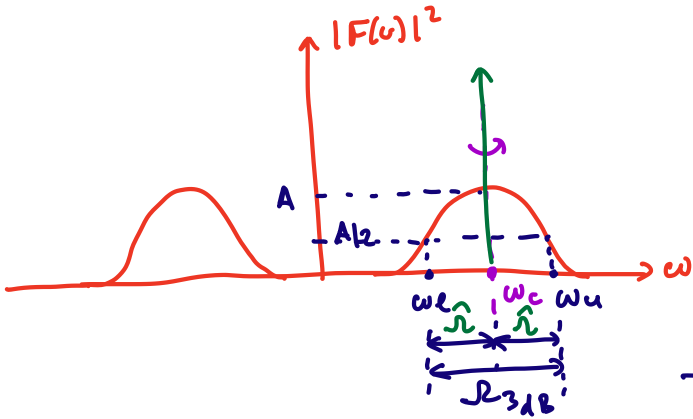
- Energy bandwidth - Band-pass signals
  - 3dB bandwidth

↳ energy is concentrated in an intermediate freq. band



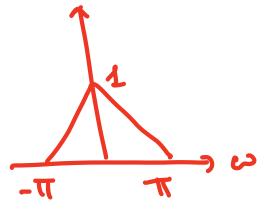
$$\Omega_{3dB} = \omega_u - \omega_l$$

Because of a local symmetry around ω_c, we can move axis and pretend it is a lowpass case, then double Ω.

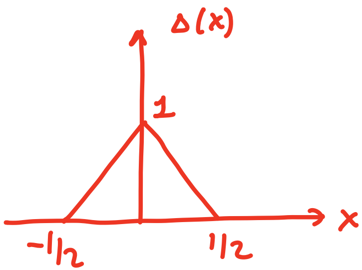
$$\Omega_{3dB} = 2\hat{\Omega}$$

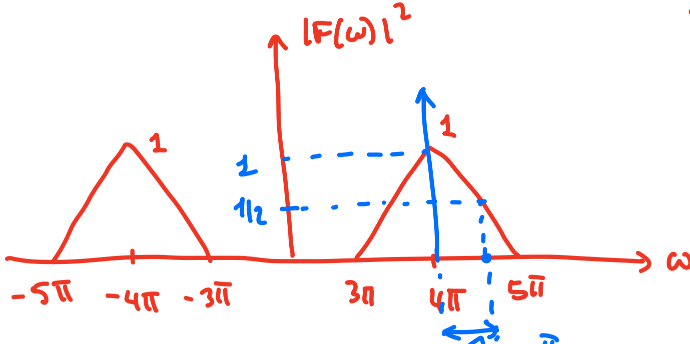
- Energy bandwidth - Band-pass signals - Example # 10
  - Determine the 3dB bandwidth of the signal with energy spectrum

$$|F(\omega)|^2 = \Delta\left(\frac{\omega - 4\pi}{2\pi}\right) + \Delta\left(\frac{\omega + 4\pi}{2\pi}\right)$$



