ECE 210/211 HWs HW 8

Varenya Jain

TOTAL POINTS

63.5 / 70

QUESTION 1

1 0 / 0

✓ **- 0 pts** Correct

QUESTION 2

11 pts

2.1 2 / 2

✓ **- 0 pts** Correct

2.2 3 / 3

+ **0 pts** Incorrect/ no solution
✓ **+ 1 pts** $$F_0 = \frac{3}{2}$$
✓ **+ 1 pts** $$F_1 = \frac{j}{\pi}$$
✓ **+ 1 pts** $$F_2 = 0$$
+ **0.5 pts** incorrect sign in one equation

2.3 4 / 6

✓ **- 2 pts** $$c_1 = \frac{2}{\pi}$$

QUESTION 3

20 pts

3.1 7.5 / 10

✓ **- 2 pts** Partially incorrect Fn
✓ **- 0.5 pts** Minus sign in exponent

3.2 8 / 10

✓ **- 2 pts** F_n not in equation/Incorrect F_n/No Constant Term

QUESTION 4

14 pts

4.1 3 / 3

✓ **- 0 pts** Correct

4.2 2 / 2

✓ **- 0 pts** Correct

4.3 3 / 3

✓ **- 0 pts** Correct

4.4 3 / 3

✓ **- 0 pts** Correct

4.5 3 / 3

✓ **- 0 pts** Correct

QUESTION 5

25 pts

5.1 10 / 10

✓ **- 0 pts** Correct

5.2 15 / 15

✓ **- 0 pts** Correct

655479542

vareya3

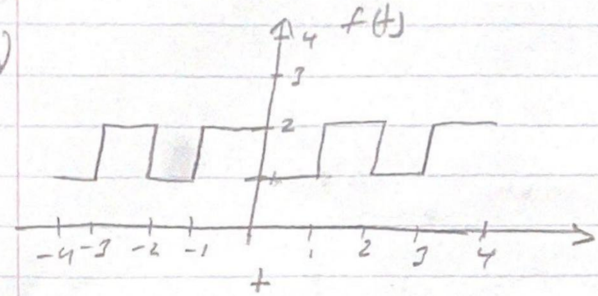
HW8

03/23/2022 Vareya Jain ECE 210 ①

1. Vareya Jain

2. $f(t)$ is periodic w/ $T=2s$. B/w $t=0$ and $t=2s$: $f(t) = \begin{cases} 1 & t \in (0,1) \\ 2 & t \in [1,2) \end{cases}$

a)



2b,c) $n=0 \to 1.5$ $c_0 = 3$

$n=1 \to j/\pi$ $c_1 = 2j/\pi$

$n=2 \to 0$ $c_2 = 0$

b) $w_0 = \frac{2\pi}{T} = \frac{2\pi}{2s} \Rightarrow w_0 = \pi \text{ rad/s}$, $f(t) = \sum_{n=-\infty}^{\infty} F_n \cdot e^{[j \cdot n \cdot w_0 \cdot t]}$

$$F_n = \frac{1}{T}\int_T f(t) \cdot e^{[-j \cdot n \cdot \pi \cdot t]} dt = \frac{1}{2}\left[\int_0^1 (1) e^{-jn\pi t} dt + \int_1^2 (2) e^{-jn\pi t} dt\right] =$$

$$= \frac{1}{2}\left[\frac{e^{-jn\pi t}}{(-jn\pi)}\right]_0^1 + \left[\frac{e^{-jn\pi t}}{(-jn\pi)}\right]_1^2$$

not relevant* $\hookrightarrow \int_a^b \left(\frac{e^{-jn\pi t}}{(-jn\pi)}\right) dt = \frac{j}{n\pi} \int_a^b \left(e^{-jn\pi t}\right) dt = \ldots = -\frac{e^{-j\pi n b} - e^{-jn\pi a}}{(\pi n)^2}$

2b $F_n \to \boxed{n=0} \to \frac{1}{2}(1+2) = \frac{3}{2} = \boxed{1.5}$

$\boxed{\hookrightarrow n=1} \to \frac{1}{2}\left(-\frac{2j}{n}\right) + \frac{2j}{\pi} = -\frac{j}{\pi} + \frac{2j}{\pi} = \boxed{j/\pi}$

