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Analog Signal Processing

Thursday, October 24, 8:45-10pm

Exam II

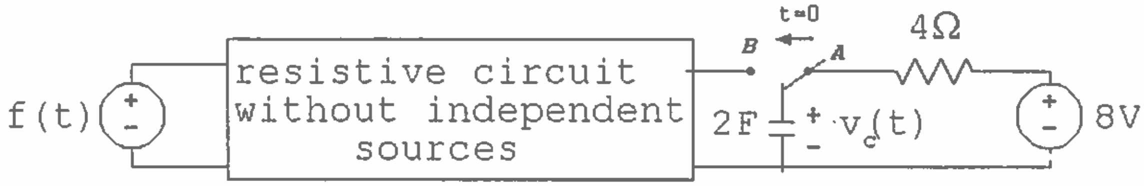
Last Name (capitalized):	Solutions
First Name:	

UIN:		**netID:**	

Course: (circle one)		ECE210	ECE211	
Section to return exam: (circle one)	11AM	12PM	2PM	3PM

Clearly PRINT your name in CAPITAL LETTERS.	**DO NOT write in these spaces.**
This is a closed book and closed notes exam.	Problem 1 (25 points):______
Calculators are not allowed.	Problem 2 (25 points):______
To get full credit, please SHOW all your work and simplify your answers.	Problem 3 (25 points):______
Write your final answers in the spaces provided.	Problem 4 (25 points):______
All answers should INCLUDE UNITS whenever appropriate.	Total: (100 points):______
The exam is printed **double-sided**.	

1. (25 pts) In the following circuit, the switch has been in position A for a long time, and then moves to position B at time $t = 0$. Several questions about this circuit will be asked, and possible answers are provided at the bottom of the page. After each question, write down the letter corresponding to your chosen answer. Answers could be used multiple times.



$v_c(t) = v_p(t) + v_h(t)$

same 'shape' of $f(t)$

$= \begin{cases} B & \text{if } f(t)=k \\ Be^{-pt} & \text{if } f(t) = ke^{-pt} \\ B\cos(\omega t+\theta) & \text{if } f(t) = H\cos(\omega t+\psi) \end{cases}$

Ae^{-at} transient

(a) [3 pts] If $f(t) = 3\sin(2t)$V, then $v_{c,ss}(t)$ is C.

$y(t) = A\cos(2t+\theta)$

(b) [3 pts] If $f(t) = 4e^{-2t}$V, then $v_{c,ss}(t)$ is K.

$y(t) = Ae^{-2t}$ transient

(c) [3 pts] If $f(t) = 2$V, then $v_{c,ss}(t)$ is J.

$y(t) = B$

(d) [3 pts] If $f(t) = 3\sin(2t) + 4e^{-2t}$V, then $v_{c,ss}(t)$ is C.

$y(t) = A\cos(2t+\theta) + Be^{-2t} \rightarrow$ transient

(e) [3 pts] If $f(t) = 3\sin(2t) + 2$V, then $v_{c,ss}(t)$ is D.

$y(t) = A\cos(2t) + B$

(f) [3 pts] If $f(t) = 4e^{-2t} + 2$V, then $v_{c,ss}(t)$ is J.

$y(t) = Ae^{-2t} + B$ $\rightarrow$ transient

(g) [3 pts] If $f(t) = 2\cos(2t) + 4\sin(2t)$V, then $v_{c,ss}(t)$ is C.

$y(t) = A\cos(2t+\theta)$

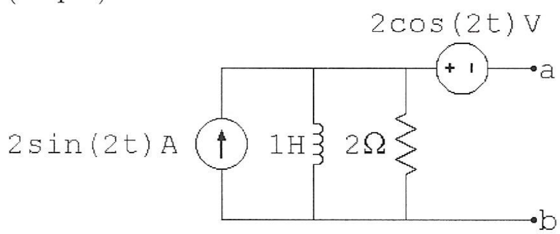
(h) [4 pts] If $f(t) = 5\cos(2(t+1))$V, then $v_{c,ss}(t)$ is C.

