#1)

a) $\alpha = ?$ (varies by student)

b) i) $f_1(t) = -10\sin\left(2t - \frac{\pi}{\alpha}\right) = -10\cos\left(2t - \frac{\pi}{\alpha} - \frac{\pi}{2}\right)$

$\Rightarrow F_1 = -10 e^{j\left(-\frac{\pi}{\alpha} - \frac{\pi}{2}\right)} = 10 e^{j\left(-\frac{\pi}{\alpha} - \frac{\pi}{2} + \pi\right)}$

$= 10 e^{j\pi\left(\frac{-2-\alpha+2\alpha}{2\alpha}\right)} = 10 e^{j\pi\left(\frac{\alpha-2}{2\alpha}\right)}$

plug in α

ii) $f_2(t) = \underbrace{-\alpha\cos(3t)}_{\text{phasor } 1} + \alpha\underbrace{\sin(3t)}_{-j} \rightarrow$ same frequency

$\Rightarrow F_2 = -\alpha(1) + \alpha(-j) = -\alpha(1+j) = \alpha\sqrt{1^2+1^2}\, e^{j\underbrace{\left(\pi + \frac{\pi}{4}\right)}_{=\frac{5\pi}{4} > \pi}}$

$= \sqrt{2}\,\alpha\, e^{-j\frac{3\pi}{4}}$

plug in α

c) $\frac{d^2}{dt^2}y + \frac{d}{dt}y + \alpha y = f(t) = \sin(\alpha t)$ $\quad \omega = \alpha$

phasors

$(j\omega)^2 Y + j\omega Y + \alpha Y = -j \Rightarrow Y = \frac{-j}{-\alpha^2 + j\alpha + \alpha} = \frac{-j}{\alpha(\underbrace{1-\alpha}_{\leq 0} + j)}$

$= \frac{e^{-j\pi/2}}{\alpha\sqrt{(1-\alpha)^2 + 1^2}\, e^{j\left(\pi + \tan^{-1}\left(\frac{1}{1-\alpha}\right)\right)}}$

$= \frac{e^{+j\left(\frac{-\pi}{2} - \pi - \tan^{-1}\left(\frac{1}{1-\alpha}\right)\right)}}{\alpha\sqrt{1+(1-\alpha)^2}}$

$\Rightarrow \boxed{y(t) = \frac{1}{\alpha\sqrt{1+(1-\alpha)^2}}\cos\left(\alpha t + \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{1-\alpha}\right)\right)}$ $\leftarrow$ plug in α

Notice that no matter what the value of α is, the calculation is simple.

Problem 2 solutions:

Note: replace every instance of the simbol β with your own numerical value obtained from your UIN.

a) β

b) Method 1: $P_{Z2} = \frac{1}{2}Re\{V_{Z2}I_{Z2}^*\} = \frac{1}{2}Re\{Z_{Z2}I_{Z2}I_{Z2}^*\} = \frac{1}{2}Re\{Z_2|I_2|^2\}$, where $|I_2|^2$ is real and $Z_2$is imagenary, so $P_{Z2} = 0$.

Method 2: $Z_2 = j2\ \Omega$, so Z_2 corresponds to in an inductor (if $\omega > 0$) or a capacitor (if $\omega < 0$). Either way, $P_{Z2} = 0$.

(Note 1: no numerical computation is required to reach this solution)

(Note 2: This part of the problem is essentially the same as problems 3c and 3d of HW #6)

c) Rectangular form: $Z_1 = \frac{R\cdot\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{R}{1+j\omega RC} = \frac{\beta}{1+j} = \frac{\beta(1-j)}{2} = \frac{\beta}{2} - j\frac{\beta}{2}$.

(Note: The computational complexity should not be affected by your β value as β is an odd number regardless of you UIN)

