- Dissipative systems

- *A dissipative system has a transient zero-input response*

$y_{zI}(t) \to 0$
$t \to \infty$

$\sin x = \cos(x - \pi/2)$

$\to$ cos or sin

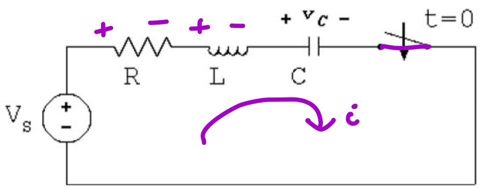
- *In a dissipative system, the steady-state response to a cosinusoidal input applied at t = −∞ will be a cosinusoidal independent of the initial state:*

$\to 0$ as $t \to \infty$

$$y(t) = Ae^{-at} + \underbrace{H\cos(\omega t + \psi)}_{y_{ss}(t)}$$

- n-th order LTI systems

- Determine the ODE governing the capacitor voltage in the following circuit:



$t>0$

$$i = C\frac{dv_c}{dt}$$

$$-V_s + V_R + v_L + v_c = 0$$

$$-V_s + i\cdot R + L\frac{di}{dt} + v_c = 0$$

$$-V_s + R\left(C\frac{dv_c}{dt}\right) + L\frac{d}{dt}\left(C\frac{dv_c}{dt}\right) + v_c = 0 \Big/ : LC$$

$$\boxed{\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{V_s}{LC}}$$

2nd order ODE with constant coeff.

• n-th order LTI systems-cont

- In general

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_n y = f(t)$$

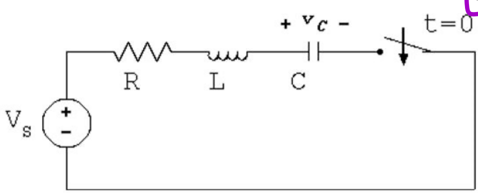
where n – number of energy storage elements (L, C)

- Solution is

same as in 1st order ODE (same shape as input)

$$y(t) = y_p(t) + y_h(t)$$

going to change



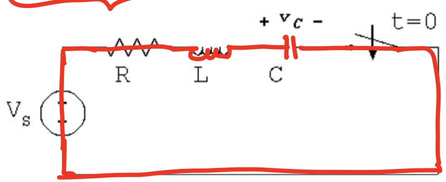
$\cancel{y_h(t) = Ae^{-at}}$

if $p_1 = p_2$

$A_1 e^{-p_1 t} + tA_2 e^{-p_2 t} + \ldots + A_n e^{-p_n t} = y_h(t)$

- Example #23:
- For t > 0, obtain v_c(t) in the following circuit if v_s = 0V, R = 0Ω, $\underbrace{i_L(0^-) = 0A}$, $\underbrace{v_c(0^-) = 1V}$, L = 1H and C = 1F:

" $i_L(0^+) = i_c(0^+)$ series circuit



- Recall

$$\frac{\cancel{V_s}^{\,0}}{LC} = \frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{\cancel{dv_c}^{\,0}}{dt} + \frac{v_c}{LC} \quad = 1$$

Solution:

$\frac{d^2 v_c}{dt^2} + v_c = 0$ ← homogenous ODE

$y_h(t) = e^{st}$

$s^2 e^{st} + e^{st} = 0$

$s^2 + 1 = 0$

$s = \pm\sqrt{-1} = \pm j$

$y_h = A_1 e^{-jt} + A_2 e^{jt}$
? ?

$0 = \frac{d^2 v_c}{dt^2} + v_c \Rightarrow v_c(t) = \frac{1}{2} e^{jt} + \frac{1}{2} e^{-jt} =$

$= \cos(t) \rightarrow$ zero-input response is not transient ⇒ system is non-dissipative

$i_c(t) = C \frac{dv_c}{dt} = -\sin(t)$

Find A_1 and A_2:

$v_c(0^-) = v_c(0^+) = 1$

$1 = A_1 + A_2$ (1)

$i_c(0^+) = 0$

$i_c(t) = C\frac{dv_c}{dt} = -jA_1 e^{-jt} + jA_2 e^{jt}$

$i_c(0^+) = -jA_1 + jA_2 = 0$ (2)

$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$

Solving (1) and (2) ⇒ $A_1 = A_2 = \frac{1}{2} \Rightarrow v_c(t) = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt}$ V

Chapter objectives

- Apply ideal op-amp approximation to do signal processing
- Understand zero-input and zero-state responses
- Understand what is linearity and how to test if a system is linear
- Understand what is time-invariance and how to test if system is TI
- Analyze first order RC and RL circuits with constant inputs:
  - How to obtain particular and homogeneous solutions
  - How to obtain zero-state and zero-input solutions
  - How to obtain transient and steady-state solutions
  - Understand the effect of the time-constant in the solution
- Analyze first order RC, RL circuits with time-varying inputs
  - How to obtain particular and homogeneous solutions
  - How to obtain zero-state and zero-input solutions
  - How to obtain transient and steady-state solutions
- Be familiar with n-th order LTI systems