

Fall 2021

## 2 Laboratory exercise

### 2.1 Frequency Response $H(\omega)$

The **frequency response**  $H(\omega)$  of a linear and dissipative time-invariant circuit contains all the key information about the circuit which is needed to predict the circuit response to arbitrary inputs. Its magnitude  $|H(\omega)|$  is known as **amplitude response** and  $\angle H(\omega)$  is usually referred to as **phase response**. In this section, you will construct an active bandpass filter circuit and measure its amplitude response over the frequency range 1-20 kHz.

1. Construct the circuit shown in Figure 4 on your protoboard. For now, do not connect it to the three-stage circuit from Lab 2. Remember the rules for wiring and using the 741 op-amp, which are repeated in Figure 3.

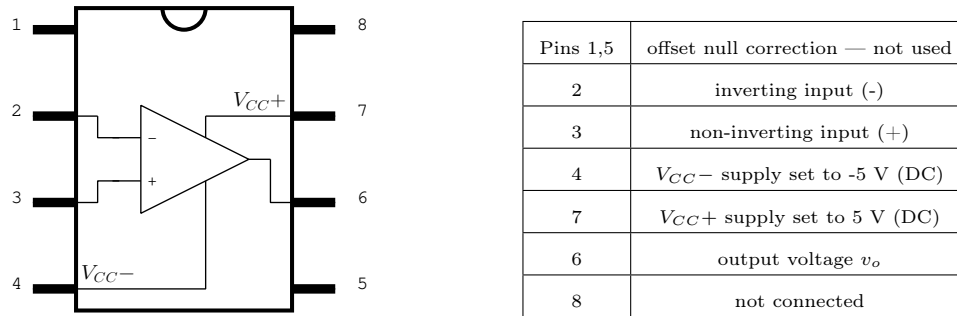


Figure 3: Pin-out diagram for the 741 op-amp.

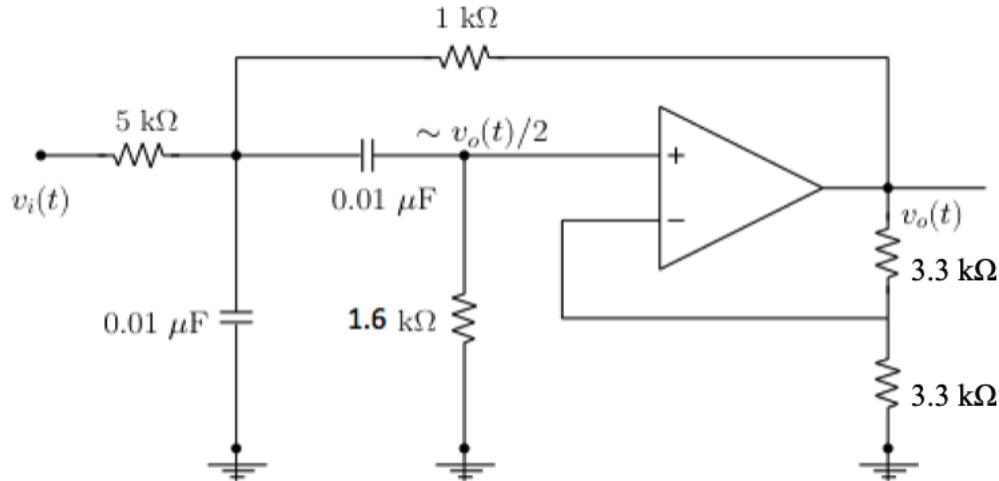
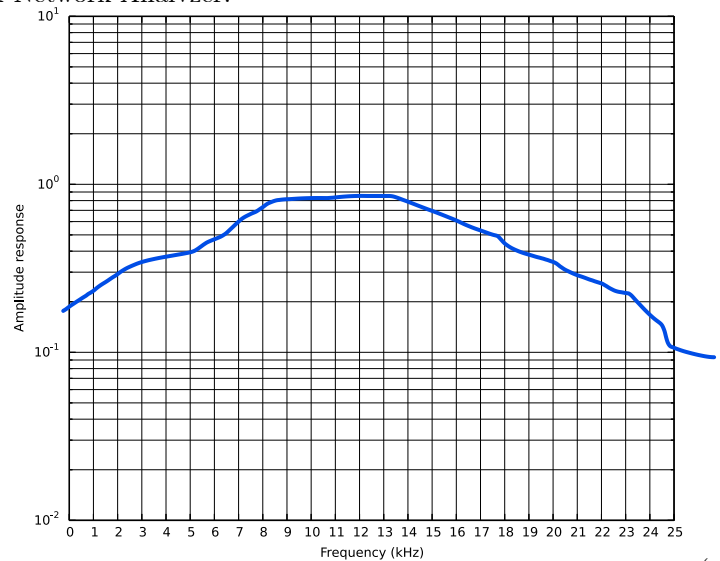


Figure 4: Circuit for analysis in prelab and lab.

