Lecture 24, Tuesday, March 1, 2022

- Frequency response of LTI systems: $H(\omega)$

  - In general

    $$f(t) = Re\left\{Fe^{j\omega t}\right\} \rightarrow \boxed{\text{LTI}} \rightarrow y(t) = Re\left\{Ye^{j\omega t}\right\},$$

    then the phasors follow

    $$F \rightarrow \boxed{H(\omega)} \rightarrow Y = FH(\omega).$$

    * The output phasor is simply the product of the input phasor, F, and the frequency response of the LTI system, $H(\omega)$, hence

    $$H(\omega) = \frac{Y}{F} = \frac{\text{output phasor}}{\text{input phasor}}$$

- General first and second order filters

  - Lowpass

    $$|H(\omega)| \approx \frac{1}{\sqrt{1+\omega^2}} \qquad |H(0)| \approx 1 \qquad |H(\infty)| \approx 0 \qquad |H(\omega)| \searrow \text{ as } \omega \rightarrow \infty$$

  - Highpass

    $$|H(\omega)| \approx \frac{\omega}{\sqrt{1+\omega^2}} \qquad |H(0)| \approx 0 \qquad |H(\infty)| \approx 1 \qquad |H(\omega)| \nearrow \text{ as } \omega \rightarrow \infty$$

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- Bandpass

$$|H(\omega)| \approx \frac{\omega}{\sqrt{\omega^2 + (1-\omega^2)^2}} \quad |H(0)| \approx 0 \quad |H(\infty)| \approx 0 \quad |H(\omega)| \nearrow \text{ and then } \searrow \text{ as } \omega \to \infty$$

- $H(\omega)$ is only meaningful for dissipative LTI systems
- Properties of $H(\omega)$ for real-valued systems:
  - Conjugate symmetry

$$H(-\omega) = H^*(\omega)$$

  - Even amplitude response

$$|H(-\omega)| = |H(\omega)|$$

  - Odd phase response

$$\angle H(-\omega) = -\angle H(\omega)$$

  - Real-valued D.C. response

$$H(0) \in \mathbb{R}$$

  - Steady-state response to complex exponential

$$f(t) = Ae^{j(\omega t+\theta)} \quad \longrightarrow \boxed{H(\omega)} \longrightarrow \quad y(t) = H(\omega)Ae^{j(\omega t+\theta)}$$

- NOTE: correction in solutions to phase response in example # 3.

