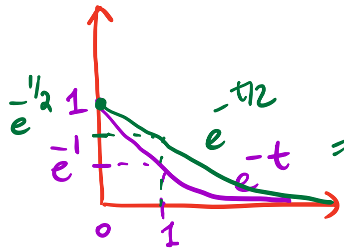
- Time-constant
  - Recall



$e^{-t/\tau} \Rightarrow \tau = 1s \quad e^{-t}$

$\tau = 2s \quad e^{-t/2}$

$\Rightarrow \tau \uparrow$, decays slower

Rate of decay is controlled by a time constant $\boxed{\tau = RC}$ [s]

$$y(t) = \frac{K}{a} + \left(y(0^+) - \frac{K}{a}\right) e^{-at}$$

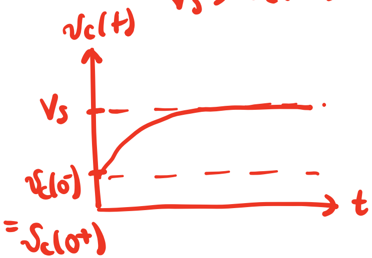
$$v_c(t) = \underbrace{\left(v_c(0^-) - V_s\right) e^{-\frac{t}{RC}}}_{\text{transient response}} + \underbrace{V_s}_{\text{steady-state response}}$$

$RC = \tau$

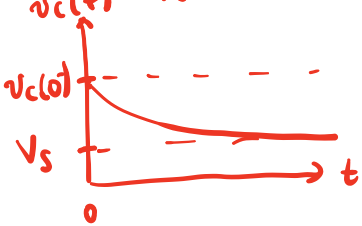
transient response (part which goes to 0 when $t \to \infty$)

steady-state response (which is left after transient response has vanished)

$V_s > v_c(0^-)$



$V_s < v_c(0^-)$



- Simple solution method (short cut) only works for 1^{st} order ODE with constant coefficients and constant inputs
  - Recall

$$\underbrace{y(t) = B + Ae^{-\overset{?}{a}t}}$$

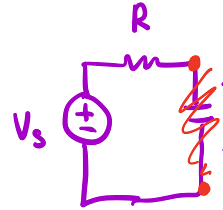
$$a = \frac{1}{RC} = \frac{1}{\tau}$$

$$v_c(t) = \overset{?}{B} + \overset{?}{A}e^{-t/\tau}$$

$$i_c = C\frac{dv_c}{dt}$$

$\Downarrow$

at the steady state $(t \to \infty)$ cap. acts as open-circuit $(i_c = 0)$



$v_c(\infty) = B = V_s$

Need to evaluate:

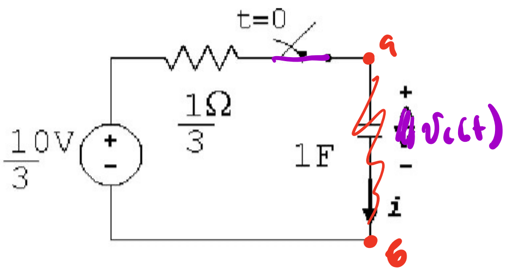
- $v_c(\infty) = B + A\underbrace{e^{-t/\tau}}_{0} = B = V_s$
- $v_c(0^+) = v_c(0^-) = B + A\underbrace{e^{-t/\tau}}_{1 \text{ if } t=0} = B + A$

$$A = v_c(0^-) - B = v_c(0^-) - V_s$$

$$\boxed{v_c(t) = V_s + (v_c(0^-) - V_s)\,e^{-t/\tau}}$$

same equation :)

- Example #14:
  - Consider the following circuit with $V_c\,(0^-) = 1V$. - I.C.
  - Determine $V_c(t)$ for $t>0$



$\underline{t>0}$

$v_c(t) = \overset{?}{B} + \overset{?}{A} e^{-t/\tau ?}$

$\tau = RC = \frac{1}{3}\, s$

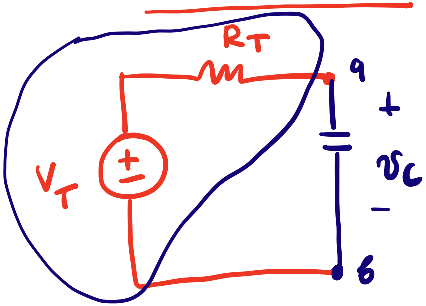
$v_c(\infty) = B = \frac{10}{3}\, V$

$v_c(0^-) = \overset{10/3}{B} + A \Rightarrow$

1 $\quad A = \left(1 - \frac{10}{3}\right) = -\frac{7}{3}$

$$\boxed{v_c(t) = \frac{10}{3} - \frac{7}{3} e^{-3t}}\ V$$

- General RC circuits
  - If a resistive circuit with a single capacitor:



τ Thevenin's eq. circuit as seen by the capacitor

$$v_c(t) = V_T + (v_c(0^-) - V_T)\, e^{-t/\tau}$$

$$\tau = R_T C$$

$$v_c(\infty) = B = V_T$$

$$v_c(0^-) = B + A$$

- Zero-input (ZI), zero-state (ZS)

$y(t) = y_{ZS}(t) + y_{ZI}(t)$

  - Recall

$$\frac{dy}{dt} + ay = \underset{\text{input}}{K}$$

$$y(t) = \underbrace{\frac{K}{a}}_{y_p} + \underbrace{\left(y(0^+) - \frac{K}{a}\right) e^{-at}}_{y_h} =$$

$$= \frac{K}{a} + y(0^+) e^{-at} - \frac{K}{a} e^{-at}$$

z-I

z-S response

$$y_{ZS}(t) = \frac{K}{a} - \frac{K}{a} e^{-at}$$

$$y_{ZI}(t) = y(0^+) e^{-at}$$

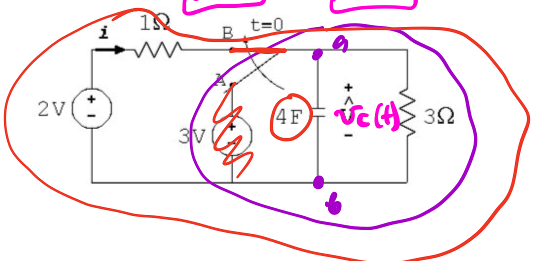
- Example #15:

- Consider the circuit below.
- Assume the switch has been in position A for a long time and it switches to position B at time t = 0

" we are in DC steady-state

- Determine $V_{ZS}(t)$ and $V_{ZI}(t)$

$v_c(t)$ for $t>0$, and



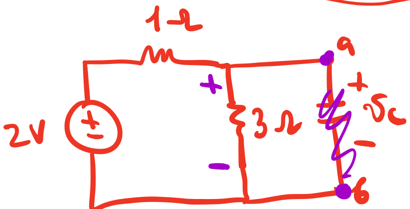
$t<0$

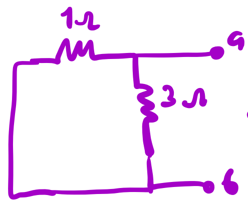
$v_c(0^-) = 3V$

$t>0$

$v_c(t) = B + A e^{-t/\tau}$? ? ?

Find τ: $\tau = R_T C = \frac{3}{4} \cdot 4 = 3\,s$





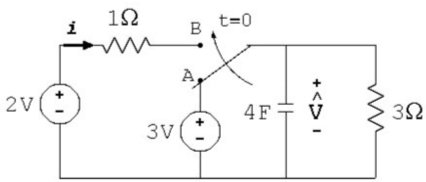
& $R_{eq} = R_T = \frac{1 \cdot 3}{1+3} = \frac{3}{4}\,\Omega$

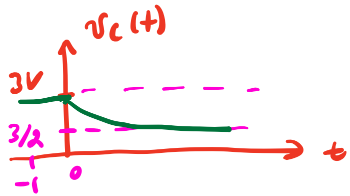
$v_c(\infty) = B = 2\left(\frac{3}{3+1}\right) = \frac{3}{2}$

$v_c(0^+) = v_c(0^-) = B + A \Rightarrow$

$A = \left(v_c(0^-) - \frac{3}{2}\right)$

- Example #15-cont:





$$v_c(t) = B + Ae^{-t/\tau} = \frac{3}{2} + \left(v_c(0^-) - \frac{3}{2}\right)e^{-t/3} = \frac{3}{2} + v_c(\underset{0}{\cancel{0^-}})e^{-t/3} - \frac{3}{2}e^{-t/3}$$

$$v_{zs}(t) = \frac{3}{2} - \frac{3}{2}e^{-t/3}\ V$$

$+$

$$v_{zi}(t) = v_c(0^-)e^{-t/3} = 3e^{-t/3}\ V \Rightarrow v_c(t) = \frac{3}{2} - \frac{3}{2}e^{-t/3} + 3e^{-t/3} =$$

$$= \frac{3}{2} + \underbrace{\frac{3}{2}e^{-t/3}}_{\text{"0 when } t\to\infty}\ V$$