2. Enable the 5V DC supplies, then connect a 14 kHz sine wave with amplitude 1V as the AC input  $v_i(t)$ . Display  $v_i(t)$  on channel 1 of the oscilloscope and  $v_o(t)$  on channel 2. Set the Time Base around 20  $\mu$ s. (You should obtain an output of approximately of 900 mV amplitude at the frequency of 14 kHz)

3. Next let's examine the frequency response using Network Analyzer.

- Go to Network Analyzer module and click the setting icon at the top right corner.
- In “REFERENCE” column, select Channel 1
- In “SWEEP” column, select Logarithmic and input 1 kHz for Start and 25 kHz for Stop, 500 samples for Samples count
- Click Single (You should have same setting as Figure 5)
- Sketch the frequency response, covering from 1 kHz to 25 kHz in Logarithmic scale



(\_\_\_\_/2)

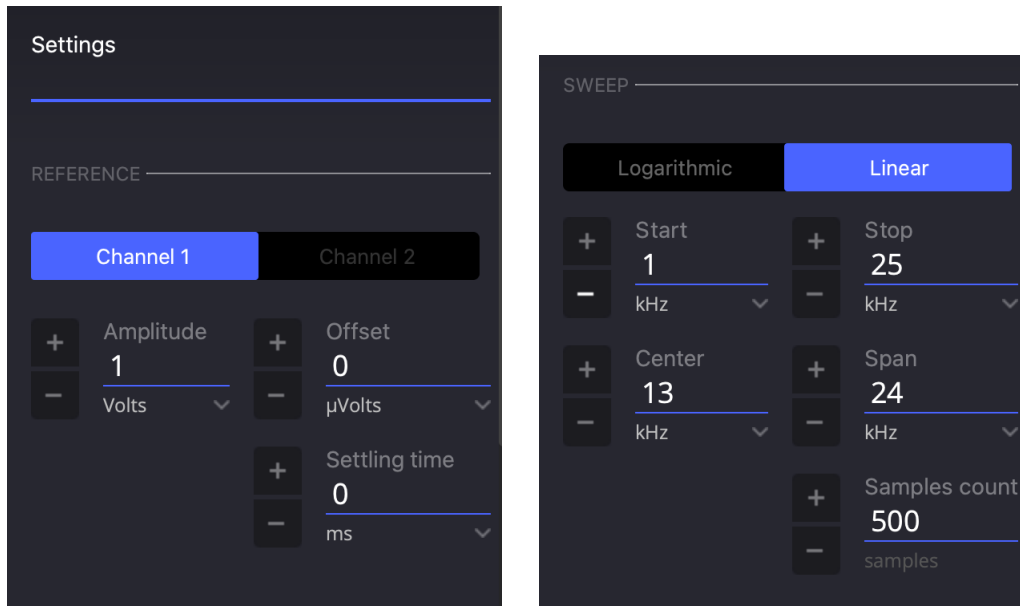


Figure 5: Network Analyzer setting.

4. The **center frequency**  $\omega_o = 2\pi f_o$  of a bandpass  $H(\omega)$  is defined as the frequency at which the amplitude response  $|H(\omega)|$  is maximized. What is the center frequency  $f_o$  in kHz units and what is the maximum amplitude response  $|H(\omega_o)|$  of the circuit? Estimate  $f_o$  and  $|H(\omega_o)|$  from your graph as accurately as you can (You may find the Cursors tool useful. To enable cursor, click the hollow square in the bottom right, next to “Cursors”. You should see the legend for frequency and magnitude of each cursor. ).

$f_o = $ <span style="color: blue;">14.5kHz</span>	(____/2)	$ H(\omega_o)  = $ <span style="color: blue;">0.649</span>	(____/2)
--	----------	--	----------

5. The **3 dB cutoff frequencies**  $\omega_u = 2\pi f_u$  and  $\omega_l = 2\pi f_l$  are the frequencies above and below  $\omega_o = 2\pi f_o$  at which the amplitude response  $|H(\omega)|$  is  $\frac{1}{\sqrt{2}} \approx 0.707$  times its maximum value  $|H(\omega_o)|$ . The same frequencies are also known as **half-power** cutoff frequencies since at frequencies  $\omega_u$  and  $\omega_l$  the output signal power is one half the value at  $\omega_o$ , assuming equal input powers at all three frequencies.

