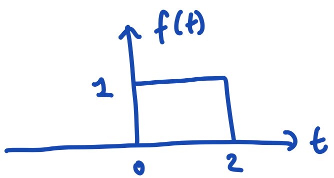
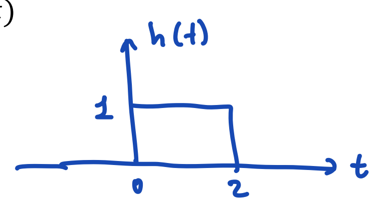
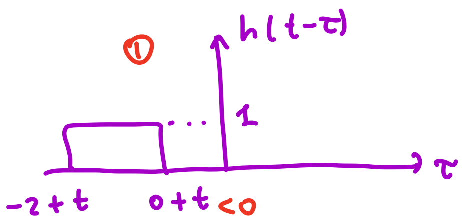
• Convolution-Example #5

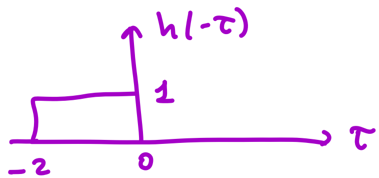
Table 9.2 $rect(\frac{t}{T}) * rect(\frac{t}{T}) = T\Delta(\frac{t}{2T})$

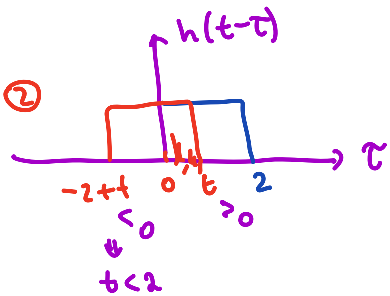
- Let $f(t) = rect(\frac{t-1}{2})$ and $h(t) = rect(\frac{t-1}{2})$
- Obtain $y(t) = f(t) * h(t)$

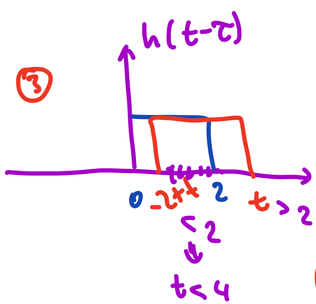


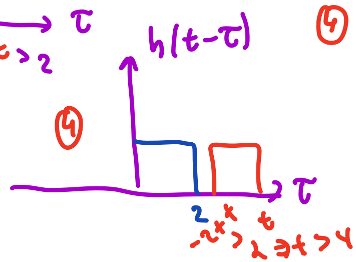








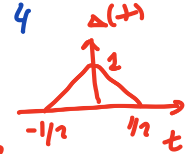




$t_{s,y} = t_{s,f} + t_{s,h} = 0+0 = 0$

$t_{e,y} = t_{e,f} + t_{e,h} = 2+2 = 4$

$T_y = T_f + T_h = 2+2 = 4$



$t<0$

① $y(t) = 0$

$0<t<2$

② $y(t) = \int_0^t (1)(1)d\tau = t$

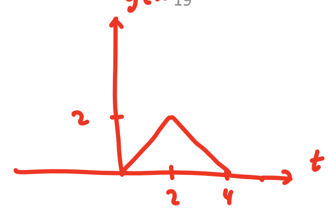
$2<t<4$

③ $y(t) = \int_{-2+t}^{2} (1)(1)\,d\tau = 4-t$

$t>4$

④ $y(t) = 0$

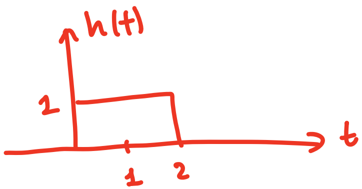
$y(t) = 2\Delta\left(\frac{t-2}{4}\right)$

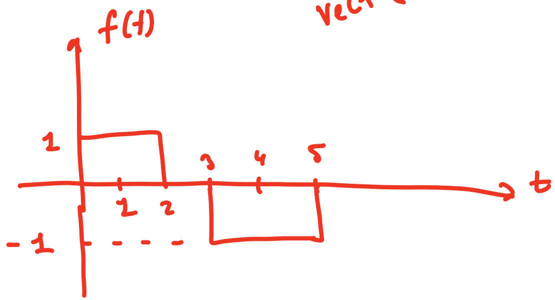


- Convolution-Example #6
  - Let $f(t) = rect\left(\frac{t-1}{2}\right) - rect\left(\frac{t-4}{2}\right)$ and $h(t) = rect\left(\frac{t-1}{2}\right)$
  - Obtain $z(t) = f(t) * h(t)$

