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Analog Signal Processing

Thursday, September 23, 8:45-10pm

Exam I

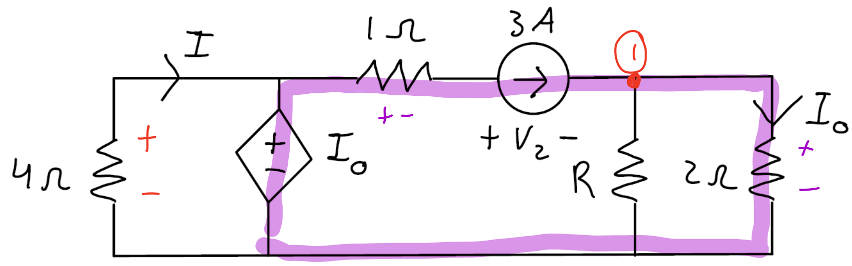
Last Name (capitalized):	Solutions
First Name:	

UIN:		**netID:**	

Clearly PRINT your name in CAPITAL LETTERS. This is a closed book and closed notes exam. Calculators are not allowed. To get credit, please SHOW all your work and simplify your answers. Write your final answers in the spaces provided. All answers should INCLUDE UNITS whenever appropriate. The exam is printed **double-sided**.	**DO NOT write in these spaces.** Problem 1 (25 points):________ Problem 2 (25 points):________ Problem 3 (25 points):________ Problem 4 (25 points):________ Total: (100 points):________

1. (25 pts) The two parts of this problem are unrelated.

(a) [20 pts] Consider the circuit below. It is desired for the output current, $I_o = 1\text{A}$. Determine the value of R, I, V_2, and the absorbed power at the current source, P_{I_s}.



$R =$ $1\,\Omega$

$I =$ $-1/4$ A

$V_2 =$ -4 V

$P_{I_s} =$ -12 W

3A are divided @ node 1 into the two resistors:

$$I_0 = 3\,\frac{R}{R+2} \rightarrow 1 = \frac{3R}{R+2} \rightarrow R+2 = 3R \rightarrow 2 = 2R \rightarrow R = 1$$

$V_{4\Omega} = I_0$ because in parallel

$$-4I = I_0 \rightarrow I = -\frac{I_0}{4} = -\frac{1}{4}$$

KVL on purple loop: $-I_0 + (1)(3) + V_2 + 2I_0 = 0$

$$\rightarrow V_2 = -3 - I_0 = -3 - 1 = -4$$

$$P_{I_s} = V_2 3 = -4(3) = -12$$

(b) [5 pts] Determine the magnitude and phase of the complex number $Z = \dfrac{e^{j\pi/2}}{e^{-j\pi/4} + e^{j\pi/4}}$.

Recall from Euler's: $\dfrac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$

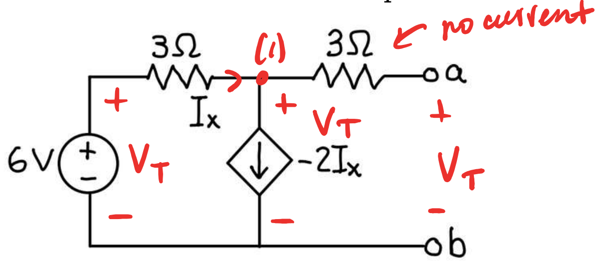
$$\rightarrow Z = \frac{e^{j\frac{\pi}{2}}}{2\cos\left(\frac{\pi}{4}\right)} = \frac{e^{j\frac{\pi}{2}}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}\,e^{j\frac{\pi}{2}} = \frac{\sqrt{2}}{2}\,e^{j\frac{\pi}{2}}$$

$|Z| =$ $\sqrt{2}/2$

$\angle Z =$ $\pi/2$ rad

2. (25 points) Parts a and b are unrelated.

(a) [10 pt] In the following circuit between a and b determine Thevenin's voltage, Thevenin's resistance and the available power of the circuit.



$V_T =$ 6 V

$R_T =$ 4 Ω

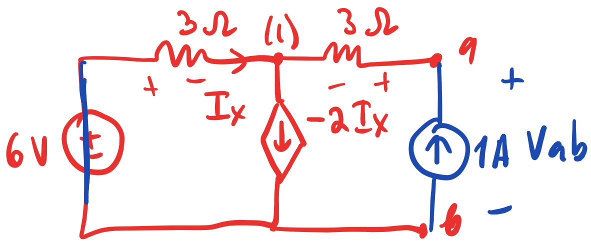
$P_a =$ 9/4 W

Find V_T:

KCL @ (1): $I_x = -2I_x$

$I_x = 0 \Rightarrow V_T = 6V$

Find R_T: test signal method:



