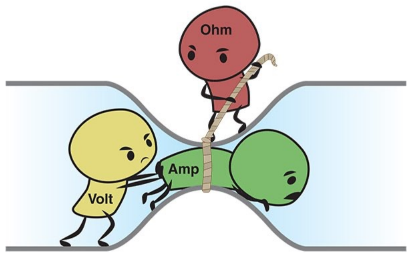
ECE 210 (AL2) - ECE 211 (E)

Chapter 1

Circuit Fundamentals



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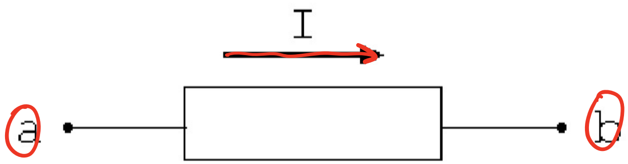
Outline

- Current and voltage
- Series and parallel configurations, SRS
- Kirchhoff's voltage and current laws (KVL and KCL)
- Ideal Resistors, Ohm's law
- Independent and dependent sources
- Absorbed power
- Ideal Capacitors and inductors
- Obtain voltages, currents and absorbed power in basic circuits using KVL, KCL and Ohm's law

- **Current**: amount of net electrical charge per unit time passing in the direction of arrow.



$$I = \frac{dq}{dt} \qquad \begin{matrix} > 0 \\ = 0 \\ < 0 \end{matrix}$$

- **Units:** Amperes (A) = $\frac{Coulomb\ (C)}{second\ (s)}$

- Voltage: energy gain per Coulomb moved from "-" terminal to "+" terminal, or energy loss per Coulomb moved from "+" terminal to "-" terminal.

$$\mathbf{V} = \frac{\mathbf{dW}}{\mathbf{dq}}$$

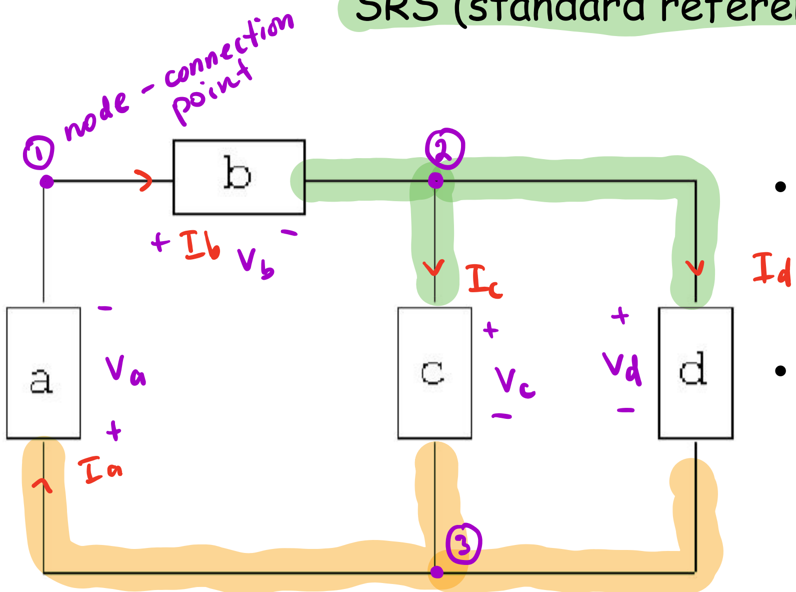
> 0 - energy gain from "-" to "+"

$= 0$

< 0 - energy loss from "-" to "+"

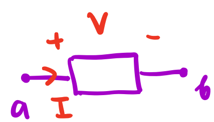
- Units: Volts (V) = $\frac{Joule\ (J)}{Coulomb\ (C)}$

- **Example #1:** assignment of polarities and current directions, SRS (standard reference system)

