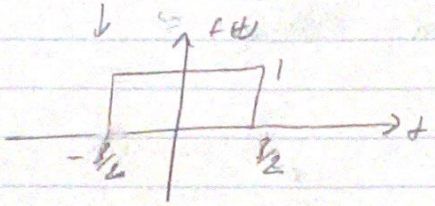
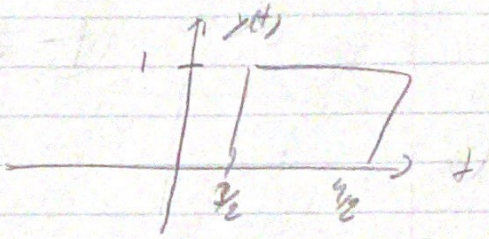
Vareya Jain - vareya3 - 655479542 - 04/18/2022 - ECE 210 - HW 12

1. Vareya Jain

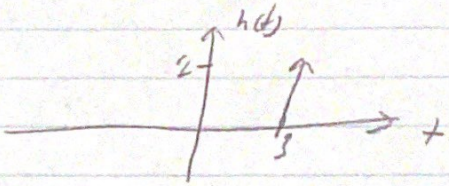
2. impulse response $h(t)$, system output, $y(t) = h(t) * f(t)$
$= \text{rect}(t - 3/2)$. Sketch $h(t)$

a) $f(t) = \text{rect}\left(\frac{t}{3}\right)$

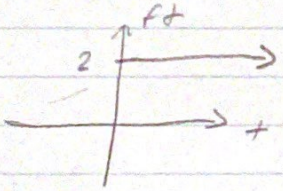


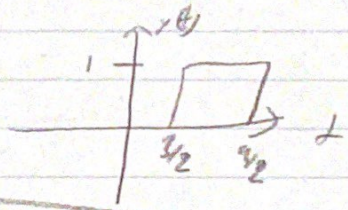


$\hookrightarrow$ $\boxed{h(t) = \delta(t-3)}$

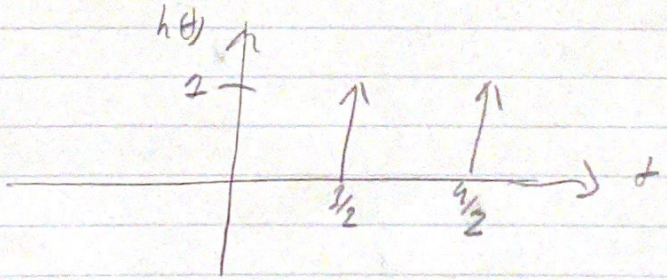


b) $f(t) = 2u(t)$



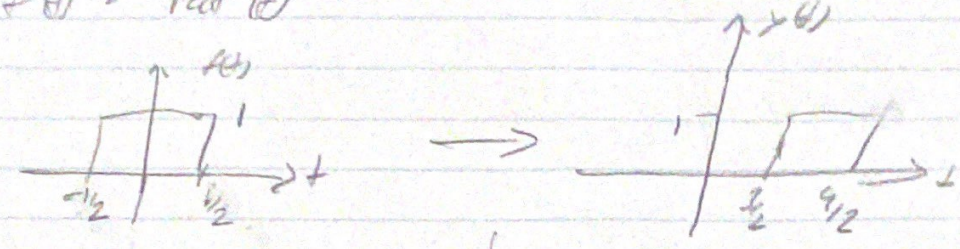


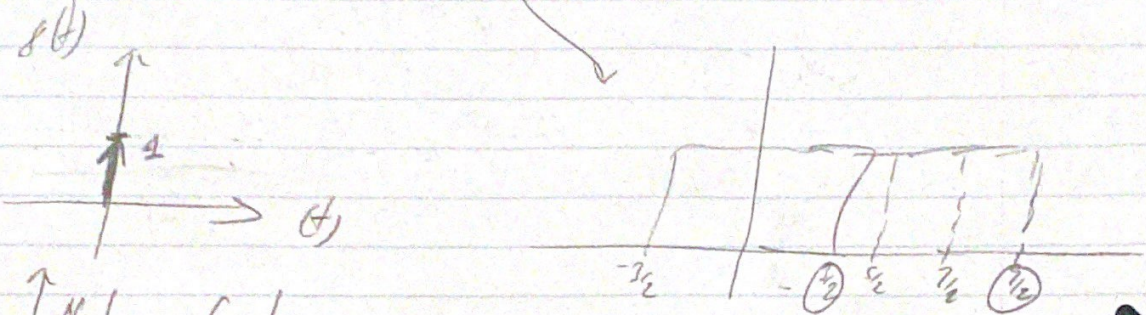
$\boxed{h(t) = \frac{1}{2}\left[\delta(t-\frac{3}{2}) + \delta(t-\frac{9}{2})\right]}$



(2)

c) $f(t) = \text{rect}(t)$





↑ Not a function
↳ cannot time-shift

$$h(t) = \delta(t-2) + \delta(t-3) + \delta(t-4)$$

3. fourier transforms

a) $f(t) = 5\cos(3t) + 3\sin(15t)$

Table 4, #18, 19

$\cos(\omega_0 t) \equiv \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

$\sin(\omega_0 t) \equiv j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$

$F(\omega) = 5\pi[\delta(\omega-3) + \delta(\omega+3)] + 3j\pi[\delta(\omega+15) - \delta(\omega-15)]$

b) $x(t) = \cos^2(6t)$

$\star\ \cos(2\alpha) = 2\cos^2\alpha - 1 \rightarrow \cos(12t) = 2\cos^2(6t) - 1$

