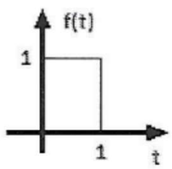
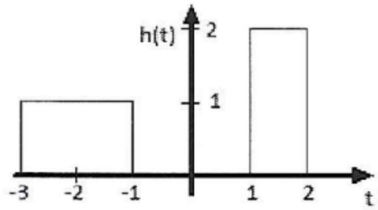
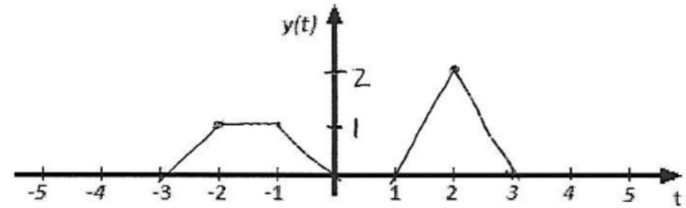
3. (25 pts) The two parts of this problem are unrelated.

(a) Consider the signal $y(t) = f(t) * h(t)$, where $f(t)$ and $h(t)$ are plotted below. Plot $y(t)$ for $-5 < t < 5$ and fill in the table for the value of $y(t)$.

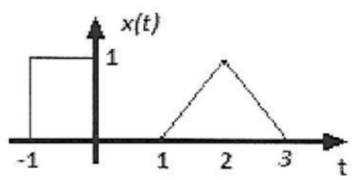


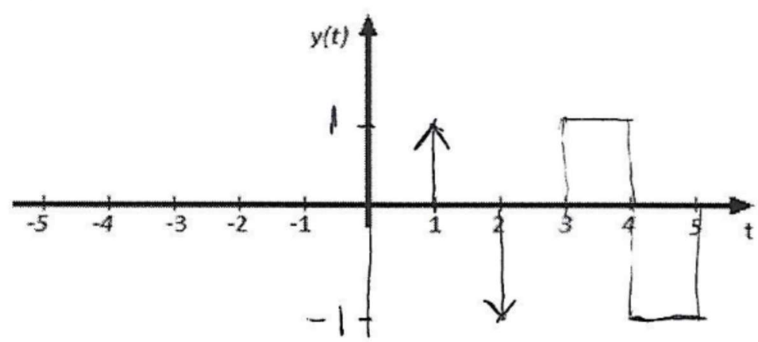




t	-5	-4	-3	-2	-1	0	1	2	3	4	5
y(t)	0	0	0	1	1	0	0	2	0	0	0

(b) Consider the signal $x(t) = m(t) * h(t)$ plotted below.
Plot the signal $y(t) = \left(\frac{d}{dt}m(t)\right) * h(t-2)$.





Problem 3 (25 points)

In each of the following subsections, **find and sketch** $y(t) = h(t) * f(t)$. Be sure that your sketch clearly shows the values of $y(t)$ at every time.

a. **(8 points)** Sketch $y(t) = f(t) * h(t)$ for

$$f(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}, \quad h(t) = \begin{cases} 1 & 3 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Solution: Sketch should show that $y(t) = 0$ for $t \le 3$, then rises linearly to a peak at $y(4) = 1$, then falls linearly to $y(5) = 0$, and is zero thereafter.

b. **(8 points)** Sketch $y(t) = f(t) * h(t)$ for

$$f(t) = \text{rect}\left(\frac{t}{2}\right), \quad h(t) = \delta(t-1) + \delta(t-2)$$

Solution: Sketch should show that $y(t) = 1$ for $0 < t < 1$ and for $2 < t < 3$, $y(t) = 2$ for $1 < t < 2$, and $y(t) = 0$ elsewhere.

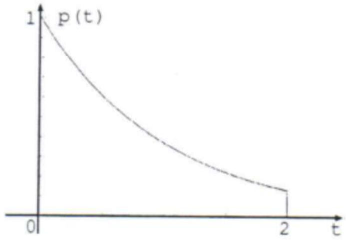
c. **(9 points)** Find and sketch $y(t) = f(t) * h(t)$ for

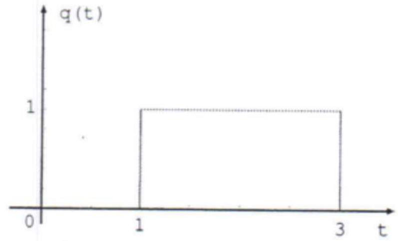
$$f(t) = \text{sinc}\,(0.1\pi t)\,, \quad h(t) = \text{sinc}\,(0.2\pi t)$$

Solution: $y(t) = 5\text{sinc}(0.1\pi t)$. Sketch should show that $y(0) = 5$, and that $y(t) = 0$ at non-zero integer multiples of $t = 10$.

3. (25 pts) The two parts in this problem are unrelated.

