Lecture 25, Friday, March 4, 2022

- LTI frequency response to multiple frequencies:

$$f(t) \;=\; \sum_n c_n \cos(\omega_n t + \theta_n) \;+\; \sum_k b_k \sin(\hat{\omega}_k t + \psi_k) \;+\; \sum_m F_m e^{j\tilde{\omega}_m t}$$

for real-valued $c_n, \; b_k, \; \omega_n, \; \hat{\omega}_k, \; \tilde{\omega}_m, \; \theta_n, \; \psi_k$ and complex-valued F_m.

$$y(t) \;=\; \sum_n c_n |H(\omega_n)| \cos(\omega_n t + \theta_n + \angle H(\omega_n))$$
$$+ \; \sum_k b_k |H(\hat{\omega}_k)| \sin(\hat{\omega}_k t + \psi_k + \angle H(\hat{\omega}_k))$$
$$+ \; \sum_m F_m H(\tilde{\omega}_m) e^{j\tilde{\omega}_m t}$$

  – Basically, if you have a sum of cosines, sines and complex exponentials, each one goes through on its own, and then the outputs all get added together.

- Decibel amplitude response

$$|H(\omega)|_{dB} \;=\; 10 \log_{10}\left(|H(\omega)|^2\right) \;=\; 20 \log |H(\omega)|$$

  – Small differences are emphasized

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- Periodic signals

  - A signal is periodic if there exists a time-shift t_o such that for all t,

  $$f(t) = f(t - t_o)$$

  - The smallest such delay is called the *period* of the signal, denoted by T.
  - The *fundamental frequency* of the signal is defined as $\omega_o = \frac{2\pi}{T}$, in rad/s.
  - T and ω_0 are inversely proportional to each other.

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