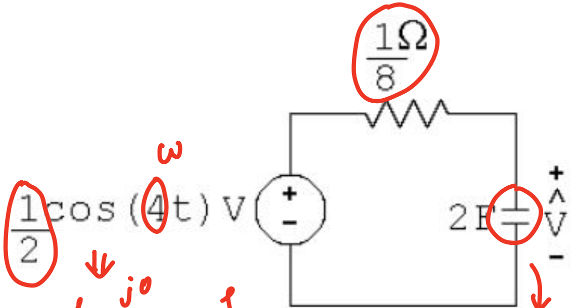
• Example #4-cont.

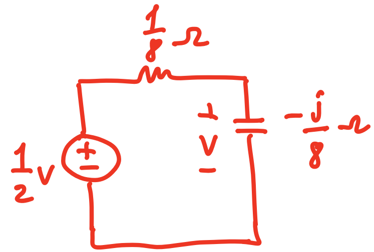
• Determine $\widehat{v_{ss}}(t)$ using phasors:



$$\frac{1}{2}\underbrace{e^{j0}}_{1} = \frac{1}{2}\,V$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j4\cdot 2} = -\frac{j}{8}\,\Omega$$

phasors $\Longrightarrow$



Voltage division:

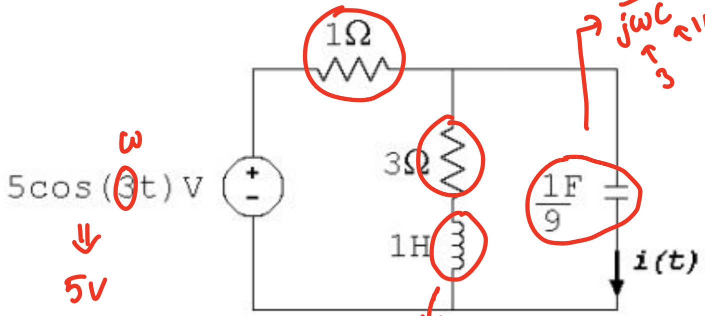
$$\hat{V} = \frac{1}{2}\left(\frac{-j/8}{-j/8+1/8}\right) = \frac{1}{2}\left(\frac{-j}{1-j}\right) = \frac{1}{2}\,\frac{e^{-j\pi/2}}{\sqrt{1^2+(-1)^2}\,e^{-j\pi/4}} = \frac{1}{2\sqrt{2}}\,e^{-j\pi/4}\ V$$

$$\boxed{\hat{v}_{ss}(t) = \frac{1}{2\sqrt{2}}\cos(4t - \pi/4)\ V}$$

same as before :)

- Example #6
- Determine $i_{ss}(t)$ using phasors:

$\frac{1}{j\omega C} = \frac{3}{j} = -j3\,\Omega$

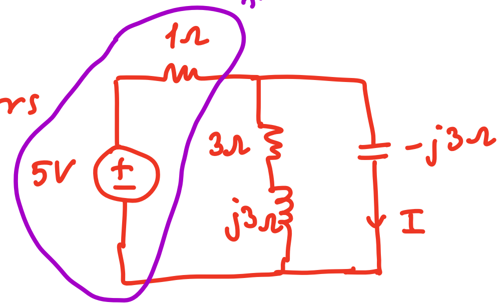


$j\omega L = j\cdot 3\cdot 1 = j3\,\Omega$

$Z_p = \frac{3(1+j)}{3+j4}$

phasors ⟹

source transformation



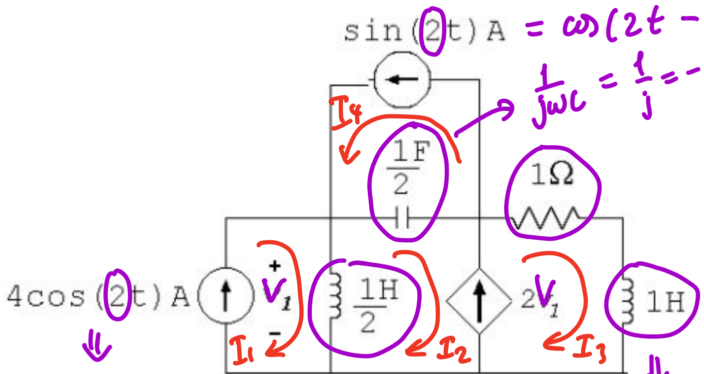
⇓

current division:

$$I = 5\left(\frac{Z_p}{-j3}\right) = \sqrt{2}\, e^{j\left(\frac{3\pi}{4} - \tan^{-1}\left(\frac{4}{3}\right)\right)}\ A$$

$$i_{ss}(t) = \sqrt{2}\cos\left(3t + \frac{3\pi}{4} - \tan^{-1}\left(\frac{4}{3}\right)\right)\ A$$