$y(t) = A\cos(2t+\theta)$

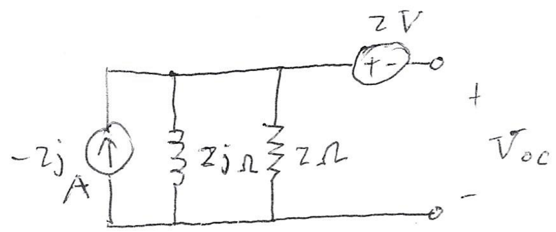
i. Below are the possible answers in units of ~~Amperes~~ Volts: α, β, γ are real-valued non-zero constants, and θ is a real-valued constant.

A. αe^{-2t}.
B. $\alpha e^{-2t} + \beta$.
C. $\alpha\cos(2t+\theta)$.
D. $\alpha\cos(2t+\theta) + \beta$.
E. $\alpha\cos(2t+\theta) + \beta + \gamma e^{-2t}$.
F. αe^{j2t}.
G. αe^{-j2t}
H. $\alpha e^{-j(2t+\theta)}$
I. $\alpha e^{-j\theta}$
J. α.
K. 0.
L. none of the above

2. (25 pts) Consider the circuit network between node a and b.



(a) [04 pts] Construct the equivalent phasor circuit.



(b) [09 pts] Determine the Thevenin phasor voltage in polar form.

From phasor circuit, apply current division or impedence combination:

$$V_T = V_{oc} = \frac{2}{2+2j}(2j)(-2j) - 2$$

$$= \frac{8}{2+2j} - 2$$

$$= -2j = 2\angle(-\tfrac{\pi}{2}) \ (V)$$

$V_T =$ $2\angle(-\frac{\pi}{2})$ (V)

(c) [09 pts] Determine the Thevenin impedance in rectangular form.

$$Z_T = (2j \,//\, 2) = \frac{1}{\frac{1}{2j} + \frac{1}{2}} = \frac{2j}{1+j} = 1+j \ \Omega$$

$Z_T =$ $1+j$ Ω

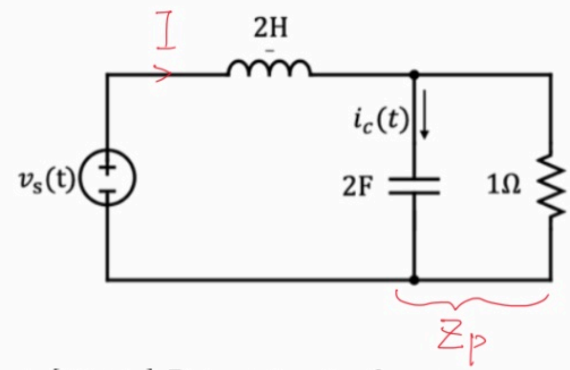
(d) [03 pts] Determine the matched load of the network.

$$Z_L = Z_T^*$$

$Z_L =$ $1-j$ Ω

3. (25 pts) The two parts of this problem are unrelated.

(a) [12 pts] Consider the following circuit.



i. [06 pts] Determine its frequency response, where $v_s(t)$ is the input and $i_c(t)$ is the output. Simplify your answer.

$$Z_p = 1 \,//\, \frac{1}{j\omega 2} = \frac{1}{1+j\omega 2}$$

$$I = \frac{V_s}{j\omega 2 + Z_p} = \frac{V_s}{j\omega 2 + \frac{1}{1+j\omega 2}}$$

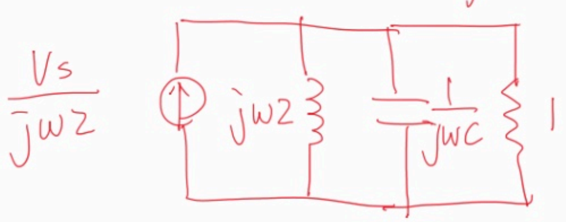
$$I_c = \frac{1}{1+\frac{1}{j\omega C}} I = \frac{j\omega 2}{1+j\omega 2} \cdot \frac{1+j\omega 2}{1+j\omega 2-\omega^2 4} V_s = \frac{j\omega 2}{1-\omega^2 4+j\omega 2} V_s$$

$$H(\omega) = \frac{I_c}{V_s}$$

$H(\omega) =$ $\frac{j\omega 2}{1-\omega^2 4+j\omega 2}$

ii. [03 pts] Determine the resonant frequency of the system, ω_0.

