ECE 210 (AL2)

Chapter 9

Convolution, Impulse, Sampling, and Reconstruction

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Chapter objectives

- Understand what convolution represents
- Understand how to convolve two signals
- Understand and be able to apply properties of convolution
- Understand what an impulse represents
- Understand and be able to apply properties of the impulse
- Understand what the impulse response of an LTI system represents
- Understand Fourier Transforms of power signals
- Understand sampling and reconstruction
- Understand Nyquist sampling frequency and aliasing
- Understand the difference between sampling bandwidth and energy bandwidth

- Convolution

$$F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow Y(\omega) = F(\omega)H(\omega)$$

$$f(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = f(t) * h(t)$$

- Recall that

$$\underbrace{f(\overset{t-\tau}{t}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega \overset{t-\tau}{t}}d\omega}_{} \longrightarrow \overset{H(\omega)}{\boxed{\text{LTI}}} \longrightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{H(\omega)F(\omega)}_{Y(\omega)}e^{j\omega t}d\omega$$

- Can we do this strictly in the time domain?

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\left(\int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}d\tau\right)e^{j\omega t}d\omega =$$

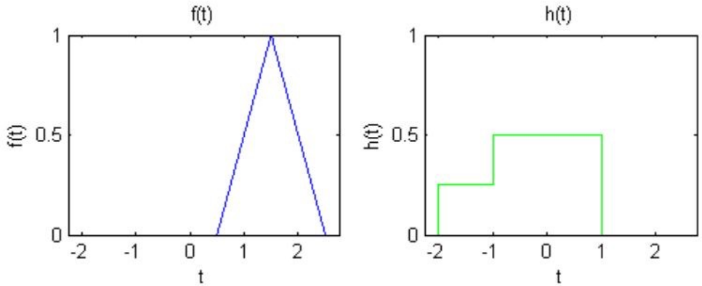
$$= \int_{-\infty}^{\infty} f(\tau)\underbrace{\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega(t-\tau)}d\omega\right)}_{\text{" } h(t-\tau)} d\tau =$$

convolution

$$= \boxed{\int_{-\infty}^{\infty} f(\tau)h(t-\tau)\,d\tau = y(t) = f(t) * h(t)}$$

- Convolution-Example #1

  - Consider f(t) and h(t) given below.



  - Let $y(t) = f(t) * h(t)$, and obtain $y(0)$.

$$y(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

look at $t=0$ $\Rightarrow$ $y(0) = \int_{-\infty}^{\infty} f(\tau)\, h(-\tau)\, d\tau$

- Convolution-Example #1-cont

$$y(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau \implies y(0) = \int_{-\infty}^{\infty} f(\tau)h(-\tau)d\tau = \frac{1}{4}$$

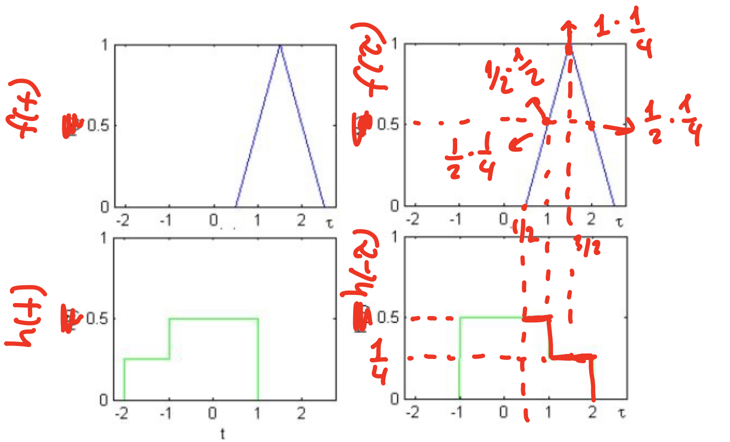
$$= \int_{-\infty}^{-1} f(\tau)\cdot 0 \, d\tau + \int_{-1}^{1} f(\tau)\cdot \frac{1}{2} \, d\tau + \int_{1}^{2} f(\tau)\cdot \frac{1}{4} \, d\tau + \int_{2}^{\infty} f(\tau)\cdot 0 \, d\tau$$