Table 9.2

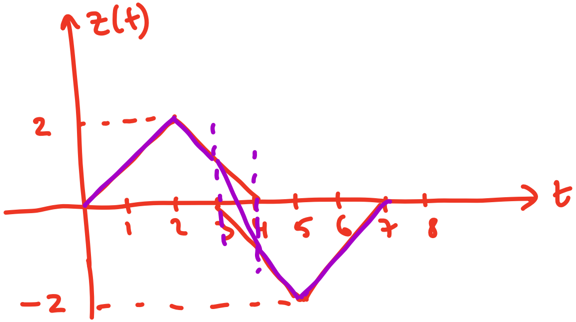
$$rect\left(\frac{t}{T}\right) * rect\left(\frac{t}{T}\right) = T\Delta\left(\frac{t}{2T}\right)$$





$$z(t) = rect\left(\frac{t-1}{2}\right) * rect\left(\frac{t-1}{2}\right) -$$

$$- rect\left(\frac{t-1}{2}\right) * rect\left(\frac{t-4}{2}\right) = 2\Delta\left(\frac{t-2}{4}\right) - 2\Delta\left(\frac{t-5}{4}\right) = z(t)$$



- Impulse
  - Recall that

$$f(t) + 0 = f(t)$$

$$f(t)(1) = f(t)$$

$$f(t)/1 = f(t)$$

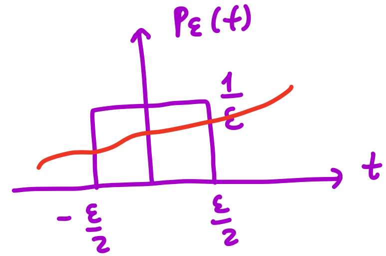
  - Is there a function *p(t)* such that

$$f(t) * p(t) = f(t)$$

  for all *f(t)*?

- Impulse-cont
  - Consider

$$p_\epsilon(t) = \frac{1}{\epsilon} rect(\frac{t}{\epsilon})$$



  - then

$$y(t) = f(t) * p_\epsilon(t) = ?$$

$$y(t) = f(t) * p_\varepsilon(t) = \int_{-\infty}^{\infty} p_\varepsilon(\tau)\, f(t-\tau)\, d\tau = \int_{-\frac{\varepsilon}{2}}^{\varepsilon/2} f(t-\tau) \cdot \frac{1}{\varepsilon}\, d\tau \approx$$

$\varepsilon \to 0$

$$\approx \frac{1}{\varepsilon} \int_{-\frac{\varepsilon}{2}}^{\varepsilon/2} f(t)\, d\tau = \frac{1}{\varepsilon} f(t) \int_{-\frac{\varepsilon}{2}}^{\varepsilon/2} d\tau = f(t)$$

$$\lim_{\varepsilon \to 0} f(t) * p_\varepsilon(t) = f(t)$$

• Impulse-cont

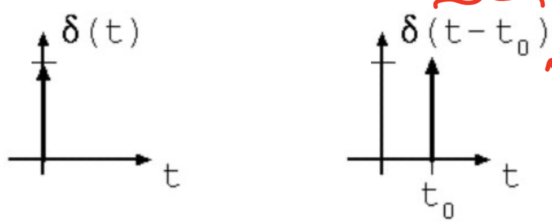
delta, Dirac delta

- $\delta(t)$ is the limit of

$$p_\epsilon(t) \; as \; \epsilon \longrightarrow 0$$

- However, $\delta(t)$:
  - is not a function
  - does not have a numerical interpretation
  - it is defined in terms of how it interacts with other functions through convolution
  - it is a distribution

=0 => @ t=to



↑ Delta is located where the argument is 0

- Impulse-cont
  - How much energy does the impulse have?

$W_{p_{\epsilon(t)}} = ?$ $\int_{-\infty}^{\infty} |p_\varepsilon(t)|^2 dt = \int_{-\varepsilon/2}^{\varepsilon/2} \left(\frac{1}{\varepsilon}\right)^2 dt = \frac{1}{\varepsilon^2}\int_{-\varepsilon/2}^{\varepsilon/2} dt = \frac{1}{\varepsilon} \xrightarrow[\varepsilon \to 0]{} \infty$

- Impulse - Properties
  - Convolution:

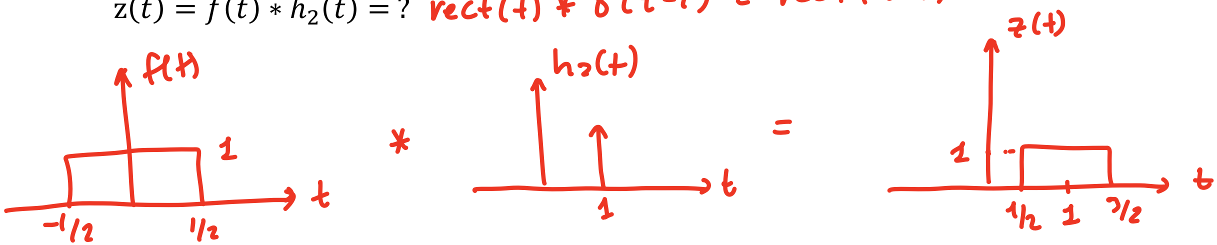
$$\underbrace{f(t) * \delta(t) = f(t)}$$

$$\underbrace{f(t) * \delta(t - t_0) = ?} \quad f(t - t_0)$$

Impulse - Properties - Examples

- Let $\mathrm{f(t)} = rect(t)$ and $h_2(t) = \delta(t-1)$

  $\mathrm{z}(t) = f(t) * h_2(t) = ?$ $rect(t) * \delta(t-1) = rect(t-1)$



- Let $\mathrm{f(t)} = rect(t)$ and $h_3(t) = \underbrace{\delta(1-t)} = \delta(-(t-1)) = \delta(t-1)$