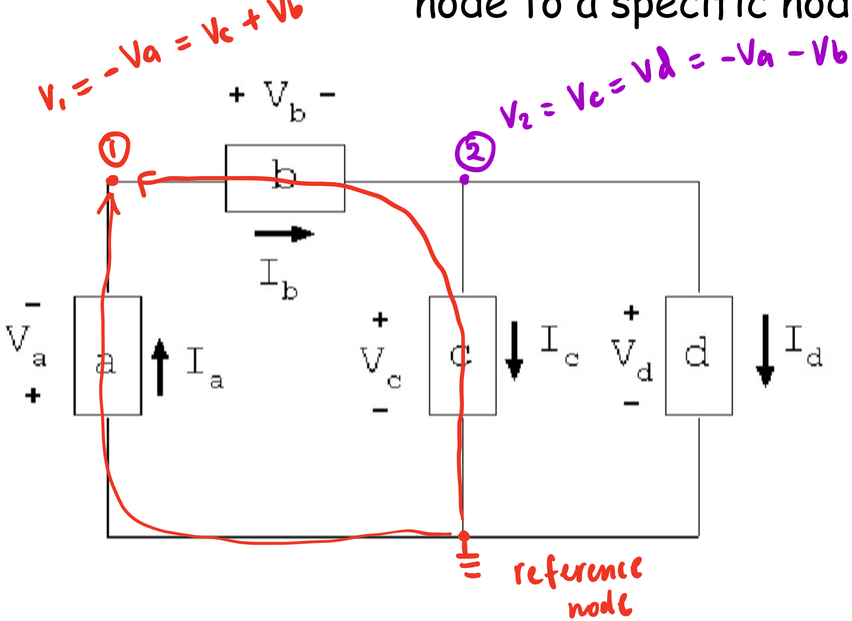


- **Series vs Parallel:**
  - Series : same current ($I_a = I_b$)
  - Parallel : connected to the same two nodes => same voltage, e.g. $V_c = V_d$

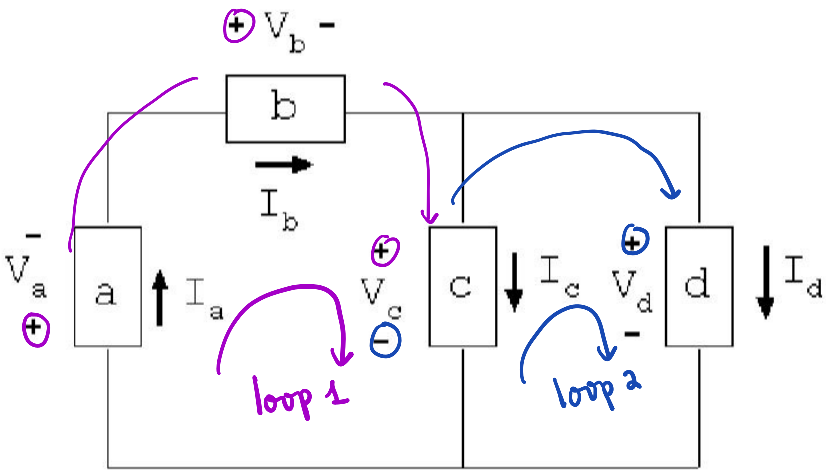
SRS: current goes into "+" terminal



- Node voltage: energy gain per Coulomb moved from a reference node to a specific node.



- Kirchhoff's voltage law (KVL): around any closed loop in a circuit,
  - $\sum V_{rise} = \sum V_{drop} \implies \underbrace{\sum V_{drop} - \sum V_{rise}}$ $= 0$