- Example #7
- Use the loop-current method using phasors to obtain the loop currents:



$\sin(2t)\,A = \cos(2t - \pi/2) = 1e^{-j\pi/2} = -j\,A$

$\frac{1}{j\omega C} = \frac{1}{j} = -j\,\Omega$

$4\cos(2t)\,A \Rightarrow 4\,A$

$\frac{1}{2}\,H \Rightarrow j\omega L = j\,\Omega$

$1\,H \Rightarrow j\omega L = j2\,\Omega$

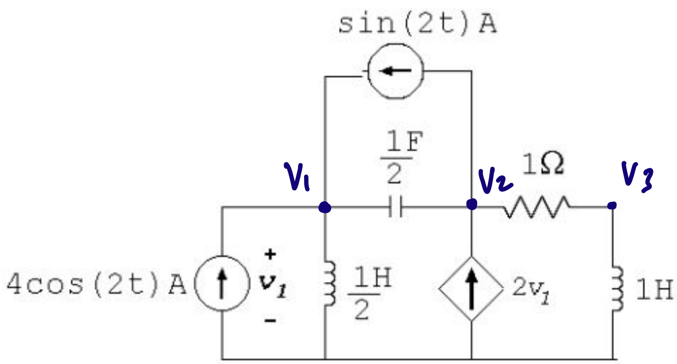
$I_1 = 4\,A$

$I_4 = -j\,A$

$I_2 = \frac{17 - j4}{5}\,A$

$I_3 = \frac{9 + j2}{5}\,A$

- Example #7-cont
  - Use the node-voltage method using phasors to obtain the node voltages:



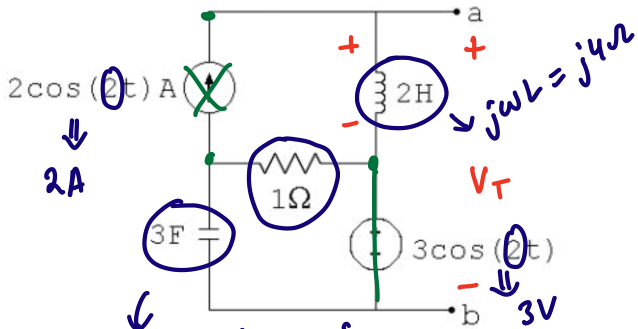
$$V_1 = \frac{3+j4}{1-5j}\ V$$

$$V_2 = 1+j4\ V$$

$$V_3 = -\frac{4+j8}{5}\ V$$

- Example #8
- Determine V_T, I_N and Z_T in phasors:

$\rightarrow$ Thevenin's impedance $Z_T = R_T + jX_T$



$$\frac{1}{j\omega C} = \frac{1}{j6} = -\frac{j}{6}\,\Omega$$

Get V_T:

$$V_T = V_{j4} + 3 = $$
$$= 2(j4) + 3 =$$
$$= 3 + j8 \text{ V}$$

Get Z_T:

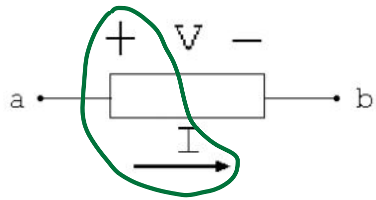
$$Z_T = j4\,\Omega$$

Get I_N:

$$I_N = \frac{V_T}{Z_T} = \frac{3+j8}{j4} \text{ A}$$

- Average power
  - Recall that absorbed power is

$$P = VI$$



  - If v and i are time-varying, then the instantaneous power is

$$p(t) = v(t)i(t)$$

  - For periodic signals with period T, the average absorbed power is

$$P = \frac{1}{T}\int_T p(t)dt = \frac{1}{T}\int_T v(t)i(t)dt$$

- Average power for phasors

$$\mathrm{P} = \frac{1}{T}\int_T p(t)dt = \frac{1}{T}\int_T v(t)i(t)dt$$

$$\boxed{Re\{z\} = \frac{z+z^*}{2}}$$

Recall: $v(t) = Re\{Ve^{j\omega t}\} = \frac{Ve^{j\omega t} + (Ve^{j\omega t})^*}{2} = \frac{Ve^{j\omega t} + V^* e^{-j\omega t}}{2}$

$i(t) = Re\{Ie^{j\omega t}\} = \frac{Ie^{j\omega t} + (Ie^{j\omega t})^*}{2} = \frac{Ie^{j\omega t} + I^* e^{-j\omega t}}{2}$

$$v(t)i(t) = \left(\frac{Ve^{j\omega t} + V^* e^{-j\omega t}}{2}\right)\left(\frac{Ie^{j\omega t} + I^* e^{-j\omega t}}{2}\right) =$$

$$= \frac{1}{4}\left(VIe^{j2\omega t} + VI^* + V^*I + V^*I^* e^{-j2\omega t}\right) =$$

$$= \frac{1}{2}\left(\frac{VIe^{j2\omega t} + (VIe^{j2\omega t})^*}{2} + \frac{VI^* + (VI^*)^*}{2}\right) =$$

$$= \frac{1}{2}\left(Re\{VIe^{j2\omega t}\} + Re\{VI^*\}\right)$$

- Average power for phasors-cont

$$P = \frac{1}{T}\int_T p(t)dt = \frac{1}{T}\int_T v(t)i(t)dt$$

$$P = \frac{1}{T}\int_T \left(\frac{1}{2}\left(Re\{VI e^{j2\omega t}\} + Re\{VI^*\}\right)\right) dt =$$

$$= \frac{1}{2}\cdot\frac{1}{T}\int_T Re\{\underset{\text{phasor}}{VI} e^{j2\omega t}\} dt + \frac{1}{2}\cdot\frac{1}{T}\int_T Re\{VI^*\} dt =$$

$$\frac{1}{2T} Re\{VI^*\}\int_T dt = \frac{1}{2} Re\{VI^*\}$$

$$= \underbrace{\frac{1}{2}\cdot\frac{1}{T}\int_T |VI| \cos(2\omega t + \angle VI)\, dt}_{\text{average of a cosine over 2 cycles is 0}} + \frac{1}{2} Re\{VI^*\}$$

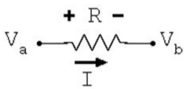
$$\boxed{P = \frac{1}{2} Re\{VI^*\}} \leftarrow$$

average power is purely real! (no j!)

- Average absorbed power

$z \cdot z^* = |z|^2$

  - Resistor



$$P = \frac{1}{2}\mathrm{Re}\{\overset{R\cdot I}{V} I^*\} = \frac{1}{2}\mathrm{Re}\{R \cdot \underbrace{I \cdot I^*}_{|I|^2}\} =$$

$$= \frac{R}{2}\mathrm{Re}\{|I|^2\} =$$

$$P = \frac{1}{2} R |I|^2 = \frac{1}{2}\frac{|V|^2}{R}$$

$$P = I_{rms}^2 \cdot R = \frac{V_{RMS}^2}{R}$$

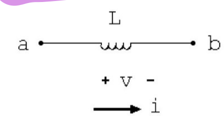
for DC:

$P = R \cdot I^2$

$I_{rms} = \frac{|I|}{\sqrt{2}}$

$V_{rms} = \frac{|V|}{\sqrt{2}}$

  - Inductor



$$P = \frac{1}{2}\mathrm{Re}\{\overset{j\omega L\cdot I}{V} I^*\} = \frac{1}{2}\mathrm{Re}\{j\omega L \cdot \underbrace{I \cdot I^*}_{|I|^2}\} =$$

$$= \frac{1}{2}\omega L |I|^2 \mathrm{Re}\{j\} = \underline{0\,W} \quad !$$

  - Capacitor

$$\underline{P = 0\,W} \quad !$$