Lecture 27, Monday, March 7, 2022

- Periodic signals
  - Sums of periodic signals
    - For a sum of periodic signals to be periodic, all possible *ratios* of the individual fundamental frequencies must be *rational* numbers $\left(\frac{\text{integer}}{\text{integer}}\right)$:

$$\frac{\omega_1}{\omega_2},\ \frac{\omega_1}{\omega_3},\ \ldots\ ,\ \frac{\omega_1}{\omega_n},\ \frac{\omega_2}{\omega_3},\ \ldots\ ,\ \frac{\omega_2}{\omega_n},\ \ldots\ ,\frac{\omega_{n-1}}{\omega_n},$$

    - Same holds for the individual periods (basically, each must complete an integer number of cycles when they meet again).
    - There must exist a frequency $\hat{\omega}$ such that all individual frequencies, ω_i, are *integer multiples* of $\hat{\omega}$. The largest such frequency is the fundamental frequency ω_0.
      - The fundamental frequency does not have to be one of the individual frequencies.
    - Similarly, there must exist a time duration $\hat{T}$ which is an *integer multiple* of all individual periods, T_i. The smallest such time duration is the period T.
  - For a fixed ω_0

    $e^{\pm j\omega_0 t}$ is periodic

    $e^{\pm j2\omega_0 t}$ is periodic

    $\vdots$

    $e^{\pm jn\omega_0 t}$ is periodic

    continued on next page....

For any integers $n,\ m$, the ratio $\frac{n}{m}$ is a rational number

$\rightarrow \displaystyle\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$ is periodic

where F_n is a complex number, independent of time

- If $f(t)$ is periodic with fundamental frequency ω_0, then we can express it as

$$f(t) \;=\; \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

  F_n is complex BUT independent of time

  * Distinct functions with the same ω_0 will have distinct sets of F_n
  * How to get the coefficients F_n?
    · If $f(t)$ consists of cosinusoidals or complex exponentials, just match coefficients after using Euler's formula