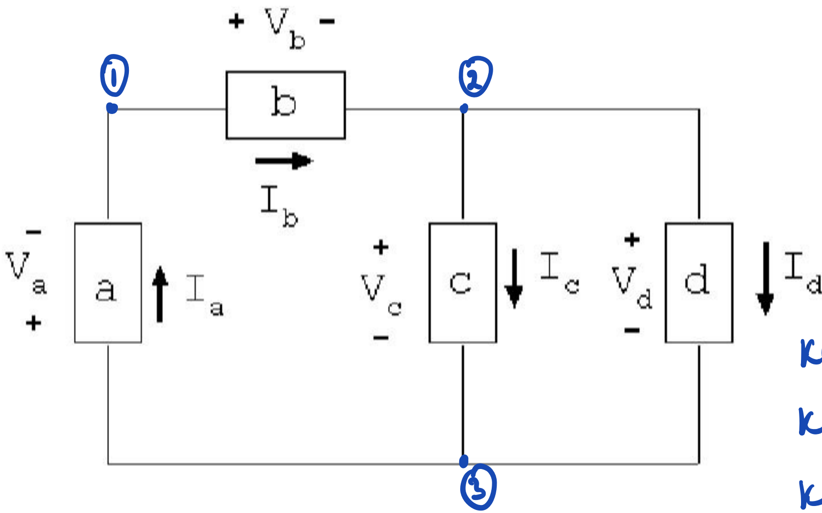


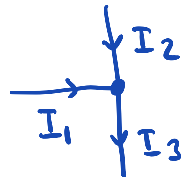
$V_a + V_b + V_c = 0$

$-V_c + V_d = 0$

- Kirchhoff's current law (KCL): at any node in a circuit,

$$\sum I_{in} = \sum I_{out}$$





$I_1 + I_2 = I_3$

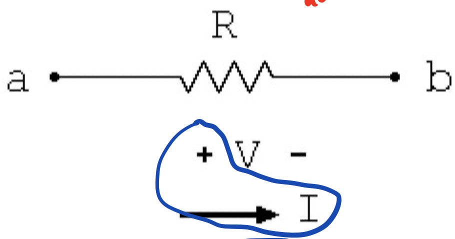
KCL @ (1): $I_a = I_b$

KCL @ (2): $I_b = I_c + I_d$

KCL @ (3): $I_c + I_d = I_a$

- Resistor

I ← add "-"



$\boxed{V = RI}$ ← Ohm's law

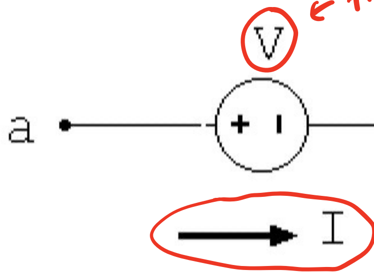
(-I)

- V: voltage drop in the direction of current I
- R: resistance
  - $R \geq 0$
  - Units: Ohms (Ω) = $\frac{V}{A}$

- Special cases of an ideal resistor:
- R = 0: short-circuit element
  - V =0, I: any

