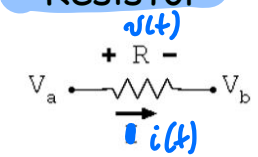
- V-I relationships in phasors

- Resistor



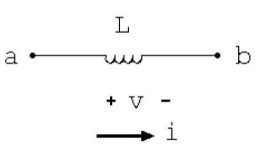
$v(t) = R \cdot i(t)$

↓ phasors

$\boxed{V = R \cdot I}$ Ohm's law

$|V| e^{j\theta_v} = R \cdot |I| e^{j\theta_i}$ ← (V and I are in-phase (V and I have the same angle))

- Inductor



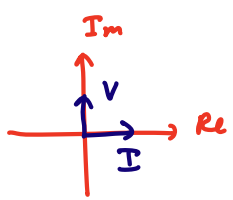
$v(t) = L \frac{di}{dt}$

↓ phasors

$V = L j\omega I = \underbrace{(j\omega L)}_{= Z \text{ - impedance } [\Omega]} I = V$

$\boxed{V = Z \cdot I}$

$|V| e^{j\theta_v} = \omega L |I| e^{j\theta_i} \cdot e^{j\pi/2}$

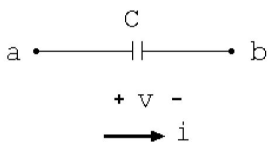


voltage leads (ahead of) current by $\pi/2$

or

current lags (behind) voltage by $\pi/2$

- Capacitor



$i = C\frac{dv}{dt}$

↓ phasors

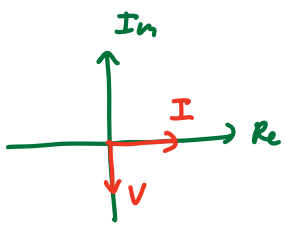
$I = C\,j\omega V \Rightarrow \underbrace{\frac{1}{j\omega C}}_{\parallel} I = V$

$z = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$\boxed{V = zI}$

$|V| e^{j\theta_v} = \frac{1}{\omega C} |I| e^{j\theta_i} \cdot e^{-j\pi/2}$

$\frac{1}{j} = -j$



Voltage lags current by $\pi/2$

or

current leads voltage by $\pi/2$

- V-I relationships in phasors – cont.

$$\text{Impedance, } Z = \begin{cases} R & \text{resistor} \\ j\omega L = X_L & \text{inductor} \\ \frac{1}{j\omega C} = \frac{-j}{\omega C} = X_C & \text{capacitor} \end{cases}$$

resistor ← purely resistive

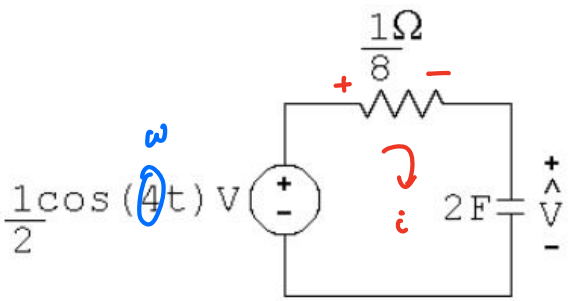
inductor, capacitor } purely reactive

More generally:

$$Z = \text{Re}\{Z\} + \text{Im}\{Z\} = R + jX$$

resistance ↙ reactance

- Example #4
- Determine $\hat{v}_{ss}(t)$ using phasors:



$\frac{d\hat{v}}{dt} \rightarrow j\omega\hat{V}$

$i_c = C\frac{d\hat{v}}{dt}$

KVL: $-\frac{1}{2}\cos(4t) + \frac{1}{8}\cdot i + \hat{v} = 0$

$-\frac{1}{2}\cos(4t) + \frac{1}{8}\left(C\frac{d\hat{v}}{dt}\right) + \hat{v} = 0$

$-\frac{1}{2}\cos(4t) + \frac{1}{4}\frac{d\hat{v}}{dt} + \hat{v} = 0$

$\frac{1}{2}\cos(4t+\pi)$

$\frac{1}{2}e^{j\pi} = -\frac{1}{2}$ ($e^{j\pi} = -1$)

↓ phasors

$-\frac{1}{2} + \frac{1}{4}j\omega\hat{V} + \hat{V} = 0$

$-\frac{1}{2} + j\hat{V} + \hat{V} = 0$

$\hat{V} = \frac{1/2}{1+j} = \frac{1/2\, e^{j0}}{\sqrt{1^2+1^2}\, e^{j\pi/4}} =$

$= \frac{1}{2\sqrt{2}} e^{-j\pi/4}\ V$

time ←

$\hat{v}_{ss}(t) = \frac{1}{2\sqrt{2}}\cos(4t - \pi/4)\ V$