Lecture 16, Monday, February 14, 2022

- For a general first order ODE with constant coefficients and constant source
  - If the conditions of the ODE change at a time $t_0 > 0$, then the solution still holds until time t_0, but it will change after that.
  - If the reference time is t_0, then

$$y(t) \;=\; \frac{K}{a} \;+\; \left(y(t_0^+) - \frac{K}{a}\right) e^{-a(t-t_0)}$$

- If the input is time-varying, the ODE

$$f(t) \;=\; \frac{dy}{dt} + ay$$

has the solution

$$y(t) \;=\; y_p(t) \;+\; y_h(t) \;=\; y_p \;+\; \overbrace{Ae^{-at}}^{=\, y_h}$$

  - The homogeneous solution is the same no matter what the input is:

$$y_h(t) \;=\; Ae^{-at}.$$

  - However the particular solution changes, depending on the input, for example:

$$y_p(t) \;=\; \begin{cases} B & \text{if } \; f(t) = K \\ Be^{-pt} & \text{if } \; f(t) = Ke^{-pt}, \; a \neq p \\ Bte^{-at} & \text{if } \; f(t) = Ke^{-at} \\ H\cos(\omega t + \psi) & \text{if } \; f(t) = K\cos(\omega t + \theta) \end{cases}$$

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- To obtain the constants :
  - Subtitute $y_p(t)$ in ODE and match coefficients.
  - Evaluate $y(t_0) = y_p(t_0) + y_h(t_0)$ to match the initial conditions.