Lecture 29, Wednesday, March 9. 2022

- Properties of Fourier series
  - Sometimes, we can determine the effect of transformations on signals by transforming the coefficients in a simple way:

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\, f(t) \leftrightarrow K\, F_n$
2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f\left(t - t_o\right) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\left(n\omega_o t\right)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin\left(n\omega_o t\right)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

Table 2: Fourier series properties

- LTI system response to periodic inputs

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = \sum_{n=-\infty}^{\infty} H(n\omega_0) F_n e^{jn\omega_0 t}$$

$$\Rightarrow \quad F_n \longrightarrow \boxed{H(\omega)} \longrightarrow Y_n = H(n\omega_0) F_n$$

  - Each frequency contribution, F_n, at frequency $n\omega_0$, gets multiplied by $H(n\omega_0)$.