ECE 210 Review Session

MIDTERM TWO

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The Path

1. Circuits!
2. Generalize circuits into arbitrary linear systems
3. Analyzing linear systems by hand is painful (so much algebra, time derivatives…)
   a. Three capacitors in a circuit = 3rd order differential equation. YUCK
4. Introducing the frequency domain!
   a. This is a pivotal moment. You will live and breathe in this domain for at least the remainder of your time here. The Fourier domain pops up in signal processing, power transfer, energy, communications, matrix/vector multiplication, antenna arrays, and so much more.
   b. So powerful that even quantum computing still uses it.
   c. The only 3-of-5 for EEs that doesn't feature the Fourier domain is ECE391. Boring class anyway.
5. Phasors - single sinusoid, steady state only
6. Fourier series: Periodic functions, steady state only
7. Fourier Transform: Almost all functions, steady state only
8. Laplace Transform: Almost all functions, transient and steady state solutions

RL, RC, RLC Circuit

- RL and RC circuits require setting up and solving a first-order ODE
- RLC circuits require setting up and solving a second-order

At the **steady state**,

- Capacitors act as open circuits
- Inductors act as wires

Continuous functions of time:

- Voltage across a capacitor
- Current through an inductor

$$i_c(t) = C\frac{d}{dt}v_c(t)$$

$$v_l(t) = L\frac{d}{dt}i_l(t)$$

Second Order Differential Equations

Given equation, where α and β are constants

$$\frac{d^2y}{dt^2} + \alpha\frac{dy}{dt} + \beta y(t) = f(t)$$

Then, $\boldsymbol{y(t) = Ae^{-r_1t} + Be^{-r_2t} + h(t)}$, where *A* and *B* are constants, and h(t) is the particular solution.

r_1 and r_2 are solutions to the equation $r^2 + \alpha r + \beta = 0$.

Homogeneous solution (f(t)=0) – the "exponential" terms (the transient solution)

Particular solution – $h(t)$

Special case: If $r_1 = r_2$, then the homogenous solution is instead $y(t) = \boldsymbol{Ae^{-rt} + Bte^{-rt}}$

First and Second Order Circuits

Zero State Response: Solution to ODE when initial state is 0

Zero Input Response: Solution to ODE when input is 0

- $y(t) = y_h(t) + y_p(t)$
- $y_p(t) = y_{ZSR}(t)$
- $y_h(t) = y_{ZIR}(t)$

y(t) expressions

$y(t) = ZI + ZS$

$y(t) = Transient + Steady\ State$

Both add up to y(t) but can be expressed differently

Tips:

Find ZI and ZS first → add to get y(t) → figure out Transient and Steady State

	Source function $f(t)$	Particular solution of $\frac{dy}{dt} + ay(t) = bf(t)$
1	constant D	constant K
2	Dt	$Kt + L$ for some K and L
3	De^{pt}	Ke^{pt} if $p \neq -a$ Kte^{pt} if $p = -a$
4	$\cos(\omega t)$ or $\sin(\omega t)$	$H\cos(\omega t + \theta)$, where H and θ depend on ω, a, and b

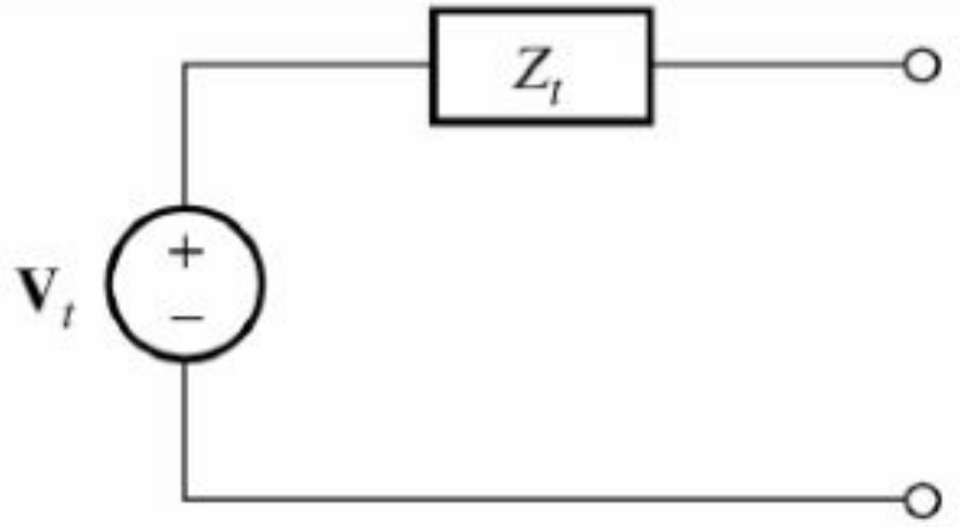
Table 3.1 Suggestions for particular solutions of $\frac{dy}{dt} + ay(t) = bf(t)$ with various source functions $f(t)$.

Conceptual Questions (30 seconds each)

A system converts its input $f(t)=10\cos(2t)$ into a steady-state output $y(t)=5\sin(2t-\frac{\pi}{3})+5\cos(4t+\frac{\pi}{4})$. Is the system LTI? Why?

A system converts its input f(t) = 64.27cos(2t+3000pi) + 20.3cos(4t) into a steady-state output y(t) = 1038sin(2t – 1.25pi). Is the system LTI? Why?

Available Power



$Z_L = Z_T^*$ for max available power

$$P_a = \frac{|V_T|^2}{8R_T}$$

Resonance

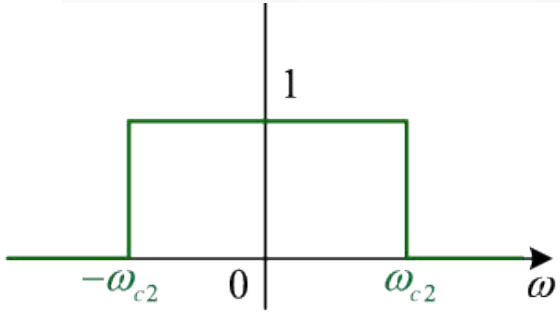
Frequency at which the circuit response is purely real.

If the input frequency equals the resonance frequency, then the impedance of the capacitor and inductor cancel each other out till the only impedance in the circuit is due to resistor.

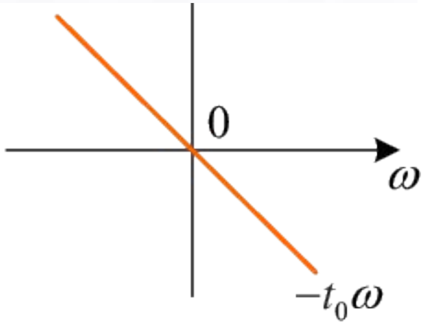
As a result the input and output are in-phase with each other and the output has its maximum amplitude.

For RLC circuit, the resonant frequency is $\omega_o = \frac{1}{\sqrt{LC}}$

Filters

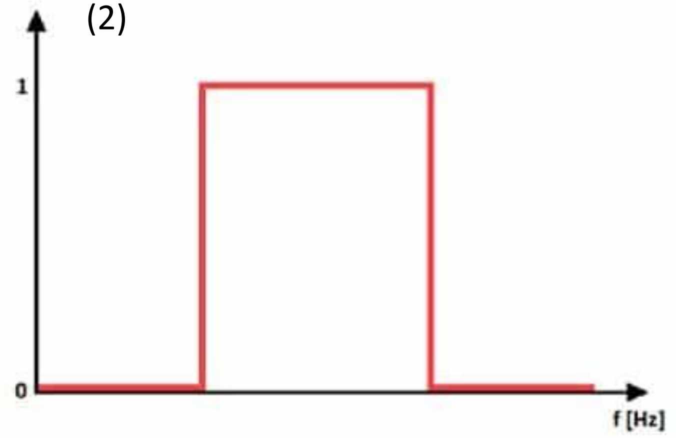


(a) Amplitude-frequency.



(b) Phase-frequency.

(2)



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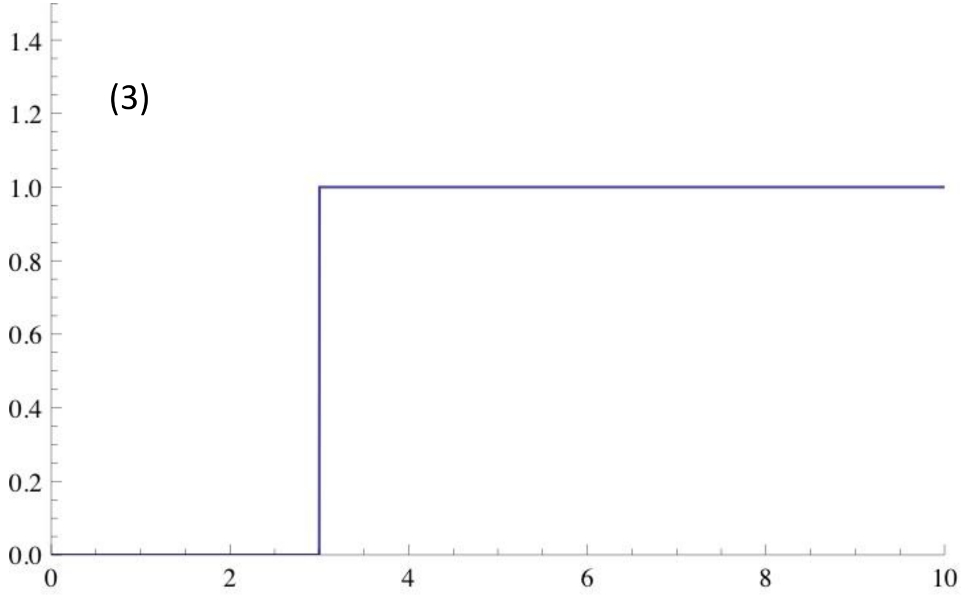


$$|H(\omega)| = \sqrt{H(\omega)H(\omega)^*}$$

$$\angle H(\omega) = \tan^{-1}\left(\frac{Im\{H(\omega)\}}{Re\{H(\omega)\}}\right)$$