unit triangle

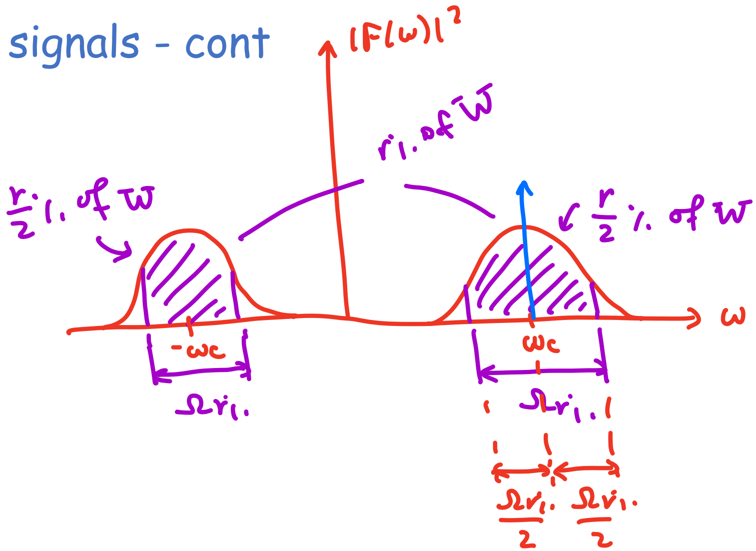




$\hat{\Omega} = \frac{\pi}{2}$ rad/s

$\Omega_{3dB} = 2 \cdot \hat{\Omega} = \pi$ rad/s

- Energy bandwidth - Band-pass signals - cont
  - r% bandwidth



$$\frac{r}{100}\cdot\bar{W} = \frac{1}{2\pi}\left[\int_{-\omega_c-\frac{\Omega_{r\%}}{2}}^{-\omega_c+\frac{\Omega_{r\%}}{2}} |F(\omega)|^2 d\omega + \int_{\omega_c-\frac{\Omega_{r\%}}{2}}^{\omega_c+\frac{\Omega_{r\%}}{2}} |F(\omega)|^2 d\omega\right]$$

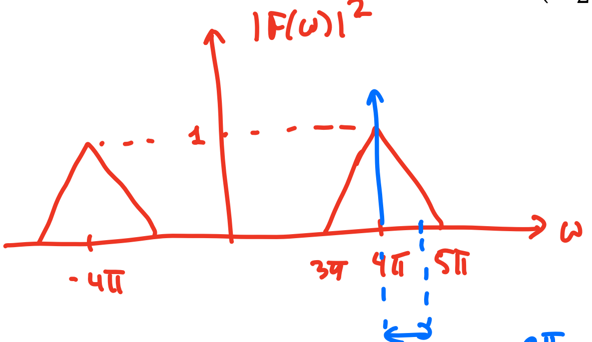
or just do right-hand side:

$$\frac{r}{100}\cdot\frac{\bar{W}}{2} = \frac{1}{2\pi}\int_{\omega_c-\frac{\Omega_{r\%}}{2}}^{\omega_c+\frac{\Omega_{r\%}}{2}} |F(\omega)|^2 d\omega$$

Also, as in 3dB case, can move axis to ω_c, solve as lowpass and double Ω!

- Energy bandwidth - Band-pass signals - Example # 11
  - Determine the 99% bandwidth of the signal with energy spectrum

$$|F(\omega)|^2 = \Delta\left(\frac{\omega - 4\pi}{2\pi}\right) + \Delta\left(\frac{\omega + 4\pi}{2\pi}\right)$$



$\hat{\Omega}_{99\%} = \frac{9\pi}{10}$ rad/s (from ex. 9)

$\Omega_{99\%} = 2\hat{\Omega}_{99\%} = \frac{2\cdot 9\pi}{10} = \frac{9\pi}{5}$ rad/s

- LTI system response to energy signals

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \rightarrow \underset{H(\omega)}{\boxed{\text{LTI}}} \rightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{H(\omega)F(\omega)}_{Y(\omega)}e^{j\omega t}d\omega$$

$$F(\omega) \rightarrow \underset{H(\omega)}{\boxed{\text{LTI}}} \rightarrow Y(\omega) = \underbrace{H(\omega)F(\omega)}$$

phasors : specific ω $F \rightarrow \boxed{H(\omega)} \rightarrow Y = F \cdot H(\omega)$

F.S. : specific $n\omega_0$ $F_n \rightarrow \boxed{H(\omega)} \rightarrow Y_n = F_n \cdot H(n\omega_0)$

F.T : ω $F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow Y(\omega) = F(\omega)H(\omega)$

- LTI system response to energy signals - Example # 12
  - Let $\underbrace{f(t) = e^{-4t}u(t)}$ be the input to an LTI system with frequency response $H(\omega) = \frac{1}{4+j\omega}$ $\Updownarrow$ $\frac{1}{4+j\omega}$ (table)
  - Determine the resulting output $\overset{zs}{y(t)}$

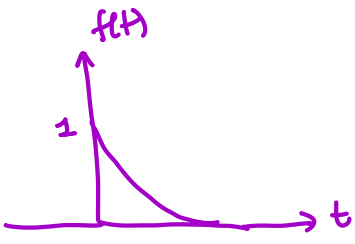
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega \rightarrow \boxed{H(\omega)} \rightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{H(\omega)F(\omega)}_{Y(\omega)} e^{j\omega t}\,d\omega$$

$$F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow Y(\omega) = F(\omega)H(\omega)$$

