ECE 210 Review Session

Knowledges after Midterm 3

- Laplace transform
  - ROC, poles, zeros
  - BIBO stability
- S-domain circuit analysis

Laplace transform

- Definition:

$$\hat{H}(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt = \int_{0^-}^{\infty} f(t) e^{-\sigma t} e^{-j\omega t} dt$$

- Extend the Fourier transform:

$$j\omega \to s = \sigma + j\omega$$

- Real part σ determines the convergence of the Laplace transform
  - e.g. $f(t) = e^{2t}$
  - Fourier transform doesn't exist
  - Laplace transform ROC $\sigma > 2$.

Laplace transform

- Poles at s when $\widehat{H}(s) \to \infty$, usually found when denominator $= 0$
  - Special cases: poles at $\mathbf{s = \pm\infty \text{ and } 0}$.

- Zeros at s when $\widehat{H}(s) = 0$. Poles and zeros can cancel each other and change the BIBO stability.

Laplace transform

- BIBO stability is determined by the location of poles
  - BIBO stable: all poles at left hand side of s-plane ($\sigma < 0$)
  - Marginally stable: purely imaginary poles ($\sigma = 0$)
  - Not BIBO stable: any pole at right hand side of s-plane ($\sigma > 0$)

- Pay attention to the special poles (on previous slides)

S-domain circuit analysis

- Transfer function $\widehat{H}(s)$ only determined by the circuit itself, not the initial condition of the circuit
- Zero-state response $\hat{Y}_{ZS}(s)$: set zero initial condition, but keep the input source, i.e.,

$$\hat{Y}_{ZS}(s) = \widehat{H}(s)\ \hat{F}(s)$$

- Zero-input response $\hat{Y}_{ZI}(s)$: non-zero initial condition, but set input source to 0. (cont. next page)

S-domain circuit analysis

- Zero-input response $\hat{Y}_{ZI}(s)$: non-zero initial condition, but set input source to 0. Two equivalent methods for $\hat{Y}_{ZI}(s)$:
  - Method 1: Find poles of $\hat{H}(s)$, follow by the characteristic modes.
  - e.g. $\hat{H}(s) = \frac{1}{(s+1)(s+2)}$ has poles at $s = -1$ and $s = -2$
  - Characteristic modes e^{-t} and e^{-2t}
  - The zero-input response takes the form
$$y_{ZI}(t) = Ae^{-t} + Be^{-2t}$$
  - Determine the coefficients A and B using $y(0)$ and $y'(0)$
  - Main thought: when $\hat{F}(s) = 0$, $\hat{H}(s) \to \infty$. The only possible output is the characteristic modes.

S-domain circuit analysis

- Zero-input response $\hat{Y}_{ZI}(s)$: non-zero initial condition, but set input source to 0. Two equivalent methods for $\hat{Y}_{ZI}(s)$:
  - Method 2: Transform ODE to s-domain, and include the initial condition

$$
\begin{aligned}
y(t) &\rightarrow \hat{Y}(s) \\
y'(t) &\rightarrow s\hat{Y}(s) - y(0^-) \\
y''(t) &\rightarrow s^2\hat{Y}(s) - sy(0^-) - y'(0^-) \\
&\vdots
\end{aligned}
$$

  - After that, solve for $\hat{Y}_{ZI}(s)$ and $y_{ZI}(t)$

Midterm 1 Knowledges

- Basic circuit analysis – KCL, KVL
- RC and RL circuits in phasor domain
- RLC circuit and frequency response $H(\omega)$

Basic circuit analysis – KCL, KVL

- KCL (a.k.a. node voltage method)
  - The current flow into each node has to equal the current flow out
  - Key point: you may choose whichever current flow direction, but stick with that through out the entire question
  - Cannot be applied to the node with voltage source, since we don't know the current flow through the voltage source (use other nodes or super-node)
- KVL (a.k.a loop current method)
  - The voltage through a current loop has to sum to 0
  - Cannot be applied to the loop with current source, since we don't know the voltage across the current source (use other loops or super-loop)

RC and RL circuits in phasor domain

- Phasor representations:

$$R \rightarrow R$$
$$C \rightarrow \frac{1}{j\omega C}$$
$$\mathrm{L} \rightarrow j\omega L$$
$$f(t) \rightarrow F$$
$$\mathrm{y}(t) \rightarrow Y$$

