• Impulse – Properties-cont

- Scaling:

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

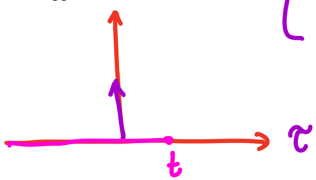
- Definite integral:

0 - changes

$$\int_{-\infty}^{t} (1)\,\delta(\tau)\,d\tau = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

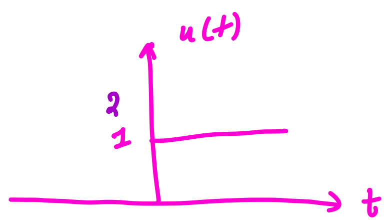
(δ is inside the integral) $= u(t)$

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

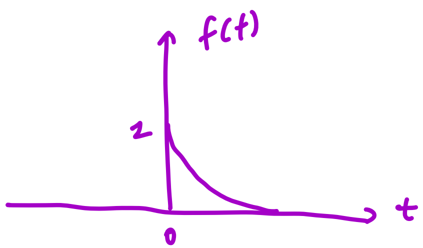


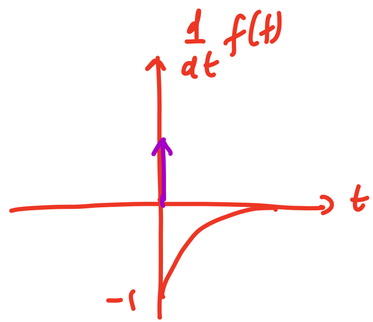
- Unit-step derivative

$$\frac{d}{dt}u(t) = 2\,\delta(t)$$



- Impulse - Properties – Examples-cont
  - Let $\mathrm{f(t)} = e^{-t}u(t)$
  - Determine $\frac{d}{dt}f(t)$





$$\frac{d}{dt}f(t) = \frac{d}{dt}\left(e^{-t}u(t)\right) = e^{-t}\frac{d}{dt}u(t) + u(t)\frac{d}{dt}e^{-t} =$$

$$= \underbrace{e^{-t}\delta(t)}_{\text{sampling property}} - e^{-t}u(t) = e^{-(0)}\delta(t) - e^{-t}u(t) = \delta(t) - e^{-t}u(t)$$

@ t=0

- Impulse – Properties-cont

$$\boxed{\begin{array}{l} \delta(t) \leftrightarrow 1 \\ 1 \leftrightarrow 2\pi\delta(\omega) \end{array}}$$

  - Fourier transform:

$$F\{\delta(t)\} = ?$$

$$\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j\omega(0)} = 1$$

sifting

  - Inverse Fourier transform:

$$F^{-1}\{\delta(\omega)\} = \frac{1}{2\pi}$$

$$f(t) \leftrightarrow F(\omega)$$

$$F(t) \leftrightarrow 2\pi f(-\omega)$$

  - Doublet

$$\underbrace{\delta'(t)} = \underbrace{\frac{d}{dt}\delta(t)} \Longrightarrow \underbrace{f(t) * \delta'(t)} = \underbrace{f'(t)}$$

$$\delta(t) \leftrightarrow 1$$

$$\frac{1}{2\pi}\; 1 \leftrightarrow \frac{1}{2\pi} 2\pi \underbrace{\delta(-\omega)}_{\delta(\omega)}$$

- # Impulse response
  - Recall

$$= \mathcal{F}^{-1}\{H(\omega)\} \qquad Y(\omega) = F(\omega)H(\omega)$$

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = f(t) * h(t)$$

  - How can we obtain h(t) if we don't know it?

$$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \delta(t) * h(t) = h(t)$$

$$\downarrow \mathcal{F}$$

$$1 \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega) = 1 \cdot H(\omega)$$

impulse response

(the response of the system to an impulse)

- Impulse response - Example # 14

  - Suppose an input $\mathrm{f(t)} = u(t)$ is applied to an LTI system with frequency response

