- Example #1-cont

- Recall $|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$ $\qquad \angle H(\omega) = -\tan^{-1}(\overset{3}{\omega} RC)$

- Determine $v_o(t)$ if $v_i(t) = 2\cos(\overset{\omega = 3}{3}t + \frac{\pi}{3})$

$$v_o(t) = 2 \cdot |H(3)| \cos\left(3t + \pi/3 + \angle H(3)\right) = 2 \frac{1}{\sqrt{1+9R^2C^2}} \cos\left(3t + \pi/3 - \tan^{-1}(3RC)\right) \text{ V}$$

- Determine $v_o(t)$ if $v_i(t) = 3\sin(\overset{\omega}{6}t + \frac{\pi}{6})$ $= 3\cos(6t + \pi/6 - \pi/2)$

$$v_o(t) = 3 \cdot |H(6)| \cos\left(6t + \pi/6 - \pi/2 + \angle H(6)\right) =$$

$$= 3 \cdot |H(6)| \sin\left(6t + \pi/6 + \angle H(6)\right)$$

$$v_o(t) = 3 \cdot |H(6)| \sin\left(6t + \pi/6 + \angle H(6)\right)$$

• Frequency response of LTI systems-cont

- More generally, for LTI systems with

input f(t) = Re $\{Fe^{j\omega t}\}$
output y(t) = Re $\{Ye^{j\omega t}\}$

We have:

output phasor input phasor

$$\underbrace{Y = FH(\omega)}$$

$$F = |F|e^{j(\angle F)}$$

$$Y = |F||H(\omega)|e^{j(\angle F+\angle H(\omega))}$$

$$f(t) = |F|\cos(\omega t + \angle F)$$

$$y(t) = |F||H(\omega)|\cos(\omega t + \angle F + \angle H(\omega))$$

$f(t) \rightarrow \boxed{LTI} \rightarrow y(t)$

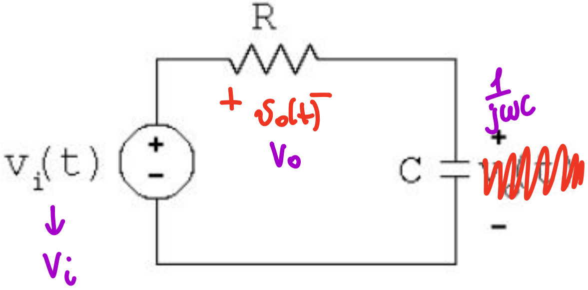
$\downarrow$

$F \rightarrow \boxed{H(\omega)} \rightarrow Y = F \cdot H(\omega)$

$\Downarrow$

$$\boxed{H(\omega) = \frac{Y}{F}}$$

- Example #2



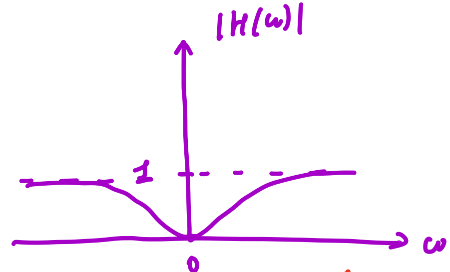
$$V_o = V_i \frac{R}{R+\frac{1}{j\omega C}} = V_i \underbrace{\frac{j\omega RC}{j\omega RC+1}}_{H(\omega)}$$

$$Y = F \cdot H(\omega)$$

- Let $v_i(t) = A\cos(\omega t + \theta)$
- Determine $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega))$

$$|H(\omega)| = \frac{|\omega| RC}{\sqrt{1+\omega^2R^2C^2}}$$

$$\measuredangle H(\omega) = \begin{cases} \pi/2 - \tan^{-1}(\omega RC) & \text{if } \omega > 0 \\ -\pi/2 - \tan^{-1}(\omega RC) & \text{if } \omega < 0 \\ 0 & \text{if } \omega = 0 \end{cases}$$

