ECE210 Laboratory
PRE-LAB#3

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Lab section:

ABH

Lab 3: Frequency Response and Fourier Series

In this lab you will build an active bandpass filter circuit with two capacitors and an op-amp, and examine the response of the circuit to periodic inputs over a range of frequencies. The same circuit will be used in Lab 4 in your AM radio receiver system as an intermediate frequency (IF) filter, but in this lab our main focus will be on the frequency response $H(\omega)$ of the filter circuit and the Fourier series of its periodic input and output signals. In particular we want to examine and gain experience about the response of linear time-invariant circuits to periodic inputs.

1 Prelab

1. Determine the compact-form Fourier series of the periodic square wave signal, $f(t)$ shown in Figure 1, with a period T and amplitude A. That is, find c_n and θ_n such that

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n), \text{ where } \omega_o = \frac{2\pi}{T}.$$

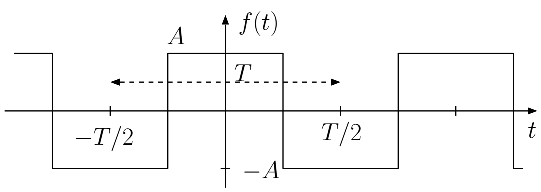


Figure 1: Square wave signal for prelab.

Notice $\frac{c_0}{2} = 0$. How could you have determined that without any calculation? (____/2)

Co/2 is the average of the function, the DC term, Because the average is zero C0 is zero, so Average = (area)/ (period length)

(____/11)

Show your work.(____/3)

Fn = 1/T Integral{ f(t)*e^(-jnt*w0) }dt

A/(T*jnw0) * [e^(jn(2pi/T)*(T/4)) - e^(jn(2pi/T)*(T/2)) - e^(-jn(2pi/T)*(T/4)) + e^(-jn(2pi/T)*(T/2)) - e^(-jn(2pi/T)*(T/4))]

Fn = A/(2jn*pi) [2e^(jnpi/2) - 2e^(ijnpi/2) - e^(jnpi) + e^(-jnpi)]

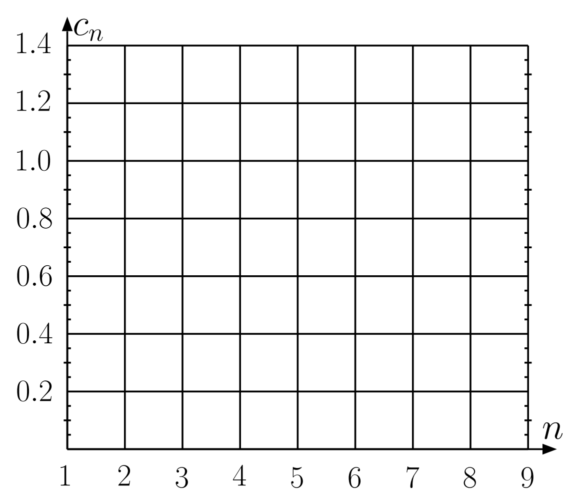
(2A/(pi*n)) * cos((n*pi/2) - (pi/2))

Cn = 2|Fn| = 2 * (2A/(n*pi)) = 4A/(n*pi)
Theta = n*pi/2 - pi/2 = {0, n =1,5,9,13...}, {pi, n= 3,7,11,15}

$F_n =$ (2A/(n*pi)) * sin(n*pi/2) (____/2)

$c_n =$ = 4A/(n*pi) when n is odd, = n when is even (____/2)

$\theta_n =$ {0, n = 1,5,9,13} , {pi, n = 3,7,11,15..} (____/2)



With $A = 1$, plot c_n over n ($n \in [1, 9]$) (____/2)

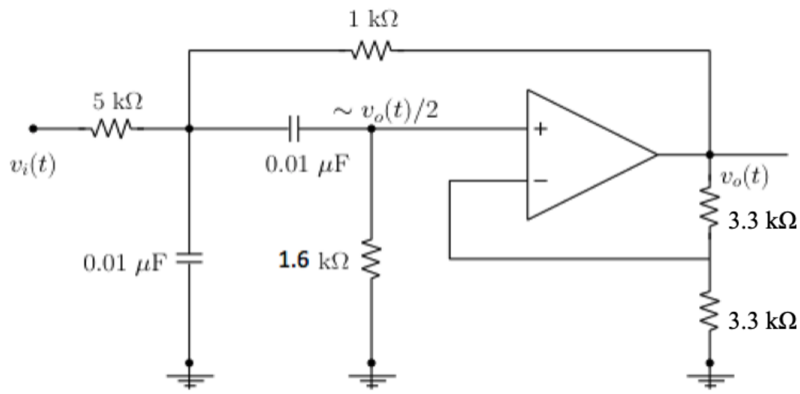


Figure 2: Circuit for analysis in prelab and lab.

