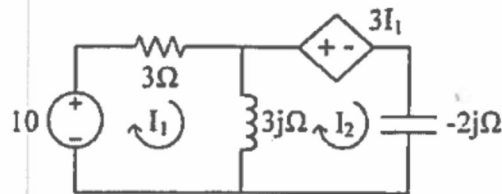


Problem 2

(a) In the following circuit, write two loop equations in terms of I_1 and I_2 .



$$10 - 3I_1 - 3j(I_1 - I_2) = 0 \quad 3I_1 + 3j(I_1 - I_2) = 10$$

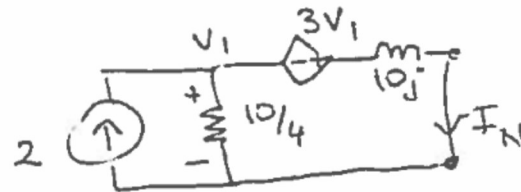
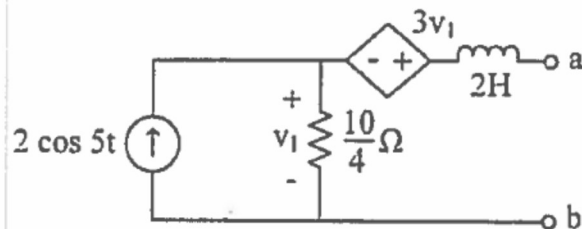
$$(3 + 3j)I_1 - 3jI_2 = 10 \quad (1)$$

$$(I_2 - I_1)3j + 3I_1 + (-2j)I_2 = 0 \quad (3 - 3j)I_1 + jI_2 = 0 \quad (2)$$

$$(3 + 3j)I_1 + (-3j)I_2 = (10)$$

$$(3 - 3j)I_1 + (j)I_2 = (0)$$

(b) For the following circuit, find I_N , Z_T and maximum available average power of the circuit between a and b. (Give I_N and Z_T in rectangular form i.e. $\alpha + j\beta$)



$$\frac{V_1}{5/2} + \frac{V_1 + 3V_1}{10j} = 2 \quad \frac{2V_1}{5} + \frac{4V_1}{10j} = 2 \rightarrow V_1 = \frac{5j}{1+j} \quad I_N = \frac{4V_1}{10j}$$

$$I_N = \frac{2}{1+j} = 1-j$$

$$I_N = 1-j$$

SOLUTION

$$Z_{eq} = j\omega L + R \parallel -\frac{j}{\omega C}$$

Because $\omega = 1 \text{ rad-s}^{-1}$, $R = 2 \Omega$,

and $L = 1 \text{ H}$,

$$Z_{eq} = j + \frac{-2j/\omega C}{2 - j/\omega C}$$

$$= j - j \left\{ \frac{2}{2C - j} \right\}$$

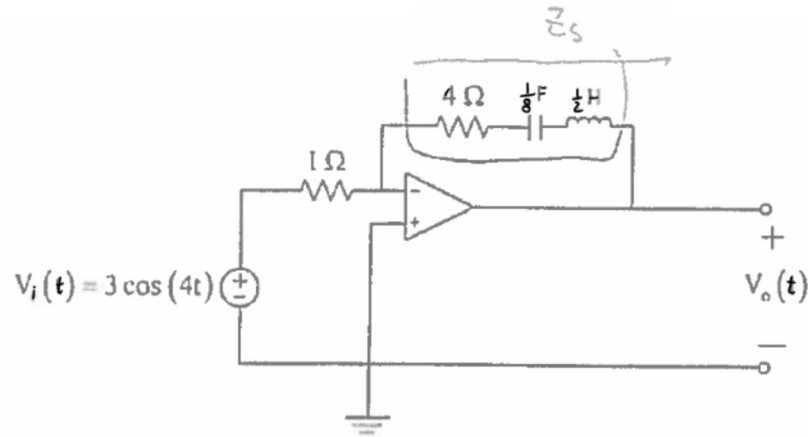
$$= j - \frac{j4C}{4C^2 + 1} + \text{REAL TERM}$$

For $i_g(t)$ to be in phase with

$v_g(t)$ requires $\text{Im}\{Z_{eq}\} = 0$.

$$\therefore \frac{4C}{4C^2 + 1} = 1 \quad \text{AND} \quad \boxed{C = \frac{1}{2} \text{ F}}$$

Problem 2



Assume the op-amp in this circuit is an ideal op-amp.

- a) (15 points) Use the phasor method to calculate the output voltage $V_o(t)$.

$$Z_s = R + j\omega L + \frac{1}{j\omega C} = 4 + j\frac{\omega}{2} - j\frac{8}{\omega} = 4 \quad \text{for } \omega = 4 \text{ rad/s}$$

$$\text{KCL at } V_- : \quad \frac{V_i - V_-}{1} = \frac{V_+ - V_o}{Z_s} \quad \text{and } V_+ = V_- = 0$$

$$\Rightarrow V_i = -\frac{V_o}{Z_s}$$

$$V_o = -4V_i = -4 \times 3 = -12V$$

$$\therefore V_o(t) = -12 \cos(4t) = 12 \cos(4t + \pi)$$

- b) (5 points) If a 1000Ω resistor is connected across the output terminals, what is the average power dissipated by the resistor?

Problem 2 (continued)

- c) (5 points) The op-amp is biased at $\pm 15\text{ V}$. How would the output voltage change as the input signal frequency is increased or decreased from 4 rad/s ? Explain your reasoning.

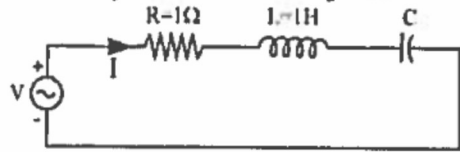
$$Z_s = 4 + \frac{j\omega}{2} \left(1 - \frac{16}{\omega^2}\right) = 4 \left(1 + \frac{j\omega}{8} \left(1 - \frac{16}{\omega^2}\right)\right)$$

$$|Z_s| = 4 \left[1 + \frac{\omega^2}{8^2} \left(1 - \frac{16}{\omega^2}\right)^2\right]^{1/2}$$

$$|V_o| = V_i |Z_s| = 12 \left[1 + \frac{\omega^2}{8^2} \left(1 - \frac{16}{\omega^2}\right)^2\right]^{1/2}$$

$|V_o|$ increases as ω is either increased or decreased from 4 rad/s until it is clipped by the bias ~~voltage~~ voltage

2. (25 pts) An RLC series circuit consisting of a 1Ω resistor R , an inductance of value $L = 1\text{ H}$ and a capacitor C is fed by a cosinusoidal voltage source.



- (a) If the cosinusoidal voltage source has angular frequency $\omega = 1\text{ rad/s}$ and the current leads (is ahead of) the voltage at the source by 45° , what is the value of the capacitor C ?

$$\frac{X}{R} = \tan(-45^\circ) \Rightarrow \frac{X}{R} = -1$$

$$R = 1\Omega, X = -1\Omega$$

$$X_L - X_C = -1 \Rightarrow X_L = 1\Omega \Rightarrow X_C = -2\Omega$$

$$\frac{1}{\omega C} = 2 \Rightarrow C = \frac{1}{2\omega} = \frac{1}{2}\text{ F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{2}}} = \sqrt{2}$$

$$C = \underline{\underline{\frac{1}{2}\text{ F}}}$$