KCL @ (1): $I_x + 1 = -2I_x$

$3I_x = -1$

$I_x = -1/3\ A$

KVL: $3I_x - 3\cdot 1 + V_{ab} = 0$

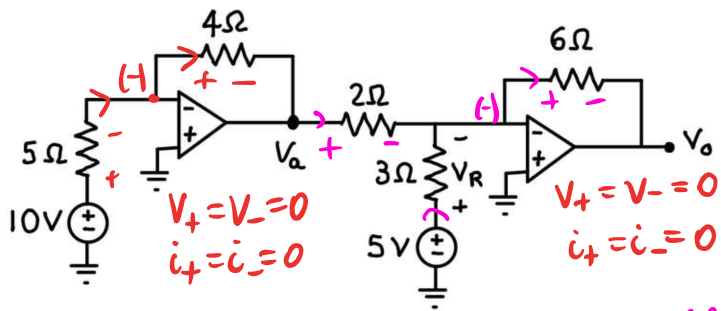
$V_{ab} = -3\cdot(-1/3) + 3 = 4V$

$R_T = 4\Omega$

Find P_a:

$$P_a = \frac{V_T^2}{4R_T} = \frac{36}{4\cdot 4} = \frac{36}{16} = \frac{9}{4}\ W$$

(b) [15 pts] Consider the ideal op amp circuit shown below: Assuming ideal op amp approximations, determine V_a ,V_R and V_o .



V_a = $-8V$

V_R = $5V$

V_o = $14V$

KCL @ (-): $\frac{10-\cancel{V_-}^{0}}{5} = \frac{\cancel{V_-}^{0}-V_a}{4}$

$\frac{10}{5} = -\frac{V_a}{4} \Rightarrow V_a = -8V$

KCL @ (-):

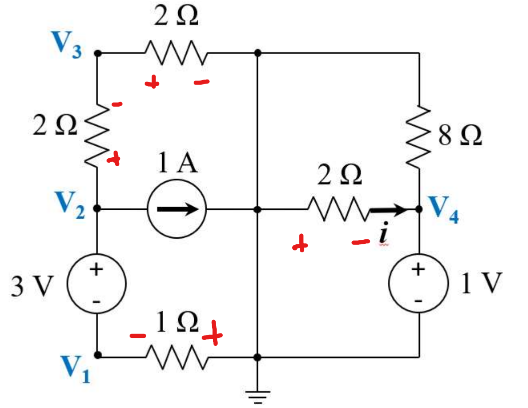
$\frac{V_a-\cancel{V_-}^{0}}{2} + \frac{5-\cancel{V_-}^{0}}{3} = \frac{\cancel{V_-}^{0}-V_o}{6}$

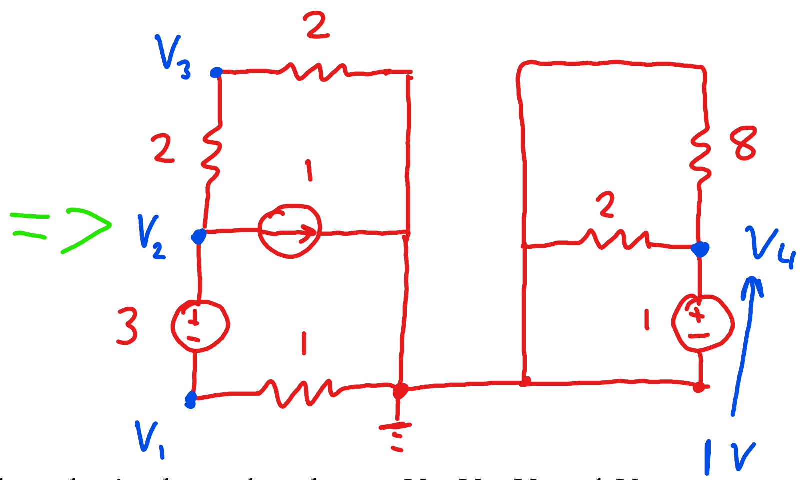
$3V_a + 10 = -V_o$

$V_o = -3\cdot(-8) - 10 = 14V$

$V_R = 5 - V_- = 5V$

3. (25 pts) Consider the following circuit.





(a) [~~??~~ 20 pts] Use the node-voltage method to obtain the node voltages V_1, V_2, V_3 and V_4.

$3V = V_2 - V_1 \; ; \quad V_1 = V_2 - 3V$

$\boxed{1V = V_4 - 0 = V_4}$

KCL @ V_2: $\frac{0 - V_1}{1} - 1 - \frac{V_2 - V_3}{2} = 0$

KCL @ V_3: $\frac{V_2 - V_3}{2} = \frac{V_3 - 0}{2}$

$V_2 - V_3 = V_3$

$V_2 = 2V_3$

$V_1 = 2 \cdot V_3 - 3V$

$-V_1 - 1 - \frac{V_2 - V_3}{2} = 0$

$-2V_3 + 3 - 1 - V_3 + \frac{V_3}{2} = 0$

$-3V_3 + \frac{V_3}{2} = -2$

$-\frac{5}{2} V_3 = -2$

$\boxed{V_3 = \frac{4}{5}}$ $\boxed{V_2 = \frac{8}{5}}$ $\boxed{V_1 = -\frac{7}{5}}$

$V_1 =$ $-\frac{7}{5}$ V

$V_2 =$ $\frac{8}{5}$ V

$V_3 =$ $\frac{4}{5}$ V

$V_4 =$ 1 V

(b) [~~??~~ 5 pts] Determine i.