Steps to Find a Filter

$$Y(\omega) = F(\omega)H(\omega)$$

- Remember, scale your input
  - So response is output / input
- In order to determine type of filter, evaluate

  |H(w)| at 0, ∞, and in between (possibly at w0)

Low Pass	High Pass	Band Pass	
$\lvert H(0)\rvert=1$	$\lvert H(0)\rvert=0$	$\lvert H(0)\rvert=0$	$\lvert H(w0)\rvert>0$
$\lvert H(\infty)\rvert=0$	$\lvert H(\infty)\rvert=1$	$\lvert H(\infty)\rvert=0$	

Conceptual Questions

A linear system has the following frequency response:

$$H(\omega) = e^{j\omega}$$

Find the output of the system when the input is

$$x(t) = 3sin(3t - 3) + 3$$

Conceptual Questions

A linear system is described by the following differential equation:

$$y(t) + \frac{d^5 y}{dt^5} + \frac{d^2 f}{dt^2} = f(t) - \frac{dy}{dt}$$

Find the frequency response H(w) of the system.

Conceptual Question Solution

$$y(t) + \frac{d^5 y}{dt^5} + \frac{d^2 f}{dt^2} = f(t) - \frac{dy}{dt}$$

$$Y + (j\omega)^5 Y + (j\omega)^2 F = F - j\omega Y$$

$$Y(1 + (j\omega)^5 + j\omega) = F(1 - (j\omega)^2)$$

Conceptual Question Solution

$$Y(1 + (jw)^5 + jw) = F(1 - (jw)^2)$$

$$Y(\omega) = H(\omega)F(\omega) \longrightarrow H(\omega) = \frac{Y(\omega)}{F(\omega)}$$

$$H(\omega) = \frac{Y(\omega)}{F(\omega)} = \frac{1 - (jw)^2}{1 + (jw)^5 + jw}$$

Phasor Definition

$$Acos(\omega t + \theta) = Re\{Ae^{j\omega t}e^{j\theta}\} \longleftrightarrow Ae^{j\theta}$$

$$sin(x) = cos(x - \frac{\pi}{2})$$

Phasors, Co-sinusoids, and Impedance

$Acos(\omega t + \theta) \rightarrow Ae^{j\theta}$

Remember $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ so

$Asin(\omega t + \theta) \rightarrow -Aje^{j\theta}$

Inductor: $Z = j\omega L$

Capacitor: $Z = \frac{1}{j\omega C}$

Resistor: $Z = R$

Once you've converted every circuit element in the phasor domain, you can analyze the circuit using all the ways covered in the first midterm!!!

- Node Voltage
- Superposition
- Loop Current
- Source Transformations

NOTE: The solution obtained after analysis with the phasor method is the *STEADY STATE SOLUTION !!!*

Formulae to Remember:

$$cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

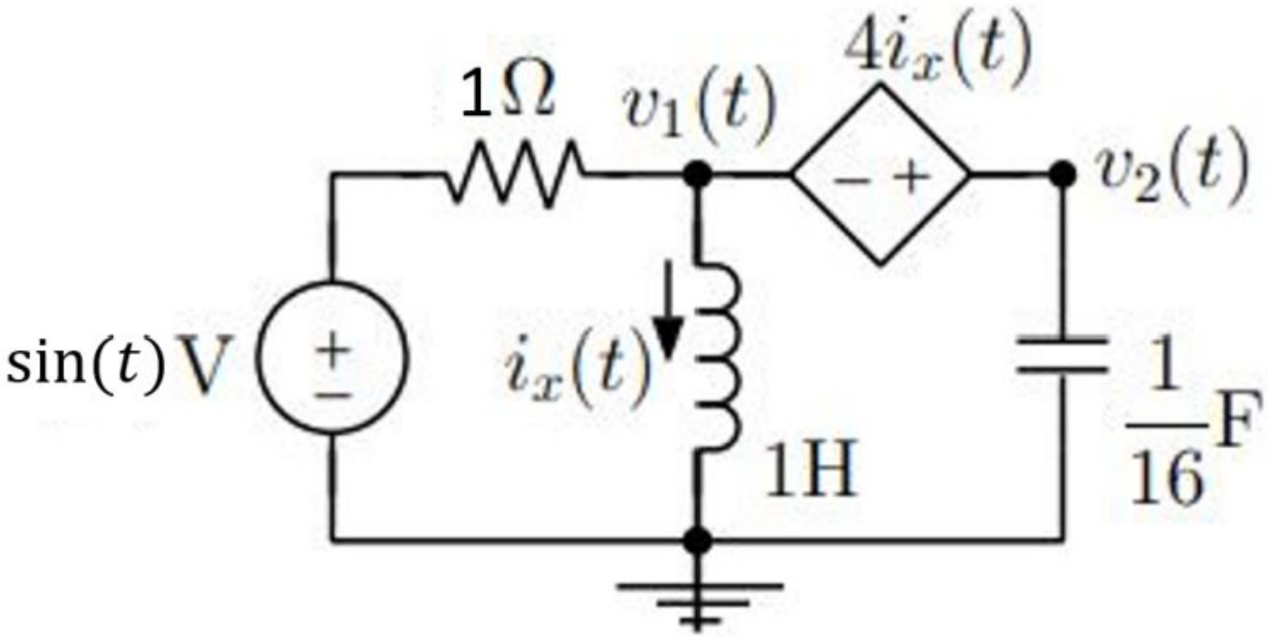
Conceptual Questions (30 seconds each)

Is sin(2t) + cos(2 pi t) periodic? Why?

Is $e^{j2t} + e^{4t}$ periodic? Why?

Conceptual Question

Convert the following circuit into the phasor domain:



Fourier Series

Getting the recipe of a periodic signal

$F_0 = \frac{1}{T}\int_T f(t)\,dt$ = average of function

$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$

$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

$\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta)$ $\quad$ *only when f(t) is real*

$c_n = 2|F_n|$

$\theta_n = \measuredangle F_n$

$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$

$a_n = F_n + F_{-n}$

$b_n = j(F_n - F_{-n})$

For trigonometric form, if function is:

- even: b_n=0
- odd: a_n=0

...onometric and compact forms of the Fourier series.

Name:	Condition:	Property:
Scaling	Constant K	$Kf(t) \leftrightarrow KF_n$
Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n, \cdots$	$f(t)+g(t)+\cdots \leftrightarrow F_n+G_n+\cdots$
Time shift	Delay t_o	$f(t-t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
Even function	$f(-t) = f(t)$	$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

6.3 Properties of Fourier series.

Example 6.5

Find the exponential and compact Fourier series of $f(t) = \lvert\sin(t)\rvert$ shown

Other properties of Fourier Series

Division by 0 IS BAD. If a coefficient depends on division by 0, find it using another method.

For discontinuities in the sequence: the Fourier series converges to the middle of the discontinuity.

Gibbs phenomenon: there are ripples that appear right before a big jump, which we can only get rid of if we let n go to infinity.

Conditions for existence and usage of Fourier Series:

- f(t) must be periodic (nonperiodic signals are dealt with using a Fourier Transform, you'll learn this very soon ;)

- f(t) must be absolutely integrable.

- There must be a finite number of maxes and mins over one period, and a finite number of finite discontinuities over one period.

Parseval's Theorem

$$P \equiv \frac{1}{T}\int_T |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |F_n|^2 = \left(\frac{c_0}{2}\right)^2 + \sum_{n=1}^{\infty} \frac{c_n^2}{2}$$

time domain exponential compact

Fourier Series:

Find the Exponential Fourier series of the following expression:

$$x(t) = cos(2t - \frac{\pi}{4}) + 3sin(5t)$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Fourier Series:

Find the Exponential Fourier series of the following expression:

$$x(t) = cos(t) + sin(\pi t)$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
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$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Old HW Question

4. The signal

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \cos\left(2n\pi t + \frac{\pi}{2}\right)$$

