- Example #13: Time-invariance
  - Consider a system with input f(t), initial state y(0) = y_0 and input-output rule given by

$$y(t) = \cancel{y_0^2} + f(t^2)$$

0

  - Determine if the system is time-invariant or not

zero-state ⇒ $y_0 = 0$ ⇒ $y_{zs}(t) = f(t^2)$

$f_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = f_1(t^2)$

$f_2(t) \rightarrow \boxed{} \rightarrow y_2(t) \overset{?}{=} y_1(t-3)$ td

$f_2(t) = f_1(t-t_d) = f_1(t-3)$ td

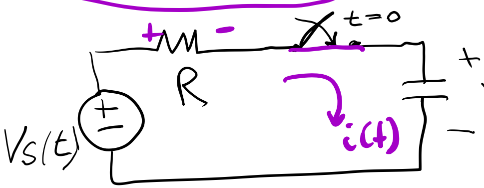
$f_2(t^2) = f_1(t^2-3)$ ✓ td

$y_1(t-3) = f_1((t-3)^2)$ ✓ td

✗ not T.I. :(

$t_d = 3s$ - used for simplisity only, has to work for any td

- First-order RC and RL circuits



$v_c(0^-) = v_c(0) = v_c(0^+)$ - I.C.

$t>0$

KVL: $-v_s(t) + i(t)\cdot R + v_c(t) = 0$

$i(t) = C\frac{dv_c}{dt}$

$RC\frac{dv_c}{dt} + v_c(t) = v_s(t) \quad | : RC$

$$\boxed{\frac{d\overset{y}{v_c}}{dt} + \underset{a}{\frac{1}{RC}} v_c(t) = \underbrace{\frac{v_s(t)}{RC}}_{f(t)}}$$

First-order linear ordinary differential equation (ODE) with constant coefficients, which governs RC circuit for $t>0$

In general:

$f(t) = \frac{dy}{dt} + ay(t)$

- First-order ODE with constant coefficients

$$f(t) = \frac{dy}{dt} + ay(t)$$

- How to solve?
  - Start with a simple case: constant input

$$f(t) = K \quad \Rightarrow \quad K = \frac{dy}{dt} + ay(t)$$

$y(t) = B$ - constant

$K = 0 + a \cdot B \Rightarrow B = \frac{K}{a}$

$y_p(t) = \frac{K}{a}$

particular solution

- First-order ODE with constant coefficients-cont

$$K = \frac{dy}{dt} + ay(t)$$

$$y(t) = \underbrace{\frac{K}{a}}_{y_p(t)} + y_h(t)$$

homogeneous solution

$$K = \frac{d}{dt}\left(\cancel{\frac{K}{a}} + y_h\right) + a\left(\frac{K}{a} + y_h\right)$$

0

$$\cancel{K} = \frac{dy_h}{dt} + \cancel{K} + ay_h$$

$$\frac{dy_h}{dt} + ay_h = 0$$

$$y_h = Ae^{-at}$$

$$-a \cdot Ae^{-at} + a \cdot Ae^{-at} = 0$$

$$= \frac{K}{a} + \overset{?}{A}\underbrace{e^{-at}}_{1 \text{ when } t=0}$$

To get A:

$$y(0^-) = y(0^+) = \frac{K}{a} + A$$

if continuous

$$\Downarrow$$

$$A = y(0^+) - \frac{K}{a}$$

$$y(t) = \frac{K}{a} + \left(y(0^+) - \frac{K}{a}\right)e^{-at}$$

- First-order ODE with constant coefficients-cont

$$K = \frac{dy}{dt} + ay(t)$$

$$y(t) = \frac{K}{a} + y_h(t)$$

- First-order ODE with constant coefficients-cont

$$K = \frac{dy}{dt} + ay(t)$$

- For t > 0

$$y(t) = y_p(t) + y_h(t) = B + Ae^{-at} = \underbrace{\overbrace{\frac{K}{a}}^{y_p} + \overbrace{\left(y(0^+) - \frac{K}{a}\right) e^{-at}}^{y_h}}$$