- R = ∞: open-circuit element
  - V: any, I=0

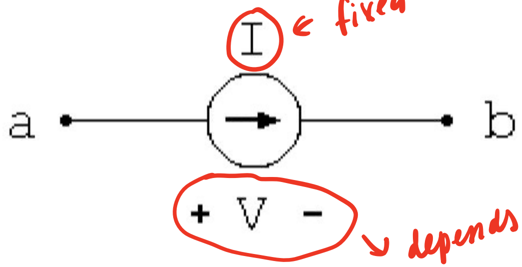
- Independent voltage source



← fixed

↘ depends on the rest of the circuit

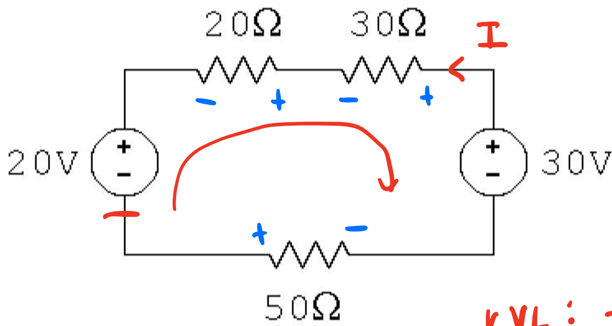
- Independent current source



← fixed

↘ depends on the circuit

- Example #2: determine the current in the following circuit



① KVL

② Polarities

KVL : $-20V - V_{20\Omega} - V_{30\Omega} + 30V - V_{50\Omega} = 0$

$-20V - I \cdot 20 - I \cdot 30 + 30V - I \cdot 50 = 0$

$100I = 10$

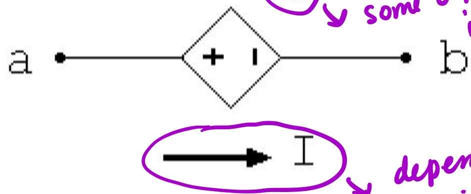
$I = 0.1\,A$ ← Units, please! ☺

- Dependent (controlled) voltage source

not fixed, it depends on Vx

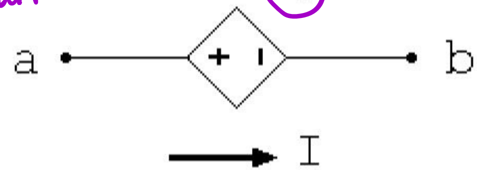
$V = kV_x$

some other voltage in a circuit



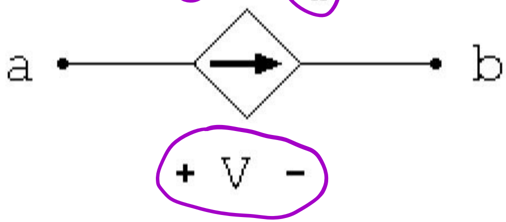
depends on a circuit

$V = kI_x$

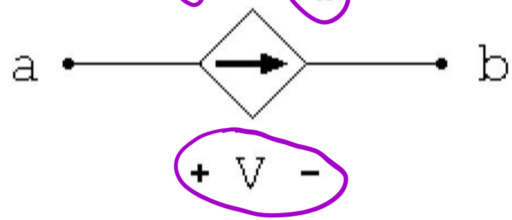


- Dependent (controlled) current source

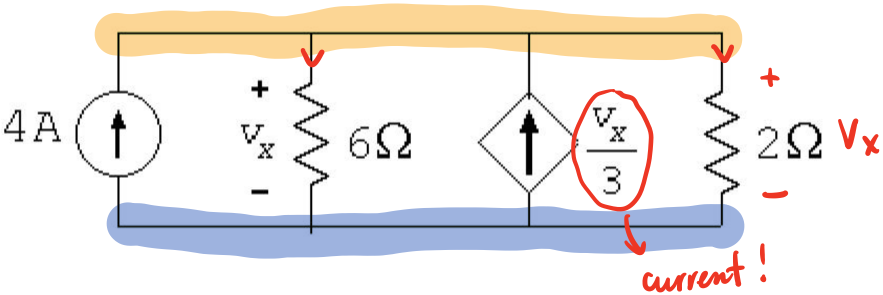
$I = kI_x$



$I = kV_x$



- Example #3: determine V_x in the following circuit

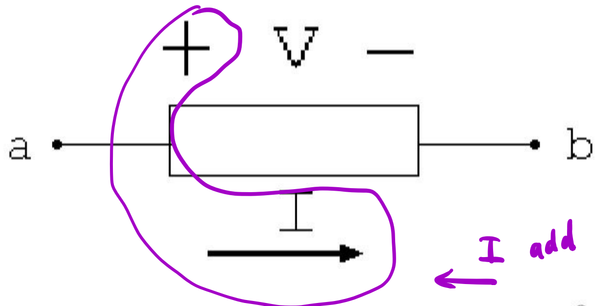


$$\text{KCL}: \sum I_{in} = \sum I_{out}$$

$$4\,A + \frac{V_x}{3} = I_{6\Omega} + I_{2\Omega} = \frac{V_x}{6} + \frac{V_x}{2}$$

$$V_x = 12V$$

- Absorbed power: total energy loss of charge carriers per unit time from "+" terminal to "-" terminal.



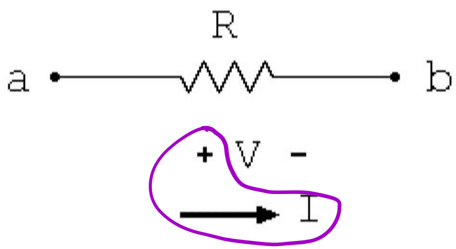
I add "-"

$$\mathbf{P} = \frac{\mathbf{dW}}{\mathbf{dt}} = \frac{\mathbf{dW}}{\mathbf{dq}}\frac{\mathbf{dq}}{\mathbf{dt}} = \mathbf{VI} \quad \begin{cases} > 0 & \text{– absorbs power} \\ = 0 & \\ < 0 & \text{– injects power} \end{cases}$$

