Lecture 18, Wednesday, February 16, 2022

- Consider an LTI dissipative system and assume initial state already dissipated:

$$f(t) = A\cos(\omega t + \theta) \longrightarrow \boxed{\text{LTI, dissipative}} \longrightarrow y_{ZS}(t) = y_{ss}(t) = B\cos(\omega t + \phi)$$

  Both input and output move at same speed. Worry about amplitude and phase differences

- Phasor:

$$f(t) = A\cos(\omega t{+}\theta) \;=\; Re\left\{Ae^{j(\omega t+\theta)}\right\} \;=\; Re\left\{Ae^{j\theta}e^{j\omega t}\right\} \;=\; Re\left\{\underbrace{Ae^{j\theta}}_{:=\,F}e^{j\omega t}\right\} \;=\; Re\left\{Fe^{j\omega t}\right\}$$

  * $F = Ae^{j\theta}$ is the phasor of $f(t)$
  * It is a complex number representing the amplitude and the constant phase
  * It represents the starting point of the cosinusoidal, which can be completely recreated from there

- Superposition principle for phasors:

$$f_3(t) = k_1 f_1(t) + k_2 f_2(t) \;=\; k_1 Re\left\{F_1 e^{j\omega t}\right\} \;+\; k_2 Re\left\{F_2 e^{j\omega t}\right\} \;\;\Rightarrow\;\; F_3 = k_1F_1 + k_2F_2$$

  Note: $f_1(t)$ and $f_2(t)$ must have the same frequency ω.

- Derivative principle of phasors:

$$g(t) = \frac{d}{dt}f(t) \;\;\Rightarrow\;\; G = j\omega F \;=\; \omega e^{j\frac{\pi}{2}}F \;=\; \omega A e^{j\left(\theta+\frac{\pi}{2}\right)}$$

  Scales the phasor amplitude by ω and shifts the phase by $\frac{\pi}{2}$