• Example #5

- What if there are multiple frequencies?
- Let $f(t) = \cos\left(\overset{\omega_1}{\frac{1}{2}}t\right) + 3\cos(\overset{\omega_2}{2}t + \frac{\pi}{4}) + 2\sin(\overset{\omega_3}{3}t)$

be the input to the LTI system with input-output rule $\underbrace{y(t) = \frac{d}{dt}f(t)}$

$\downarrow$ phasors

$Y = j\omega F$

- Determine $y_{ss}(t)$

① $H(\omega)$, $|H(\omega)|$, $\measuredangle H(\omega)$

$H(\omega) = \frac{Y}{F} = j\omega$

$|H(\omega)| = |\omega|$

$$\measuredangle H(\omega) = \begin{cases} \pi/2 & \omega > 0 \\ -\pi/2 & \omega < 0 \\ 0 & \omega = 0 \end{cases}$$

• Example #5-cont

$$f(t) = \underbrace{1\cos\left(\frac{1}{2}t\right)}_{} + \underbrace{3\cos\left(2t + \frac{\pi}{4}\right)}_{} + \underbrace{2\sin(3t)}_{}$$

$\omega_1 \qquad \omega_2 \qquad \omega_3$

$$y(t) = \frac{d}{dt} f(t)$$

$$|H(\omega)| = |\omega|$$

$$\measuredangle H(\omega) = \begin{cases} \pi/2 & \omega > 0 \\ -\pi/2 & \omega < 0 \\ 0 & \omega = 0 \end{cases}$$

• Determine $y_{ss}(t)$

$$y_{ss}(t) = 1 \cdot |H(\omega_1)| \cos\left(\frac{1}{2}t + \measuredangle H(\omega_1)\right) +$$

$$+ 3 \cdot |H(\omega_2)| \cos\left(2t + \frac{\pi}{4} + \measuredangle H(\omega_2)\right) +$$

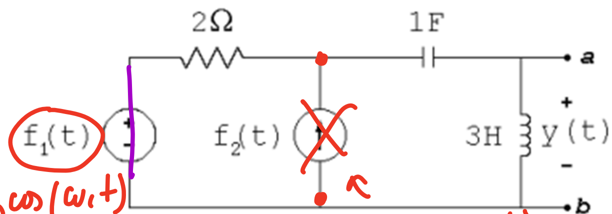
$$+ 2 \cdot |H(\omega_3)| \sin\left(3t + \measuredangle H(\omega_3)\right) =$$

$$= 1 \cdot \frac{1}{2} \cos\left(\frac{1}{2}t + \frac{\pi}{2}\right) + 3 \cdot 2 \cos\left(2t + \frac{\pi}{4} + \frac{\pi}{2}\right) + 2 \cdot 3 \sin\left(3t + \frac{\pi}{2}\right)$$

- Example #6

$\omega_1 = \frac{1}{3}$ rad/s $\qquad \omega_2 = 1$ rad/s

Consider the circuit below, where $f_1(t) = 2\cos\left(\frac{1}{3}t\right)\ V$ and $f_2(t) = 3\sin(t)\ A$.



$2\cos(\omega_1 t)$

$3\sin(\omega_2 t) =$
$= 3\cos(\omega_2 t - \pi/2)$

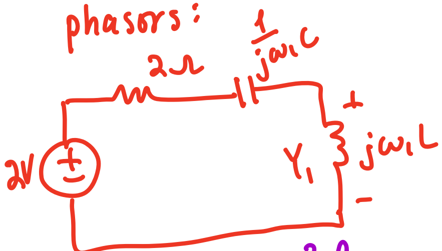
look at effect of each source individually, and add in time domain!

$y_{ss}(t) = y_{ss_1}(t) + y_{ss_2}(t)$

- Determine $y_{ss}(t)$

Only $f_1(t)$:

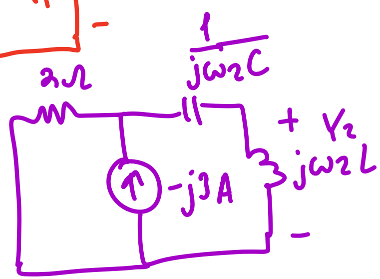
phasors:



$Y_1 = \frac{1}{\sqrt{2}} e^{j\frac{3\pi}{4}} \rightarrow y_{ss_1}(t) = \ldots$

Only f_2:

phasors:



$Y_2 = \frac{9}{\sqrt{2}} e^{-j\pi/4} \rightarrow y_{ss_2}(t) = \ldots$

$y_{ss}(t) = y_{ss_1}(t) + y_{ss_2}(t)$