$\boxed{n=2} \to \frac{1}{2}(0) + 0 = \boxed{0}$

* $\int_a^b e^{-jn(\pi)t} dt = \frac{j\left(e^{-ibn(\pi)} - e^{-ian(\pi)}\right)}{\pi n}$

1 0 / 0

✓ - 0 pts Correct

(2)

2c) Compact Fourier Series: for real-valued function only

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cdot \cos(n\omega_0 t + \theta_n)$$

$\downarrow$

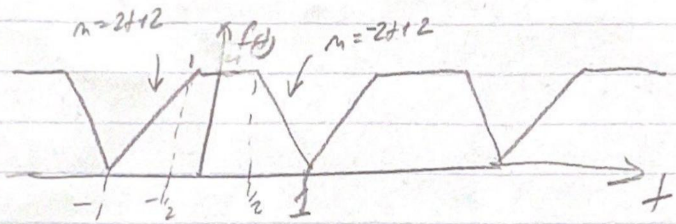
$c_0 = 2F_0 \rightarrow c_0 = 2\,(\tfrac{3}{2}) = \boxed{3 = c_0}$

$c_n = 2|F_n|$

$\hookrightarrow c_1 = 2\,(j/\pi) = \boxed{2j/\pi = c_1}$

$\hookrightarrow c_2 = 2\,(0) = \boxed{0 = c_2}$

3.



a) exponential Fourier Series of $f(t)$ $\rightarrow f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$

$T = 2 \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \omega_0 = \pi$ rad

$\hookrightarrow F_n = \frac{1}{T}\int_T f(t)\, e^{-jn\omega_0 t}\,dt$ $\begin{cases} 2t+2, & t \in (-1, -\frac{1}{2}) \\ 1, & t \in [-\frac{1}{2}, \frac{1}{2}) \\ -2t+2, & t \in [\frac{1}{2}, 1) \end{cases}$

general integral: $\int t e^{-jn\pi t} = \frac{e^{-jn\pi t}}{n\pi}\left(j + \frac{1}{n\pi}\right)$

$\hookrightarrow$

2.1 2 / 2

✓ **- 0 pts** Correct

(2)

2c) Compact Fourier Series: for real-valued functions only

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cdot \cos(n\omega_0 t + \theta_n)$$

↓

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$c_n = 2|F_n|$

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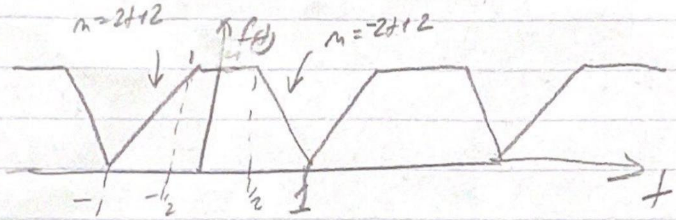
$\hookrightarrow c_2 = 2(0) = \boxed{0 = c_2}$

3.



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$T = 2 \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \omega_0 = \pi$ rad

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general integral: $\int t e^{-jn\pi t} = \frac{e^{-jn\pi t}}{n\pi}\left(j + \frac{1}{n\pi}\right)$

↪

2.2 3 / 3

- \+ **0 pts** Incorrect/ no solution
- ✓ + **1 pts** $$F_0 = \frac{3}{2}$$
- ✓ + **1 pts** $$F_1 = \frac{j}{\pi}$$
- ✓ + **1 pts** $$F_2 = 0$$
- \+ **0.5 pts** incorrect sign in one equation

(2)

2c) Compact Fourier Series: for real-valued functions only

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cdot \cos(n\omega_0 t + \theta_n)$$

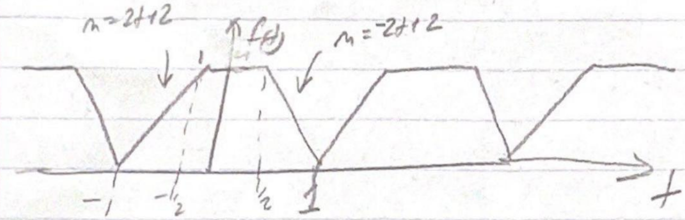
$$c_0 = 2F_0 \rightarrow c_0 = 2\left(\tfrac{3}{2}\right) = \boxed{3 = c_0}$$

$$c_n = 2|F_n|$$

$$\hookrightarrow \quad c_1 = 2\left(\tfrac{j}{\pi}\right) = \boxed{\tfrac{2j}{\pi} = c_1}$$

$$\hookrightarrow \quad c_2 = 2(0) = \boxed{0 = c_2}$$

3.



