- Example #10: Linearity-cont
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = y_0 e^{-t} - \int_0^t f(\cancel{x})\,dx$$

0

  - Determine if the system is linear or not

zero-input => $f(t) = 0$ => $y_{ZI}(t) = y_0 e^{-t}$

is this a line going through the origin?

$y = kx$

$\boxed{y_0} \to y_{ZI,0}(t) = y_0 e^{-t}$

$\boxed{y_1} \to y_{ZI,1}(t) = y_1 e^{-t}$

$\boxed{y_3 = k_1 y_0 + k_2 y_1} \to y_{ZI,3}(t) \overset{?}{=} k_1 y_{ZI,0} + k_2 y_{ZI,1}$ (*)

$y_3 e^{-t} = (k_1 y_0 + k_2 y_1) e^{-t} = k_1 \underbrace{(y_0 e^{-t})}_{y_{ZI,0}} + k_2 \underbrace{(y_1 e^{-t})}_{y_{ZI,1}}$

(*) Z-I linear :)

• Example #11: Linearity

- Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = y_0^2 + f^2(t)$$

NO → z-S linear + Z-I linear ⇒ not linear ∴

- Determine if the system is linear or not

zero-state linear? ⇒ $y_0 = 0$ ⇒ $y_{zs}(t) = f^2(t)$

← not a line going through the origin ⇒ not z-s linear ∴

$$f_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = f_1^2(t)$$

$$f_2(t) \rightarrow \boxed{} \rightarrow y_2(t) = f_2^2(t)$$

$$f_3(t) \rightarrow \boxed{} \rightarrow y_3(t) \overset{?}{=} k_1 y_1 + k_2 y_2 \quad (*)$$

$$f_3 = k_1 f_1 + k_2 f_2$$

$$y_3(t) = f_3^2(t) = (k_1 f_1 + k_2 f_2)^2 \quad (*)$$

$\neq$ not z-S linear ∴

$y(t) = f(t)$

is the system zero-state linear?

Yes! line through the origin

- Time invariance: delayed inputs cause equally delayed outputs for zero initial state and any delay t_d. So,

if

$$f_1(t) \rightarrow \boxed{\text{time-invariant}} \rightarrow y_1(t)$$

then

$$f_2(t) = f_1(t - t_d) \rightarrow \boxed{\text{time-invariant}} \rightarrow y_2(t) = y_1(t - t_d)$$

- Example #12: Time-invariance
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = \cancel{y_0^2} + 2f(t)$$

0

  - Determine if the system is time-invariant or not

zero-state $\Rightarrow y_0 = 0 \Rightarrow y_{zs}(t) = 2f(t)$

$f_1(t) \rightarrow \square \rightarrow y_1(t) = 2f_1(t)$

$f_2(t) \rightarrow \square \rightarrow y_2(t) \stackrel{?}{=} y_1(t-t_d)$

$\| $

$f_1(t-t_d)$

$\| \; 2f_2(t) = 2f_1(t-t_d)$ ✓ same ☺ $\Rightarrow$ T.I. system ☺

$y_1(t-t_d) = 2f_1(t-t_d)$ ✓

- Example #13: Time-invariance
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = \underbrace{\cancel{y_0^2}}_{0} + f(t^2)$$

  - Determine if the system is time-invariant or not

zero-state $\Rightarrow y_0 = 0 \Rightarrow y_{zs}(t) = f(t^2)$

$f_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = f_1(t^2)$

$f_2(t) \rightarrow \boxed{} \rightarrow y_2(t) \overset{?}{=} y_1(t-3)$

$\parallel$

$f_1(t-t_d) =$

$= f_1(t-3)$

td = 3s - used for simplisity only, has to work for any td