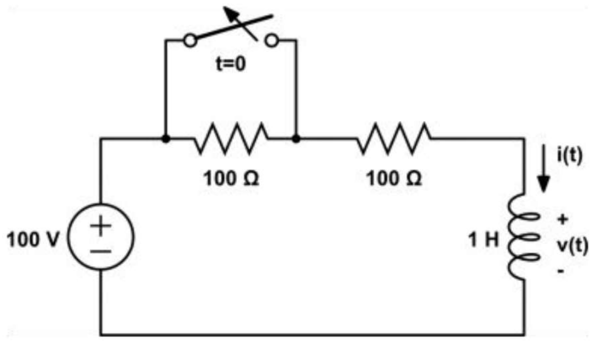
KVL (right bottom square)

$i \cdot 2\Omega + 1V = 0$

$i = -\frac{1V}{2\Omega} = -\frac{1}{2}$ A

$i =$ $-\frac{1}{2}$ A

4. (25 pts) In the circuit above, the switch has been closed for a very long time prior to $t = 0$.



(a) [~~??~~ 12 pts] Determine an expression for $i(t)$ for $t > 0$, and identify its zero-state and zero-input parts.

KVL: for $t>0$ $\quad 100 = \underbrace{R_T}_{200\,\Omega}\, i(t) + L \frac{di}{dt}$

$$\frac{100}{L} = \frac{di}{dt} + \frac{R_T}{L} i(t) \qquad \tau = \frac{L}{R_T} = \frac{1}{200} = 5\,ms$$

$$i(t) = B + Ae^{-t/\tau}$$

$$i(\infty) = B = 0.5A = I_N = f$$

$$1A = i(0^+) = i(0^-) = B + A$$

$$A = i(0^+) - B$$

Simple solutions method:

$$\boxed{i(t) = 0.5 + 0.5e^{-t/\tau}, \quad \tau = 0.005s}$$

$$i(t) = f + (i(0^+) - f)e^{-t/\tau}$$

$i(0^+) = 0 \rightarrow$ $i_{ZS}(t) = f - fe^{-t/\tau} = 0.5 - 0.5e^{-t/\tau}$

$f = 0 \rightarrow$ $i_{ZI}(t) = i(0^+)e^{-t/\tau} = e^{-t/\tau}$

$\tau = 0.005s$

$i_{ZI}(t) =$ $e^{-t/\tau}$ A

$i_{ZS}(t) =$ $0.5 - 0.5e^{-t/\tau}$ A

$i(t) =$ $0.5 + 0.5e^{-t/\tau}$ A $\quad \tau = 0.005s$

(b) [~~??~~ 7 pts] Determine an expression for $v(t)$ for $t > 0$, and identify its zero-state and zero-input parts.

$$v(t) = L\frac{di(t)}{dt}$$

$$L = 1$$

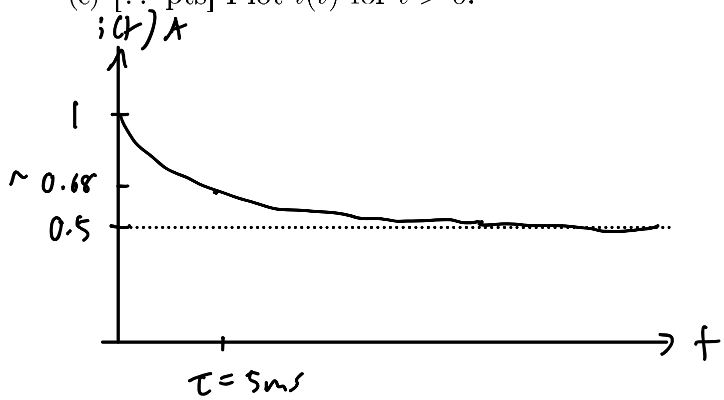
$$\tau = \frac{1}{200}s = 5\,ms$$

$v_{ZI}(t) = \frac{di_{ZI}}{dt} = -\frac{1}{\tau}e^{-t/\tau} = -200e^{-t/\tau}$

$v_{ZS}(t) = \frac{di_{ZS}}{dt} = -\frac{1}{\tau}\cdot -0.5\,e^{-t/\tau} = 100e^{-t/\tau}$

$v(t) = V_{ZI}(t) + V_{ZS}(t) = -100e^{-t/\tau}$

(c) [~~??~~ 3 pts] Plot $i(t)$ for $t > 0$.



(d) [~~??~~ 3 pts] Plot $v(t)$ for $t > 0$.