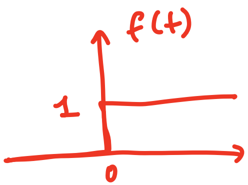
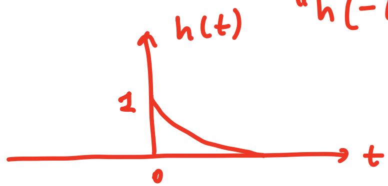
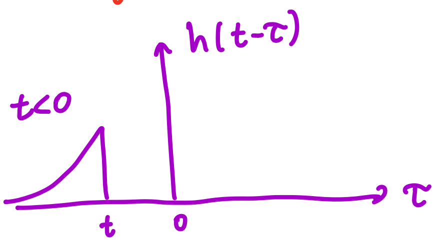
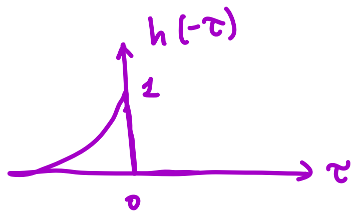
• Convolution-Example #2

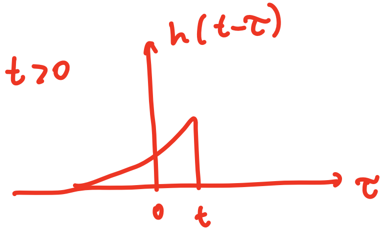
- Let $f(t) = u(t)$ and $h(t) = e^{-t}u(t)$
- Obtain $y(t) = f(t) * h(t)$ $= \int_{-\infty}^{\infty} f(\tau)\, \underbrace{h(t-\tau)}_{h(-(\tau-t))}\, d\tau$

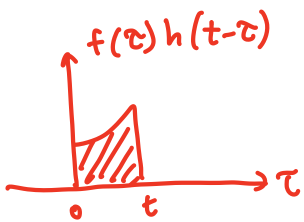












$\underline{t<0}$

$$y(t) = \int_{-\infty}^{\infty} 0 \, d\tau = 0$$

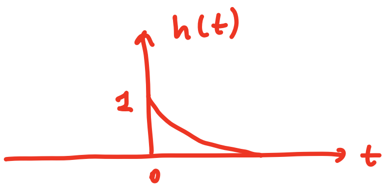
$\underline{t>0}$

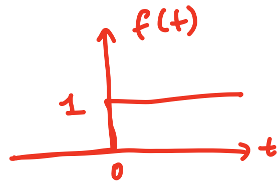
$$y(t) = \int_0^t (1)\, e^{-(t-\tau)}\, d\tau = \frac{e^{-(t-\tau)}}{1}\Bigg|_0^t = 1 - e^{-t}$$

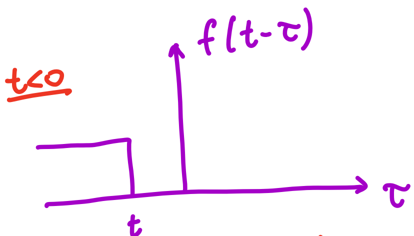
$$y(t) = \begin{cases} 0 & t<0 \\ 1-e^{-t} & t>0 \end{cases} = (1-e^{-t})\,u(t)$$

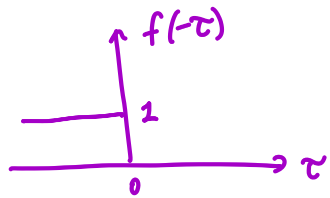
• Convolution-Example #2-cont

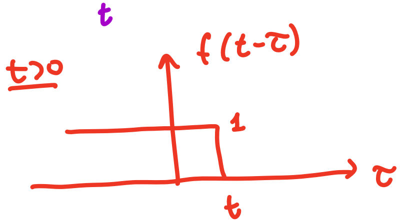
- Let $f(t) = u(t)$ and $h(t) = e^{-t}u(t)$
- Obtain $y(t) = f(t) * h(t)$ $= \int_{-\infty}^{\infty} h(\tau) f(t-\tau)\, d\tau$

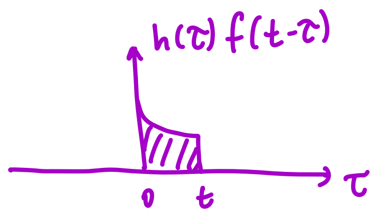


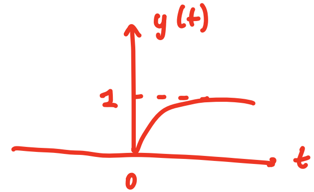












$\underline{t<0}$

$y(t) = 0$

$\underline{t>0}$

$$y(t) = \int_0^t e^{-\tau} \cdot (1)\, d\tau = $$

$$= 1 - e^{-t}$$

$$y(t) = (1 - e^{-t})\, u(t)$$

- Convolution – Properties-cont
  - Time shift:

Convolution is time invariant operation

$$\underbrace{y(t) = f(t) * h(t)}_{Y(\omega) = F(\omega)\cdot H(\omega)} \Longrightarrow \underbrace{f(t-t_0) * h(t)}_{F(\omega)e^{-j\omega t_0}\cdot H(\omega)} = ? \quad y(t-t_0) = f(t) * h(t-t_0)$$

$$F(\omega)e^{-j\omega t_0}\cdot H(\omega) =$$

$$= \underbrace{F(\omega)H(\omega)}_{Y(\omega)}e^{-j\omega t_0} = Y(\omega)e^{-j\omega t_0}$$

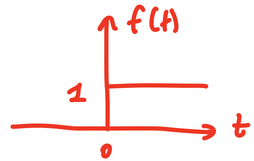
$$Y(\omega)e^{-j\omega t_0} \xrightarrow{\mathcal{F}^{-1}} y(t-t_0)$$

$$f(t-t_0) * h(t-t_1) = y(t-t_0-t_1)$$

• Convolution-Example #3

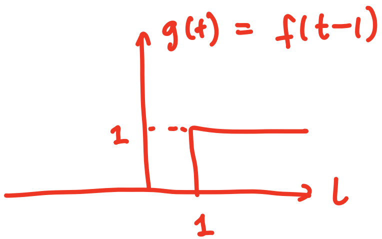
- Let $\mathrm{g(t)} = u(t-1)$ and $h(t) = e^{-t}u(t)$
- Obtain $\mathrm{x}(t) = g(t) * h(t)$ $= f(t-1) * h(t) = y(t-1) =$

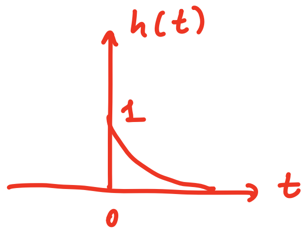
$$= (1-e^{-(t-1)})\,u(t-1)$$

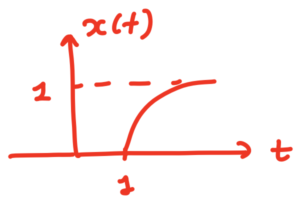


$f(t) * h(t) =$

$y(t) = (1-e^{-t})\,u(t)$







- Convolution – Properties-cont
  - Distributive:

$$f(t) * [g(t) + h(t)] = \underbrace{f(t) * g(t)}_{F(\omega)\cdot G(\omega)} + \underbrace{f(t) * h(t)}_{F(\omega)\cdot H(\omega)}$$

$\downarrow \mathcal{F}$ $\qquad \mathcal{F}^{-1}$

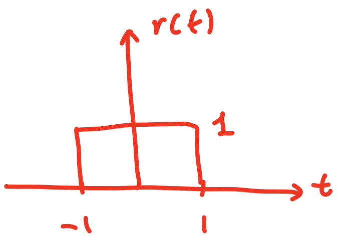
$$F(\omega)\cdot[G(\omega) + H(\omega)] = F(\omega)\cdot G(\omega) + F(\omega)\cdot H(\omega)$$

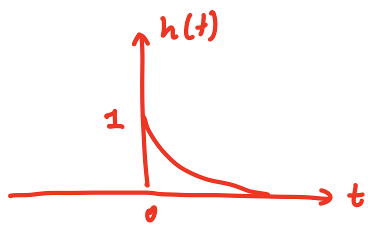
- Convolution-Example #4

$$u(t) * h(t) = (1-e^{-t})u(t) = y(t)$$

$u(t) = f(t)$

  - Let $r(t) = rect(\frac{t}{2})$ and $h(t) = e^{-t}u(t)$
  - Obtain $z(t) = r(t) * h(t) = [u(t+1) - u(t-1)] * h(t)$





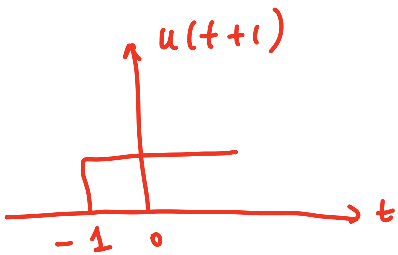
$$z(t) = \underbrace{u(t+1)}_{= f(t+1)} * h(t) - \underbrace{u(t-1)}_{f(t-1)} * h(t) =$$

$$= f(t+1) * h(t) - f(t-1) * h(t) =$$

$$= y(t+1) - y(t-1) =$$

$$= (1-e^{-(t+1)})u(t+1) - (1-e^{-(t-1)})u(t-1)$$

=



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