a) exponential Fourier Series of f(t) $\rightarrow f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$

$T = 2 \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \omega_0 = \pi$ rad

$$\hookrightarrow F_n = \frac{1}{T}\int_T f(t)\, e^{-jn\omega_0 t}\, dt \quad \begin{cases} 2t+2, & t \in (-1, -\tfrac{1}{2}) \\ 1, & t \in [-\tfrac{1}{2}, \tfrac{1}{2}) \\ -2t+2, & t \in [\tfrac{1}{2}, 1) \end{cases}$$

general integral: $\int t e^{-jn\pi t} = \frac{e^{-jn\pi t}}{n\pi}\left(j + \frac{1}{n\pi}\right)$

$\hookrightarrow$

2.3 **4 / 6**

✓ **- 2 pts** **$$c_1 = \frac{2}{\pi}$$**

(2)

2c) Compact Fourier Series: for real-valued functions only

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cdot \cos(n\omega_0 t + \theta_n)$$

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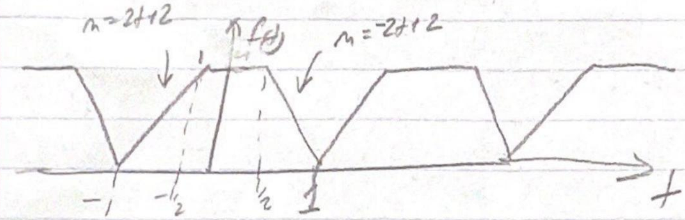

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general integral: $\int t e^{-jn\pi t} = \frac{e^{-jn\pi t}}{n\pi}\left(j + \frac{1}{n\pi}\right)$

$\hookrightarrow$

3a + cont.

$$f_n = \frac{1}{2}\left[\int_{-1}^{-1/2}(2t+2)e^{-jn\pi t}\,dt + \int_{-1/2}^{1/2} e^{-jn\pi t}\,dt + \int_{1/2}^{1}(-2t+2)e^{-jn\pi t}\,dt\right]$$

$$f_n = \left[\frac{e^{-jn\pi t}}{n\pi}\left(j(t+1) + \frac{1}{n\pi}\right)\right]_{-1}^{-1/2}$$

$$+ \frac{j}{2n\pi}\left[e^{-jn\pi t}\right]_{-1/2}^{1/2} + \left[\frac{e^{-jn\pi t}}{n\pi}\left(j(1-t) + \frac{1}{n\pi}\right)\right]_{1/2}^{1}$$

$$\hookrightarrow F_n \quad \frac{(1+j)}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{2}{n^2\pi^2}\cos(n\pi)$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{e^{-jn\pi t}}{n\pi}\left((1+j)\sin\left(\tfrac{1}{2}n\pi\right) - \frac{2\cos(n\pi)}{n\pi}\right)}$$

b) $g(t)$: $\underline{T = 4s} \rightarrow \omega_0 = \frac{2\pi}{T} \Rightarrow \underline{\frac{\pi}{2}\text{ rad/s} = \omega_0}$

$$G_n = \frac{1}{T}\int_T\left(g(t)\cdot e^{-jn\omega_0 t}\right)dt \qquad \begin{cases} 2t+4, & t\in[-2,-1) \\ 4, & t\in[-1,1) \\ -2t+4, & t\in[1,2) \end{cases}$$

$$g(t) = 4f\left(\tfrac{t}{2}\right)$$

$$\hookrightarrow \boxed{g(t) = \sum_{n=-\infty}^{\infty} \frac{4e^{-jnt\frac{\pi}{2}}}{n\pi}\left((1+j)\sin\left(\tfrac{1}{2}n\pi\right) - \frac{2\cos(n\pi)}{n\pi}\right)}$$