  $\mathrm{x}(t) = f(t) * h_3(t) = ?$ $rect(t) * \delta(t-1) = rect(t-1)$

• Impulse - Properties-cont

$$y(t) = \underbrace{f(t) * \delta(t)}_{= f(t)} = \int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$

- Symmetry:

$$\delta(t) = \delta(-t)$$

$$y(0) = f(0) = \int_{-\infty}^{\infty} f(\tau)\,\delta(0-\tau)\,d\tau = \underbrace{\int_{-\infty}^{\infty} f(\tau)\,\delta(\tau)\,d\tau = f(0)}\ *$$

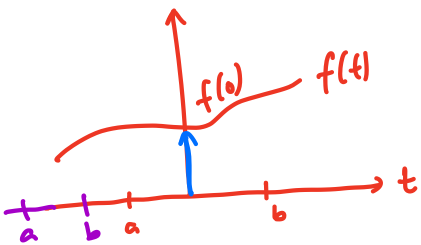
- Sifting:

at the location of δ

$$* \ \overbrace{\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt} = f(0)$$

@ $t=0$

$$\int_{-\infty}^{\infty} f(t)\,\underbrace{\delta(t-t_0)}_{@\ t=t_0}\,dt = f(t_0)$$

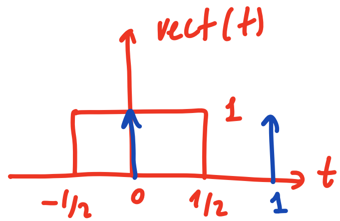


δ must be inside integral limits to get something

$$\int_a^b f(t)\,\delta(t)\,dt = \begin{cases} f(0) & a<0<b \\ 0 & \text{else} \end{cases}$$

• Impulse - Properties - Examples-cont

- Let $\mathrm{f(t)} = rect(t)$
- Determine



a) $\int_{-\infty}^{\infty} f(t)\delta(t)\, dt = f(0) = 1$

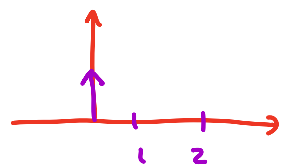
@ t=0

b) $\int_{-\infty}^{\infty} f(t)\delta(t-1)\, dt = f(1) = 0$

@ t=1

c) $\int_{1}^{2} f(t)\delta(t)\, dt = \cancel{f(0)} = 0$

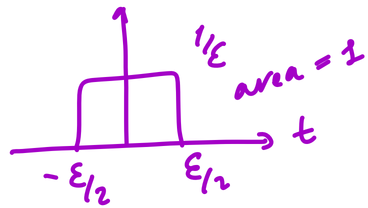
δ is outside of interval of integration



• Impulse – Properties-cont

- Area:

$$\int_{-\infty}^{\infty} (1)\ \delta(t)\ dt = 1$$



- Sampling:

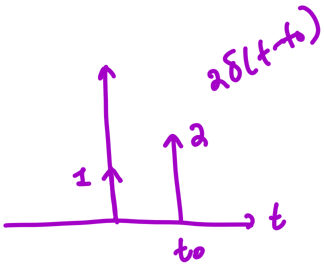
@ location of δ

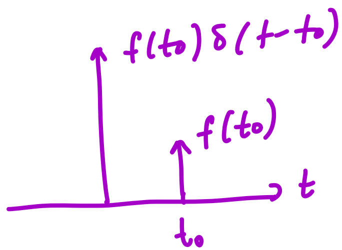
$$f(t)\delta(t) = f(0)\delta(t)$$

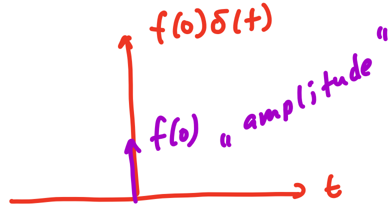
delta stays!

$$f(t)\delta(t-t_0) = ?$$

$$= f(t_0)\,\delta(t-t_0)$$







$$g(t) * \left(f(0)\,\delta(t)\right) =$$

$$= f(0)\left(g(t) * \delta(t)\right)$$

rescale convolution by f(0)

- Impulse - Properties - Examples-cont
  - Determine

a) $2\sin(t)\delta(t)$ $= 2\sin(0)\,\delta(t) = 0\,\delta(t) = 0$

@ $t=0$

a) $2\sin(t)\delta(t-\frac{\pi}{2})$ $= \underbrace{2\sin\left(\frac{\pi}{2}\right)}_{1}\,\delta\left(t-\frac{\pi}{2}\right) = 2\,\delta(t-\pi/2)$

@ $t=\frac{\pi}{2}$