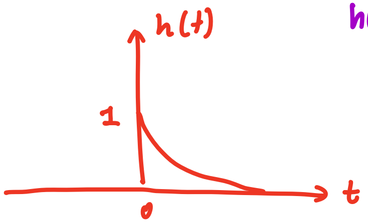
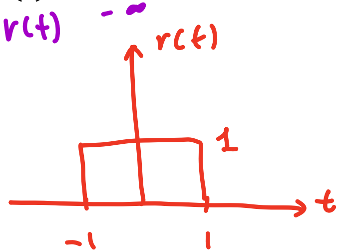
• Convolution-Example #4-cont

- Let $r(t) = rect(\frac{t}{2})$ and $h(t) = e^{-t}u(t)$
- Obtain $z(t) = r(t) * h(t)$ $= \int_{-\infty}^{\infty} h(\tau)\, r(t-\tau)\, d\tau$

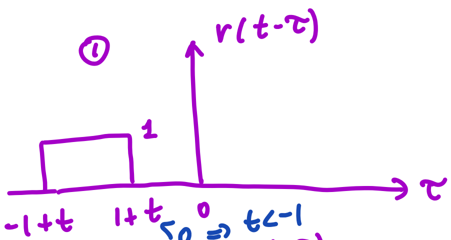
$h(t) * r(t)$

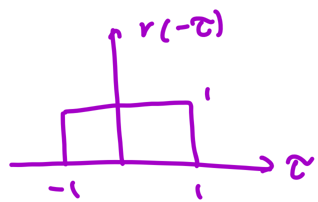




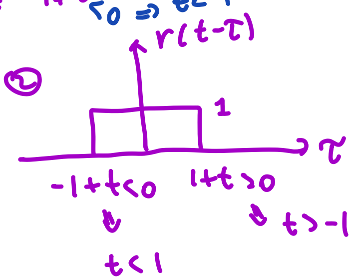
$$z(t) = \begin{cases} 0 & t<-1 \quad ① \\ 1-e^{-(t+1)} & -1<t<1 \\ e^{-(t-1)} - e^{-(t+1)} & t>1 \end{cases}$$

①

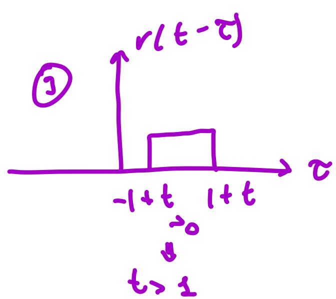




②



③



② $z(t) = \int_{0}^{1+t} e^{-\tau} \cdot (1)\, d\tau =$

$= 1 - e^{-(t+1)}$

$\underline{-1 < t < 1}$

③ $z(t) = \int_{-1+t}^{1+t} e^{-\tau} \cdot (1)\, d\tau =$

$= e^{-(t-1)} - e^{-(t+1)}$ $\quad \underline{t>1}$

- Convolution-Example #4-cont
  - Let $\mathrm{r(t)} = rect(\frac{t}{2})$ and $h(t) = e^{-t}u(t)$
  - Obtain $\mathrm{z}(t) = r(t) * h(t)$

$$z(t) = \underbrace{(1-e^{-(t+1)})u(t+1)}_{=0 \text{ for } t<-1} - \underbrace{(1-e^{-(t-1)})u(t-1)}_{=0 \text{ for } t<1}$$

$$z(t) = \begin{cases} 0 & t<-1 \\ 1-e^{-(t+1)} & -1<t<1 \\ e^{-(t-1)} - e^{-(t+1)} & \underline{t>1} \end{cases}$$

- Convolution – Properties-cont
  - Start-point:

If $f(t) = 0$ for $t < t_{s,f}$ and $h(t) = 0$ for $t < t_{s,h}$, then

$y(t) = f(t) * h(t) = 0$ for $t <?$ $\quad t_{s,y} = t_{s,f} + t_{s,h}$

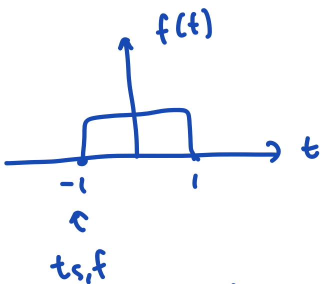
$y(t) = f(t) * h(t) = 0$

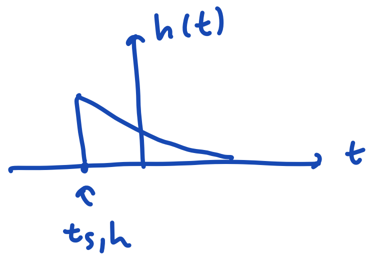
when

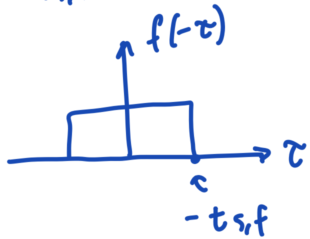
$-t_{s,f} + t < t_{s,h}$

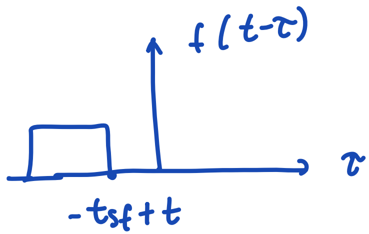
$\Downarrow$

$t < t_{s,h} + t_{s,f}$









- Convolution – Properties-cont
  - End-point:

    If $f(t) = 0$ for $t > t_{e,f}$ and $h(t) = 0$ for $t > t_{e,h}$, then

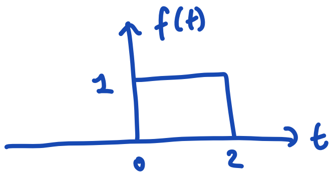
    $y(t) = f(t) * h(t) = 0$ for $t >?$ $t_{e,y} = t_{e,f} + t_{e,h}$

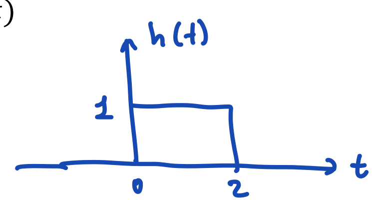
- Convolution – Properties-cont
  - Width:

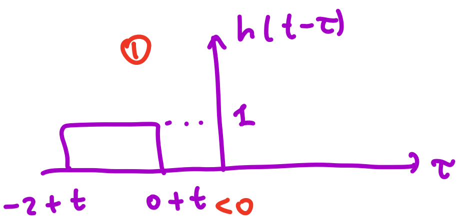
    If $f(t)$ has width T_f and $h(t)$ has width T_h then

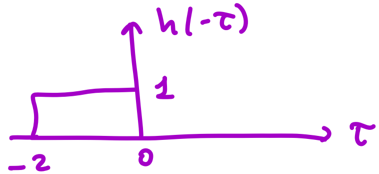
    $y(t) = f(t) * h(t)$ has width T_y =? $T_f + T_h$

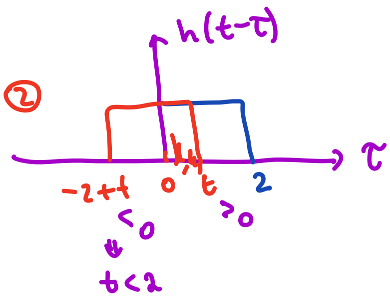
- Convolution-Example #5
  - Let $f(t) = rect(\frac{t-1}{2})$ and $h(t) = rect(\frac{t-1}{2})$
  - Obtain $y(t) = f(t) * h(t)$

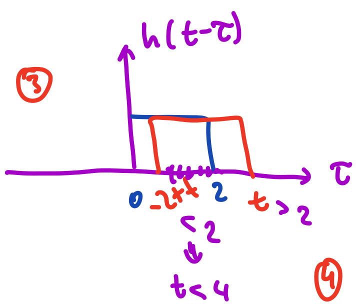


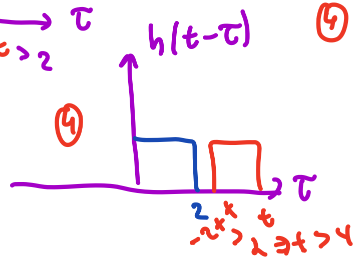












$t_{s,y} = t_{s,f} + t_{s,h} = 0+0=0$

$t_{e,y} = t_{e,f} + t_{e,h} = 2+2=4$

$T_y = T_f + T_h = 2+2=4$

$t<0$

① $y(t)=0$

$0<t<2$

② $y(t) = \int_0^t (1)(1)\,d\tau = t$

$2<t<4$

③ $y(t) = \int_{-2+t}^{2} (1)(1)\,d\tau = 4-t$

$t>4$

④ $y(t)=0$