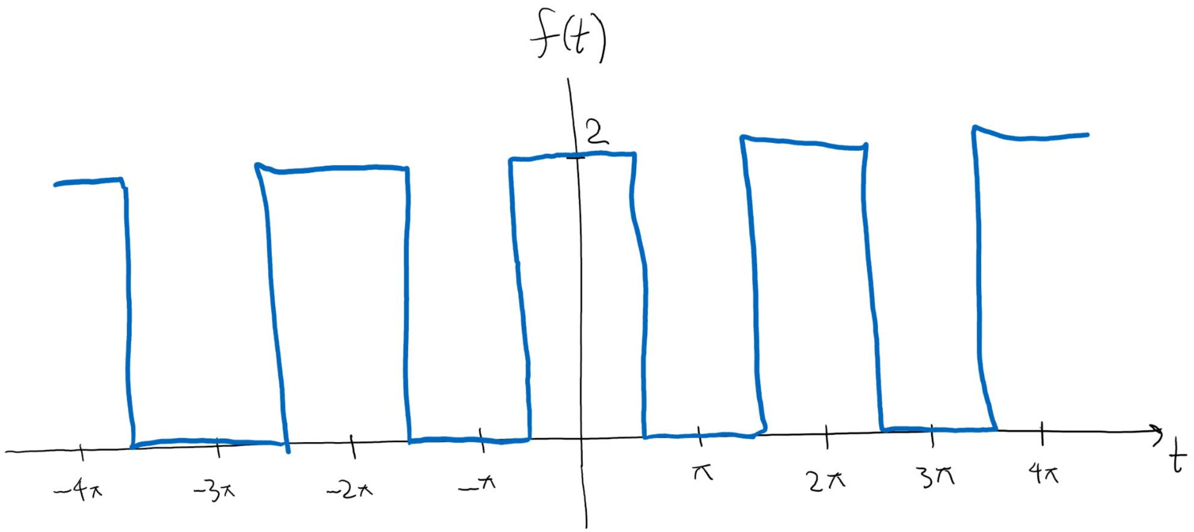
is the input to a linear system with

$$H(\omega) = \begin{cases} 1 & \omega \in [-5\pi, 5\pi] \\ 0 & otherwise \end{cases}$$

What is the system output $y(t)$

Monster Problem

Find the exponential Fourier series of the following signal:



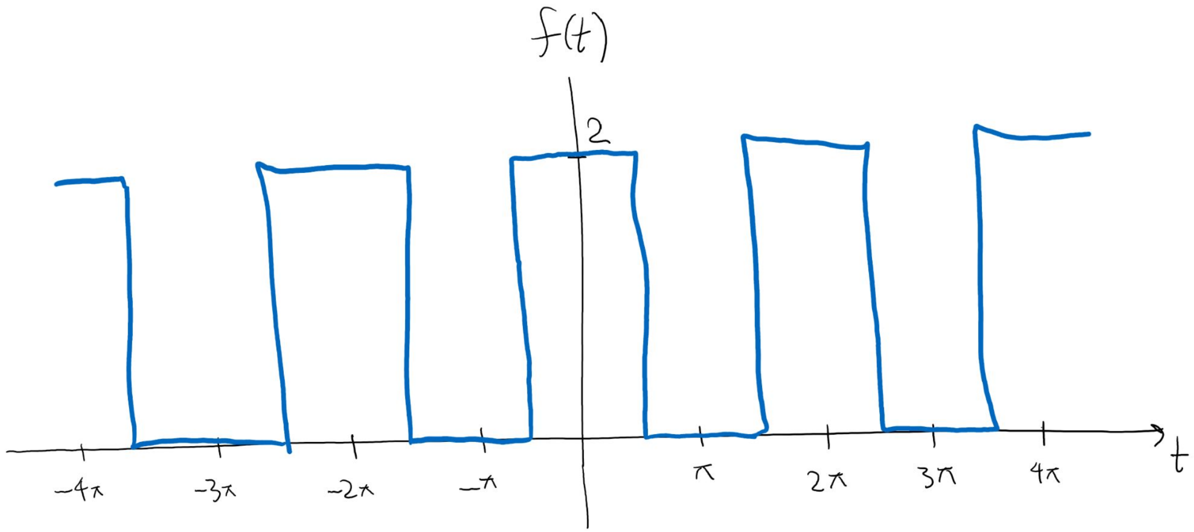
$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution

Step 1: Find T and w_0.

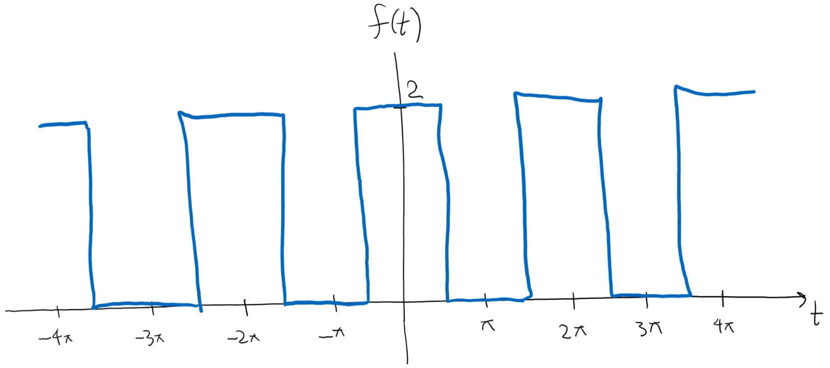
$$T = 2\pi$$

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$



$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution



Step 2: Set up the integral

$$F_n = \frac{1}{2\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{jn1t}dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0e^{jn1t}dt\right)$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution

Step 3: Evaluate said integral

$$F_n = \frac{1}{2\pi}(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{jn1t}dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0e^{jn1t}dt)$$

$$F_n = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{jnt}dt$$

$$F_n = \frac{1}{2\pi}\frac{1}{jn}2e^{jnt}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

Monster Problem Solution

Step 3: Evaluate said integral

$$F_n = \frac{1}{2\pi}\frac{1}{jn}2e^{jnt}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$F_n = \frac{2}{\pi n}\frac{1}{2j}\left(e^{jn\frac{\pi}{2}} - e^{jn\frac{-\pi}{2}}\right)$$

Can we simplify?

Monster Problem Solution

Step 3: Evaluate said integral

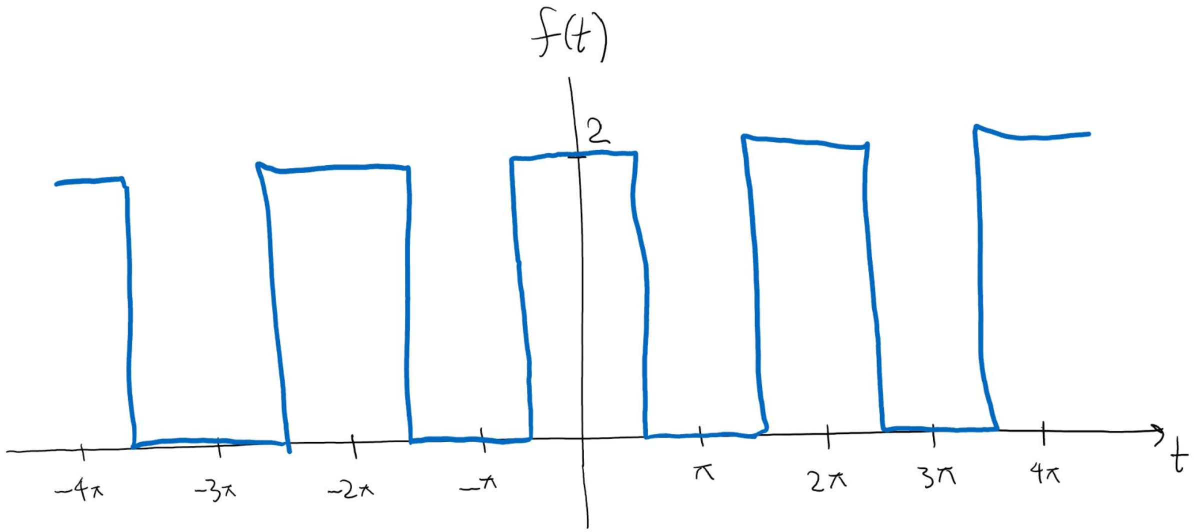
$$F_n = \frac{2}{\pi n}\frac{1}{2j}(e^{jn\frac{\pi}{2}} - e^{jn\frac{-\pi}{2}})$$

$$F_n = \frac{2sin(\frac{n\pi}{2})}{\pi n}$$

Monster Problem Solution

Step 4: Done!

$$F_n = \frac{2sin(\frac{n\pi}{2})}{\pi n}$$



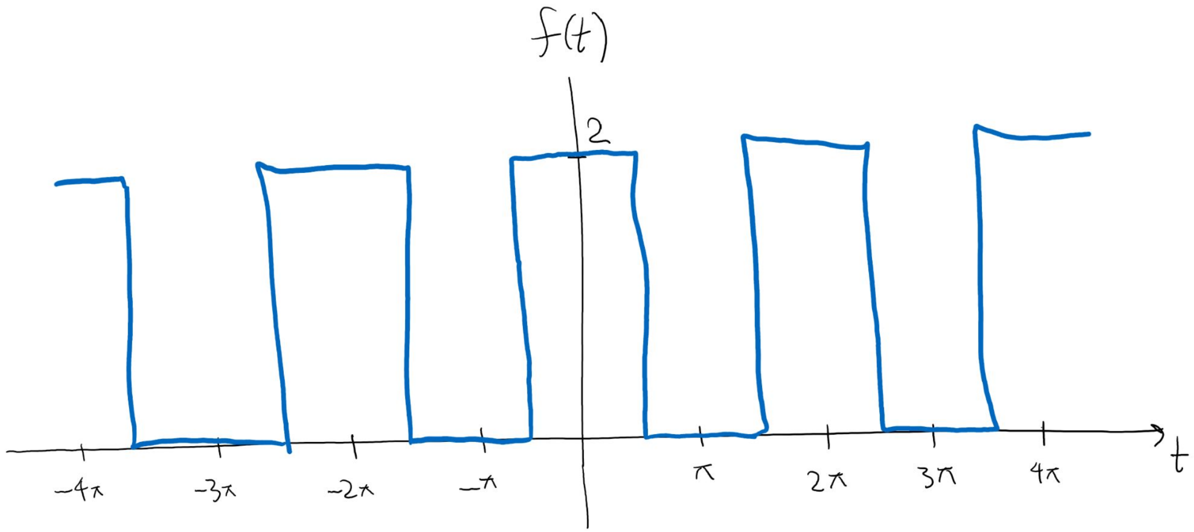
Or are we…

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution

What if n = 0?

