Analog Signal Processing

Thursday, September 26, 8:45-10pm

Exam I

Last Name:	Solutions
First Name:	
UIN:	
netID	
Course: (circle one)	ECE210 ECE211
Section to return exam: (circle one)	11AM 12PM 2PM 3PM

Clearly PRINT your name in CAPITAL LETTERS.

This is a closed book and closed notes exam.

Calculators are not allowed.

To get full credit, please SHOW all your work and simplify your answers.

Write your final answers in the spaces provided.

All answers should INCLUDE UNITS whenever appropriate.

The exam is printed **double-sided**.

DO NOT write in these spaces.

Problem 1 (30 points):________

Problem 2 (25 points):________

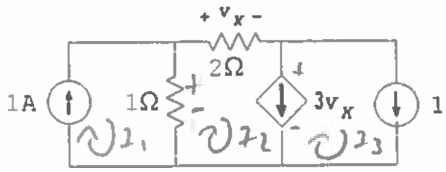
Problem 3 (25 points):________

Problem 4 (20 points):________

Total: (100 points):________

1. (30 pts) The two parts of this problem are unrelated.

(a) [15 pts] Consider the circuit below. Determine the voltage V_X and the absorbed power at the dependent source, P_{3V_X}.



$V_X =$ $-2/5$ V

$P_{3V_X} =$ $-48/25$ W

$I_1 = 1$

$I_3 = 1$

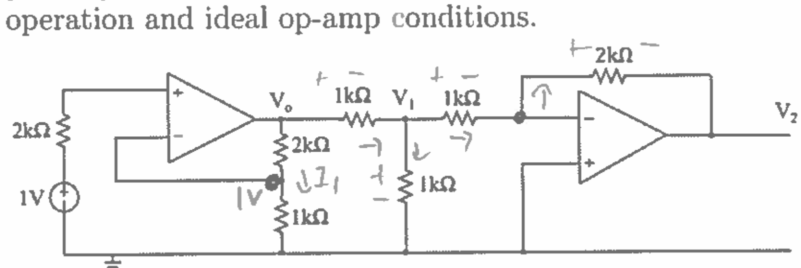
$3V_X = I_2 - I_3 = I_2 - 1 = 3(2I_2) = 6I_2 \Rightarrow 5I_2 = -1 \Rightarrow I_2 = \frac{-1}{5}$

$V_X = 2I_2$

$= 2\left(\frac{-1}{5}\right) = \frac{-2}{5}$

$P_{3V_X} = 3V_X\left[1(I_1 - I_2) - 2I_2\right] = 3\left(\frac{-2}{5}\right)\left[1 - \left(\frac{-1}{5}\right) - 2\left(\frac{-1}{5}\right)\right] = \frac{-6}{5}\left[\frac{5+1+2}{5}\right] = \frac{-48}{25}$

(b) [15 pts] Determine the voltages V_0, V_1 and V_2 in the following circuit, assuming linear operation and ideal op-amp conditions.



$V_0 =$ 3 V

$V_1 =$ 1 V

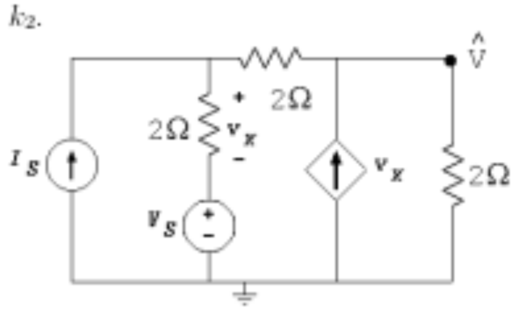
$V_2 =$ -2 V

$I_1 = \frac{1}{1K} \Rightarrow V_0 = (2K + 1K)I_1 = \frac{3K}{1K} = 3$

KCL @ V_1: $\frac{V_0 - V_1}{1K} = \frac{V_1 - 0}{1K} + \frac{V_1 - 0}{1K} \Rightarrow 3V_1 = V_0 \Rightarrow V_1 = 1V$

