

## Exam II - Thursday, March 26, 8:45-10pm

- Show all your work and simplify your answers. Answers should include units if appropriate.
- **Important:** Throughout the exam, you will use a parameter  $\alpha$ , whose numerical value is equal to the last non zero digit of your UIN. Use that numerical value whenever  $\alpha$  is referenced, so the symbol  $\alpha$  should not appear in any answers. You will automatically get zero points for any problem where the parameter is not correctly recorded and used.
- The last page includes Fourier series tables that might be useful.

1. (25 pts) Parts (b) and (c) of this problem are unrelated.

(a) [00 pts] Write down the numerical value of  $\alpha$ .  $\alpha = ?$

(b) [10 pts] Convert each of the following two time signals into phasors in exponential form:

i. [05 pts]  $f_1(t) = -10 \sin\left(2t - \frac{\pi}{\alpha}\right)$ .  $F_1 = ?$

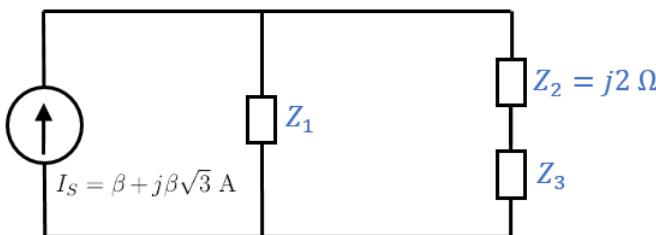
ii. [05 pts]  $f_2(t) = -\alpha \cos(3t) + \alpha \sin(3t)$ .  $F_2 = ?$

(c) [15 pts] An LTI system is given by the following differential equation.

$$\frac{d^2}{dt^2}y + \frac{d}{dt}y + \alpha y = f(t).$$

Determine the steady-state solution if  $f(t) = \sin(\alpha t)$ .  $y_{ss}(t) = ?$

2. (25 pts) Consider the following phasor network. The individual parts of the problem are independent from each other, except for part (a). A parameter  $\beta = 10\alpha + 7$  will be used throughout this problem.



(a) [00 pts] Write down the numerical value of  $\beta$ .  $\beta = ?$

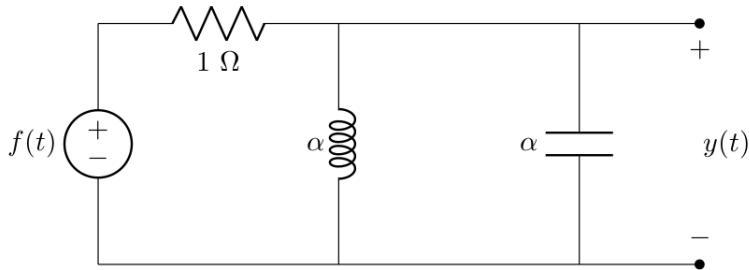
(b) [05 pts] Let  $Z_1 = 2.72 + j2\beta \Omega$ ,  $Z_3 = 3\beta + j4\beta \Omega$ . Determine the average absorbed power of  $Z_2$ .  $P_{Z_2} = ?$

(c) [06 pts] Let  $\omega = 2$  rad/s, and  $Z_1$  be composed of a resistor (resistance =  $\beta \Omega$ ) and a capacitor (capacitance =  $\frac{0.5}{\beta}$  F) in parallel. Determine the value of  $Z_1$ .  $Z_1 = ?$

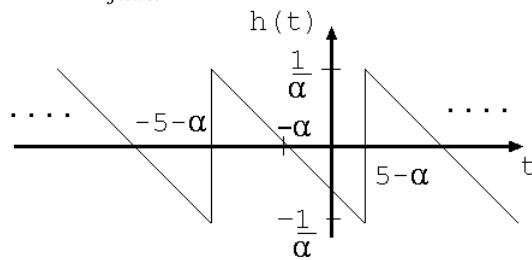
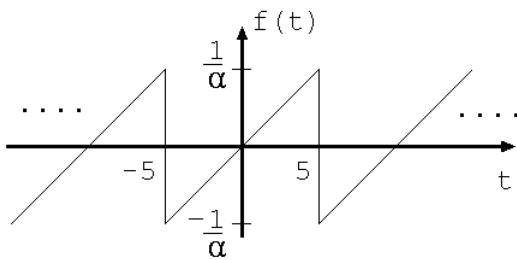
(d) [06 pts] Let  $Z_1 = 2\beta + j\beta$ , and  $Z_3 = Z_1 - Z_2$ . Determine the average absorbed power of  $Z_1$ .  $P_{Z_1} = ?$

(e) [08 pts] Let  $Z_1 = \beta + j3.14 \Omega$ , and  $P_{Z_2}$  and  $P_{Z_3}$  represent the average absorbed power of  $Z_2$  and  $Z_3$ , respectively. Determine the value of  $Z_3$  that maximizes  $P_{Z_2} + P_{Z_3}$ .  $Z_3 = ?$

3. (25 pts) Consider the circuit below.



- (a) [00 pts] Write down the numerical value of  $\alpha$ .  $\alpha = ?$
- (b) [03 pts] What is the natural (resonant) frequency of the circuit?  $\omega_0 = ?$
- (c) [12 pts] What is the frequency response of the circuit?  $H(\omega) = ?$
- (d) [10 pts] What is the output  $y(t)$  if the input is  $f(t) = 2 + 2 \cos(2t)$ ?  $y(t) = ?$
4. (25 pts) Consider the periodic signal  $f(t)$  shown below, which can be expressed as a Fourier series with exponential coefficients  $F_0 = 0$  and  $F_n = \frac{(-1)^{n+1}}{jn\pi\alpha}$  for  $n \neq 0$ .



- (a) [00 pts] Write down the numerical value of  $\alpha$ .  $\alpha = ?$
- (b) [10 pts] Determine the exponential Fourier series coefficients of  $h(t)$ .  $H_n = ?$
- (c) [15 pts]  $f(t)$  is the input to an LTI system with frequency response
- $$H(\omega) = \begin{cases} \alpha^2 \sin\left(\frac{5\omega}{2} - \frac{\pi}{2}\right) & |\omega| < \pi \\ 0 & \text{else.} \end{cases}$$
- Determine the output,  $y(t)$ , and its fundamental frequency,  $\omega_{0,y}$ .  $y(t) = ?$   
 $\omega_{0,y} = ?$