Lecture 48, Monday, April 25, 2022

- Inverse Laplace transform
  - Partial fraction expansion for proper rational functions with distinct poles:

$$\hat{F}(s) = \frac{\text{polynomial of degree } < n}{\text{polynomial of degree } = n} = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p_1)(s-p_1)\cdots(s-p_n)}$$
$$= \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \cdots + \frac{A_n}{s-p_n}, \quad \text{where } p_i \neq p_j \text{ if } i \neq j$$

    - To obtain the coefficient A_k:

$$A_k = \left[\hat{F}(s)(s-p_k)\right]\Big|_{s=p_k}$$

    - Works for real or complex poles.
    - If poles are complex
      - its poles are complex conjugates:

$$p_1 = p_2^* = \alpha + j\beta$$

      - their partial fraction expansion coefficients are also complex conjugates:

$$A_1 = A_2^* = re^{j\theta}$$

      - can combine the two terms into a real-valued cosinusoidal:

$$2re^{\alpha t}\cos(\beta t + \theta)u(t)$$

continued on next page....

– Partial fraction expansion for proper rational functions with repeated poles:

$$\begin{aligned}\hat{F}(s) &= \frac{\text{polynomial of degree } < n}{\text{polynomial of degree } = n} = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p_1)^n} \\ &= \frac{A_1}{s-p_1} + \frac{A_2}{(s-p_1)^2} + \cdots + \frac{A_n}{(s-p_1)^n}\end{aligned}$$

  * To obtain the coefficient A:

$$A_{n-m} = \frac{1}{m!}\left[\frac{d^m}{ds^m}\left(\hat{F}(s)(s-p_1)^n\right)\right]\Bigg|_{s=p_1}$$

– In both cases, distinct and repeated poles, could also obtain the coefficients by matching the coefficients of the polynomials via a linear system of equations. That might be faster than taking derivatives when there are just 1-2 coefficients missing.

