Lecture 47, Wednesday, April 20, 2022

- Laplace transform
  - BIBO stability: an LTIC system with impulse response $h(t)$ is BIBO stable *if and only if* $\hat{H}(s)$ has all of its poles on the left-half plane, where $\hat{H}(s)$ is a rational function $\frac{N(s)}{D(s)}$ in minimal form (after zero-pole cancellation).
  - The Laplace transform integral ignores the function before time zero, so two functions that are identical for $t \geq 0$ but different for $t < 0$ will have the same Laplace transform.
  - Properties of Laplace transform:
    * Note: $\longleftrightarrow$ applies to causal signals only, while $\longrightarrow$ applies to all signals.
    * Time-shift: if $f(t)$ is causal and $t_0 \geq 0$

$$g(t) = f(t - t_0) \overset{\mathcal{L}}{\longleftrightarrow} \hat{G}(s) = \hat{F}(s)e^{-st_0}$$

      This actually applies for $t_0 < 0$ if $g(t)$ is also causal.
  - Properties:
    * Time-derivative:

$$g(t) = \frac{d}{dt}f(t) \overset{\mathcal{L}}{\longrightarrow} \hat{G}(s) = s\hat{F}(s) - f\left(0^-\right)$$

      More generally,

$$x(t) = \frac{d^n}{dt^n}f(t) \overset{\mathcal{L}}{\longrightarrow} \hat{X}(s) = s^n\hat{F}(s) - s^{n-1}f\left(0^-\right) - s^{n-2}f'\left(0^-\right) - \cdots - f^{(n-1)}\left(0^-\right)$$

continued on next page....

  * Look at all the properties in the table, and pay attention to which apply to all signals and which apply only to causal signals

- Common inverse Laplace transform pairs:

$\delta(t) \leftrightarrow 1$
$e^{pt}u(t) \leftrightarrow \frac{1}{s-p}$
$te^{pt}u(t) \leftrightarrow \frac{1}{(s-p)^2}$
$t^n e^{pt}u(t) \leftrightarrow \frac{n!}{(s-p)^{n+1}}$