3.1 7.5 / 10

- ✓ **- 2 pts** Partially incorrect Fn
- ✓ **- 0.5 pts** Minus sign in exponent

3a + cond. (3)

$$f_n = \frac{1}{2}\left[\int_{-1}^{-\frac{1}{2}} (2t+2) e^{-jn\pi t} dt + \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-jn\pi t} dt + \int_{\frac{1}{2}}^{1} (-2t+2) e^{-jn\pi t} dt\right]$$

$$f_n = \left[\frac{e^{-jn\pi t}}{n\pi}\left(j(t+1) + \frac{1}{n\pi}\right)\right]_{-1}^{-\frac{1}{2}}$$

$$+ \frac{j}{2n\pi}\left[e^{-jn\pi t}\right]_{-\frac{1}{2}}^{\frac{1}{2}} + \left[\frac{e^{-jn\pi t}}{n\pi}\left(j(1-t) + \frac{1}{n\pi}\right)\right]_{\frac{1}{2}}^{1}$$

$$\hookrightarrow F_n = \frac{(1+j)}{n\pi} \sin\left(\frac{n\pi}{2}\right) - \frac{2}{n^2\pi^2} \cos(n\pi)$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{e^{-jn\pi t}}{n\pi}\left((1+j)\sin\left(\tfrac{1}{2}n\pi\right) - \frac{2\cos(n\pi)}{n\pi}\right)}$$

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$$G_n = \frac{1}{T}\int_T \left(g(t) \cdot e^{-jn\omega_0 t}\right) dt \qquad \begin{cases} 2t+4, & t \in [-2,-1) \\ 4, & t \in [-1,1) \\ -2t+4, & t \in [1,2) \end{cases}$$

$$g(t) = 4 f\left(\tfrac{t}{2}\right)$$

$$\hookrightarrow \boxed{g(t) = \sum_{n=-\infty}^{\infty} \frac{4e^{-jnt\frac{\pi}{2}}}{n\pi}\left((1+j)\sin\left(\tfrac{1}{2}n\pi\right) - \frac{2\cos(n\pi)}{n\pi}\right)}$$

3.2 **8 / 10**

✓ **- 2 pts** F_n not in equation/Incorrect F_n/No Constant Term

(4)

4. a) $\boxed{T = 3\text{seconds}} \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \boxed{\omega_0 = \frac{2\pi}{3} \text{ rad/s}}$

b) DC-term $\rightarrow f_0 =$ avg. $(f(t)) = \frac{1}{T}\int_T f(t)dt = \frac{1}{3}\int_0^3 f(t)dt = \frac{1}{3}(2\cdot 2) =$

$\hookrightarrow 2\cdot 2 = \frac{4}{3sec} \rightarrow \boxed{F_0 = \frac{4}{3}}$

c) $F_n = \frac{1}{T}\int_T f(t)\cdot e^{-jn\omega_0 t}dt = \frac{1}{3}\left[\int_0^2 (2)e^{-jn\frac{2\pi}{3}t}dt + \int_2^3 (0)e^{-jn\frac{2\pi}{3}t}dt\right]$

$\hookrightarrow f_1 = \frac{2}{3}\left[\int_0^2 e^{-jn\frac{2\pi}{3}t}dt\right] + \frac{1}{3}\left[\int_2^3 (0)e^{-jn\frac{2\pi}{3}t}dt\right]$

* General Integral, $\omega_0 = c/d$

$$\frac{1}{T}\int_a^b fe^{-jnt\left(\frac{c}{d}\right)}dt = \frac{jdf}{cnT}\cdot\left(e^{\frac{-jcbn}{d}} - e^{\frac{-jcan}{d}}\right)$$

$$F_1 = \frac{2}{3}\left(\frac{e^{-jn\frac{2}{3}\cdot\pi\cdot 2}}{-j\cdot\frac{2}{3}\pi} - \frac{e^{-j\cdot\frac{2}{3}\cdot\pi\cdot 0}}{-j\cdot\frac{2}{3}\cdot\pi}\right) = \frac{2}{3}\left(\frac{e^{-j\frac{4\pi}{3}} - 1}{-j\cdot 2\pi/3}\right)$$

$\downarrow$

$$f_1 = \frac{2}{3}\left(\frac{-\frac{3}{2} + j\sqrt{3}/2}{-2\pi j/3}\right) \rightarrow \boxed{F_1 = \frac{-\sqrt{3} - 3j}{2\pi}}$$

4.1 3 / 3

✓ **- 0 pts** Correct

(4)

4. a) $\boxed{T = 3\text{seconds}} \Rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \boxed{\omega_0 = \frac{2\pi}{3}\text{ rad/s}}$

b) DC-term $\rightarrow f_0 =$ avg. $(f(t)) = \frac{1}{T}\int_T f(t)dt = \frac{1}{3}\int_0^3 f(t)dt = \frac{1}{3}(2 \cdot 2) =$