$$H(\omega) = \frac{j2\omega - 1}{1 + j\omega}$$

$\mathcal{F}^{-1}\{H(\omega)\} - ?$

$\frac{d}{dt} f(t) \Leftrightarrow j\omega F(\omega)$

  - Determine $y_{zs}(t)$ via the impulse response

$$= f(t) * h(t) =$$

$$= u(t) * (2\delta(t) - 3e^{-t}u(t)) =$$

$$= 2u(t) - 3(1 - e^{-t})u(t)$$

$$y_{zs}(t) = (3e^{-t} - 1)u(t)$$

$$H(\omega) = \frac{j\omega 2}{1+j\omega} - \frac{1}{1+j\omega}$$

$$\downarrow \mathcal{F}^{-1}$$

$$h(t) = \frac{d}{dt} 2(e^{-t}u(t)) - e^{-t}u(t) =$$

$$= 2(\underbrace{e^{-t}\delta(t)} - e^{-t}u(t)) - e^{-t}u(t) =$$

$$= 2(\delta(t) - e^{-t}u(t)) - e^{-t}u(t) =$$

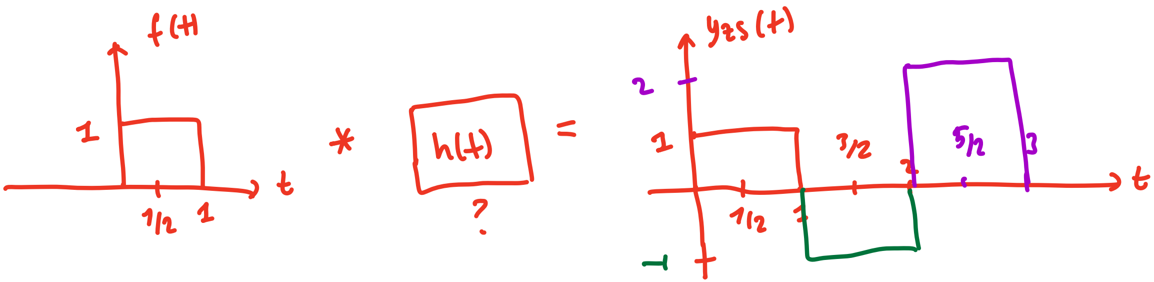
$$= 2\delta(t) - 3e^{-t}u(t)$$

• Impulse response - Example # 14

- Suppose an input $f(t) = rect(t - \frac{1}{2})$ is applied to an LTI system with the corresponding zero-state output is

$$y_{zs}(t) = rect\left(t - \frac{1}{2}\right) - rect\left(t - \frac{3}{2}\right) + 2rect(t - \frac{5}{2})$$

- Determine the system's impulse response *h(t)*



$$h(t) = \delta(t) - \delta(t-1) + 2\delta(t-2)$$

Note: For an LTI system, there is an impulse response $h(t)$, such that $y_{zs}(t) = f(t) * h(t)$ for any $f(t)$

- Fourier transform of power signals $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$
  - Recall that if f(t) is absolutely integrable then its Fourier transform exists
  - There are signals that are not absolutely integrable, but whose Fourier transform exists.
    energy signals have finite energy
  - Some signals have infinite energy, but finite instantaneous power:

$$|f(t)|^2 < \infty$$

power signals (such as cos, sin, unit step)

  - These are power signals, like sin(t).
  - We can use the impulse to represent their Fourier transform.

- Fourier transform of power signals - Example # 16
  - Determine the Fourier transform of $\cos(\omega_c t)$

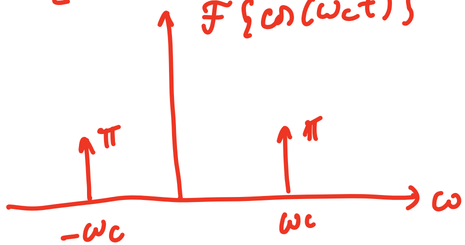
$$f(t)\, e^{j\omega_0 t} \leftrightarrow F(\omega-\omega_0)$$

$$\mathcal{F}\{\cos(\omega_c t)\} = \mathcal{F}\left\{\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right\} =$$

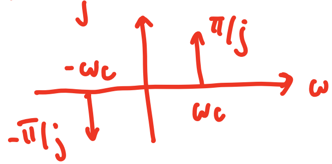
$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$= \frac{1}{2}\mathcal{F}\{1 e^{j\omega_c t}\} + \frac{1}{2}\mathcal{F}\{1 e^{-j\omega_c t}\} =$$

$$= \frac{2\pi}{2}\delta(\omega-\omega_c) + \frac{2\pi}{2}\delta(\omega+\omega_c) = \pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)$$



$$\mathcal{F}\{\sin(\omega_c t)\} = \frac{\pi}{j}\delta(\omega-\omega_c) - \frac{\pi}{j}\delta(\omega+\omega_c) = -j\pi\delta(\omega-\omega_c) + j\pi\delta(\omega+\omega_c)$$



$\measuredangle\, \mathcal{F}\{\sin(\omega_c t)\}$