(-I)

- Units: Watts (W) = $\frac{Joule\ (J)}{second\ (s)}$

- Absorbed power in resistor

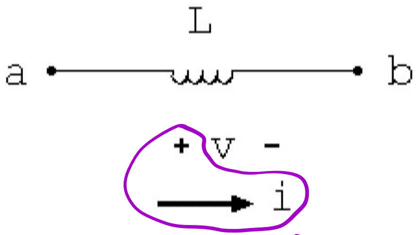


Always absorbs power!
selfish ☺

$$P = VI = (RI)I = \underline{I^2 R \geq 0}$$

$$P = VI = \left(\frac{V}{R}\right)V = \frac{V^2}{R} \geq 0$$

- Inductor



$$v(t) = L\frac{di}{dt}$$ → const I => V=0 → acts as a short circuit

- v: voltage drop in the direction of current I
- L: inductance
  - $L \geq 0$
  - Units: Henries (H)

- Absorbed power in inductor

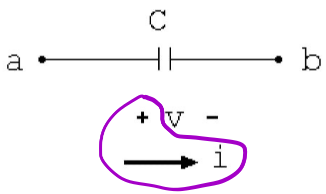
$$P = vi = \left(L\frac{di}{dt}\right) i = \underbrace{\frac{d}{dt}\left(\overbrace{\frac{1}{2}Li^2}^{\text{energy stored}}\right)}$$

> 0 – energy ↑, taking energy

= 0

< 0 – energy ↓, giving energy.

- Capacitor



$$i(t) = C\frac{dv}{dt}$$

↘ constant V ⇒ I=0 → acts as open-circuit

- v: voltage drop in the direction of current I
- C: capacitance
  - $C \geq 0$
  - Units: Farads (F)

- Absorbed power in capacitor

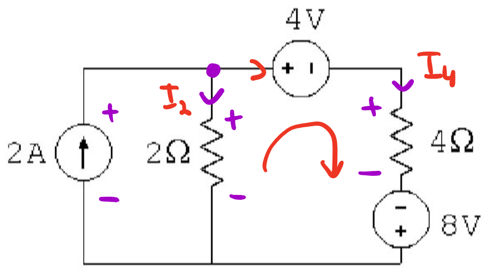
$$P = vi = v\underbrace{\left(C\frac{dv}{dt}\right)}_{i} = \frac{d}{dt}\left(\frac{1}{2}Cv^2\right)$$

> 0 – taking energy

$= 0$

< 0 – giving energy

• Example #4: determine the absorbed power in each element in the following circuit



KCL : $2 = I_{2\Omega} + I_{4\Omega}$ ①

KVL : $-V_{2\Omega} + 4V + V_{4\Omega} - 8V = 0$

$-I_{2\Omega} \cdot 2 + 4 + I_{4\Omega} \cdot 4 - 8 = 0$ ②

$I_{2\Omega} = \frac{2}{3} A \quad I_{4\Omega} = \frac{4}{3} A$

$P_{2\Omega} = R \cdot I_2^2 = 2 \cdot \left(\frac{2}{3}\right)^2 = \frac{8}{9} W$

$P_{4\Omega} = R \cdot I_4^2 = 4 \cdot \left(\frac{4}{3}\right)^2 = \frac{64}{9} W$

$\Sigma P = 0$

$P_{4V} = V_{4V} \cdot I_4 = 4 \cdot \frac{4}{3} = \frac{16}{3} W > 0 \rightarrow$ absorbing

$P_{8V} = V_{8V} \cdot (-I_4) = 8 \cdot \left(-\frac{4}{3}\right) = -\frac{32}{3} W < 0$

$P_{2A} = V_{2\Omega} \cdot (-2) = (I_2 \cdot 2) \cdot (-2) = -\frac{8}{3} W < 0$

injecting ☺

Summary

- Current and voltage
- Series and parallel configurations, SRS
- Absorbed power
- Kirchhoff's voltage and current laws (KVL and KCL)
- Ideal Resistors, Ohm's law
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