Lecture 27, Tuesday, October 12, 2021

- Fourier series

  - If $f(t)$ is periodic with fundamental frequency ω_0, then we can express it as

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

    F_n is complex BUT independent of time

    * Distinct functions with the same ω_0 will have distinct sets of F_n
    * How to get the coefficients F_n?
      · Use $\left\{e^{jn\omega_0 t}\right\}_{n=-\infty}^{\infty}$ as an orthonormal basis set to project onto

$$F_n = \left\langle f(t), e^{jn\omega_0 t}\right\rangle = \frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}dt$$

$$\left\langle e^{jn\omega_0 t}, e^{jm\omega_0 t}\right\rangle = \begin{cases} 1 & n = m \\ 0 & \text{else.} \end{cases} \rightarrow \quad \text{orthonormal basis}$$

  - For the existence of the Fourier series representation of a periodic signal $f(t)$, we need
    * Absolute integrability of $f(t)$:

$$\int_T |f(t)|dt < \infty$$

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- Plottability:
  - finite number of maxima and minima over one period
  - finite number of discontinuities over one period

– At discontinuities, the series converges to the midpoint

– Fourier series forms:

- Exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

where $F_n \in \mathbb{C}$

- Trigonometric Fourier series:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right)$$

where $a_n,\ b_n \in \mathbb{C}$

$$a_0 = 2F_0 \qquad a_n = F_n + F_{-n} \qquad b_n = j\left(F_n - F_{-n}\right)$$

$$F_0 = \frac{a_0}{2} \qquad F_n = \frac{a_n - jb_n}{2} \qquad F_{-n} = \frac{a_n + jb_n}{2}$$

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* Compact Fourier series, for real-valued functions only:

$$f(t) \;=\; \frac{c_0}{2} \;+\; \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

where $c_n,\ \theta_n \in \mathbb{R}$

$$c_0 = 2F_0 \qquad\qquad c_n = 2|F_n| \qquad\qquad \theta_n = \angle F_n$$

$$F_0 = \frac{c_0}{2} \qquad\qquad F_n = \frac{c_n}{2}e^{j\theta_n} = F^*_{-n}$$