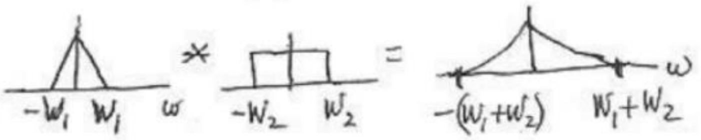
Problem 1

a) The signal y(t) = f(t) · g(t). If f(t) has a bandwidth W_1 rad/s and g(t) has a bandwidth W_2 rad/s, what is the bandwidth of y(t)?

$F(\omega) * G(\omega) = Y(\omega)$

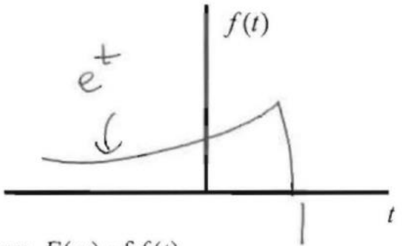


BW = $W_1 + W_2$

Problem 2

a) Consider the function $f(t) = e^{t}u(1-t)$.

i) Sketch $f(t)$.



ii) Find the Fourier transform $F(\omega)$ of $f(t)$.

$$\int_{-\infty}^{1} e^{t} e^{-j\omega t} dt = \frac{e\, e^{-j\omega}}{1-j\omega}$$

$F(\omega) = \frac{e \cdot e^{-j\omega}}{1-j\omega}$

b) Consider the convolution $y(t) = \underset{f_1}{Sinc(2t)} * \underset{f_2}{Sinc(5t)}$. ⇔ $F_1(\omega) \cdot F_2(\omega)$

i) Find $y(t)$.

$\text{Sinc}\, 2t \leftrightarrow \frac{\pi}{2} \text{rect}\left(\frac{\omega}{4}\right)$

$\text{Sinc}\, 5t \leftrightarrow \frac{\pi}{5} \text{rect}\left(\frac{\omega}{10}\right)$

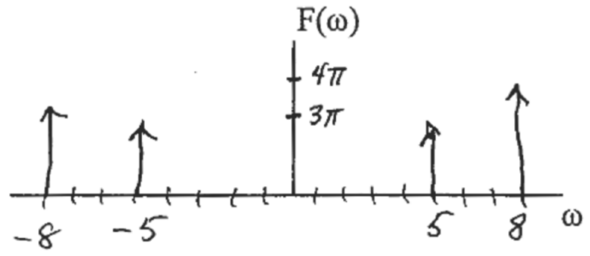
$y(t) = \frac{\pi}{5} \text{Sinc}\, 2t$

ii) Specify the complete set of times t when $y(t) = 0$.

$2t = n\pi$ $\quad$ $t = n\frac{\pi}{2}$

Problem 3

a) Given $f(t) = 3\cos 5t + 4\cos 8t$, find and plot $F(\omega)$. Clearly label axes.

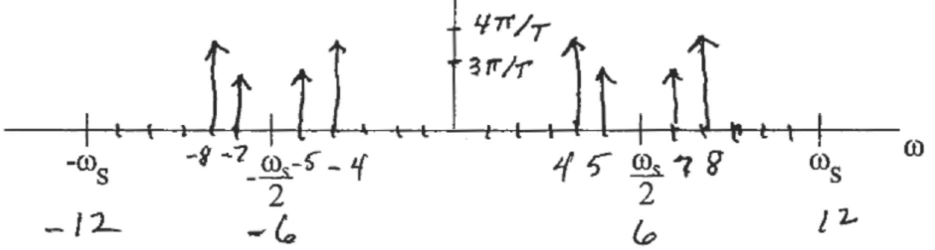


$F(\omega) =$

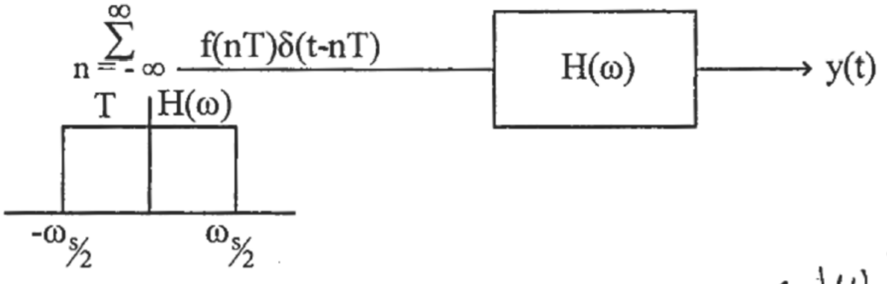
$= 3\pi[\delta(\omega-5)+\delta(\omega+5)] + 4\pi[\delta(\omega-8)+\delta(\omega+8)]$

b) $f(t)$ is sampled at a sampling rate of $\omega_S=12$ rad/s. Plot the frequency spectrum $F_T(\omega)$ of the sampled signal for $-\omega_S \le \omega \le \omega_S$. Clearly label all values.

$F_T(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_s)$



c) An analog signal is reconstructed from the above samples as shown below. What is the output $y(t)$.

