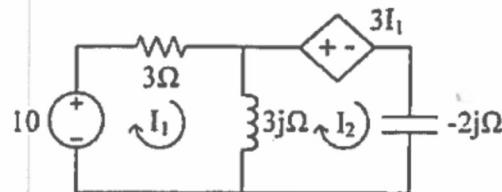


**Problem 2**

(a) In the following circuit, write two loop equations in terms of  $I_1$  and  $I_2$ .



$$10 - 3I_1 - 3j(I_1 - I_2) = 0 \quad 3I_1 + 3j(I_1 - I_2) = 10 \quad (1)$$

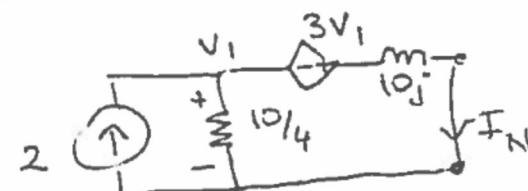
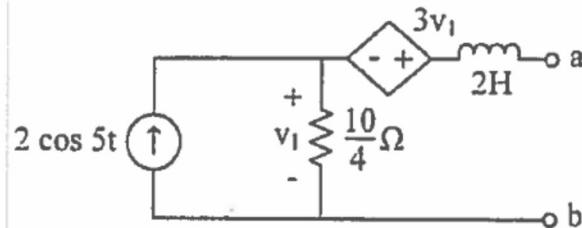
$$(3+3j)I_1 - 3jI_2 = 10 \quad (3+3j)I_1 - 3jI_2 = 10$$

$$(I_2 - I_1)3j + 3I_1 + (-2j)I_2 = 0 \quad (3-3j)I_1 + jI_2 = 0 \quad (2)$$

$$(3+3j)I_1 + (-3j)I_2 = (10)$$

$$(3-3j)I_1 + (j)I_2 = (0)$$

(b) For the following circuit, find  $I_N$ ,  $Z_T$  and maximum available average power of the circuit between a and b. (Give  $I_N$  and  $Z_T$  in rectangular form i.e.  $\alpha + j\beta$ )



$$\frac{V_1}{5j/2} + \frac{V_1 + 3V_1}{10j} = 2 \quad \frac{2V_1}{5} + \frac{4V_1}{10j} = 2 \rightarrow V_1 = \frac{5j}{1+j} \quad I_N = \frac{4V_1}{10j}$$

$$I_N = \frac{2}{1+j} = 1-j$$

$$I_N = \frac{1-j}{1+j}$$

## SOLUTION

$$Z_{eq} = j\omega L + R \parallel -j\frac{1}{\omega C}$$

Because  $\omega = 1 \text{ rad-s}^{-1}$ ,  $R = 2\Omega$ ,

and  $L = 1 \text{ h}$ ,

$$Z_{eq} = j + \frac{-2j/\omega C}{2 - j/\omega C}$$

$$= j - j \left\{ \frac{2}{2C - j} \right\}$$

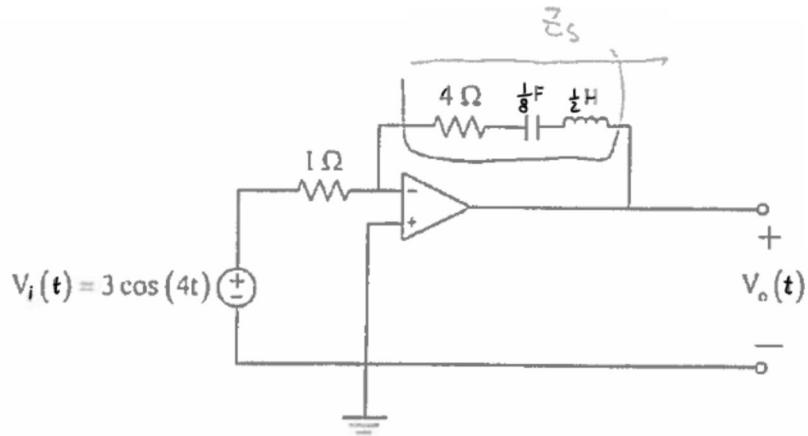
$$= j - \frac{j4C}{4C^2 + 1} + \frac{\text{REAL TERM}}{4C^2 + 1}$$

For  $i_g(t)$  to be in phase with

$v_g(t)$  requires  $\text{Im}\{Z_{eq}\} = 0$ .

$$\therefore \frac{4C}{4C^2 + 1} = 1 \text{ AND } C = \frac{1}{2} \text{ F}$$

Problem 2



Assume the op-amp in this circuit is an ideal op-amp.

- a) (15 points) Use the phasor method to calculate the output voltage  $V_o(t)$ .

$$Z_s = R + j\omega L + \frac{1}{j\omega C} = 4 + j\frac{\omega}{2} - j\frac{8}{\omega} = 4 \quad \text{for } \omega = 4 \text{ rad/s}$$

$$\text{KCL at } V_- : \frac{V_i - V_-}{1} = \frac{V_+ - V_o}{Z_s} \quad \text{and} \quad V_f = V_- = 0$$

$$\Rightarrow V_i = - \frac{V_o}{Z_s}$$

$$V_o = -4V_i = -4 \times 3 = -12V$$

$$\therefore V_o(+) = -12 \cos(4t) = 12 \cos(4t + \pi)$$

- b) (5 points) If a  $1000 \Omega$  resistor is connected across the output terminals, what is the average power dissipated by the resistor?

$$P = \frac{V_o^2}{R}$$

**Problem 2 (continued)**

- c) (5 points) The op-amp is biased at  $\pm 15V$ . How would the output voltage change as the input signal frequency is increased or decreased from 4 rad/s? Explain your reasoning.

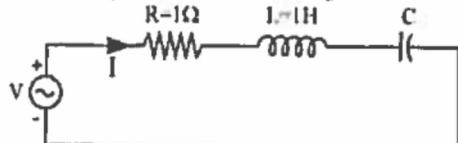
$$\vec{Z}_S = 4 + \frac{j\omega}{2} \left(1 - \frac{16}{\omega^2}\right) = 4 \left(1 + \frac{j\omega}{8} \left(1 - \frac{16}{\omega^2}\right)\right)$$

$$|Z_S| = 4 \left[1 + \frac{\omega^2}{8^2} \left(1 - \frac{16}{\omega^2}\right)^2\right]^{1/2}$$

$$|V_o| = V_i |Z_S| = 12 \left[1 + \frac{\omega^2}{8^2} \left(1 - \frac{16}{\omega^2}\right)^2\right]^{1/2}$$

$|V_o|$  increases as  $\omega$  is either increased or decreased from 4 rad/s until it is clipped by the bias ~~voltage~~

2. (25 pts) An RLC series circuit consisting of a  $1\Omega$  resistor  $R$ , an inductance of value  $L = 1 \text{ H}$  and a capacitor  $C$  is fed by a cosinusoidal voltage source.



- (a) If the cosinusoidal voltage source has angular frequency  $\omega = 1 \text{ rad/s}$  and the current leads (is ahead of) the voltage at the source by  $45^\circ$ , what is the value of the capacitor  $C$ ?

$$\frac{X}{R} = \tan(-45^\circ) \Rightarrow \frac{X}{R} = -1$$

$$R = 1\Omega, X = -1\Omega$$

$$X_L - X_C = -1 \Rightarrow X_L = 1\Omega \Rightarrow X_C = -2\Omega$$

$$\frac{1}{\omega C} = 2 \Rightarrow C = \frac{1}{2\omega} = \frac{1}{2} F$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{2}}} = \sqrt{2}$$

$$C = \frac{1}{2} F$$