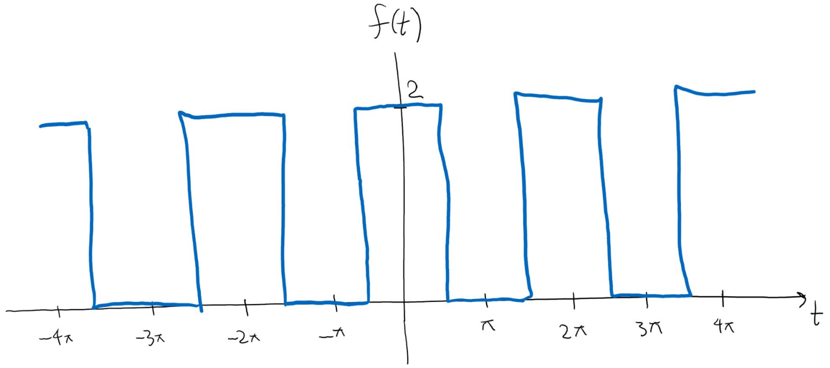
$$F_n = \frac{2sin(\frac{n\pi}{2})}{\pi n}$$



Yikes. We need a different equation.

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution



Step 2: Set up the integral, but now n = 0

$$F_n = \frac{1}{2\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{jn1t}dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0e^{jn1t}dt\right)$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution

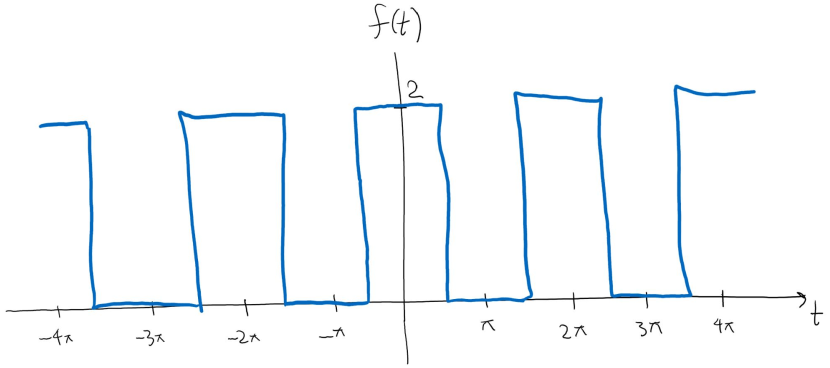
Step 2: Evaluate the integral, but now n = 0

$$F_n = \frac{1}{2\pi}(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{jn1t}dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0e^{jn1t}dt)$$

$$F_0 = \frac{2}{2\pi}t|_{t=-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$F_0 = \frac{1}{\pi}(\frac{\pi}{2} - (-\frac{\pi}{2})) = 1$$

Monster Problem Solution



Now we're done!

$$F_0 = 1$$

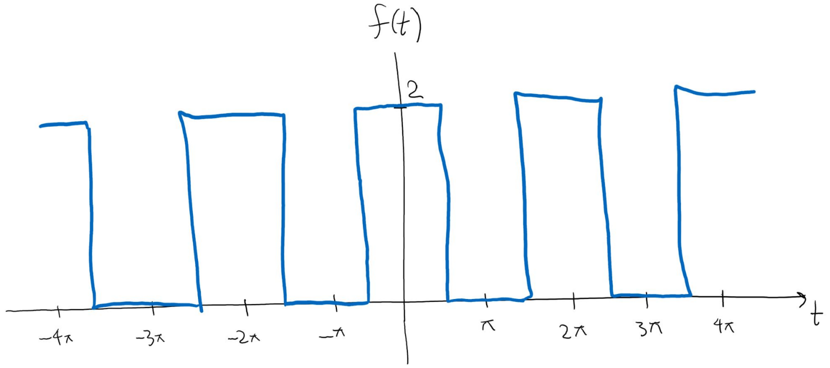
$$F_n = \frac{2sin(\frac{n\pi}{2})}{\pi n}$$

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Solution

Now we're done!



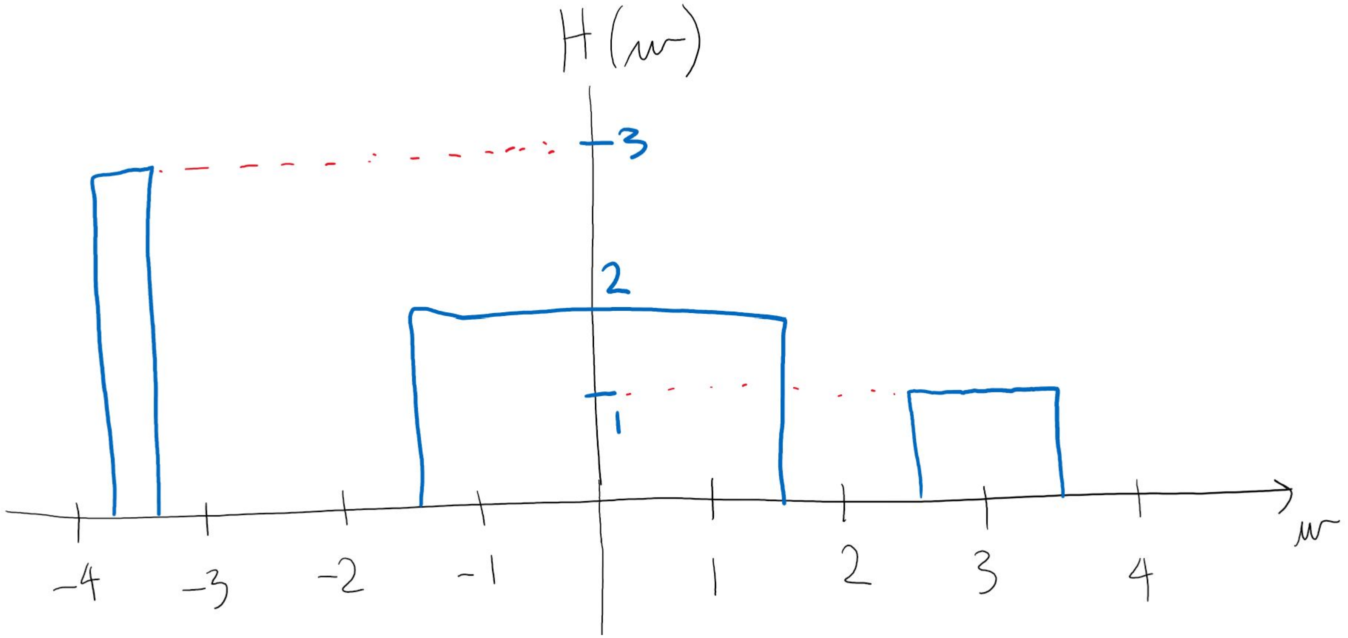
$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

Now we pass f(t) through a linear system with the following frequency response:



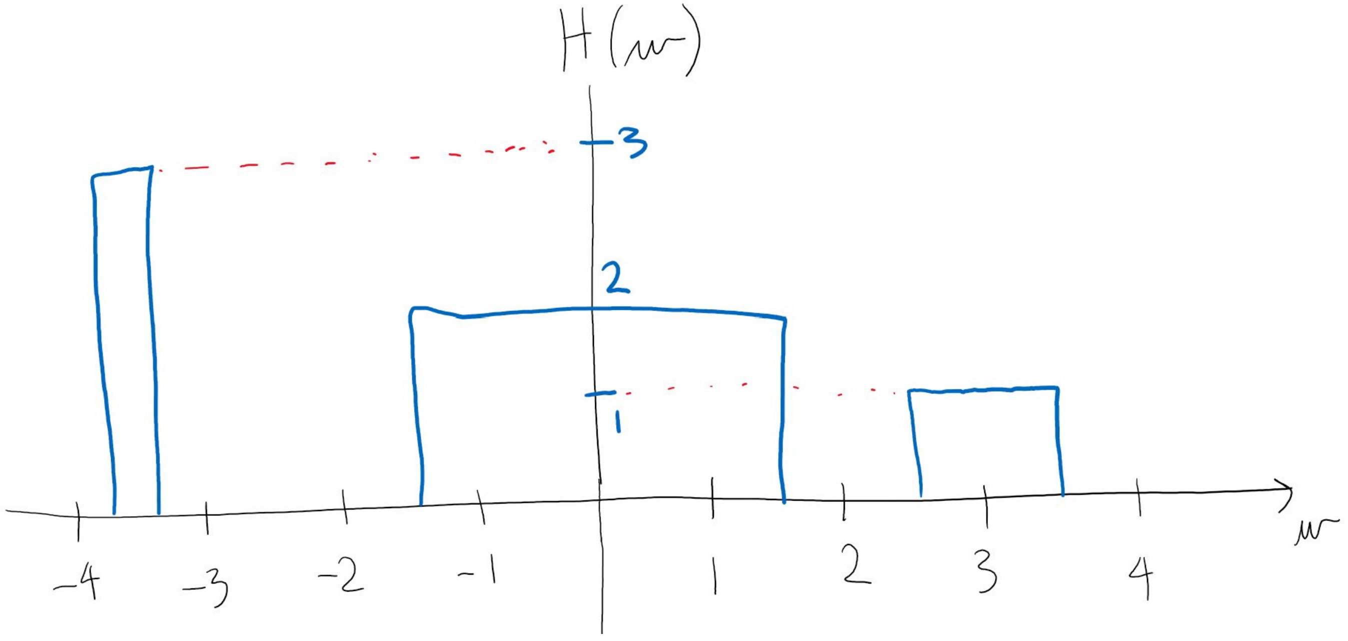
What is the output y(t)?