$\hookrightarrow \frac{2 \cdot 2 = 4}{3\text{sec}} \rightarrow \boxed{F_0 = \frac{4}{3}}$

c) $F_n = \frac{1}{T}\int_T f(t) \cdot e^{-jn\omega_0 t}dt = \frac{1}{3}\left[\int_0^2 (2) e^{-jn\frac{2\pi}{3}t}dt + \int_2^3 (0) e^{-jn\frac{2\pi}{3}t}dt\right]$

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* General Integral, $\omega_0 = c/d$

$\frac{1}{T}\int_a^b f e^{-jnt\left(\frac{c}{d}\right)}dt = \frac{jdf}{cnT} \cdot \left(e^{\frac{-jcbn}{d}} - e^{\frac{-jcan}{d}}\right)$

$f_1 = \frac{2}{3}\left(\frac{e^{-jn\frac{2}{3}\cdot\pi\cdot 2}}{-j\cdot\frac{2}{3}\pi} - \frac{e^{-j\cdot\frac{2}{3}\cdot\pi\cdot 0}}{-j\cdot\frac{2}{3}\cdot\pi}\right) = \frac{2}{3}\left(\frac{e^{-j\frac{4\pi}{3}} - 1}{-j\cdot 2\pi/3}\right)$

$\downarrow$

$f_1 = \frac{2}{3}\left(\frac{-3/2 + j\sqrt{3}/2}{-2\pi j/3}\right) \rightarrow \boxed{F_1 = \frac{-\sqrt{3} - 3j}{2\pi}}$

4.2 2 / 2

✓ **- 0 pts** Correct

(4)

4. a) $\boxed{T = 3\text{seconds}} \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \boxed{\omega_0 = \frac{2\pi}{3} \text{ rad/s}}$

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* General Integral , $\omega_0 = \frac{c}{d}$

$$\frac{1}{T}\int_a^b f e^{-jnt\left(\frac{c}{d}\right)} dt = \frac{jdf}{cnT} \cdot \left(e^{\frac{-jcbn}{d}} - e^{\frac{-jcan}{d}}\right)$$

$$F_1 = \frac{2}{3}\left(\frac{e^{-jn\frac{2}{3}\cdot\pi\cdot 2}}{-j\cdot\frac{2}{3}\pi} - \frac{e^{-j\cdot\frac{2}{3}\cdot\pi\cdot 0}}{-j\cdot\frac{2}{3}\cdot\pi}\right) = \frac{2}{3}\left(\frac{e^{-j\frac{4\pi}{3}} - 1}{-j\cdot 2\pi/3}\right)$$

$\downarrow$

$$f_1 = \frac{2}{3}\left(\frac{-\frac{3}{2} + j\sqrt{3}/2}{-2\pi j/3}\right) \rightarrow \boxed{F_1 = \frac{-\sqrt{3} - 3j}{2\pi}}$$

$-\pi + \arctan()$

$\begin{matrix} -a \\ -b \end{matrix} \rightarrow$ Q_3 $\begin{matrix} (-a) & +a \\ (-a) & +a \end{matrix}$ (5)

4c) $F_1 = \dfrac{-\sqrt{3} - 3j}{2\pi}$

$C_1 = 2 \cdot |F_1| = 2\sqrt{\left(\frac{\sqrt{3}}{2\pi} - \frac{3j}{2\pi}\right)\left(\frac{\sqrt{3}}{2\pi} + \frac{3j}{2\pi}\right)}$

$= 2\sqrt{\frac{3}{4\pi^2} + \frac{9}{4\pi^2}} = 2\sqrt{\frac{12}{4\pi^2}} = \boxed{\frac{2\sqrt{3}}{\pi} = C_1}$

$\angle\theta_1 = \angle F_1 = -\pi + \left(\frac{-3}{-\sqrt{3}}\right) \pm\pi$ Q_3

$a<0,\ b<0 \rightarrow -\pi + \arctan()$

$-\pi + \pi/3 = \boxed{-2\pi/3 = \theta_1}$

4d) $F_2 = 1/3\left(\int_0^2 2 \cdot e^{-j\frac{4\pi}{3}t}\,dt + \int_2^3 0 \cdot e^{-j\frac{4\pi}{3}t}\,dt\right)$

