Lecture 41, Monday, April 11, 2022

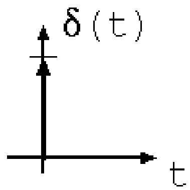
- The impulse, $\delta(t)$, is such that

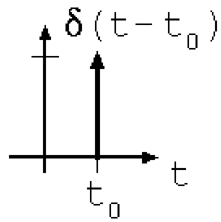
$$f(t) * \delta(t) \;=\; f(t)$$

  – More generally,

$$f(t) * \delta(t - t_0) = f(t - t_0)$$

  $\Rightarrow$ convolving with an impulse shifts $f(t)$ sideways to the location of the impulse





- However, $\delta(t)$:

  – is **not** a function, it is a *distribution*

  – it is defined in terms of how it interacts with other functions through convolution;

  – does **not** have a numerical interpretation;

- Properties of impulse:

  – Energy:

$$W_{\delta(t)} = \infty$$

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– Symmetry:

$$\delta(t) = \delta(-t)$$

– Sifting:

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt \;=\; f(0)$$

* More generally

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt \;=\; f(t_0)$$

integrating against an impulse gives the value of the function at the location of the impulse

* Even more generally

$$\int_{a}^{b} f(t)\delta(t-t_0)dt \;=\; \begin{cases} f(t_0) & a < t_0 < b \\ 0 & else. \end{cases}$$

integrating against an impulse gives the value of the function at the location of the impulse only if the impulse is inside the integration limits

– Area

$$\int_{-\infty}^{\infty} \delta(t)dt \;=\; 1$$

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– Sampling

$$f(t)\delta(t) = f(0)\delta(t)$$

notice that the impulse remains because there is no integral involved yet

– Time scaling

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

– Definite integral

$$\int_{-\infty}^{t} \delta(\tau)d\tau \;=\; u(t) \quad \text{unit-step}$$

– Unit-step derivative

$$\frac{d}{dt}u(t) = \delta(t)$$