Source transformation



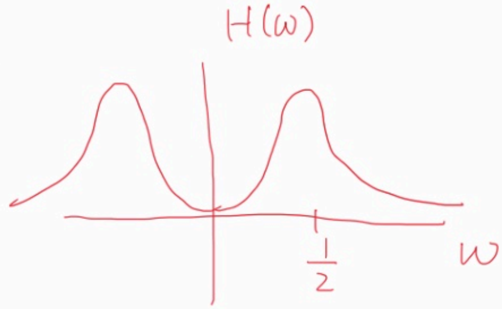
L & C in parallel

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2}$$

$\omega_0 =$ $\frac{1}{2}$ rad/s

iii. [03 pts] What kind of filter is the circuit? Write letter A, B, C or D in this line: C.

A. Low Pass Filter
B. High Pass Filter
C. Band Pass Filter
D. Band Stop Filter



(b) [13 pts] A linear system with input $f(t)$ and output $y(t)$ is described by the following ODE:

$\frac{dy(t)}{dt} - y(t) = -\frac{df(t)}{dt} - f(t)$

i. [04 pts] Determine the frequency response of the system.

$$jwY - Y = -jwF - F$$

$$H = \frac{Y}{F} = \frac{-jw-1}{jw-1} = \frac{1+jw}{1-jw}$$

$H(\omega) = \underline{\frac{1+jw}{1-jw}}$

ii. [09 pts] Given an input $f(t) = 2 + \cos\left(t - \frac{\pi}{6}\right) + 2\sin\left(2t + \frac{\pi}{3}\right)$, determine the steady-state response of the system.

$$2 \longrightarrow \boxed{LTI} \xrightarrow{2H(0) = 2}$$

$\cos(t - \frac{\pi}{6})$ $\quad H(1) = \frac{1+j}{1-j}$ $\quad |H(1)| = 1$ $\quad \angle H(1) = \frac{\pi}{2}$

$$= \frac{(1+j)^2}{2} = \frac{2j}{2} = j \Rightarrow \cos(t - \frac{\pi}{6} + \frac{\pi}{2})$$

$$= \cos(t + \frac{\pi}{3})$$

$$2\sin(2t + \frac{\pi}{3}) = 2\cos(2t + \frac{\pi}{3} - \frac{\pi}{2}) = 2\cos(2t - \frac{\pi}{6})$$

$$H(2) = \frac{1+2j}{1-2j} = \frac{(1+2j)^2}{5} = \frac{1-4+4j}{5} = \frac{-3+4j}{5}$$

$|H(2)| = 1$ $\quad \angle H(2) = \pi - \tan^{-1}\frac{4}{3}$ $\quad$ or $\angle H(2) = \tan^{-1} 2 - \tan^{-1}(-2) = 2\tan^{-1} 2$

$$\Rightarrow 2\cos(2t - \frac{\pi}{6} + \pi - \tan^{-1}\frac{4}{3})$$

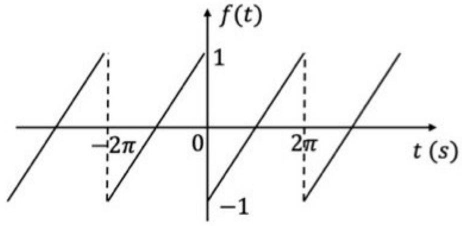
$y_{ss}(t) = \underline{2 + \cos(t + \frac{\pi}{3}) + 2\cos(2t + \frac{5}{6}\pi - \tan^{-1}\frac{4}{3})}$

or

$2\sin(2t + \frac{4}{3}\pi - \tan^{-1}\frac{4}{3})$

$2\sin(2t + \frac{\pi}{3} + 2\tan^{-1}(2))$

4. (25 pts) Consider the periodic signal $f(t)$ shown below, which can be expressed as a Fourier series with exponential coefficients $F_0 = 0$ and $F_n = \frac{j}{n\pi}$ for $n \neq 0$.



(a) [6 pts] Write the trigonometric and compact Fourier series forms of $f(t)$.