$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

What is the output y(t)?

Step 1: What actually are the frequencies in f(t)?



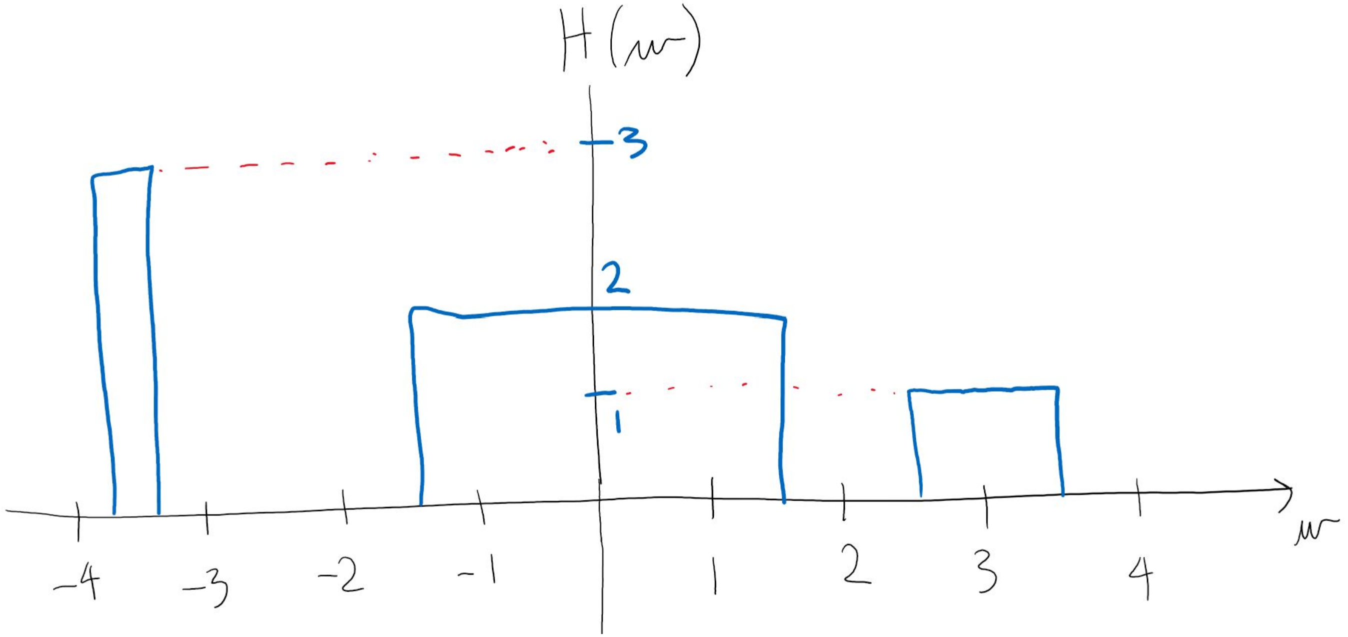
$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

What is the output y(t)?

Step 1: What actually are the frequencies in f(t)?

Answer: w = all integers!

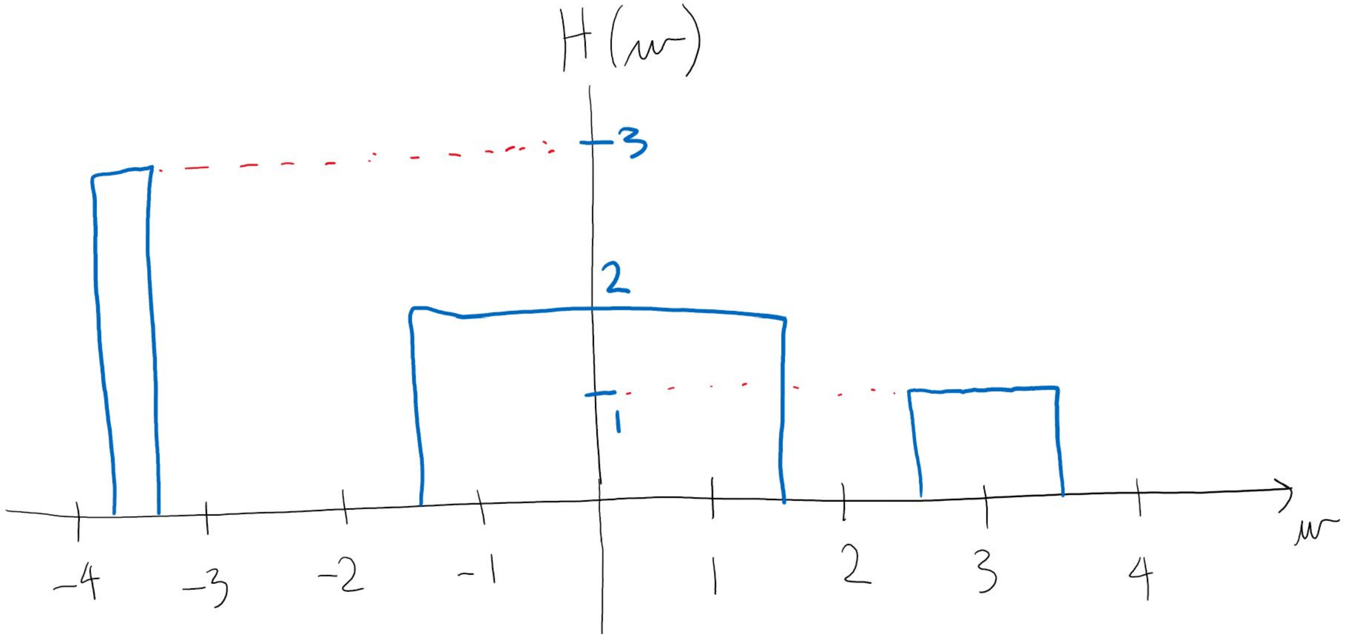


$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

What is the output y(t)?

Step 2: What frequencies will survive the filter?



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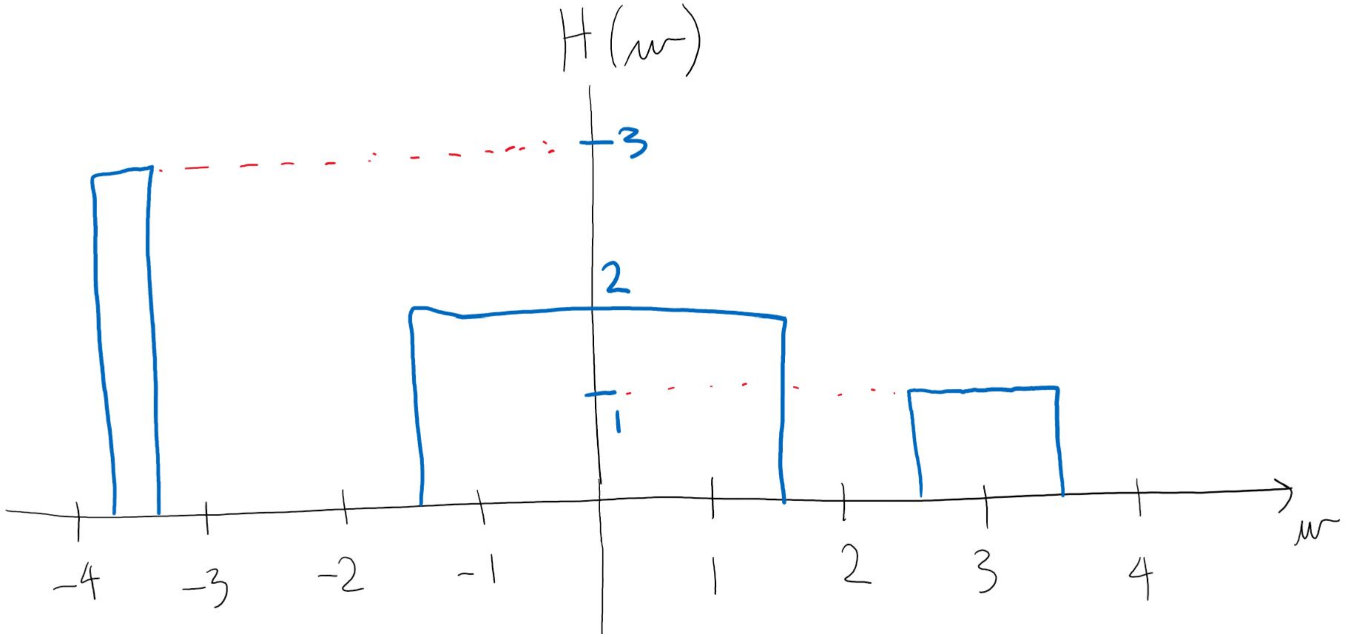
Monster Problem Part 2

What is the output y(t)?

