Lecture 50, Wednesday, April 27, 2022

$$\hat{Y}(s) \;=\; \underbrace{\hat{F}(s)\frac{(b_0 s^m + \cdots + b_m)}{(s^n + a_1 s^{n-1} + \cdots + a_n)}}_{=\,\hat{Y}_{ZS}(s)\,=\,\hat{F}(s)\hat{H}(s)} + \underbrace{\frac{y(0^-)\left(s^{n-1} + \cdots + a_{n-1}\right) \;+\; \ldots \;+\; y^{(n-1)}(0^-)}{(s^n + a_1 s^{n-1} + \cdots + a_n)}}_{=\,\hat{Y}_{ZI}(s)}$$

$$\Rightarrow\; \hat{H}(s) \;=\; \frac{(b_0 s^m + \cdots + b_m)}{(s^n + a_1 s^{n-1} + \cdots + a_n)}$$

- The common denominator is called the *characteristic polynomial*:

$$s^n + a_1 s^{n-1} + \cdots + a_n$$

  - Its roots are called the *characteristic poles*:

$$p_1,\; p_2,\; \ldots,\; p_n$$

    Important: these are before any cancellation from possible zeros.

  - Each pole gets transformed back into time into a *characteristic mode*:

$$e^{p_1 t},\; e^{p_2 t},\; \ldots,\; e^{p_n t}$$

    Note: if pole p_r has multiplicity m, it gets transformed back into *characteristic modes*:

$$e^{p_r t},\; te^{p_r t},\; \ldots,\; t^{m-1}e^{p_r t}$$