$\frac{1}{\sqrt{2}}|H(\omega_0)| = 0.4586$

(\_\_\_\_/2)

$f_l = 12.6\text{kHz}$

(\_\_\_\_/1)

$f_u = 17.4\text{kHz}$

(\_\_\_\_/1)

6. Determine the **3 dB bandwidth**  $B \equiv f_u - f_l$  of the bandpass filter in kHz units and calculate the **quality factor** of the circuit defined as  $Q \equiv \frac{\omega_o}{2\pi B} = \frac{f_o}{B}$ .

$B = 4.8 \text{ kHz}$

(\_\_\_\_/2)

$Q = 3.02$

(\_\_\_\_/2)

## 2.2 Displaying Fourier coefficients

In order to display the Fourier coefficients of a periodic signal on the oscilloscope we can use the built in FFT function<sup>1</sup>. On the “FFT screen” of your scope the horizontal axis will represent frequency  $\omega$  (like in the frequency response plot of the last section) normalized by  $2\pi$ , and you will see narrow spikes positioned at values  $f = \frac{\omega}{2\pi}$  equal to harmonic frequencies  $n\frac{\omega_o}{2\pi}$  of the periodic input signal; spike amplitudes will be proportional to compact Fourier coefficients  $c_n = 2|F_n|$  in dB. The next set of instructions tells you how to view the single Fourier coefficient (namely  $c_1$ ) of a co-sinusoidal signal:

The Signal Generator module can create an AM signal for you:

- Enable the 5V DC supplies. Set the Signal Generator to a 13 kHz sine with amplitude 500 mV.
- Go to the Spectrum Analyzer module. Select “Linear”, and set input 1 kHz for Start frequency and 50 kHz for Stop frequency.
- Select dBV as units and choose the largest Resolution BW (with the setting above, choose 195.31 Hz or above for Resolution BW to avoid long processing time)
- Click Run (You should have same setting as Figure 6)

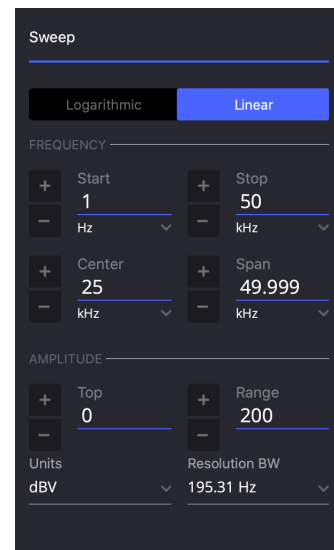


Figure 6: Spectrum Analyzer module setting.

Observe the output signal’s Fourier coefficient. How does the FFT display change as you sweep the frequency of the input from 1 kHz to 20 kHz ? (To sweep the frequency, you have to switch back and forth from Signal Generator module to Spectrum Analyzer module to update the frequency and see the change) Describe how the signal changes in frequency domain and explain why the signal changes as a function of the frequency. .

Describe how the signal changes in the frequency domain and explain why.

There is a spike in the output signal (in the frequency domain / oscilloscope ) around the value of the input signal. This shows that there is a remnant signal present.

(\_\_\_\_/4)

<sup>1</sup>FFT stands for *fast Fourier transform* and it is a method for calculating Fourier transforms with sampled signal data — see Example 9.26 in Section 9.3 of Chapter 9 to understand the relation of windowed Fourier transforms to Fourier coefficients.

## 2.3 Fourier coefficients of a square wave

Now you will introduce a periodic signal with a more interesting set of Fourier coefficients — a square-wave:

1. Change the Signal Generator setting to create a 15 kHz square wave with amplitude 0.5 V as the input to your circuit.
2. Display the FFT of the square wave at the filter input by enabling the CH1 and disabling CH2 in the Spectrum Analyzer module. Set the Start frequency to 1 kHz and the Stop frequency to 120 KHz. Change the Resolution BW to 234.75 Hz or larger.
3. Use “Markers” tool to find the peak

- Click the setting icon next to “Markers” in the bottom right corner and you will see panel shown in Figure 7.
- Double click the number in the hollow square to enable markers.
- You may move the marker manually by adjusting the Frequency Position or you may move the marker automatically to find peaks by clicking “Peak ->” or “<- Peak” in the panel.
- To add multiple markers, double click the hollow squares with different numbers.
- Enable the Marker Table under the General column to see the measurement.
- You may have to stop the continuous displaying by clicking “Stop” in order to obtain steady results.