Step 2: What frequencies will survive the filter?

Answer: -3.7 to -3.3, -1.5 to 1.5, 2.5 to 3.5.

(Note that these are just guesses)



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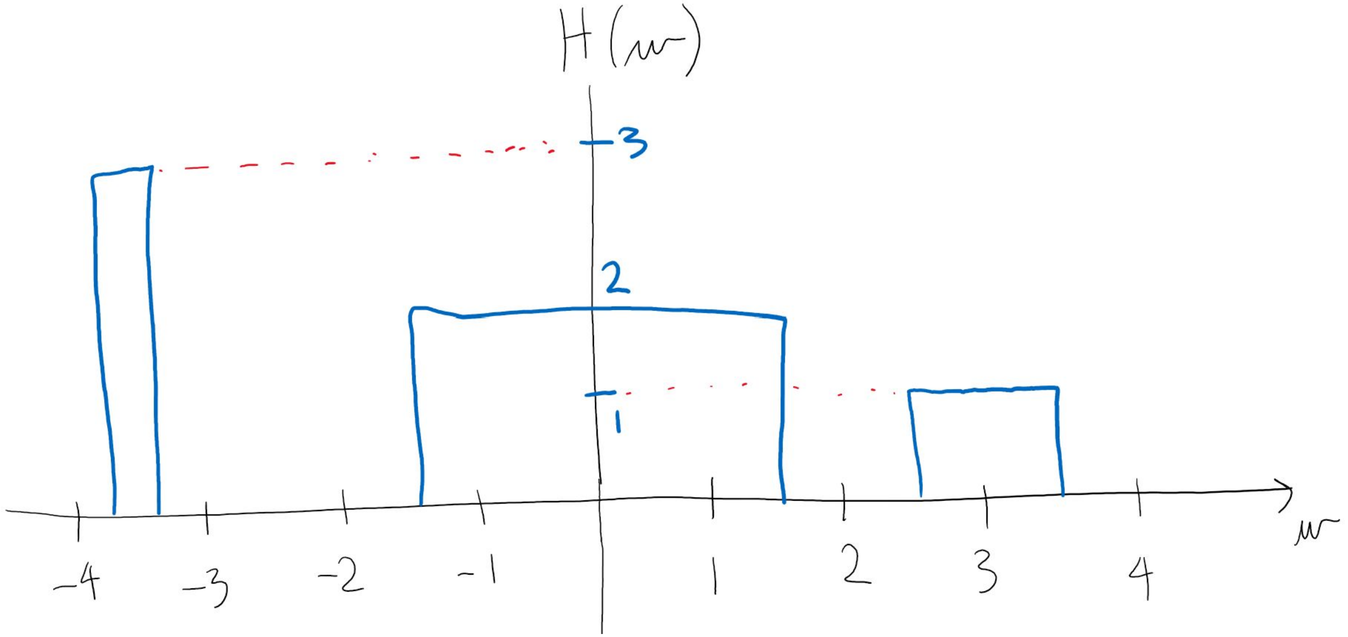
Monster Problem Part 2

What is the output y(t)?

Step 3: What values of n survive the filter?



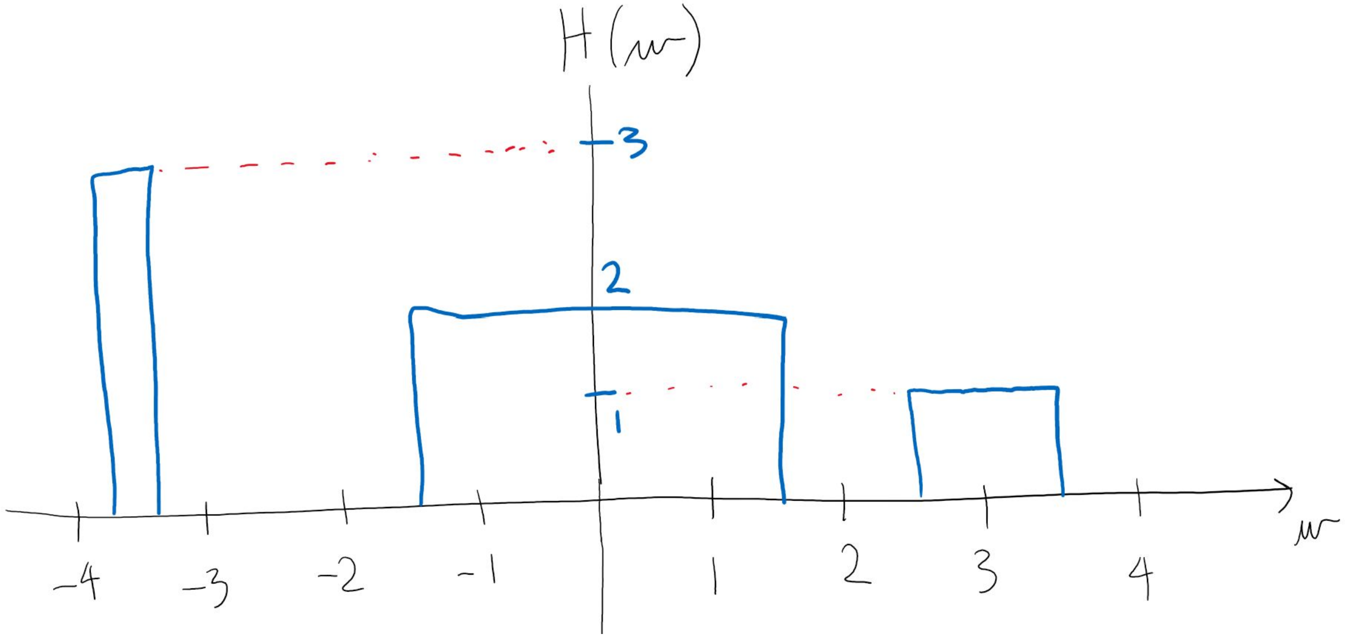
$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

What is the output y(t)?

Step 3: What values of n survive the filter?

n = -1, 0, 1, 3. Everything else is multiplied by 0 and thus goes away!

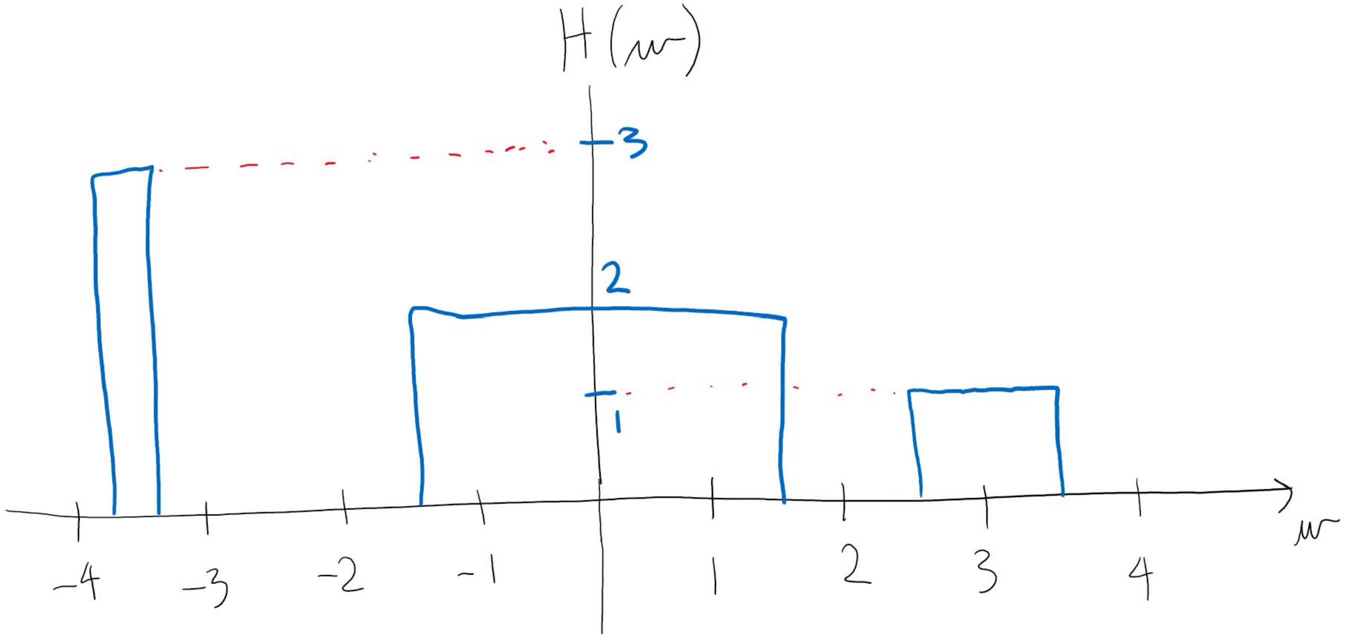


$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

What is the output y(t)?

Step 4: Now calculate Y = HF only at n = -1, 0, 1, and 3.