d) $Z_3 = Z_1 - Z_2 \implies Z_1 = Z_2 + Z_3$

According to current division, the current through Z_1 is $I_1 = I_S/2$

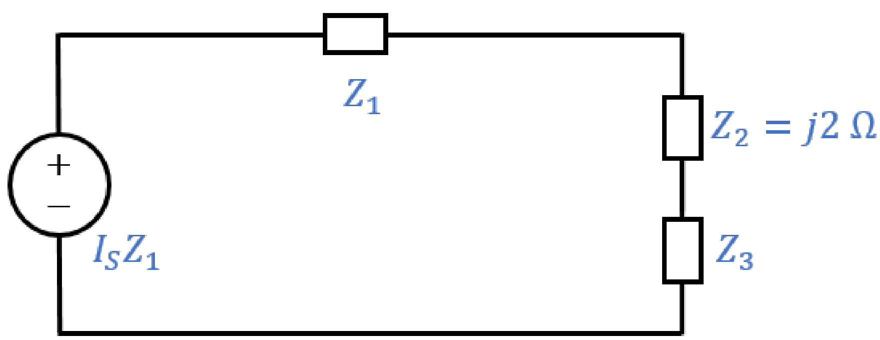
$P_{Z1} = \frac{1}{2}Re\{V_1I_1^*\} = \frac{1}{2}Re\{Z_1I_1I_1^*\} = \frac{1}{2}Re\{Z_1|I_1|^2\} = \frac{1}{2}|I_1|^2Re\{Z_1\} = \frac{1}{8}|I_S|^2Re\{Z_1\} = \frac{1}{8}4\beta^2 2\beta = \beta^3$

(Note: Leaving your solution to be something like 47^3 is acceptable.)

e) $P_{Z2} = 0$, so the problem is converted to finding Z_3 that maximizes P_3.

According to the concept of matched load, P_3 is maximized when $Z_3 = Z_T^*$

To find Z_T, we convert the original circuit into the following via source transformation:



$Z_T = Z_1 + Z_2 = \beta + j\cdot 3.14 + j2 = \beta + j5.14$

$Z_3 = Z_T^* = \beta - j5.14$

(Note: this part does not involve any numerical computations on β)

3) a) $\alpha =$ ______

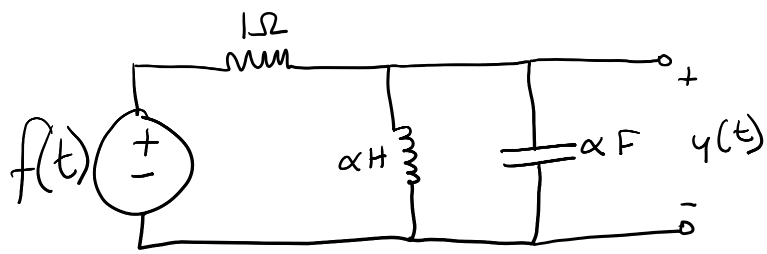
b) LC circuit

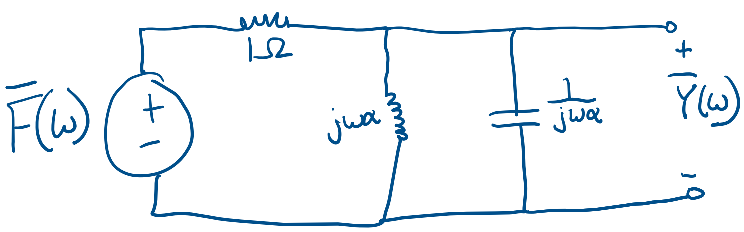
$$\omega_o = \frac{1}{\sqrt{LC}} \Rightarrow \omega_o = \frac{1}{\sqrt{\alpha\alpha}} = \frac{1}{\sqrt{\alpha^2}}$$

$$\boxed{\omega_o = \frac{1}{\alpha}}$$

$L = \alpha H$
$C = \alpha F$

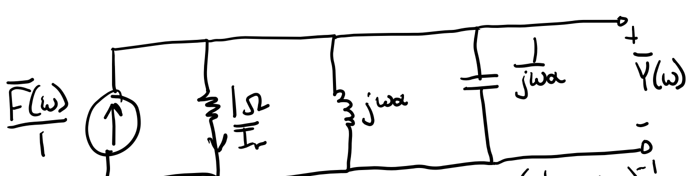
c)





* There are Many ways to find $H(\omega) = \frac{\bar{Y}(\omega)}{\bar{F}(\omega)}$. Below are a few methods.