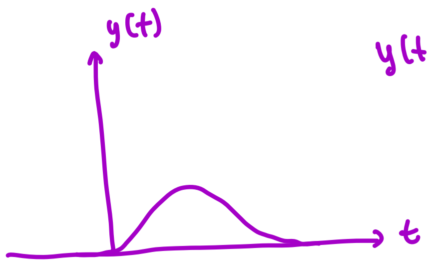
$$\frac{1}{4+j\omega} \rightarrow \boxed{\frac{1}{4+j\omega}} \rightarrow \frac{1}{4+j\omega}\cdot\frac{1}{4+j\omega} = \frac{1}{(4+j\omega)^2}$$

$$\Big\downarrow \mathcal{F}^{-1} \text{ via tables}$$

$$y(t) = t e^{-4t} u(t)$$



transient



- LTI system response to energy signals - Example # 13
  - Let $f(t) = sinc(t)$ be the input to an LTI system with frequency response $H(\omega) = rect(\omega)e^{-j3\omega}$
  - Determine the resulting output *y(t)*

$$\mathrm{sinc}(Wt) \leftrightarrow \frac{\pi}{W}\,\mathrm{rect}\left(\frac{\omega}{2W}\right) \quad W = 1$$

table

$$\mathrm{sinc}(t) \leftrightarrow \pi\,\mathrm{rect}\left(\frac{\omega}{2}\right)$$

$$F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow Y(\omega) = F(\omega)H(\omega) =$$

$$= \underbrace{\pi\,\mathrm{rect}\left(\frac{\omega}{2}\right)}_{F(\omega)} \cdot \underbrace{\mathrm{rect}(\omega)\,e^{-j3\omega}}_{H(\omega)}$$

$$Y(\omega) = |Y(\omega)|\,e^{j\angle Y(\omega)}$$

$$Y(\omega) = \underbrace{\pi\,\mathrm{rect}(\omega)}\,e^{-j3\omega}$$

$\updownarrow$ $\frac{1}{2}\mathrm{sinc}\left(\frac{t}{2}\right)$

$\downarrow$ time shift of -3

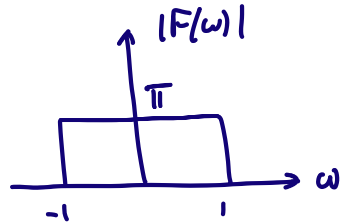
time shift property:

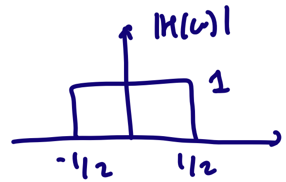
$$f(t) \leftrightarrow F(\omega)$$

$$f(t-t_0) \leftrightarrow F(\omega)\,e^{j\omega(-t_0)}$$

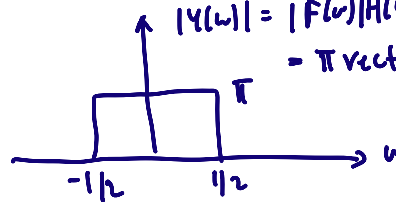
$$\frac{1}{2}\mathrm{sinc}(Wt) \leftrightarrow \frac{1}{2}\frac{2\pi}{W}\,\mathrm{rect}\left(\frac{\omega}{2W}\right) \quad W = 1/2$$

$$y(t) = \frac{1}{2}\mathrm{sinc}\left(\frac{t-3}{2}\right)$$





$$|Y(\omega)| = |F(\omega)||H(\omega)| = \pi\,\mathrm{rect}(\omega)$$



Chapter objectives

- Understand the significance and interpretation of Fourier transform and the inverse Fourier transform
- Apply properties of Fourier transform to determine effect of basic signal processing
- Be able to calculate the energy of a signal both in time and frequency
- Be able to determine energy bandwidth of a signal
- Understand the effect of LTI systems, via $H(\omega)$, on signals via their Fourier transform