$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

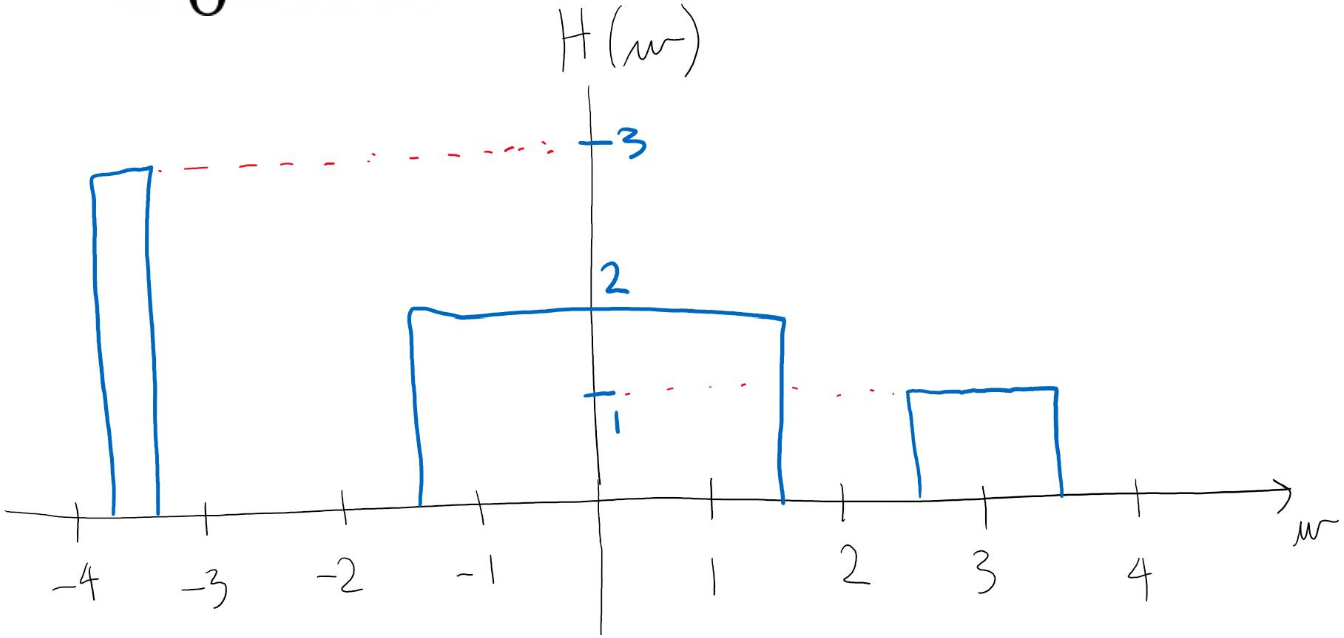
Step 4: Now calculate Y = HF only at n = -1, 0, 1, and 3.

$$F_{-1} = \frac{2sin(\frac{-1*\pi}{2})}{-1*\pi} = \frac{2}{\pi}$$

$$F_0 = 1$$

$$F_1 = \frac{2sin(\frac{1*\pi}{2})}{1*\pi} = \frac{2}{\pi}$$

$$F_3 = \frac{2sin(\frac{3*\pi}{2})}{3*\pi} = -\frac{2}{3\pi}$$



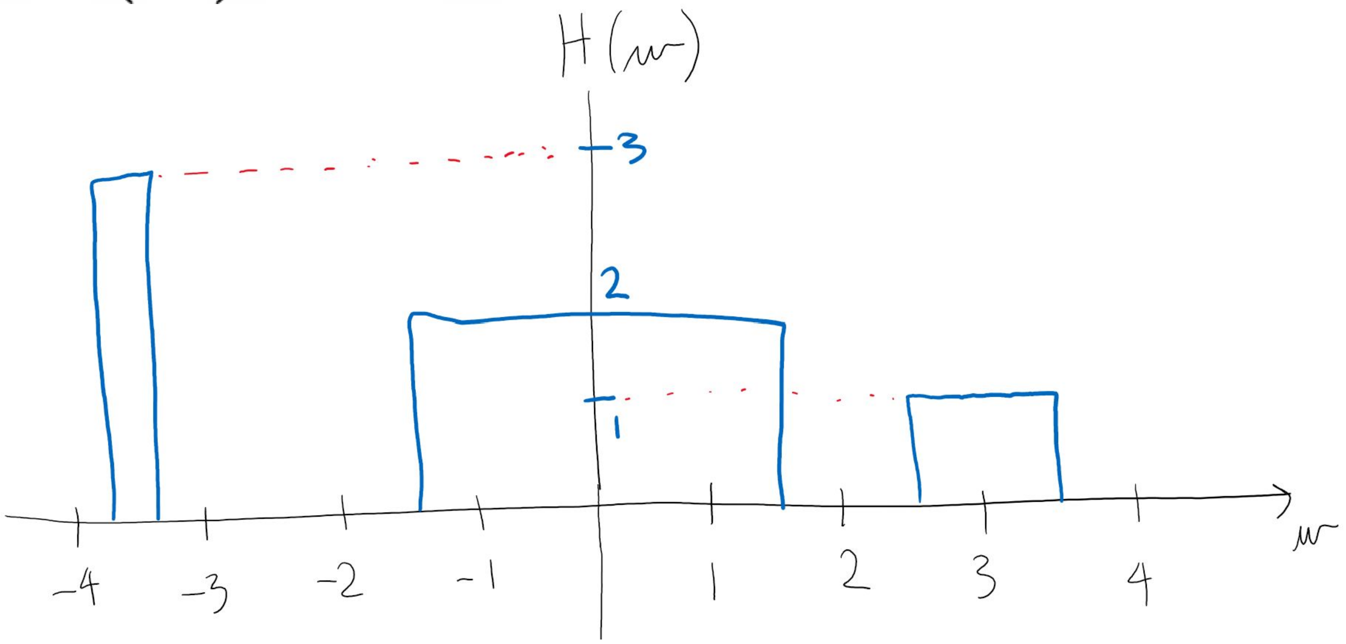
$$f(t) = 1 + \sum_{n=-\infty}^{-1} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt} + \sum_{n=1}^{\infty} \frac{2sin(\frac{n\pi}{2})}{\pi n} e^{jnt}$$

Monster Problem Part 2

Step 4: Now calculate Y = HF only at n = -1, 0, 1, and 3.

$$H(-1) = H(0) = H(1) = 2$$

$$H(3) = 1$$



$$F_{-1} = \frac{2sin(\frac{-1*\pi}{2})}{-1*\pi} = \frac{2}{\pi} \quad F_0 = 1 \quad F_1 = \frac{2sin(\frac{1*\pi}{2})}{1*\pi} = \frac{2}{\pi} \quad F_3 = \frac{2sin(\frac{3*\pi}{2})}{3*\pi} = -\frac{2}{3\pi}$$

Monster Problem Part 2

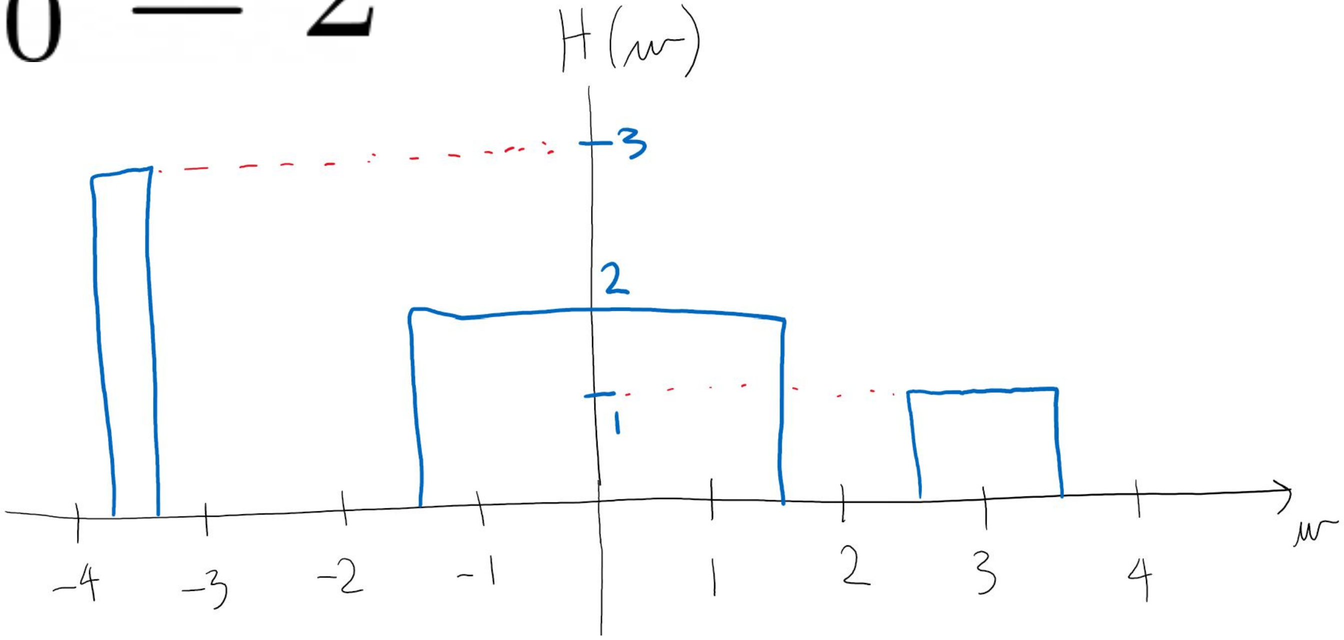
Step 4: Now calculate Y = HF only at n = -1, 0, 1, and 3.

$$Y_{-1} = 2\frac{2}{\pi} = \frac{4}{\pi}$$

$$Y_0 = 2$$

$$Y_1 = 2\frac{2}{\pi} = \frac{4}{\pi}$$

$$Y_3 = -\frac{2}{3\pi}$$



Monster Problem Part 2

All together now…

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$

$$Y_{-1} = 2\frac{2}{\pi} = \frac{4}{\pi} \qquad Y_0 = 2 \qquad Y_1 = 2\frac{2}{\pi} = \frac{4}{\pi} \qquad Y_3 = -\frac{2}{3\pi}$$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$

Monster Problem Part 3

Calculate the average power of y(t).