- Solution to RC circuit with constant source
  - Recall

$$\underbrace{K = \frac{dy}{dt} + ay(t)}$$

$$\underbrace{\overset{K}{\frac{V_s}{RC}} = \frac{dV_c}{dt} + \overset{a}{\frac{1}{RC}} V_c(t)}$$

for $\underline{t>0}$ $y(t) = \underbrace{\frac{K}{a}}_{y_p} + \underbrace{\left(y(0^+) - \frac{K}{a}\right) e^{-at}}_{y_h}$

$y_p = \frac{K}{a} = \frac{V_s}{RC} \cdot RC = V_s$ ← constant source

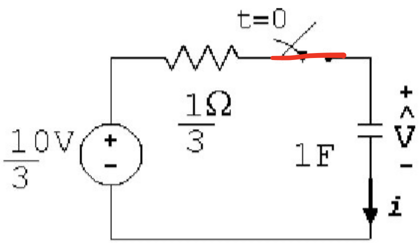
For a capacitor:

$y(0^+) = y(0^-) = v_c(0^-)$

$y_h = \left(y(0^+) - \frac{K}{a}\right) e^{-at} = \left(v_c(0^-) - V_s\right) e^{-\frac{t}{RC}}$

$$\boxed{v_c(t) = \left(v_c(0^-) - V_s\right) e^{-\frac{t}{RC}} + V_s}$$ for $\underline{t>0}$

- Example #14:
  - Consider the following circuit with $V_c(0^-) = 1V$.
  - Determine $V_c(t)$ and $i_c(t)$.



for $t>0$:

① $\dfrac{dv_c}{dt} + \dfrac{1}{RC} v_c(t) = \dfrac{V_s}{RC}$ $a=3$ $=10$

② $\dfrac{dv_c}{dt} + 3v_c(t) = 10$

③ $y(t) = y_p + y_h = B + Ae^{-at}$

$0 + 3B = 10 \Rightarrow B = \frac{10}{3}$

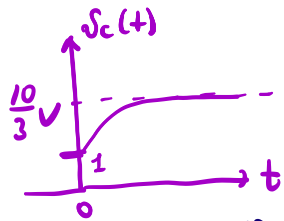
④ $v_c(t) = \frac{10}{3} + \overset{?}{A}\,\underbrace{e^{-3t}}_{1 \text{ if } t=0}$

⑤ Find A:

$v_c(0^-) = v_c(0^+) = 1 = \frac{10}{3} + A \Rightarrow A = 1 - \frac{10}{3} = -\frac{7}{3}$

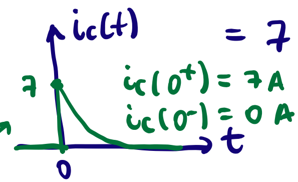
⑥ $v_c(t) = \frac{10}{3} \underbrace{- \frac{7}{3} e^{-3t}}_{\to 0 \text{ when } t \to \infty}$ (V)

units, please :)



$v_c(0^+) = \frac{10}{3} - \frac{7}{3} = 1 = v_c(0^-)$

$i_c(t) - ?$ $i_c(t) = C\dfrac{dv_c}{dt} = 7e^{-3t}$ A

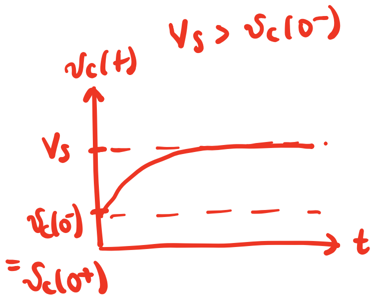


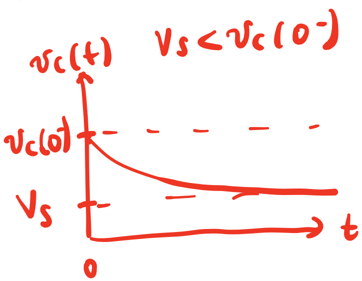
can jump!

- Time-constant
  - Recall

$$y(t) = \frac{K}{a} + \left(y(0^+) - \frac{K}{a}\right) e^{-at}$$

Rate of decay is controlled by a time constant $\boxed{\tau = RC}$





$$v_c(t) = \underbrace{\left(v_c(0^-) - V_s\right) e^{-\frac{t}{RC}}}_{\text{transient response}} + \underbrace{V_s}_{\text{steady-state response}}$$

τ

transient response (part which goes to 0 when $t \to \infty$)

steady-state response (which is left after transient response has vanished)