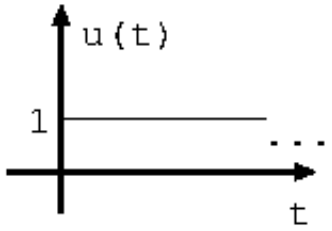
Lecture 32, Tuesday, March 22, 2022

- Unit-step function:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{else.} \end{cases}$$



- Fourier transform has a lot of properties, some similar to Fourier series, others different.
  - Get familiar with tables 7.1 and 7.2 from textbook, or 3 and 4 from handout.
  - Time-shift:

$$f(t) \leftrightarrow F(\omega) \qquad f(t - t_0) \leftrightarrow F(\omega)\, e^{-j\omega t_0}$$

  - Time-scaling: for real valued a,

$$f(t) \leftrightarrow F(\omega) \qquad f(at) \leftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

  - Symmetry:

$$f(t) \leftrightarrow F(\omega) \qquad F(t) \leftrightarrow 2\pi f(-\omega)$$

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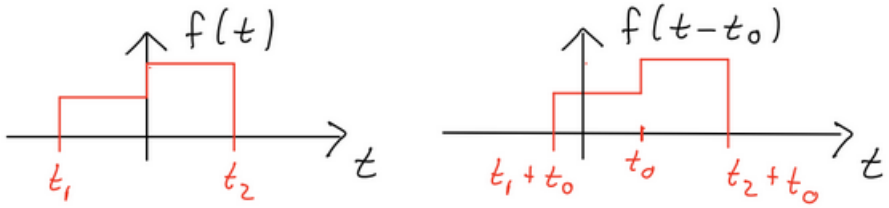
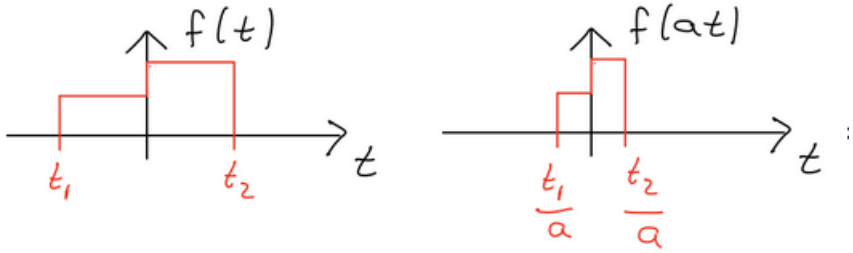
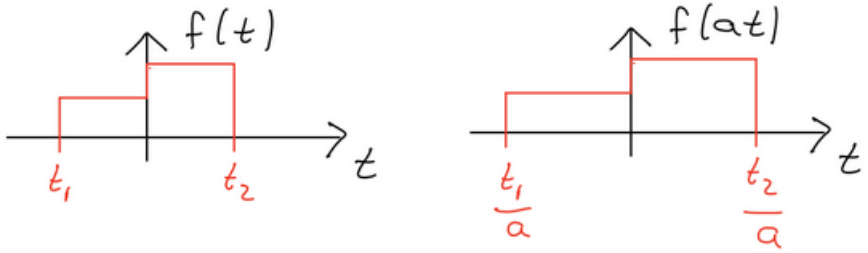
- Time-scaling and time-shifting of functions and their plots:
  - If $b > 0$, then $f(t-b)$ is shifted to the right and $f(t+b)$ is shifted to the left.



  - If $a > 1$, then $f(at)$ is shrunk, narrower, by a factor of a.



  - If $0 < a < 1$, then $f(at)$ is expanded, wider, by a factor of a.



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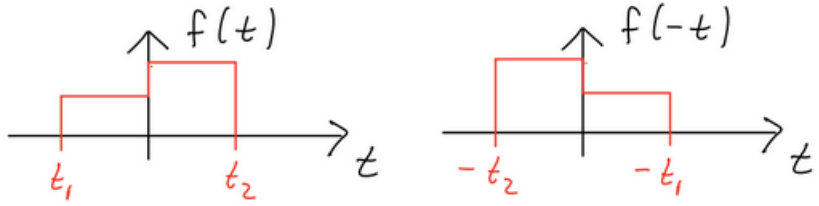
– If $a < 0$, then $f(at)$ is also reversed in time, flipped around the y-axis.

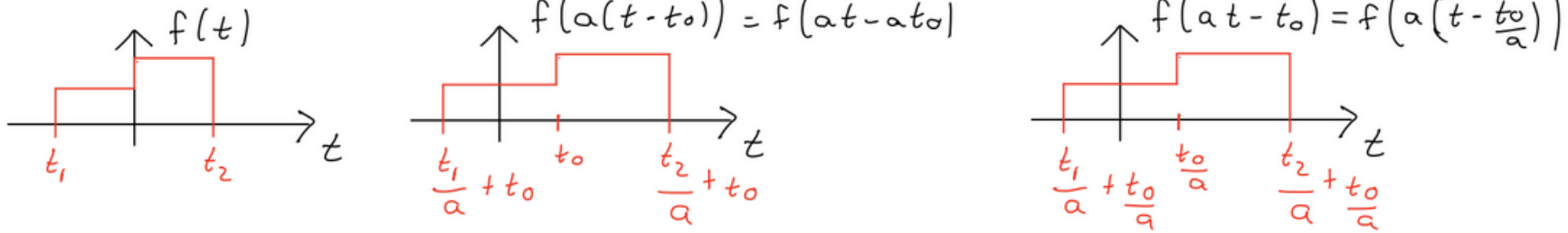


– Time-scaling takes precedence over time-shifting:

$$f\left(at+b\right) \;=\; f\left(a\left(t+\frac{b}{a}\right)\right)$$

Can be done in two ways:

* Scale then shift relative to the new scale: $f(t) \to g(t) = f(at) \to h(t) = g\left(t+\frac{b}{a}\right)$
* Shift then rescale everything: $f(t) \to g(t) = f(t+b) \to h(t) = g\left(at\right)$



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– This is also true if the time-scaling is negative, so be careful with the time-shifts

  * If $b > 0$, then $f(-t-b) = f(-(t+b))$ is shifted to the left and $f(-t+b) = f(-(t-b))$ is shifted to the right