ECE 210 (AL2) – ECE 211 (E)

Chapter 4

Phasors and Sinusoidal Steady State

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Chapter objectives

- Understand the representation of co-sinusoids as phasors
- Understand and apply the principles of superposition and derivative in phasors
- Understand the representation of resistors, inductors and capacitors as impedances
- Carry out co-sinusoidal steady-state analysis of dissipative LTI systems through differential equations and directly through circuits
- Calculate the average absorbed power of basic circuit elements
    - For inductors and capacitors P = 0W
- Understand the meaning of available power and matched impedance, as well as how to calculate them
- Understand the concept of resonance

- Dissipative LTI systems

$$f(t) \rightarrow \boxed{\text{LTI}} \rightarrow y(t) = \underbrace{\cancel{y_{ZI}}}_{0}(t) + y_{ZS}(t) = \underbrace{\cancel{y_{tr}}}_{0}(t) + y_{ss}(t)$$

- If LTI system is dissipative, then

$$y_{ZI}(t) \xrightarrow{t \to \infty} 0$$

y_{ZI} is transient

- Assume we wait long enough for initial state to dissipate

amplitude

phase shift

$$f(t) = A\cos(\omega t + \theta) \rightarrow \boxed{\text{LTI, dissipative}} \rightarrow y_{ss}(t) = B\cos(\omega t + \psi)$$

frequency rad/s

part of $y_{ZS}(t)$

same!

- Phasors

$$f(t) = A\cos(\omega t + \theta) = \underbrace{\text{Re}\left\{Ae^{j(\omega t+\theta)}\right\}} = \text{Re}\left\{Ae^{j\theta}e^{j\omega t}\right\} = \text{Re}\left\{\boldsymbol{F}e^{j\omega t}\right\}$$

$Ae^{j\theta}$ → constant

$e^{j\omega t}$ → time varying

$Ae^{j\theta} = F$

phasor (complex constant)

$$\text{Re}\{A e^{j(\omega t+\theta)}\} =$$

$$= \text{Re}\{A(\cos(\omega t+\theta) + j\sin(\omega t+\theta))\} =$$

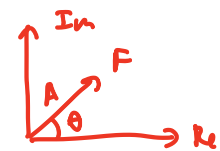
$$= A\cos(\omega t+\theta)$$

$$F = A e^{j\theta} = \underbrace{A\angle\theta}_{\text{polar}}$$

A ↑ magnitude

θ ↙ phase

$$F = |F| e^{j\theta}$$



input

$$f(t) = \text{Re}\{F e^{j\omega t}\}$$

output

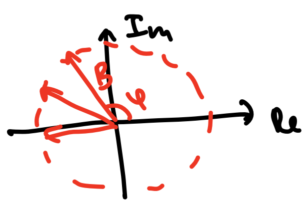
$$y(t) = \text{Re}\{Y e^{j\omega t}\}$$

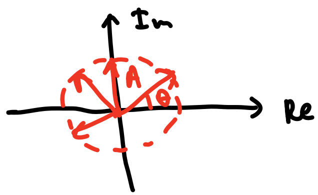
same!

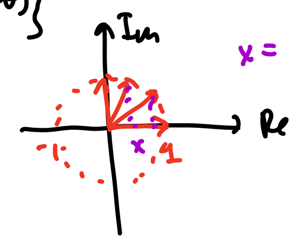
different

$$1e^{j\omega t}$$

$$y(t) = \text{Re}\{B e^{j\varphi} e^{j\omega t}\} \quad f(t) = \text{Re}\{A e^{j\theta} e^{j\omega t}\} = \text{Re}\{A e^{j(\omega t+\theta)}\}$$

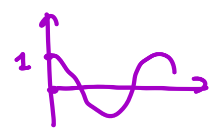






$x = \cos(\omega t)$

$\cos(\omega t)$



- Phasors

$$f(t) = \underbrace{A\cos(\omega t + \theta)} = \text{Re}\left\{Ae^{j(\omega t+\theta)}\right\} = \text{Re}\left\{Ae^{j\theta}e^{j\omega t}\right\} = \text{Re}\left\{\boldsymbol{F}e^{j\omega t}\right\}$$

↓ time to phasors

$$F = Ae^{j\theta}$$

phasors to time

$$f(t) = A\cos(\omega t + \theta)$$

Assuming A is positive.

If $-A$, fix it by shifting by π

$$f(t) = -A\cos(\omega t + \theta) = A\cos(\omega t + \theta + \pi)$$

↓ phasors

$$F = Ae^{j(\theta+\pi)}$$