$= 2/3 \cdot \frac{j}{4\pi/3}\left(e^{-j\frac{4\pi}{3}\cdot 2} - e^{-j\frac{4\pi}{3}\cdot 0}\right) = 2/3 \cdot \left(\frac{e^{-j\frac{8\pi}{3}} - 1}{-j\frac{4\pi}{3}}\right)$

$= \dfrac{-\frac{1}{2} - j\frac{\sqrt{3}}{2}}{-j2\pi} \rightarrow \boxed{F_2 = \frac{\sqrt{3} - 3j}{4\pi}}$

$C_2 = 2 \cdot |F_2| = 2\sqrt{\left(\frac{\sqrt{3}}{4\pi} - \frac{3j}{4\pi}\right)\left(\frac{\sqrt{3}}{4\pi} + \frac{3j}{4\pi}\right)} = 2\sqrt{\frac{3+9}{16\pi^2}}$

$= 2\sqrt{\frac{3}{4\pi^2}} \Rightarrow \boxed{C_2 = \frac{\sqrt{3}}{\pi}}$

$\theta_2 = \angle F_2 = \tan^{-1}\left(\frac{-3/4\pi}{\sqrt{3}/4\pi}\right) = \arctan(-\sqrt{3}) \rightarrow \boxed{\theta_2 = -\pi/3}$

4.3 3 / 3

✓ **- 0 pts** Correct

$-\pi + \arctan()$

$\begin{matrix} -a \\ -b \end{matrix} \rightarrow$ Q_3 $\begin{array}{c|c} (-a) & +a \\ \hline (-a) & +a \end{array}$ 5

4c) $F_1 = \dfrac{-\sqrt{3} - 3j}{2\pi}$

$C_1 = 2 \cdot |F_1| = 2\sqrt{\left(\frac{\sqrt{3}}{2\pi} - \frac{3j}{2\pi}\right)\left(\frac{\sqrt{3}}{2\pi} + \frac{3j}{2\pi}\right)}$

$= 2\sqrt{\frac{3}{4\pi^2} + \frac{9}{4\pi^2}} = 2\sqrt{\frac{12}{4\pi^2}} = \boxed{\frac{2\sqrt{3}}{\pi} = C_1}$

$\angle\theta_1 = \angle F_1 = -\pi + \left(\frac{-3}{-\sqrt{3}}\right) \pm \pi$ Q_3

$a<0,\ b<0 \rightarrow -\pi + \arctan()$

$-\pi + \pi/3 = \boxed{-2\pi/3 = \theta_1}$

4d) $F_2 = \frac{1}{3}\left(\int_0^2 2 \cdot e^{-j\frac{4\pi}{3}t}\,dt + \int_2^3 0 \cdot e^{-j\frac{4\pi}{3}t}\,dt\right)$

$= \frac{2}{3} \cdot \frac{j}{4\pi/3}\left(e^{-j\frac{4\pi}{3}\cdot 2} - e^{-j\frac{4\pi}{3}\cdot 0}\right) = \frac{2}{3} \cdot \left(\frac{e^{-j\frac{8\pi}{3}} - 1}{-j\frac{4\pi}{3}}\right)$

$= \dfrac{-\frac{1}{2} - j\frac{\sqrt{3}}{2}}{-j2\pi} \rightarrow \boxed{F_2 = \frac{\sqrt{3} - 3j}{4\pi}}$

$C_2 = 2 \cdot |F_2| = 2\sqrt{\left(\frac{\sqrt{3}}{4\pi} - \frac{3j}{4\pi}\right)\left(\frac{\sqrt{3}}{4\pi} + \frac{3j}{4\pi}\right)} = 2\sqrt{\frac{3+9}{16\pi^2}}$

$= 2\sqrt{\frac{3}{4\pi^2}} \Rightarrow \boxed{C_2 = \frac{\sqrt{3}}{\pi}}$

$\theta_2 = \angle F_2 = \tan^{-1}\left(\frac{-3/4\pi}{\sqrt{3}/4\pi}\right) = \arctan(-\sqrt{3}) \rightarrow \boxed{\theta_2 = -\pi/3}$

4.4 3 / 3

✓ **- 0 pts** Correct

⑥

4e) $P_f = \frac{1}{T} \int_T |f(t)|^2 \, dt = \frac{1}{3} \left(\int_0^2 2^2 \, dt + \int_2^3 0^2 \, dt \right)$