KCL @ ⊖ $\frac{V_1 - 0}{1k} = \frac{0 - V_2}{2k} \Rightarrow V_2 = -2V_1 = -2$

2. (25 pts) Consider the resistive circuit shown in the diagram below. Node voltage $\hat{V}$ can be expressed as $\hat{V} = k_1 V_S + k_2 I_S$. Use superposition to obtain the value of k_1 and the value of k_2.



suppress "I_S":

$$\text{KCL}: \quad \frac{V - V_S}{4} + \frac{V}{2} = V_x \quad \cdots (1)$$

$$\text{and} \quad V_x = \frac{V - V_S}{2} \quad \cdots (2)$$

solve (1) and (2) for V.

$$V = -V_S$$

$$k_1 = -1$$

suppress "V_S"

$$\text{KCL}: \quad I_S + \frac{V - V_x}{2} = \frac{V_x}{2} \quad \cdots (1)$$

$$V_x = \frac{V}{2} + \frac{V - V_x}{2} \quad \cdots (2)$$

solve (1) and (2) for V.

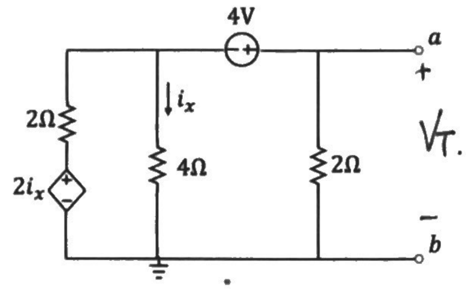
$$V = 6 I_S$$

$$k_2 = 6.$$

$K_1 = -1$ ______

$K_2 = 6$ ______

3. (25 pts) Consider the circuit below. Determine Thevenin's voltage, V_T, and Norton's current, I_N, between nodes a and b. Also draw the corresponding Thevenin equivalent circuit and Norton equivalent circuit.



$V_T =$ 2 V

$I_N =$ 2 A.

Supernode KCL @ Top.

$$\frac{(V_T-4)-2i_x}{2} + \frac{(V_T-4)-0}{4} + \frac{V_T}{2} = 0$$

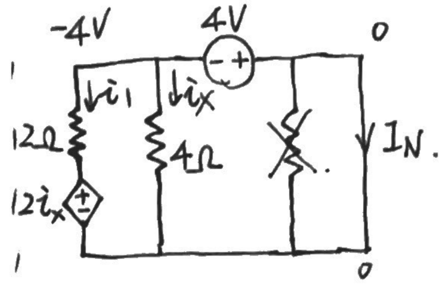
$$\frac{(V_T-4)-0}{4} = i_x$$

$$\frac{V_T-4}{2} - \cancel{\frac{V_T-4}{2}} + \cancel{\frac{V_T-4}{4}} + \frac{V_T}{2} = 0$$

$$\frac{V_T-4}{2} + \frac{V_T}{2} = 0$$

$\Rightarrow$ $V_T = 2V.$

$i_x = -\frac{1}{2}A.$



$$i_x = \frac{-4-0}{4} = -1A.$$

Voltage of $2i_x$ = $-2V$.