- After everything is in phasor domain, solve for the circuit using whichever method you like
- Time domain solution $y(t) = Re\{Ye^{j\omega t}\}$

RC and RL circuits in phasor domain

- Homogeneous solution $y_h(t)$: always take the form

$$e^{-t/\tau}, \tau = RC \text{ or } \tau = R/L$$

- Particular solution $y_p(t)$: solution corresponds to the input
  - Constant input $f(t) = C$, output takes the form C
  - Exponential input $f(t) = e^{-at}$, output takes the form e^{-at}
  - Cosinudal input $f(t) = \cos(\omega_0 t)$, output takes the form $\cos(\omega_0 t + \theta)$ or $\cos(\omega_0 t) + \sin(\omega_0 t)$
- Complete solution

$$y(t) = y_h(t) + y_p(t) = y_{ZI}(t) + y_{ZS}(t) = y_{tr}(t) + y_{ss}(t)$$

RLC circuit and frequency response $H(\omega)$

- Similar to RC and RL circuits, change everything into phasor domain
- Frequency response

$$H(\omega) = \frac{Y(\omega)}{F(\omega)}$$

- Output $Y(\omega)$ can be either current or voltage
- Given $H(\omega)$, output can be calculated using $\mathrm{Y}(\omega) = H(\omega)F(\omega)$
- Special case: single frequency input $f(t) = e^{j\omega t}$ or $\cos(\omega t)$

$$y(t) = |H(\omega)|e^{j(\omega t + \angle H(\omega))} \text{ or } |H(\omega)|\cos(\omega t + \angle H(\omega))$$

Midterm 2 Knowledges

- Fourier series – periodic time domain signal
- Fourier transform – non-periodic time domain signal

Fourier series & Fourier transform

Fourier series

- Periodic signal $f(t)$
- Discrete frequency spectrum $F(n\omega_0)$
- $F(n\omega_0)$ only non-zero at $n\omega_0$ (integer multiples of fundamental frequency)
- Coefficients $F_n = F(n\omega_0)$

Fourier transform

- Non-periodic signal $f(t)$
- Continuous frequency spectrum $F(\omega)$

Fourier series & Fourier transform

- Be familiar with:
  - Fourier series/transform pairs table
  - Fourier series/transform property table

Midterm 3 Knowledges

- Convolution
- Impulse function
- Sampling and Reconstruction
- LTIC systems

Convolution

- Definition

$$y(t) = \int_{-\infty}^{\infty} h(\tau) f(t-\tau) d\tau$$

- Input domain τ, output domain t
- Flip one of the input $f(\tau) \rightarrow f(-\tau)$
  - This implies $t = 0$ because $y(0) = \int_{-\infty}^{\infty} h(\tau) f(-\tau) d\tau$
- Important knowledge:
  - Find appropriate t ranges for your integral
  - After find the range, the integrand is $h(\tau) \cdot f(t-\tau)$, not the area overlap

Impulse function

- Important properties of impulse function $\delta(t)$:
  - Multiplication with $\delta(t)$
  - Convolution with $\delta(t)$
  - Sampling property (same as multiplication)
  - Fourier transform of $\delta(t)$
- Table 9.3 in the textbook

Sampling and Reconstruction

- Intuition of sampling:

  Sampling in time domain with frequency f_s

  $\updownarrow$

  Generating copies in frequency domain,

  Each copy is separated by f_s

- Perfect reconstruction of the original signal from samples
  - No aliasing, i.e., neighboring copies in frequency domain cannot overlap
  - Requires large f_s, minimum $f_s =$ twice the highest frequency in the signal
  - Nyquist rate

Linearity, Time Invariant, Causality

- Linear system: sum of inputs → sum of outputs
  - If every operation within a function is linear, the system is linear
  - e.g. linear system: $\mathrm{y}(t) = 2f(t^2)$
  - e.g. non-linear system: $\mathrm{y}(t) = 2f^2(t)$
- Time Invariant: input and output should have same delay
  - If time is scaled, then the system is not TI
  - e.g. TI system: $\mathrm{y}(t) = 2f^2(t)$
  - e.g. non-TI system: $\mathrm{y}(t) = 2f(t^2)$
- Causal system: output cannot depend on future input
  - $h(t)$ is given: $h(t) = 0$ for $h(t) < 0$
  - $\mathrm{y}(t)$ and $f(t)$ given: check input and output relation