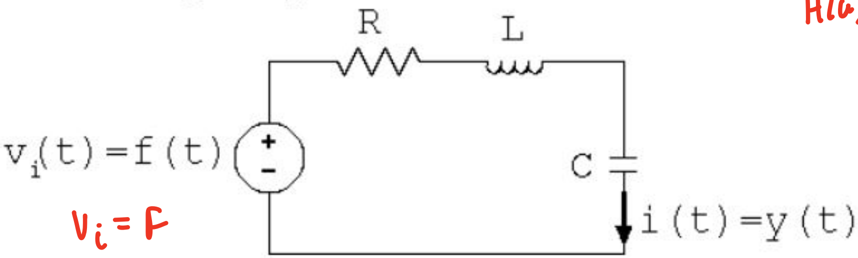


↑ high frequencies pass through almost untouched

⇓

high-pass filter

- Example #3



$V_i = F$

$Y = I$

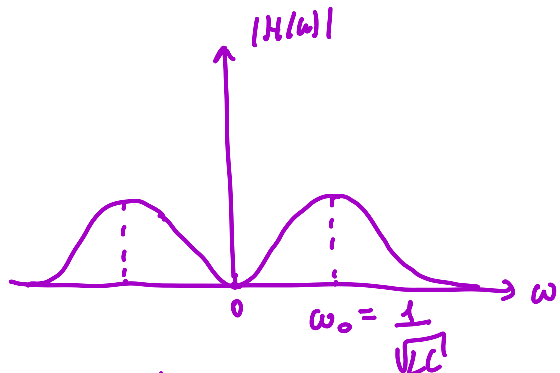
$$H(\omega) = \frac{Y}{F} = \frac{I}{V_i}$$

$$I = \frac{V_i}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega C V_i}{j\omega RC - \omega^2 LC + 1}$$

$H(\omega)$

- Let $v_i(t) = A\cos(\omega t + \theta)$
- Determine $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega)$)

$$|H(\omega)| = \frac{|\omega| C}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$



band-pass filter

• Example #4

- Consider the ODE

$$\frac{dy}{dt} + y = 3f(t)$$

$\downarrow$ phasors

$$j\omega Y + Y = 3F$$

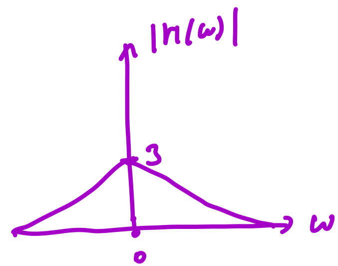
$$Y = \frac{3F}{j\omega + 1}$$

- Let $\mathrm{f}(t) = A\cos(\omega t + \theta)$
- Determine $H(\omega)$, $|H(\omega)|$ and $\angle H(\omega))$

$$H(\omega) = \frac{Y}{F} = \frac{3}{1 + j\omega}$$

$$|H(\omega)| = \frac{3}{\sqrt{1 + \omega^2}}$$

$$\angle H(\omega) = 0 - \tan^{-1}(\omega) = -\tan^{-1}(\omega)$$



low-pass filter

- General first-order filters

  and second

  - Low-pass

    $|H(0)| = 1$

    $|H(\infty)| = 0$

    $$|H(\omega)| \approx \frac{1}{\sqrt{1+\omega^2}}$$

  - High-pass

    $|H(0)| = 0$

    $|H(\infty)| = 1$

    $$|H(\omega)| \approx \frac{\omega}{\sqrt{1+\omega^2}}$$

  - Band-pass

    $|H(0)| = 0$

    $|H(\infty)| = 0$

    in between $\neq 0$

    $$|H(\omega)| \approx \frac{\omega}{\sqrt{\omega^2+(1-\omega^2)^2}}$$

  - $H(\omega)$ is only meaningful for dissipative LTI systems

- Properties of $H(\omega)$ for real-valued LTI systems

- Conjugate symmetry

$$H(\omega) = H^*(-\omega)$$

$j\omega L$ $\quad (j(-\omega)L)^* = -j\cdot(-\omega)L = j\omega L$

- Even amplitude response

$$|H(\omega)| = |H(-\omega)|$$

- Odd phase response

$$\angle H(\omega) = -\angle H(-\omega)$$

$-\angle H(\omega) = \angle H(-\omega)$

• Properties of $H(\omega)$-cont

- Real-valued DC response

$$H(0) \in \mathbb{R}$$

- Steady-state response to complex exponential

$$f(t) = A\, e^{j(\omega t+\theta)} \qquad H(\omega)\, A e^{j(\omega t+\theta)} = y(t)$$

$$\boxed{H(\omega)}$$