(Write down y(t), then I'll give you the tables on the next slide)

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\,f(t) \leftrightarrow K\,F_n$
2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2\,dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

Table 2: Fourier series properties

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 3

Average power: In the tables!

8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2\,dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

$$P = 2^2 + (\frac{4}{\pi})^2 + (\frac{4}{\pi})^2 + (\frac{2}{3\pi})^2$$

$$P = 4 + \frac{32}{\pi^2} + \frac{4}{9\pi^2}W$$

Monster Problem Part 4

Find the trigonometric and compact forms of y(t).

(Write down y(t), then I'll give you the tables on the next slide)

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
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2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2\,dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

Table 2: Fourier series properties

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

Trigonometric form… What was the formula again?

$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$

Ok. Let's go!

$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$

$$a_0 = 2 + 2 = 4$$

$$a_1 = F_1 + F_{-1} = \frac{4}{\pi} + \frac{4}{\pi} = \frac{8}{\pi}$$

$$a_3 = (F_3 + F_{-3}) = (-\frac{2}{3\pi} + 0) = -\frac{2}{3\pi}$$

$$b_1 = j(F_1 - F_{-1}) = j(\frac{4}{\pi} - \frac{4}{\pi}) = 0$$

$$b_3 = j(F_3 - F_{-3}) = j(-\frac{2}{3\pi} - 0) = -\frac{2j}{3\pi}$$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

Altogether now…

$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$

$$y(t) = 2 + \frac{8}{\pi}cos(t) - \frac{2}{3\pi}cos(3t) - \frac{2j}{3\pi}sin(3t)$$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

Compact form… What was the formula again?

$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n \rvert$ $\theta_n = \angle F_n$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

Ok. Let's go!

$$c_0 = 2F_0 = 4$$

$$c_1 = 2F_1 = \frac{8}{\pi}$$

$$c_2 = 0$$

$$c_3 = \frac{4}{3\pi}\angle\pi$$

$$c_n = 2\,|F_n|$$
$$\theta_n = \angle F_n$$

$$y(t) = 2 + \frac{4}{\pi}e^{-jt} + \frac{4}{\pi}e^{jt} - \frac{2}{3\pi}e^{j3t}$$

Monster Problem Part 4

But wait… What's the full formula?

$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\|F_n\|$ $\theta_n = \angle F_n$

$$y(t) = 2 + \frac{8}{\pi}cos(t) - \frac{2}{3\pi}e^{j3t}$$

Oh no. y(t) isn't completely real.

COMPACT FORM FOR y(t) DOESN'T EXIST!!!!

Monster Problem Part 5

Ha I’m kidding.

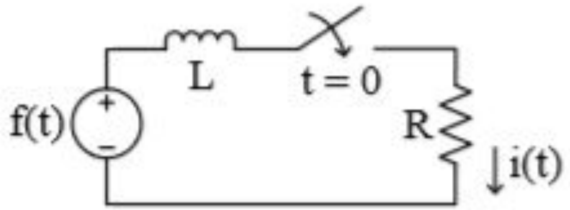
Give yourself a pat on the back.

That was a hard one.

Feedback! Please please fill it out!

Spring 2014 Question 1

Problem 1 Consider the circuit below, where the switch is initially open, and then closes at time $t = 0$.



(a) Determine the ODE satisfied by $i(t)$ for $t > 0$. ODE = ______________________________

(b) For $t > 0$, the input $f_1(t) = \sin(2t)$ volts yields the current $i_1(t) = \frac{1}{8}e^{-Rt/2} + \frac{\sqrt{2}}{8}\cos\left(\omega t - \frac{3\pi}{4}\right)$ Amperes, while the input $f_2(t) = 2e^{-3t}$ volts yields the current $i_2(t) = e^{-4t/L} - e^{-3t}$ Amperes.

i) Obtain the values of ω= ____________, R = ____________, and L = ____________.

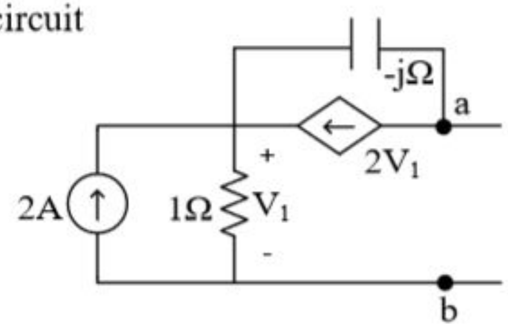
ii) Obtain the zero-input response $i_{ZI}(t)$ = ____________

iii) Obtain the phasor of the steady-state current, $i_{SS}(t)$, when the input is $f(t) = f_1(t) + f_2(t)$.

Spring 2014 Question 2

Problem 2

In the following phasor circuit



(a) Find the open circuit phasor voltage V_T between a-b.

(b) Find the Thévenin impedance Z_T between a-b.

(c) Find the average available power of this circuit.

Spring 2018 Question 4

4. (26 pts) Consider a periodic signal $f(t)$ with period $T = 2\text{s}$, given by:

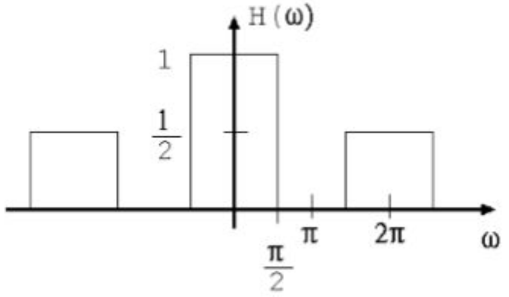
$$f(t) = \begin{cases} e^t & 0 \le t < 1 \text{ s} \\ e^{2-t} & 1 \le t < 2 \text{ s}. \end{cases}$$

(a) Sketch the signal $f(t)$ from -2s to 2s.

(b) The function $f(t)$ can be expressed as a Fourier series with exponential coefficients

$$F_n = \frac{e^{1-jn\pi} - 1}{1 + n^2\pi^2}.$$

Let $f(t)$ be the input to an LTI system with frequency response $H(\omega)$, given in the plot below. Determine the steady-state output, $y_{ss}(t)$ in compact form.



Spring 2018 Question 4 Continued

(c) Recall that $f(t)$ has period 2s, and is given by:

$$f(t) = \begin{cases} e^t & 0 \leq t < 1 \text{ s} \\ e^{2-t} & 1 \leq t < 2 \text{ s.} \end{cases}$$

Consider the periodic signal $g(t)$ with period $T = 2$s, given by:

$$g(t) = \begin{cases} e^t & -1 \leq t < 0 \text{ s} \\ e^{-t} & 0 \leq t < 1 \text{ s.} \end{cases}$$

Determine its exponential Fourier coefficients G_n using the coefficients F_n and the relation between $g(t)$ and $f(t)$.

Fall 13 Question 8

8. (13 pts) A linear system with input $f(t)$ and output $y(t)$ is decided by the ODE

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = f(t)$$

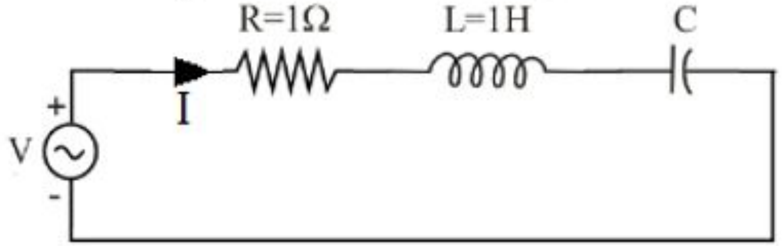
(a) (4 pts) Find the frequency response , $H(\omega) = \frac{Y}{F}$.

(b) (2 pts) Sketch $|H(\omega)|$.

(c) (7 pts) Find the steady state response, $y_{ss}(t)$, to the input: $f(t) = (1+j)e^{-jt} + (1-j)e^{jt} + 2e^{-j3t} - 2e^{j3t} + 1$.

Fall 2017 Question 2

2. (25 pts) An RLC series circuit consisting of a 1Ω resistor R, an inductance of value $L = 1$ H and a capacitor C is fed by a cosinusoidal voltage source.



(a) If the cosinusoidal voltage source has angular frequency $\omega = 1$ rad/s and the current leads (is ahead of) the voltage at the source by 45°, what is the value of the capacitor C?

.

(b) What is the angular resonant frequency of the circuit, ω_0?

Spring 2016 Question 5

5. (24 pts) Consider the periodic signal $f(t) = \frac{3}{2} + 4cos^2(\frac{\pi}{4}t) + sin(\frac{3\pi}{4}t + \frac{\pi}{4})$ being the input to an LTI system having frequency response $H(\omega) = \frac{j\omega}{1+j\omega}$.

   (a) Obtain the fundamental frequency ω_0 and period T of the input $f(t)$.

   (b) For $n = 0, 1, -1, 2, -2$, obtain the exponential Fourier series coefficients of the input, F_n, and of the output Y_n.