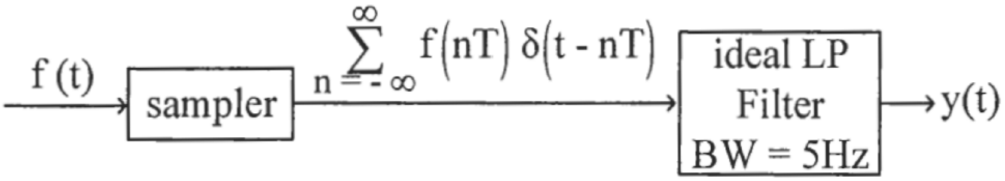


$y(t)$ contains only frequencies $< \left|\frac{\omega_s}{2}\right|$

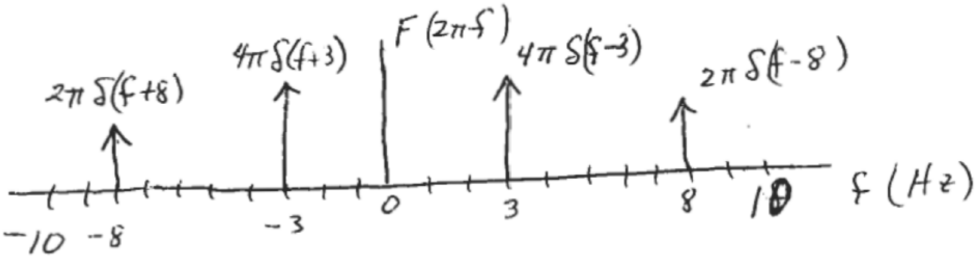
$y(t) = 3\cos 5t + \underbrace{4\cos 4t}_{\text{aliased component of } 4\cos 8t}$

Problem 4 *(25 points)*

(a) *(10 pts)* The signal $f(t) = 4\cos 2\pi \cdot 3t + 2\cos 2\pi \cdot 8t$ is sampled producing the output $\sum_{n=-\infty}^{\infty} f(nT)\,\delta(t-nT)$ where T = 0.1 second. Next the sampled signal is passed through an ideal LP filter with unity gain and a BW = 5Hz. See diagram below.

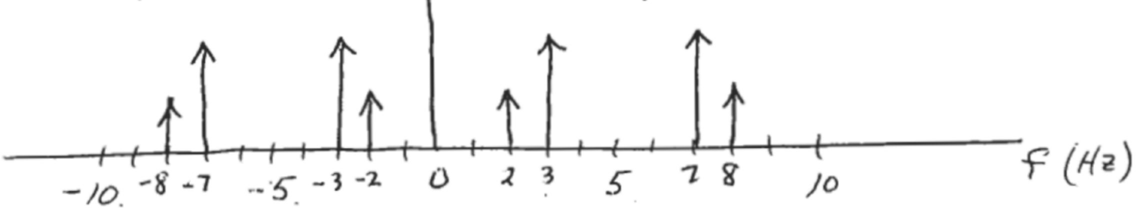


(i) Sketch $F(2\pi f)$ for $-10\text{Hz} \le f \le 10\text{Hz}$.

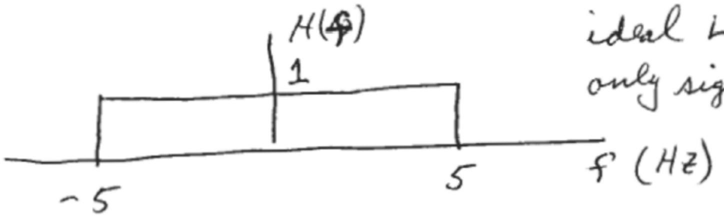


(ii) Repeat Part (a) for the frequency spectrum $F_T(2\pi f)$ of the sampled signal. $f_s = \frac{1}{T} = 10Hz$

$F_T(2\pi f) = \frac{1}{T} \Sigma F[2\pi(f - nf_s)]$



(iii) The output $y(t) = \frac{4}{T}\cos 2\pi \cdot 3t + \frac{2}{T}\cos 2\pi \cdot 2t$



ideal LP filter passes only signals with $|f| \le 5\,Hz$

(b) *(5 pts)* The step response of a LTI system is $g(t) = \left[3te^{-2t} + e^{-2t}\right]u(t)$ and its impulse response is $h(t) = [e^{-2t} - 6e^{-2t}]u(t) + \delta(t)$

$$h(t) = \frac{dg}{dt} = \left[3e^{-2t} - 6te^{-2t} - 2e^{-2t}\right]u(t) + \underbrace{\left[3te^{-2t} + e^{-2t}\right]}_{=1 \text{ at } t=0}\delta(t)$$

Simplifying $h(t) = \left[e^{-2t} - 6te^{-2t}\right]u(t) + \delta(t)$

(c) *(5 pts)* Given the impulse response $h(t) = \left(e^{-t} - te^{-t}\right)u(t)$ find $H(\omega)$. Use the table provided. Simplify and classify $|H(\omega)|$ as

Note: $|H(0)| = 0$, $|H(\infty)| = 0$ and $|H(1)| = \frac{1}{2}$

LP [✓ BP] HP (circle the correct answer)

From Table 7.2.1 and 7.2.5

$$H(\omega) = \frac{1}{1+j\omega} - \frac{1}{(1+j\omega)^2} = \frac{\cancel{1}+j\omega-\cancel{1}}{(1+j\omega)^2} \qquad |H(\omega)| = \left|\frac{j\omega}{(1+j\omega)^2}\right| = \frac{|\omega|}{1+\omega^2}$$