$a_0 = 0 \qquad C_0 = 0$

$a_n = F_n + F_{-n} = \frac{j}{n\pi} + \frac{j}{-n\pi} = 0$

$b_n = j(F_n - F_n) = j\left(\frac{j}{n\pi} + \frac{j}{n\pi}\right) = \frac{-2}{n\pi}$

$C_n = 2|F_n| = \frac{2}{n\pi}$

$\angle\theta_n = \angle F_n = \frac{\pi}{2}$

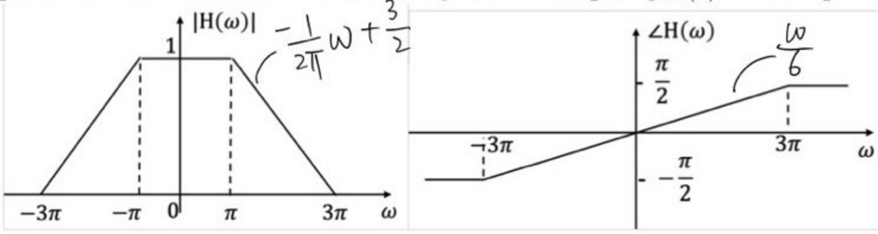
$T = 2\pi. \qquad \omega_0 = \frac{2\pi}{T} = 1$ rad/s

trigonometric form	$\sum_{n=1}^{\infty} \frac{-2}{n\pi} \sin(nt)$
compact form	$\sum_{n=1}^{\infty} \frac{2}{n\pi} \cos\left(nt + \frac{\pi}{2}\right)$

(b) [4 pts] What are the period and fundamental frequency of $f(t)$? Suppose a new function $g(t) = f(2t - 2\pi)$, what are the period and fundamental frequency of $g(t)$?

	Period	Fundamental frequency
$f(t)$	2π s	1 rad/s
$g(t)$	π s	2 rad/s

(c) [7 pts] Let $f(t)$ be the input to an LTI system with frequency response $H(\omega)$ given in the plot below. Determine the steady-state output $y_{ss}(t)$ in compact form.



$\omega_0 = 1.$

$|n\omega_0| \leq \pi \qquad n = 0, 1, 2, 3 \qquad |H(n\omega_0)| = 1. \qquad \angle H(n\omega_0) = \frac{n\omega_0}{6} = \frac{n}{6}.$

$$\sum_{n=1}^{3} \frac{2}{n\pi} \cos(nt + \frac{\pi}{2}) \longrightarrow \boxed{H(\omega)} \longrightarrow \sum_{n=1}^{3} \frac{2}{n\pi} \cos(nt + \frac{\pi}{2} + \frac{n}{6})$$

$\pi \leq |n\omega_0| \leq 3\pi. \qquad n = 4, 5, 6, 7, 8, 9.$

$|H(n\omega_0)| = \frac{3}{2} - \frac{n}{2\pi} \qquad \angle H(n\omega_0) = \frac{n}{6}$

$$\sum_{n=4}^{9} \frac{2}{n\pi} \cos(nt + \frac{\pi}{2}) \longrightarrow \boxed{H(\omega)} \longrightarrow \sum_{n=4}^{9} \left(\frac{2}{n\pi}\right)\left(\frac{3}{2} - \frac{n}{2\pi}\right) \cos(nt + \frac{\pi}{2} + \frac{n}{6})$$

$$\boxed{\sum_{n=1}^{3} \frac{2}{n\pi} \cos(nt + \frac{\pi}{2} + \frac{n}{6}) + \sum_{n=4}^{9} \left(\frac{2}{n\pi}\right)\left(\frac{3}{2} - \frac{n}{2\pi}\right) \cos(nt + \frac{\pi}{2} + \frac{n}{6})}$$

(d) [8 pts] Obtain the exponential Fourier series of $g(t) = f(2t - 2\pi)$.

$g(t) = f(2(t - \pi))$

$g'(t) = f(2t). \qquad G_n' = F_n \qquad g'(t) = \sum_{n=-\infty}^{\infty} \frac{j}{n\pi} e^{jn2t}$

$g(t) = g'(t - \pi). \qquad G_n = G_n' \cdot \underbrace{e^{-jn2\cdot\pi}}_{= 1 \text{ for all } n} = G_n' = \frac{j}{n\pi}$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} \frac{j}{n\pi} e^{jn2t}}$$