Method 1: Source Transformation



$$\bar{I}_r = \frac{\left(\frac{1}{j\omega\alpha} + j\omega\alpha\right)^{-1}}{1 + \left(\frac{1}{j\omega\alpha} + j\omega\alpha\right)^{-1}} \bar{F}(\omega) \Rightarrow \frac{\left(\frac{-j + j(\omega\alpha)^2}{\omega\alpha}\right)^{-1}}{1 + \left(\frac{-j + j(\omega\alpha)^2}{\omega\alpha}\right)^{-1}} \bar{F}(\omega)$$

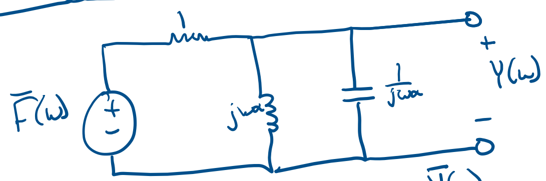
$$= \frac{\frac{\omega\alpha}{j((\omega\alpha)^2 - 1)}}{1 + \frac{\omega\alpha}{j((\omega\alpha)^2 - 1)}} \bar{F}(\omega) \Rightarrow \frac{\omega\alpha}{j((\omega\alpha)^2 - 1) + \omega\alpha} \bar{F}(\omega)$$

$$\bar{Y}(\omega) = 1\bar{I}_r$$

$$\bar{Y}(\omega) = \frac{\omega\alpha}{j((\omega\alpha)^2 - 1) + \omega\alpha} \bar{F}(\omega)$$

$$\boxed{H(\omega) = \frac{\bar{Y}(\omega)}{\bar{F}(\omega)} = \frac{\omega\alpha}{j((\omega\alpha)^2 - 1) + \omega\alpha}}$$

Method 2: Node Voltage



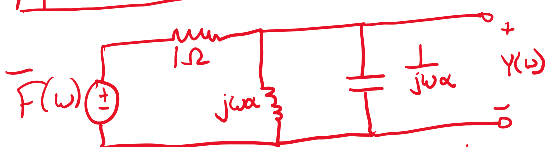
$$\frac{\bar{F}(\omega)-\bar{Y}(\omega)}{1} = \frac{\bar{Y}(\omega)}{j\omega\alpha} + j\omega\alpha\,\bar{Y}(\omega) \Rightarrow \bar{F}(\omega) = \bar{Y}(\omega) + \frac{1}{j\omega\alpha}\bar{Y}(\omega) + j\omega\alpha\,\bar{Y}(\omega)$$

$$\bar{Y}(\omega) = \frac{1}{1+j\omega\alpha+\frac{1}{j\omega\alpha}}\bar{F}(\omega)$$

$$\boxed{H(\omega) = \frac{\bar{Y}(\omega)}{\bar{F}(\omega)} = \frac{1}{1+j\omega\alpha+\frac{1}{j\omega\alpha}}}$$

$$H(\omega) = \frac{1}{\frac{\omega\alpha + j(\omega\alpha)^2 - j}{\omega\alpha}} \Rightarrow \boxed{\frac{\omega\alpha}{\omega\alpha + j\left((\omega\alpha)^2-1\right)}}$$

Method 3: Voltage Division



$$\bar{Y}(\omega) = \frac{\left(\frac{1}{j\omega\alpha}+j\omega\alpha\right)^{-1}}{1+\left(\frac{1}{j\omega\alpha}+j\omega\alpha\right)^{-1}}\bar{F}(\omega) \Rightarrow \frac{\left(\frac{-j+j(\omega\alpha)^2}{\omega\alpha}\right)^{-1}}{1+\left(\frac{-j+j(\omega\alpha)^2}{\omega\alpha}\right)^{-1}}\bar{F}(\omega)$$

$$= \frac{\frac{\omega\alpha}{j\left((\omega\alpha)^2-1\right)}}{1+\frac{\omega\alpha}{j\left((\omega\alpha)^2-1\right)}}\bar{F}(\omega) \Rightarrow \bar{Y}(\omega) = \frac{\omega\alpha}{\omega\alpha + j\left((\omega\alpha)^2-1\right)}\bar{F}(\omega)$$

$$\boxed{H(\omega) = \frac{\bar{Y}(\omega)}{\bar{F}(\omega)} = \frac{\omega\alpha}{\omega\alpha + j\left((\omega\alpha)^2-1\right)}}$$

d) $f(t) = 2 + 2\cos(2t)$

$$Y(\omega) = \sum H(\omega) F(\omega)$$

$$y(t) = 2|H(\omega=0)| + 2|H(\omega=2)| \cos(2t + \angle H(\omega=2))$$

$$H(\omega) = \frac{\omega\alpha}{\omega\alpha + j((\omega\alpha)^2 - 1)} = \frac{\omega\alpha(\omega\alpha - j((\omega\alpha)^2-1))}{(\omega\alpha)^2 + ((\omega\alpha)^2-1)^2}$$

$$= \frac{(\omega\alpha)^2 - j\omega\alpha((\omega\alpha)^2-1)}{(\omega\alpha)^2 + ((\omega\alpha)^2-1)^2}$$