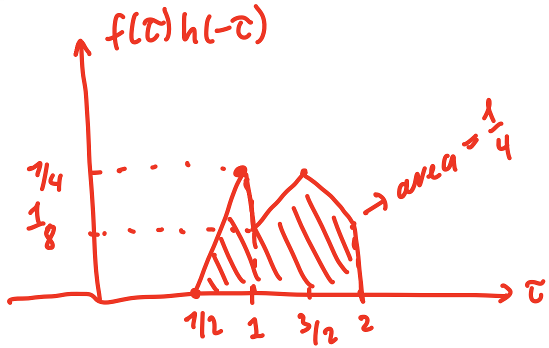
$h(-\tau)$

$\Downarrow$

$h(-\tau)$ is assigning weights to all of $f(\tau)$

$y(0)$ is a linear superposition of all of the values of f with different weights.



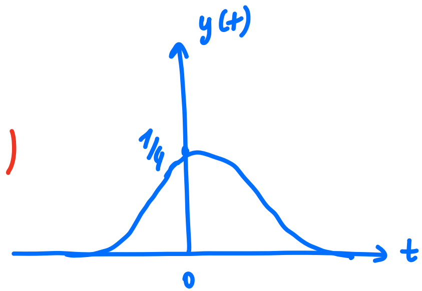


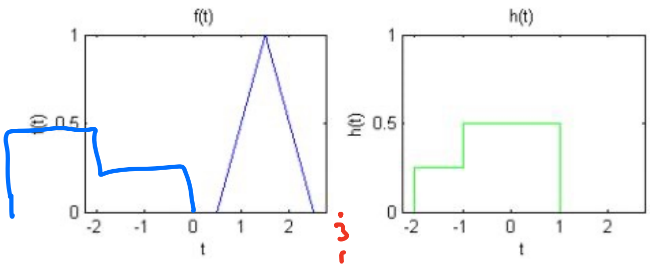
- Convolution-Example #1-cont
  - What about other values of *t*?

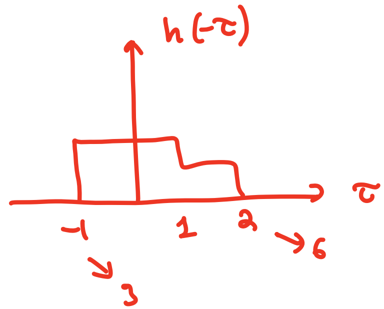
$$y(4) = \int_{-\infty}^{\infty} f(\tau)h(\underbrace{4-\tau}) d\tau = 0$$

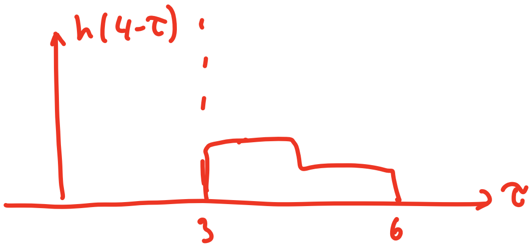
  - E.g., *t* = 4

$h(4-\tau) = h(-(\tau-4))$









h(4-τ) is giving f(τ) different weights then h(-τ) =>
the value of t in y(t) changes the location of the weights given by h(t)

- Convolution - Properties

Table 9.1

- Commutative:

$Y(\omega) = F(\omega)H(\omega) = H(\omega)F(\omega)$

$$y(t) = f(t) * h(t) = h(t) * f(t) =$$

$$= \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, f(t-\tau)\, d\tau$$

table 7.1

#13 : time convolution

multip. in freq. domains corresponds to a convolution in time domain

- Convolution-Example #2
  - Let $f(t) = u(t)$ and $h(t) = e^{-t}u(t)$
  - Obtain $y(t) = f(t) * h(t)$ $= \int_{-\infty}^{\infty} f(\tau)\, \underbrace{h(t-\tau)}_{"h(-(\tau-t))} \, d\tau$