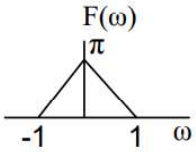
Problem 2

An f(t) signal is given by its Fourier Transform $F(\omega)$.



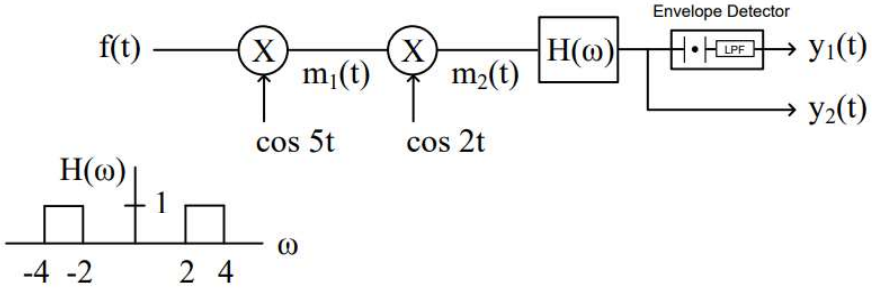
(a) Find its energy ε_f.

$\varepsilon_f =$ _____

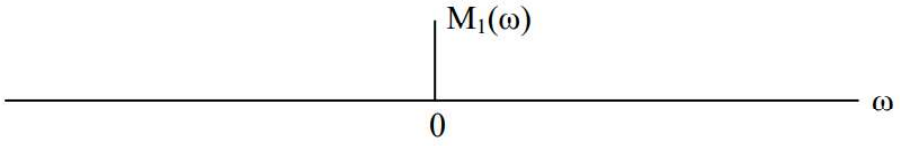
(b) Find its 3dB bandwidth.

$\Omega_{3dB} =$ _____

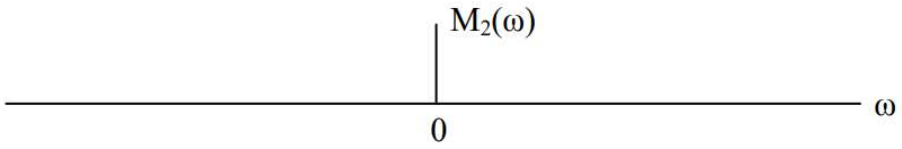
(c) This f(t) signal goes through the following system



i) Sketch $M_1(\omega)$. (Label axes carefully)



ii) Sketch $M_2(\omega)$. (Label axes carefully)



iii) Find the signals $y_1(t)$ and $y_2(t)$.

$y_1(t) =$ _____ $y_2(t) =$ _____

2. (25 pts) The radio station WILL transmits with a carrier frequency of 580kHz. For transmission, suppose that a real valued signal $f(t)$ with a bandwidth of 20kHz is used to modulate a carrier cosine wave with frequency 580kHz. The resulting signal is $g(t) = f(t)\cos(\omega_c t)$, with $\omega_c = 2\pi(580 \times 10^3)$ rad/s.

   (a) What is the largest possible positive frequency component in the modulated signal $g(t)$?

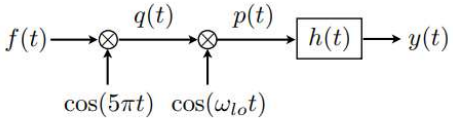
   (b) Consider the signal $m(t) = g(t)\cos(\omega_c t)$, which is passed through a filter with frequency response $H(\omega) = 1$ for $|\frac{\omega}{2\pi}| \leq 20 \times 10^3$, and zero elsewhere, to give a signal $y(t) = m(t) * h(t)$. What percentage of the energy of $f(t)$ is present in the output $y(t)$? Why?

   (c) Give an expression for the Fourier transform of the delayed signal $g_1(t) = g(t - t_0)$. You should express $G_1(\omega)$ in terms of $F(\omega)$, the Fourier transform of $f(t)$.

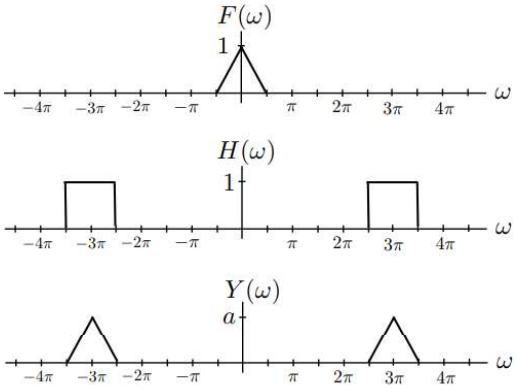
(d) On a certain day, a technical problem at WILL produces a signal $f(t)$ with Fourier transform given by $F(\omega) = 1$ for $|\frac{\omega}{2\pi}| \leq 1000$, and zero else. Sketch the Fourier transform for $G(\omega)$ on this day.

(e) Suppose that WILL modifies its signal to be $g_2(t) = [f(t) + \alpha]\cos(\omega_c t)$, with $\alpha > |f(t)|$. If a heterodyne AM receiver has intermediate frequency (IF) of 455kHz, give a value for ω_{LO}, the frequency of the local oscillator that will allow reception of the WILL broadcast.

3. (20 pts) Consider the system



where the Fourier transforms of $f(t)$, $h(t)$, and $y(t)$ are as follows:



(a) (4 pts) What is ω_{lo}? $\omega_{lo} =$ _____

(b) (4 pts) Is your answer to part (a) unique? If so, briefly explain. If not, give another possible value for ω_{lo}.

(c) (4 pts) Plot $P(\omega)$, the Fourier transform of $p(t)$, below. Be sure to mark all relevant locations on the axes.

(d) (4 pts) What is $Y(\omega)$ in terms of $F(\omega)$? You do not need to write the expression for $F(\omega)$ explicitly but your answer should specify the maximum value of $Y(\omega)$, which is denoted as a in the figure.

$Y(\omega) =$ _____

(e) (4 pts) What is $y(t)$ in terms of $f(t)$? $y(t) =$ _____