$\hookrightarrow \frac{4}{3}(2-0) \Rightarrow \boxed{8/3 = P_f}$

5. $f(t) = \cos^4(t) = (\cos^2(t))^2 = ((\frac{1}{2}) \cdot (1 + \cos(2t)))^2$

$= \frac{1}{4} (\cos^2(2t) + 2\cos(2t) + 1) = \underline{\frac{1}{8}\cos(4t) + \frac{1}{2}\cos(2t) + \frac{3}{8} = f(t)}$

$f(t) = \frac{1}{8} \cdot \frac{1}{2}(e^{j4t} + e^{-j4t}) + \frac{1}{2} \cdot \frac{e^{j2t} + e^{-j2t}}{2} + \frac{3}{8}$

$= \underline{\frac{3}{8} + \frac{1}{4}e^{j2t} + \frac{1}{4}e^{-j2t} + \frac{1}{16}(e^{j4t} + e^{-j4t})}$ exponential

1) $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jnt} \rightarrow F_0 = \frac{3}{8}, F_2 = \frac{1}{4}, F_{-2} = \frac{1}{4}$

$F_4 = \frac{1}{16}, F_{-4} = \frac{1}{16}, F_n = 0$ for everything else

2) $f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nt) + b_n \sin(nt)$, $a_0 = \frac{3}{4}, a_2 = \frac{1}{2}$

$\hookrightarrow a_4 = \frac{1}{8}$, for everything else: $a_n = 0 = b_n$

3) $f(t) = \frac{1}{2}c_0 + \sum_{n=1}^{\infty} c_n \cdot \cos(nt + \theta_n)$ $\quad c_0 = \frac{3}{4} \quad c_4 = \frac{1}{8}$
$c_2 = \frac{1}{2}$

$\hookrightarrow$ for everything else
$\hookrightarrow c_n = 0 \quad \rightarrow \theta_n = 0$

4.5 3 / 3

✓ **- 0 pts** Correct

(6)

4e) $P_f = \frac{1}{T}\int_T |f(t)|^2\,dt = \frac{1}{3}\left(\int_0^2 2^2\,dt + \int_2^3 0^2\,dt\right)$

$\hookrightarrow \frac{4}{3}(2-0) \Rightarrow \boxed{\frac{8}{3} = P_f}$

5. $f(t) = \cos^4(t) = (\cos^2(t))^2 = ((\frac{1}{2})\cdot(1+\cos(2t)))^2$

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$f(t) = \frac{1}{8}\cdot\frac{1}{2}(e^{j4t} + e^{-j4t}) + \frac{1}{2}\cdot\frac{e^{j2t}+e^{-j2t}}{2} + \frac{3}{8}$

$= \underline{\frac{3}{8} + \frac{1}{4}e^{j2t} + \frac{1}{4}e^{-j2t} + \frac{1}{16}(e^{j4t} + e^{-j4t})}$ exponential

1) $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jnt} \rightarrow F_0 = \frac{3}{8},\ F_2 = \frac{1}{4},\ F_{-2} = \frac{1}{4}$

$F_4 = \frac{1}{16},\ F_{-4} = \frac{1}{16},\ F_n = 0$ for everything else

2) $f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos(nt) + b_n\sin(nt),\ a_0 = \frac{3}{4},\ a_2 = \frac{1}{2}$

$\hookrightarrow a_4 = \frac{1}{8}$, for everything else: $a_n = 0 = b_n$

3) $f(t) = \frac{1}{2}c_0 + \sum_{n=1}^{\infty} c_n\cos(nt+\theta_n)$ $\quad c_0 = \frac{3}{4}$ $\quad c_4 = \frac{1}{8}$

$c_2 = \frac{1}{2}$

$\hookrightarrow$ for everything else

$\hookrightarrow c_n = 0 \quad \rightarrow \theta_n = 0$

5.1 10 / 10

✓ **- 0 pts** Correct

5b) $y(t) = \left\{ 2 \cdot \frac{3}{8} + 2 \cdot \frac{1}{2} \cos(2t - 3), \quad \text{in } [-3, 3], \quad 0 \text{ for other cases} \right\}$

$H(\omega) = 2e^{-j\omega t_0}$

5.2 **15 / 15**

✓ **- 0 pts** Correct