University of Illinois **Alvarez, Lee, Mironenko, Mironov**

Analog Signal Processing

Thursday, October 21, 8:45-10pm

Exam II

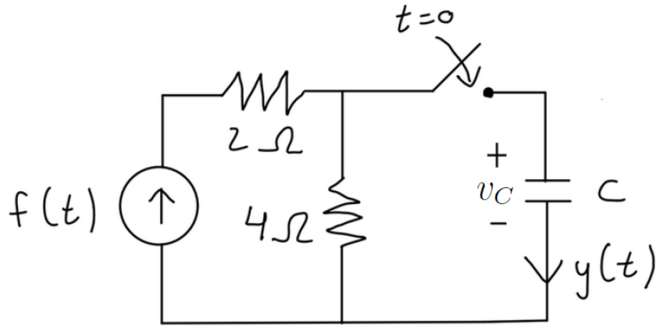
Last Name (capitalized):	

First Name:	

UIN:		**netID:**	

Clearly PRINT your name in CAPITAL LETTERS. This is a closed book and closed notes exam. Calculators are not allowed. To get credit, please SHOW all your work and simplify your answers. Write your final answers in the spaces provided. All answers should INCLUDE UNITS whenever appropriate. The exam is printed **double-sided**.	**DO NOT write in these spaces.** Problem 1 (25 points):________ Problem 2 (25 points):________ Problem 3 (25 points):________ Problem 4 (25 points):________ Total: (100 points):________

1. (25 pts) Consider the LTI circuit below with current $f(t) = \cos(\omega t) + 3\sin(\omega t)$ A and voltage $v_C(0^-) = v_0$ V.



It is known that the current $y(t) = 2\cos(2t) + \sin(2t) + Be^{-2t}$ A for $t > 0$.

(a) [5 pts] Write the ODE that governs this LTI system for $t > 0$ in terms of $f(t)$, C, $v_c(t)$ and ω.

ODE = ____ .

(b) [3 pts] Determine the value of C = ______________

(c) [3 pts] Determine the value of ω = ______________

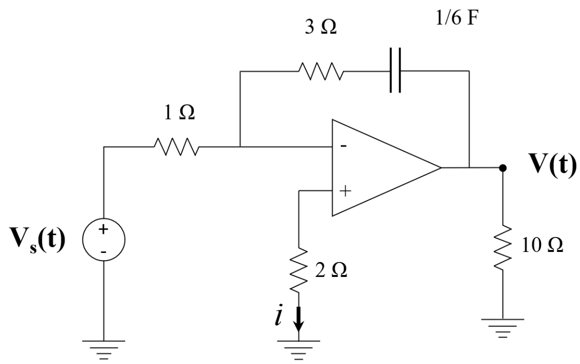
(d) [4 pts] If $v_0 = 0$V, determine the value of B = ____________

(e) [4 pts] Determine steady-state current phasor Y = __________________

(f) [5 pts] If instead of $f(t) = \cos(\omega t) + 3\sin(\omega t)$ A, the input current source in this LTI system is given by $f_1(t) = 2\cos(\omega t - 1) + 6\sin(\omega t - 1)$ A, determine the steady-state response, $y_1(t)$.

$y_1(t)$ = ____

2. (25 pts) The output voltage in the circuit below is given by $V(t) = 4cos(2t)$. Using the phasor method and ideal op-amp approximations determine:



(a) [18 pts] The input voltage $V_s(t)$ and express it in terms of real-valued functions only.

$V_s(t) =$ ____

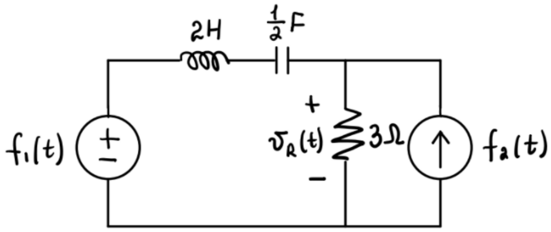
(b) [3 pts] The current in the 2Ω resistor and express it in terms of real-valued functions only .

$i_{2\Omega}(t) =$ ____ .

(c) [2 pts] The average absorbed power in the 1Ω resistor, $P_{1\Omega} =$____________________

(d) [2 pts] The average absorbed power in the capacitor, $P_C =$____________________

3. (25 points) Consider the circuit below, where the output $y(t) = v_R(t)$.



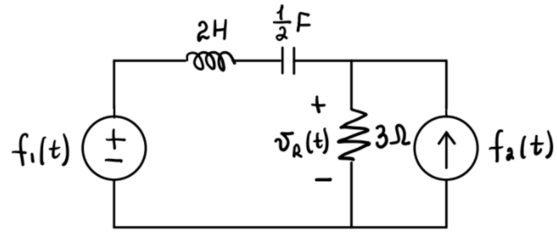
(a) [6 pts] Determine the frequency response $H_1(\omega)$, considering only the input voltage source, $f_1(t)$.

$H_1(\omega) =$ ________________________

(b) [5 pts] Determine the steady-state output voltage $v_{R_1}(t)$ given $f_1(t) = 3\cos(2t) + 2\sin\left(\frac{t}{2}\right)$ V.

$v_{R_1}(t) =$ ________________________

Recall



(c) [6 pts] Determine the frequency response $H_2(\omega)$, considering only the input current source, $f_2(t)$.

$H_2(\omega) =$ ______________________________

(d) [5 pts] Determine the steady-state output voltage $v_{R_2}(t)$ given $f_2(t) = 3\cos(2t) + 2\sin\left(\frac{t}{2}\right)$ A.

$v_{R_2}(t) =$ ______________________________

(e) [3 pts] Determine the steady-state output voltage $v_R(t)$.

$v_R(t) =$ ______________________________

4. (25 points) The two parts of this problem are unrelated.

(a) Let the periodic signal $x(t)$ be defined over a period as

$$x(t) = \begin{cases} 2t+4 & -2 < t \leq -1 \\ 2 & -1 < t \leq 1 \\ -2t+4 & 1 < t \leq 2 \\ 0 & 2 < t \leq 4 \end{cases}$$

.

i. [2 pts] The period of $x(t)$ is T = __________________

ii. [3 pts] The average value of $x(t)$ is x_{av} = __________________

iii. [2 pts] Now let us define a different periodic signal $y(t) = dx(t)/dt$. Express $y(t)$ as a piecewise function like it is done for $x(t)$ above.

$y(t) =$ ____

iv. [4 pts] Determine the Fourier coefficients Y_n for all n. You do not need to simplify.

$Y_n =$ ____

v. [4 pts] Use the derivative property to determine the Fourier coefficients X_n for all n.

$X_n =$ ____

(b) Consider the function $q(t) = 2sin(2t) + 3cos(4t)$,

i. [2 pts] Its fundamental frequency is $\omega_0 =$ ________________

ii. [5 pts] Express the function in exponential Fourier series form.

$q(t) =$ ____

.

iii. [3 pts] Its average power is $P_q =$ ________________

You may use this sheet for additional calculations but **do not** separate this sheet from the rest of the exam.

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$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\|F_n\|$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\,f(t) \leftrightarrow K\,F_n$
2	Addition	$f(t) \leftrightarrow F_n$, $g(t) \leftrightarrow G_n$, $\ldots$	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \|f(t)\|^2\,dt = \sum_{n=-\infty}^{\infty} \|F_n\|^2$

Table 2: Fourier series properties