$x(t) = \frac{1}{2}(\cos(12t) + 1)$

$x(t) = \frac{1}{2}\cos(12t) + \frac{1}{2}$

Table 4 #15: $1 \leftrightarrow 2\pi\delta(\omega)$

#14: $\delta(t) \leftrightarrow 1$

$X(\omega) = \frac{1}{2}[\pi(\delta(\omega-12) + \delta(\omega+12)) + 2\pi\delta(\omega)]$

c) $y(t) = e^{-2t}u(t) * \cos(2t)$

Table 4, # 1

$e^{-at}\cdot u(t) \equiv \frac{1}{a+j\omega}$, $a>0$

$\pi[\delta(\omega-2) + \delta(\omega+2)]$

$\delta(\omega-2)$ $\quad \delta(\omega+2) \Rightarrow \delta(\omega-(-2))$

$\frac{1}{2+j\omega} \rightarrow \frac{1}{2+j\omega} + \frac{1}{2-j\omega}$

$\hookrightarrow Y(\omega) = \pi\left[\frac{\delta(\omega-2)}{2+j\omega} + \frac{\delta(\omega+2)}{2-j\omega}\right]$

(4)

d) $z(t) = (1 + \cos(3t))\, e^{-t} u(t)$

$= e^{-t} u(t) + (e^{-t} u(t) \cdot \cos(3t))$

$\mathcal{L}\left\{ \frac{1}{1+jw} + \frac{1+jw}{(1+jw)^2 + 9} \right\}$

Table 4, #12

$e^{-at} \cdot \cos(\omega_0 t) \cdot u(t)$

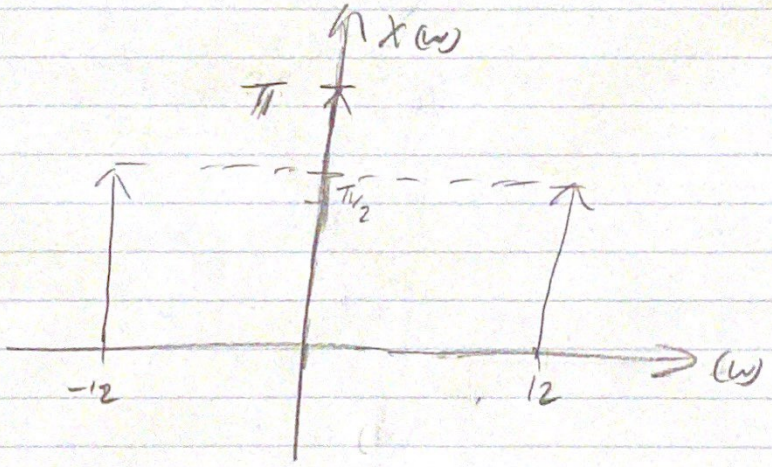
$\mathcal{L} \frac{a+jw}{(a+jw)^2 + \omega_0^2}, \quad a > 0$

$= \frac{1}{1+jw} + \frac{1+jw}{1+jw^2+9} = Z(w)$

$= \frac{10+jw + (1+jw)^2}{(1+jw)(10+jw)} = \frac{10+jw+1+2jw-w^2}{10+11jw-w^2}$

$= \frac{11+3jw-w^2}{10+11jw-w^2}$

3d)



a, c, and d are not wholly real

4. minimum sampling frequencies in kHz needed to sample analog signals w/h aliasing error

a) $f_s \geq 2\Omega \rightarrow f \geq 2(40\,kHz)$

↳ $\boxed{f_{s\,min} = 80\,kHz}$

b) $f(t) = sinc(40\pi t)$

↳ $\frac{2(\omega_0\ rad/s)}{2\pi\ rad/s} \geq f_s$

$f_s = \frac{2(40\pi)}{2\pi} \rightarrow \boxed{f_s = 40\,Hz}$

c) $g(t) = sinc(40\pi t)\cos(200\pi t)$

↳ $f_s = \frac{2(40\pi + 200\pi)}{2\pi} \rightarrow f_s = 0.24 \times 1000$

↳ $\boxed{f_s = 0.24\,kHz}$

(4)

5. Impulse response $h(t)$

a) $H(\omega) = \frac{1}{2+j\omega}$ →

Table 4, #1
$e^{-at}u(t) \equiv \frac{1}{a+j\omega}$, $a>0$

a) → $h(t) = e^{-2t}\cdot u(t)$

b) $H(\omega) = \frac{1}{(4+j\omega)^2}$

Table 4, #5
$te^{-at}u(t) \equiv \frac{1}{(a+j\omega)^2}$, $a>0$

b) → $h(t) = te^{-4t}u(t)$

c) $H(\omega) = \frac{j\omega}{2+j\omega}$

$= \frac{2+j\omega-2}{2+j\omega} = \frac{2+j\omega}{2+j\omega} - \frac{2}{2+j\omega}$

$= 1 - \frac{2}{2+j\omega} = \delta(t) - 2e^{-2t}\cdot u(t)$

Table 4, #14 #15
$\delta(t) \Leftrightarrow 1$
$1 \leftrightarrow 2\pi\delta(\omega)$

c) → $h(t) = \delta(t) - (2e^{-2t}\cdot u(t))$

d) $H(\omega) = \frac{1}{2+j\omega}e^{-2j\omega}$

Table 4, #1
$e^{-at}u(t) \Leftrightarrow \frac{1}{a+j\omega}$, $a>0$
$a = 2$

Table 4, #16
$\delta(t-t_0) \Leftrightarrow e^{-j\omega t_0}$
$t_0 = 2$

$= e^{-2t}u(t)\cdot\delta(t-2) \Rightarrow e^{-2(t-2)}\cdot u(t-2) = e^{4-2t}\cdot u(t-2)$

d) → $h(t) = e^{4-2t}\cdot u(t-2)$