Lecture 38, Monday, April 4, 2022

- Convolution
  - Recall that
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \longrightarrow \boxed{H(\omega)} \longrightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t}d\omega$$
$$F(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega) = F(\omega)H(\omega)$$
  - Now, convolution of $f(t)$ and $h(t)$:
$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = f(t) * h(t)$$
  - At any point t, the output $y(t)$ is a linear combination of $f(\tau)$ across all time, where the weights are given by $h(t-\tau)$
  - To convolve $f(t)$ and $h(t)$:
    * Change the parameter of $f(t)$ and $h(t)$ from t to τ to get $f(\tau)$ and $h(\tau)$:
$$f(t) \to f(\tau) \qquad\qquad h(t) \to h(\tau)$$
    * Time-reverse $h(\tau)$ and then shift it to the right by t (left if $t < 0$) to get $h(t-\tau)$:
$$h(\tau) \to h(t-\tau) = h(-(\tau - t))$$
    * Multiply $f(\tau)$ and $h(t-\tau)$ for each τ:
$$f(\tau)h(t-\tau)$$

continued on next page....

- Integrate the resulting product for all time τ:

$$\int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau$$

– Properties of convolution:

- Commutative:

$$y(t) = f(t) * h(t) = h(t) * f(t)$$
$$y(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)f(t-\tau)d\tau$$

  · It does not matter which signal is time-reversed.