$$i_1 = \frac{-4-(-2)}{2} = -1A.$$

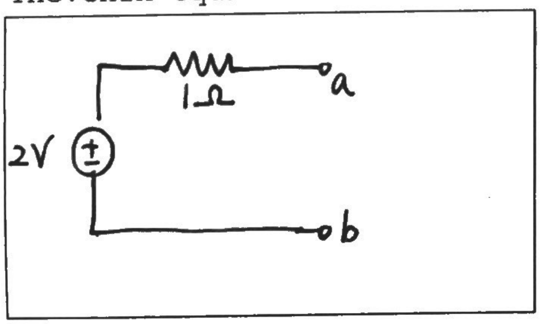
Supernode KCL @ Top

$I_N + i_1 + i_x = 0$

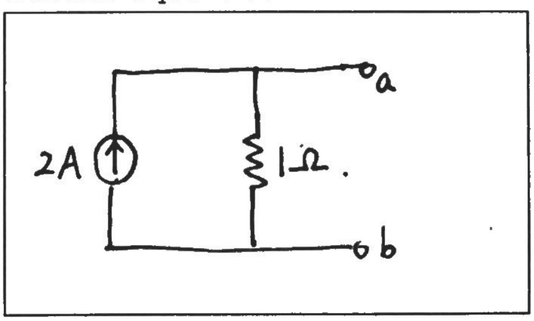
$I_N = -(i_1 + i_x) = 2A.$

$$R_T = \frac{V_T}{I_N} = 1\Omega.$$

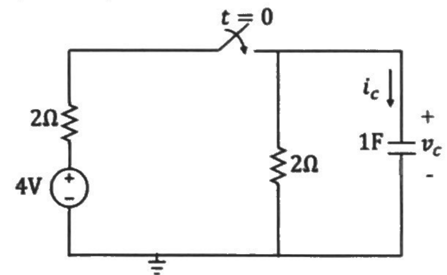
Thevenin equivalent circuit



Norton equivalent circuit

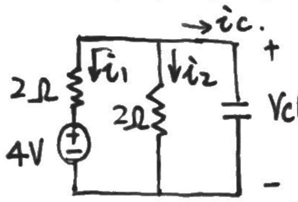


4. (20 pts) Consider the circuit below, assuming that ~~$v(0^-) = 2V$.~~ $V_c(0^-) = 1V$.



(a) [3 pts] Determine $i_C(0^+)$ and $v_C(0^+)$. Explain.

$V_c(0^+) = V_c(0^-) = 1V$. because voltage across capacitor doesn't jump (is continuous).



at $t = 0^+$. KCL @ Top node. $V_c(0^+) = 2V$ at top node

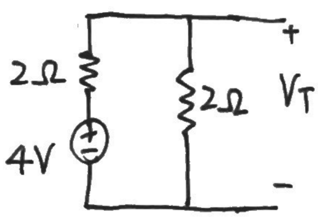
$$\frac{1-4}{2} + \frac{1-0}{2} + i_c(0^+) = 0$$

$i_c(0^+) = 1A$

$i_C(0^+) =$ 1A.

$v_C(0^+) =$ 1V.

(b) [2 pts] For $t > 0$, determine time constant . Show your work.

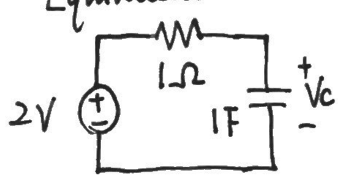


$V_T = \frac{2}{2+2} \cdot 4 = 2V$.

$R_T = 2 \| 2 = 1\Omega$.

$\tau = RC = 1s$.

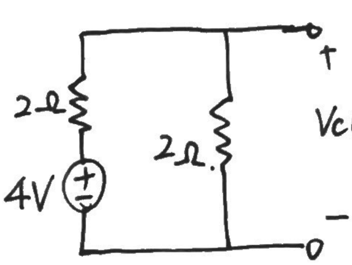
Equivalent ckt after $t > 0$.



$\tau =$ 1s.

(c) [3 pts] Determine the steady state values of i_C and v_C. Explain.

at steady state, capacitor behave like a open circuit



Use voltage divider rule

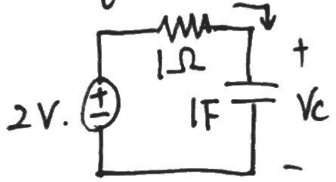
$V_c = 4 \cdot \frac{2}{2+2} = 2V$.

$i_c(\infty) = 0$ since open ckt $i_C(\infty) =$ 0 A

$v_C(\infty) =$ 2V

1. [8 pts] Express $v_C(t)$ for $t > 0$.

Equivalent ckt from problem 4.(b).



use KVL.

$$2 = R \cdot C \frac{dV_c}{dt} + V_c.$$

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_T. \quad \text{(or directly write ODE)}$$

$$\frac{dV_c}{dt} + V_c = 2.$$

$V_c(t) = Ae^{-t} + 2.$ with initial condition $V_c(0^+) = 1\,V$.

$V_c(0^+) = A + 2 = 1 \Rightarrow A = -1.$

$\therefore V_c(t) = -1e^{-t} + 2$ for $t > 0$

$v_C(t) =$ $-1e^{-t} + 2$ for $t > 0$.

(a) [4 pts] Below sketch $v_C(t)$ for $t > 0$. Label numerical values on the axes.