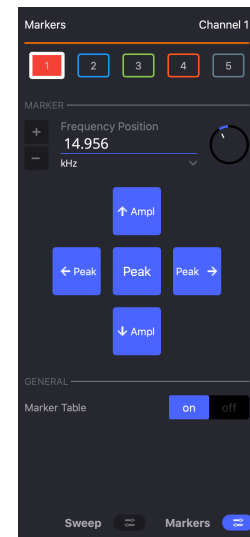


Figure 7: Markers setting.

4. Display the FFT of the square wave at the filter input. Fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power. Keeping in mind the result for Problem 1 in the Prelab, describe the signal in the FREQUENCY domain:

(You should measure the amplitude in dBV and convert dBV to V using the formula from prelab)

freq.	Ampl.(dBV)	Ampl.(V)
15.02	-13	0.224
45.07	-23	0.0708
74.89	-28	0.0398
104.93	-30	0.0316

This function is a series of sins beginning with 15kHz and increasing by 30 kHz each term. This is predicted in the prelab by the specification of  $C_n$  being zero when  $n$  is even and decaying  $4A/(n\pi)$  with when  $n$  is odd

(\_\_\_\_/4)

5. Display the FFT of the filter’s output. In addition, display on the scope both the input and output signals in the time domain. Describe the output in TIME domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

freq.	Ampl.(dBV)	Ampl.(V)
15.02	-15	0.178
45.07	-39	0.011
74.89	-49	0.0035
104.93	-52	0.0025

Describe the output signal in the time domain.

This output in the time domain is simply a sinusoid with a fundamental frequency of 15kHz

(\_\_\_\_/4)

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain:

(\_\_\_\_/4)

Total harmonic distortion is calculated by the formula  $(1/V_1) * \sqrt{V_2^2 + V_3^2 + V_4^2 \dots}$  but sometimes does not contain a root in the numerator depending on where you get your information. Using the formula provided, our total harmonic distortion is 0.06635.

6. Repeat for a 10 kHz square wave. Describe the output signal in the TIME domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

freq.	Ampl.(dBV)	Ampl.(V)
10.3	-21	0.089
30.0	-35	0.0178
49.77	-46	0.005012
70.4	-50	0.00316

Describe the output signal in the time domain.

The output signal in the time domain resembles a triangular wave with a fundamental frequency of 10 kHz

(\_\_\_\_/4)

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain:

$$(1/V_1) * \sqrt{V_2^2 + V_3^2 + V_4^2 \dots} = 0.2108$$

The THD is significantly higher in this case and thus can explain the distorted look of the output in the time domain. All of the frequencies present in the input are filtered out by the band pass filter pretty well.

(\_\_\_\_/4)

7. **(Demo required)** Repeat for a 5 kHz square wave. Describe the output signal in TIME domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

freq.	Ampl.(dBV)	Ampl.(V)
4.7	-30	0.0316
15.0	-24	0.0631
25.4	-37	0.0141
34.74	-43	0.00708

Describe the output signal in the time domain.

This output signal in the time domain looks mangled. It is periodic with a frequency near 5kHz, but it is not the most significant frequency. This might imply a very large THD.

(\_\_\_\_/4)

Based on the amplitude of the harmonics of the output signal, explain the shape of the output signal seen in time domain:

$$(1/V_1) * \sqrt{V_2^2 + V_3^2 + V_4^2 \dots} = 2.06$$

This very large total harmonic distortion would explain the very mangled shape of the output signal in the time domain

(\_\_\_\_/3)

8. In terms of the amplitude response of the system (  $|H(\omega)|$  ) explain the change in amplitude of the harmonics from the input to the output (Hint: what frequencies are being attenuated, and what frequencies are inside the bandwidth of the filter):

(\_\_\_\_/5)

Frequencies around 15kHz are allowed to pass through the filter with minimal attenuation. As frequencies get farther from that, they get more and more attenuated. These square waves are an infinite sum of increasingly higher frequencies and they all eventually get attenuated. Those with fundamental frequencies closer to 15kHz can pass through easier.

**Important!**

Leave your active filter assembled on your protoboard! You will need it in the next lab session.

## The Next Step

The active filter is the last component you will build for the AM radio receiver. In Lab 4, you will combine your components from Labs 1 through 3 to create a working AM radio receiver. The frequency-domain techniques you learned this week will be essential to following the AM signal through each stage of the receiver system.