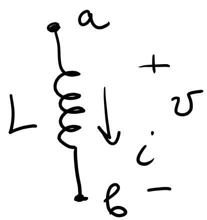
- Differentiators and integrators



$v = L\frac{di}{dt}$ ← current in inductor is continuous! for all practical sources

↑ can jump

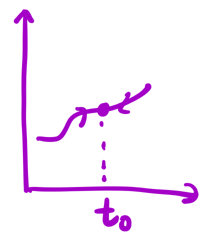
$i_L(0^-) = i_L(0^+) = i_L(0)$

If we want to know i ⇒

$$\int_{-\infty}^{t} v(s)\,ds = Li(t) \Rightarrow i(t) = \frac{1}{L}\int_{-\infty}^{t} v(s)\,ds =$$

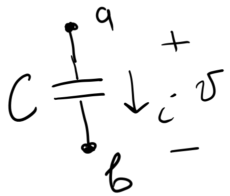
$$= \frac{1}{L}\int_{-\infty}^{0} v(s)\,ds + \frac{1}{L}\int_{0}^{t} v(s)\,ds = i(t)$$

i_0 initial state



$i_L(t_0^-) = i_L(t_0^+) = i_L(t_0)$

- Differentiators and integrators-cont



$i = C \dfrac{dv}{dt}$ → voltage in a capacitor is continuous ! for all practical sources

can jump

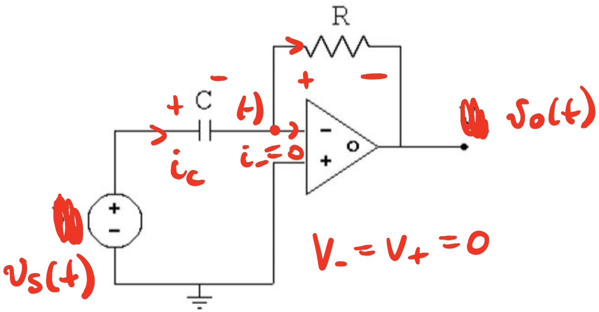
$\int_{-\infty}^{t} i(s)\,ds = Cv(t)$

$v(0^-) = v(0) = v(0^+)$

$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(s)\,ds =$

$= \underbrace{\frac{1}{C}\int_{-\infty}^{0} i(s)\,ds}_{v_0 \text{ initial state}} + \frac{1}{C}\int_{0}^{t} i(s)\,ds = v(t)$

- Example #8: Obtain Vo in the following circuit assuming the ideal op-amp approximation



$V_- = V_+ = 0$

KCL @ (−):

$i_c = \dfrac{\cancel{V_-}^{\,0} - V_o}{R}$

$i_c = C \dfrac{dv_c}{dt}$

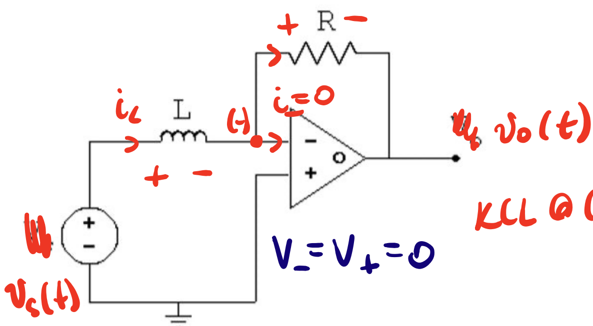
$v_o(t) = -R i_c(t) = -R\left(C \dfrac{dv_c}{dt}\right) =$

$= -RC \dfrac{d}{dt}\left(v_s - \cancel{V_-}^{\,0}\right)$

$v_o(t) = -RC \dfrac{dv_s}{dt}$

Voltage differentiator

• Example #9: Obtain Vo in the following circuit assuming the ideal op-amp approximation



$V_- = V_+ = 0$

$$v_L = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(x)\,dx$$

KCL @ (-): $i_L = \dfrac{\cancel{V_-}^{\,0} - V_o}{R}$

$$v_o(t) = -R\,i_L(t) =$$

$$= -R\left(\frac{1}{L}\int_{-\infty}^{t} v_L(x)\,dx\right) =$$

$$= -\frac{R}{L}\int_{-\infty}^{t}\left(v_s - \cancel{V_-}_{\,0}\right)dx$$

$$v_o(t) = -\frac{R}{L}\int_{-\infty}^{t} v_s(x)\,dx$$

integrator

- Linearity, time-invariance and LTI systems

  ↓ linear and time-invariant

  - Recall from previous example:

$v_s(t)$ - input

$v_{out}(t)$ - output

$$f(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

$$v_{out}(t) = -\frac{R}{L}\int_{-\infty}^{t} v_s(x)dx =$$

$$= -\frac{R}{L}\int_{-\infty}^{0} v_s(x)dx - \frac{R}{L}\int_{0}^{t} v_s(x)dx =$$

$$= v_0 - \frac{R}{L}\int_{0}^{t} v_s(x)dx =$$

$$= v_{ZI}(t) + v_{ZS}(t)$$

zero-input response (when $v_s(t)=0$ no input)

zero-state response (when $v_0 = 0$ no initial state)

- Linearity, time-invariance and LTI systems-cont

→ no initial state

In a *zero-state* linear system, a weighted sum of inputs produces a similarly weighted sum of corresponding *zero-state* outputs, consistent with the superposition principle. So,

if

$$f_1(t) \rightarrow \boxed{\text{Zero-state linear}} \rightarrow y_{zs,1}(t)$$

and

$$f_2(t) \rightarrow \boxed{\text{Zero-state linear}} \rightarrow y_{zs,2}(t)$$

then

$$f_3(t) = k_1 f_1(t) + k_2 f_2(t) \rightarrow \boxed{\text{Zero-state linear}} \rightarrow y_{zs,3}(t) = k_1 y_{zs,1}(t) + k_2 y_{zs,2}(t)$$

- Linearity, time-invariance and LTI systems-cont

→ no input

  - Similarly, in a *zero-input* linear system, a weighted sum of initial states produces a similarly weighted sum of corresponding *zero-input* outputs, consistent with the superposition principle.

  - A system is linear, if it is **both** *zero-state* linear and *zero-input* linear and the output is the sum of the *zero-state* and the *zero-input* responses.

- Example #10: Linearity
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$\underbrace{y(t) = \cancel{y_0}^{\,0} e^{-t} - \int_0^t f(x)dx}$$

  - Determine if the system is linear or not

zero-state linear + zero-input linear ⇒ linear :)

yes (zero-state linear), yes (zero-input linear)

zero-state linear ⇒

$y_0 = 0 \Rightarrow y_{zs}(t) = -\int_0^t f(x)dx$

$f_1(t) \rightarrow \square \rightarrow y_1(t) = -\int_0^t f_1(x)dx$

$f_2(t) \rightarrow \square \rightarrow y_2(t) = -\int_0^t f_2(x)dx$

$f_3(t) \rightarrow \square \rightarrow y_3(t) \overset{?}{=} k_1 y_1 + k_2 y_2$ (*)

$f_3 = k_1 f_1 + k_2 f_2$

$= -\int_0^t f_3(x)dx = -\int_0^t (k_1 f_1 + k_2 f_2)dx =$

$= k_1 \underbrace{\left(-\int_0^t f_1 dx\right)}_{y_1} + k_2 \underbrace{\left(-\int_0^t f_2 dx\right)}_{y_2}$ (*)

same ⇒ z-s linear :)

- Example #10: Linearity-cont
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = y_0 e^{-t} - \int_0^t f(x)\,dx$$

0

  - Determine if the system is linear or not

zero-input $\Rightarrow f(t) = 0 \Rightarrow y_{ZI}(t) = y_0 e^{-t}$

$\boxed{y_0} \rightarrow y_{ZI,0}(t) = y_0 e^{-t}$

$\boxed{y_1} \rightarrow y_{ZI,1}(t) = y_1 e^{-t}$

Z-I linear :)

$\boxed{y_3 = k_1 y_0 + k_2 y_1} \rightarrow y_{ZI,3}(t) \overset{?}{=} k_1 y_{ZI,0} + k_2 y_{ZI,1}$ ⊛

$\text{"} \; y_3 e^{-t} = (k_1 y_0 + k_2 y_1) e^{-t} = k_1 \underbrace{(y_0 e^{-t})}_{y_{ZI,0}} + k_2 \underbrace{(y_1 e^{-t})}_{y_{ZI,1}}$ ⊛