2. Consider the circuit in Figure 2 where $v_i(t)$ is a co-sinusoidal input with some radian frequency ω.

   (a) What is the phasor gain $\frac{V_o}{V_i}$ in the circuit as $\omega \to 0$? (Hint: How does one model a capacitor at DC — open or short?)

   Show your work

   If there is a DC current due to w -> 0, the phasor gain will be zero. This is caused by the capacitors acting as open circuits and input to the op amp being ground. Therefore, making a KVL loop at the 3.3kOhm resistor results in v0 = ground voltage, regardless of the amplitude of the high-frequency input waveform.

   (____/3)

   (b) What is the gain $\frac{V_o}{V_i}$ as $\omega \to \infty$? (Hint: think of capacitor behavior in $\omega \to \infty$ limit)

   Show your work

   If w -> infinity, there will be a phasor gain of zero. This is due to the capacitors acting as short circuits and the fact that the op-amp input is ground. Therefore, making a KVL loop at the 3.3kOhm resistor results in v0 = ground voltage, regardless of the amplitude of the high-frequency input waveform.

   (____/3)

   (c) In view of the answers to part (a) and (b), and the fact that the circuit is 2nd order (it contains two energy storage elements), try to guess what kind of a filter the system frequency response $H(\omega) \equiv \frac{V_o}{V_i}$ implements — lowpass, highpass, or bandpass? The amplitude response $|H(\omega)|$ of the circuit will be measured in the lab.

   (____/2)

   Give your answer and explain your reasoning.

   Since both low and high frequencies are obstructed, this circuit is probably a band-pass filter.

3. Decibels (dB) is a unit of measurement widely used in science and engineering to compare power or intensity quantities. A decibel (dB) is one-tenth of a Bel (B), which is the name given to $\log_{10}\left(\frac{P_1}{P_0}\right)$, where $\log_{10}$ is the base 10 logarithm, and $\frac{P_1}{P_0}$ is the ratio of two power quantities. The formula for calculating decibels is : $10\log_{10}\left(\frac{P_1}{P_0}\right)$. and for comparing voltages we can use: $20\log_{10}\left(\frac{V_1}{V_0}\right)$, which is derived from $10\log_{10}\left(\frac{V_1^2/R}{V_0^2/R}\right)$. Complete the following table of useful ratios: (use accuracy of 1 decimal in dB row)

P_1/P_0	1000	100	10	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
V_1/V_0	$\sqrt{1000}$	10	$\sqrt{10}$	2sqrt(2)	2	$\sqrt{2}$	1	1/2 * sqrt(2)	1/2	sqrt(1000) / 1000	1/10	$\frac{1}{\sqrt{1000}}$
Decibels (dB)	30	20	10	9	6	3	0	-3	-6	-10	-20	-30

(____/2)

4. In the case of the Fourier analysis, the oscilloscope compares the signals with a reference of $V_{\text{rms}} = 1\,\text{V}$. Recall that a sine wave with rms (root mean square) amplitude of $V_{\text{rms}} = 1\,\text{V}$ corresponds to a sine wave with a peak amplitude of $\sqrt{2}\,\text{V}$, which is to say a peak-to-peak amplitude of $2\sqrt{2}\,\text{V}$. To specify that the reference is $V_{\text{rms}} = 1\,\text{V}$, the decibel symbol is modified with the suffix "V" becoming "dBV". Convert the voltages in the table below to dBV units (use accuracy of 1 decimal).

v	$V_{\text{rms}} =$ 1 V	$V_{\text{rms}} =$ 2 V	$V_{\text{rms}} = \sqrt{10}\,\text{V}$	$V_{\text{rms}} = \sqrt{2}\,\text{V}$	$V_{\text{rms}} =$ 1/10 V	$2\sqrt{2}$ Vpeak-to-peak	$4\sqrt{5}$ Vpeak-to-peak	2 Vpeak-to-peak	$\frac{2\sqrt{2}}{\sqrt{10}}$ Vpeak-to-peak
v (dBV)	0.0	6	10	3	-20	0	10	-3	-10

(____/2)