$$|H(\omega)| = \sqrt{\frac{(\omega\alpha)^4 + (\omega\alpha)^2((\omega\alpha)^2-1)^2}{((\omega\alpha)^2 + ((\omega\alpha)^2-1)^2)^2}} \Rightarrow \sqrt{\frac{(\omega\alpha)^2((\omega\alpha)^2 + ((\omega\alpha)^2-1)^2)}{((\omega\alpha)^2 + ((\omega\alpha)^2-1)^2)^2}}$$

$$|H(\omega)| = \sqrt{\frac{(\omega\alpha)^2}{(\omega\alpha)^2 + ((\omega\alpha)^2-1)^2}}$$

$$|H(\omega=0)| = 0$$

$$|H(\omega=2)| = \sqrt{\frac{4\alpha^2}{4\alpha^2 + (4\alpha^2-1)^2}}$$

$$\angle H(\omega) = \tan^{-1}\left(\frac{-\omega\alpha((\omega\alpha)^2-1)}{(\omega\alpha)^2}\right) = \tan^{-1}\left(\frac{-((\omega\alpha)^2-1)}{\omega\alpha}\right)$$

$$\angle H(\omega=2) = \tan^{-1}\left(\frac{1-4\alpha^2}{2\alpha}\right)$$

$$\boxed{y(t) = 2(0) + 2\sqrt{\frac{4\alpha^2}{4\alpha^2 + (4\alpha^2-1)^2}} \cos\left(2t + \tan^{-1}\left(\frac{1-4\alpha^2}{2\alpha}\right)\right)}$$

#4 a) $\alpha = ?$ (varies by student)

b) Notice $h(t) = f(-(t+\alpha))$

Recall that if $g(t) = f(-t)$

$\Rightarrow G_n = F_{-n}$

And if $h(t) = g(t+\alpha)$

$\Rightarrow H_n = G_n e^{jn\omega_0 \alpha} = F_{-n} e^{jn\omega_0\alpha}$

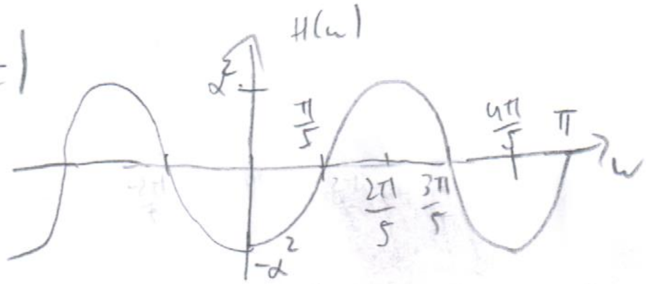
$= F_{-n} e^{jn\frac{\pi}{5}\alpha}$

$T = 10 \Rightarrow \omega_0 = \frac{2\pi}{10} = \frac{\pi}{5}$

$$\boxed{= \begin{cases} 0 & n = 0 \\ \frac{(-1)^{-n}}{jn\pi\alpha} e^{jn\frac{\pi}{5}\alpha} & n \neq 0 \end{cases}}$$

↑ plug in α

c)



↑ ↑ ↑ ↑ ↑

$n=1$ $n=2$ $n=3$ $n=4$ $n=5$

$\sin\left(\frac{5\omega}{2} - \frac{\pi}{2}\right) = -\cos\left(\frac{5\omega}{2}\right)$

→ only $n = \pm 2, \pm 4$ go through

$$y(t) = \sum_{n=-\infty}^{\infty} F_n H(n\omega_0) e^{jn\omega_0 t}$$

$$= \frac{-1\alpha^2}{j2\pi\cancel{\alpha}} e^{j\frac{2\pi}{5}t} + \frac{+1\alpha^2}{j2\pi\cancel{\alpha}} e^{-j\frac{2\pi}{5}t} + \frac{-1(-\alpha^2)}{j4\pi\cancel{\alpha}} e^{j\frac{4\pi}{5}t} + \frac{+1(-\alpha^2)}{j4\pi\cancel{\alpha}} e^{-j\frac{4\pi}{5}t}$$

$$\boxed{= -\frac{\alpha}{\pi}\sin\left(\frac{2\pi}{5}t\right) + \frac{\alpha}{2\pi}\sin\left(\frac{4\pi}{5}t\right)}$$

↑

Notice $\omega_{orig} = \frac{2\pi}{5}$ rad/s