example # 8-cont



$F_n \rightarrow \boxed{H(\omega)} \rightarrow Y_n =$

$= F_n \cdot H(n\omega_0)$

$\Downarrow$

$\omega > 4\pi$ are blocked

$\Downarrow$

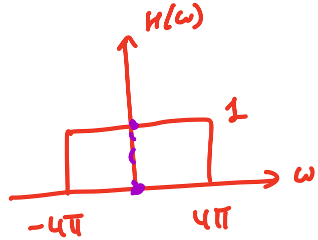
for $\omega_0 = \pi$ rad/s and $\omega = n\omega_0$

only $n = 0, \pm 1, \pm 3$ go through

- Average power - Example # 8-cont

$$f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n\ \text{odd}}}^{\infty} \frac{1}{jn\pi} e^{jn\pi t} \qquad \omega_0 = \pi \text{ rad/s}$$

$$H(\omega) = rect(\tfrac{\omega}{8\pi})$$



$$y(t) = \frac{1}{2}\cdot \underbrace{H(0)}_{1} + \sum_{\substack{n=-\infty \\ n\ \text{odd}}}^{\infty} \frac{1}{jn\pi}\cdot H(n\pi)\cdot e^{jn\pi t} =$$

$$H(n\pi) \rightarrow \begin{cases} 1 & n = \pm 1, \pm 3 \\ 0 & \text{else} \end{cases}$$

$$= \frac{1}{2}\cdot 1 + \sum_{\substack{n=-3 \\ n\ \text{odd}}}^{3} \frac{1}{jn\pi}\cdot(1)\, e^{jn\pi t} =$$

$$= \frac{1}{2} + \frac{1}{j\pi} e^{j\pi t} + \frac{1}{j(-1)\pi} e^{-j\pi t} + \frac{1}{j3\pi} e^{j3\pi t} + \frac{1}{j(-3)\pi}\cdot e^{-j3\pi t} =$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

$$\frac{1}{\pi}\,\frac{e^{j(\pi t)} - e^{-j(\pi t)}}{j}\cdot\frac{2}{2} = \sin(\pi t)$$

$$= \frac{1}{2} + \frac{2}{\pi}\sin(\pi t) + \frac{2}{3\pi}\sin(3\pi t)$$

$$P_y = \sum_{n=-\infty}^{\infty} |F_n|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\pi}\right)^2 + \left(\frac{1}{\pi}\right)^2 + \left(\frac{1}{3\pi}\right)^2 + \left(\frac{1}{3\pi}\right)^2 =$$

$$\approx 0.48$$

Chapter objectives

- Identify periodic signals and obtain their periods and fundamental frequencies
- Understand the significance and interpretation of Fourier series and its coefficients
- Apply properties of Fourier series to determine effect of basic signal processing
- Understand the effect of LTI systems, via $H(\omega)$, on periodic signals via their Fourier series and its coefficients
- Be able to calculate the average power of a periodic signal both in time and frequency