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For $t < 0$, inductor is a short, so

$$v_L(t) = 0 \text{ V},$$

which implies, by the current divider rule, that

$$i_1(0^-) = 10 \text{ A} \left(\frac{1 \Omega}{1 \Omega + 1.5 \Omega} \right) = 4 \text{ A},$$

and, again by the current divider rule,

$$i_x(0^-) = i_L(0^-) = 2 \text{ A}.$$

For time $t \geq 0$:

The short created by the new connection at $t = 0$ effectively removes the 10 A source, and the 1Ω resistor through which i_x flows, from the circuit, as all of the current from the source is diverted through the short, which implies that $i_x(t) = 0 \text{ A}$.

KVL:

$$\begin{aligned} v_L(t) + i_L(t) &= 0 \\ L \frac{di_L(t)}{dt} + i_L(t) &= 0 \\ \frac{di_L(t)}{dt} + \frac{1}{L} i_L(t) &= 0 \end{aligned}$$

The solution to this differential equation is of the form $Ke^{-\frac{1}{L}t} = Ke^{-t}$.

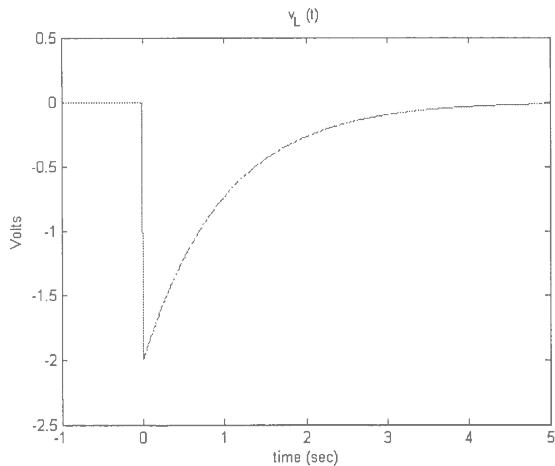
Applying the initial condition that $i_L(0^-) = 2 \text{ A}$,

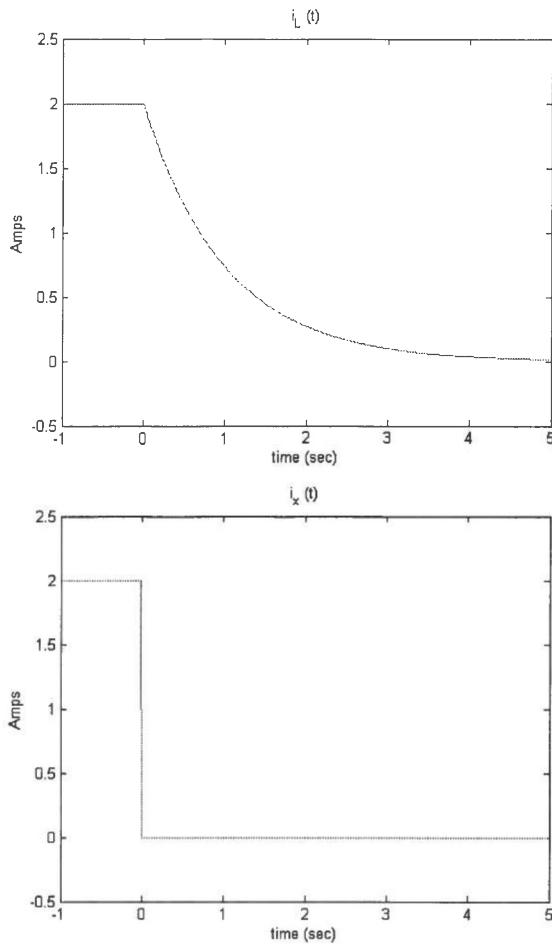
$$i_L(0) = 2 = K.$$

Therefore,

$$i_L(t) = 2e^{-t} \text{ A}, \quad i_x(t) = 0 \text{ A},$$

$$v_L(t) = -i_L(t) = -2e^{-t} \text{ V}.$$





3. Solution:

For time $t < 0$ s:

The inductor acts as a short circuit, since it has been in this state for a long time,

$$v_L = 0 \text{ V},$$

which implies, by the current divider rule (also noting that $v_R(t) = i_R(t)(1 \Omega) = i_R(t)$), that

$$i_R(0^-) = 2 \left(\frac{R_1}{1 + R_1} \right) = v_R(0^-)$$

For time $t \geq 0$ s:

The short created by the new connection at $t = 0$ effectively removes the 2 A source from the circuit, as all of the current from the source is diverted through the short, as well as R_1 . Note that $i_R(t) = i_L(t)$.

KVL:

$$\begin{aligned} 0 &= v_R(t) + v_L(t) \\ 0 &= i_R(t) + L \frac{di_R(t)}{dt} \\ 0 &= \frac{1}{L} i_R(t) + \frac{di_R(t)}{dt} \end{aligned}$$

The solution to this differential equation is of the form $i_R(t) = Ke^{-\frac{1}{R_1}t} = Ke^{-t} = v_R(t)$.

Applying the initial condition that $v_R(0^-) = v_R(0) = 2 \left(\frac{R_1}{1+R_1} \right)$,

$$v_R(0) = 2 \left(\frac{R_1}{1+R_1} \right) = K$$

Therefore,

$$v_R(t) = 2 \left(\frac{R_1}{1+R_1} \right) e^{-t},$$

which implies that $R_1 = \frac{1}{3} \Omega$.

4. Solution:

For time $t < 0$ s:

The capacitor is fully discharged and acts like an open circuit, since it has been in position A for a long time.

$$v(t) = 0 \text{ V}$$

For time $0 < t < 3$ s:

The capacitor begins charging from the 1 V source.

KVL:

$$\begin{aligned} 1 &= R_1 i_c(t) + i_c(t) + v(t) \\ \frac{1}{C(1+R_1)} &= \frac{1}{C(1+R_1)} v(t) + \frac{dv(t)}{dt} \end{aligned}$$

The solution, $v(t) = v_h(t) + v_p(t)$, to the above differential equation has a homogenous solution of the form $Ke^{-\frac{1}{C(1+R_1)}t} = Ke^{-\frac{1}{1+R_1}t}$.

For the particular solution, we guess that the solution is of the form of a constant, $v_p(t) = A$:

$$\frac{1}{C(1+R_1)} = \frac{1}{C(1+R_1)} A,$$

which implies that $A = 1$. Therefore,

$$v(t) = 1 + Ke^{-\frac{1}{1+R_1}t}.$$

Applying the initial condition that $v(0) = 0$ V,

$$\begin{aligned} v(t) = 0 &= 1 + K \\ K &= -1. \end{aligned}$$

Applying the constraint that $v(1) = 1 - e^{-1/3}$ implies that

$$R_1 = 2 \Omega$$

For time $t > 3$ s:

The capacitor is now discharging across the 1Ω resistor and R_2 .

KVL:

$$\begin{aligned} v(t) &= -(1 + R_2)i_c(t) \\ \frac{dv(t)}{dt} + \frac{1}{C(1 + R_2)}v(t) &= 0 \end{aligned}$$

The solution will be of the form:

$$v(t) = Ke^{-\frac{1}{C(1+R_2)}(t-3)} = Ke^{-\frac{1}{1+R_2}(t-3)}$$

$$v(3^-) = 1 - e^{-\frac{1}{3}(3)} = 1 - e^{-1} = K$$

Therefore,

$$v(4) = (1 - e^{-1})e^{-1/4} = (1 - e^{-1})e^{-\frac{1}{1+R_2}(1)}$$

implies

$$R_2 = 3 \Omega$$

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a) $I = \frac{1}{2} v(t) + \frac{dv}{dt}$

b) $v(0^-) = 4V$

c) $v(t) = 2 + 2e^{-t/2}$

d) $v_{2s}(t) = 4e^{-t/2}$

e) $v_{2s}(t) = 2 - 2e^{-t/2}$