(a) Given $p(t) = e^{-t} rect(\frac{t-1}{2})$ and $q(t) = rect(\frac{t-2}{2})$, determine the following convolutions.





10 pts

i. $y(t) = p(t) * q(t)$

for $t-1<0$, $y(t)=0$

for $0 \le t-1 < 2$, $y(t) = \int_0^{t-1} e^{-\tau} d\tau = 1 - e^{-(t-1)}$

for $2 \le t-1 < 4$, $y(t) = \int_{t-3}^{2} e^{-\tau} d\tau = e^{-(t-3)} - e^{-2}$

for $t-1 \ge 4$ $y(t)=0$.

so, $$y(t) = \begin{cases} 1-e^{-(t-1)} & \text{for } 1 \le t < 3 \\ e^{-(t-3)} - e^{-2} & \text{for } 3 \le t < 5 \\ 0 & \text{otherwise.} \end{cases}$$

$y(t) =$ $\begin{cases} 1-e^{-(t-1)} & \text{for } 1 \le t < 3 \\ e^{-(t-3)} - e^{-2} & \text{for } 3 \le t < 5 \\ 0 & \text{otherwise.} \end{cases}$

5 pts

ii. $z(t) = \frac{dp(t)}{dt} * q(t)$

$$\begin{aligned} z(t) &= p(t) * \frac{dq(t)}{dt} \\ &= p(t) * (\delta(t-1) - \delta(t-3)) \\ &= p(t-1) - p(t-3) \\ &= e^{-(t-1)} rect(\tfrac{t-2}{2}) - e^{-(t-3)} rect(\tfrac{t-4}{2}) \end{aligned}$$

$z(t) =$ $e^{-(t-1)} rect(\frac{t-2}{2}) - e^{-(t-3)} rect(\frac{t-4}{2})$

(b) For a LTI system, if input $f_0(t) = u(t)$ produces output $y_0(t) = rect(\frac{t-1}{2})$, determine the output $y(t)$ to the input $f(t) = rect(t)$.

10 pts

$f_0(t) = u(t)$

$y_0(t) = h(t) * u(t)$

$= rect\left(\frac{t-1}{2}\right)$

$= u(t) - u(t-2)$

$= \left(\delta(t) - \delta(t-2)\right) * u(t)$

So, $h(t) = \delta(t) - \delta(t-2)$

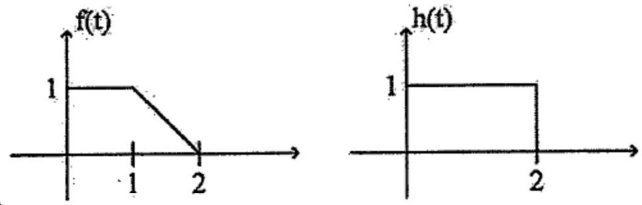
$y(t) = h(t) * f(t) = (\delta(t) - \delta(t-2)) * rect(t)$

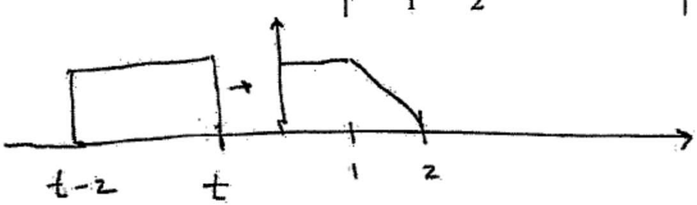
$= rect(t) - rect(t-2)$

$y(t) =$ $rect(t) - rect(t-2)$

Problem 3

(a) For h(t) and f(t) shown below, compute the specified values for $y(t) = f(t) * h(t)$





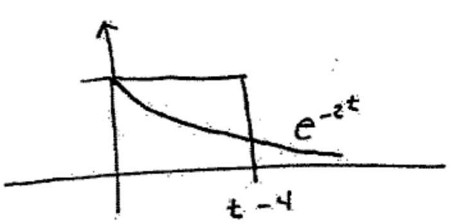
$y(-0.5) =$ 0 $\qquad$ $y(0.5) =$ $1/2$

$y(1.5) =$ $11/8$ $\qquad$ $y(2.5) =$ 1

$y(3.5) =$ $1/8$ $\qquad$ $y(4.5) =$ 0

(b) Let $h(t) = e^{-2t}u(t)$ and $f(t) = u(t-4)$. Find $y(t) = f(t) * h(t)$ for all values of t.

$$y(t) = \begin{cases} 0 & t < 4 \\ \frac{1}{2}\left(1 - e^{8-2t}\right) & t \geq 4 \end{cases}$$



for $t \geq 4$: $\displaystyle\int_0^{t-4} e^{-2\tau}\,d\tau = -\frac{1}{2}e^{-2\tau}\Big|_0^{t-4} = -\frac{1}{2}\left(e^{-2t+8} - 1\right)$