ECE210 Laboratory
PRE-LAB#5

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Lab Section: ABH

Lab 5: Sampling, reconstruction, and software radio

Until this point, your study of signals and systems has concerned only the continuous-time case[1], which dominated the early history of signal processing. About 60 years ago, however, the development of the modern computer generated research interest in digital signal processing (DSP), a type of discrete-time signal processing. Although hardware limitations made most real-time DSP impractical at the time, the continuing maturation of the computer has been matched with a continuing expansion of DSP. Much of that expansion has been into areas previously dominated by continuous-time systems: our telephone network, medical imaging, music recordings, wireless communications, and many more.

You do not need to worry whether the time and effort you have invested in studying continuous-time systems will be wasted because of the growth of DSP — digital systems are practically always hybrids of analog and digital subsystems. Furthermore, many DSP systems are linear and time-invariant, meaning that the same analysis techniques apply, although with some modifications. In this lab, you will explore some of the parallels between continuous-time systems and DSP with a "software radio" designed to the same specifications as the receiver circuit you developed on your protoboard.

1 Prelab

Our software radio is typical of many DSP systems in that both the available input and required output are continuous-time signals. The conversion of a continuous-time input signal to a discrete-time signal is called **sampling** (or A/D conversion), and the conversion of a discrete-time signal to a continuous-time output signal is called **reconstruction** (or D/A conversion). As discussed in class, **samples** $f(nT)$ of a **band-limited** analog signal $f(t)$ can be used to reconstruct $f(t)$ exactly when the **sampling interval** T and **signal bandwidth** $\Omega = 2\pi B$ satisfy the **Nyquist criterion** $T < \frac{1}{2B}$.

This is illustrated by the hypothetical system shown in Figure 1, where the analog signal $f(t)$ defined at the output stage of a low-pass filter $H_1(\omega)$ has a bandwidth $\Omega = 2\pi B$ limited by the bandwidth $\Omega_1 = 2\pi B_1$ of the filter. A/D converter extracts the samples $f(nT)$ from $f(t)$ with a sampling interval of T, and D/A conversion of samples $f(nT)$ into an analog signal $y(t)$ can be envisioned as low-pass filtering of a hypothetical signal $f_T(t) = \sum_n f(nT)\delta(t - nT)$ using the filter $H_2(\omega)$. With an appropriate choice of $H_2(\omega)$, the system output $y(t)$ will be identical to $f(t)$ in all its details as long as $T < \frac{1}{2B_1}$. The reason for that can be easily appreciated after comparing the Fourier transforms $F(\omega)$ and $F_T(\omega)$ of signals $f(t)$ and $f_T(t)$ with the help of Figure 2.

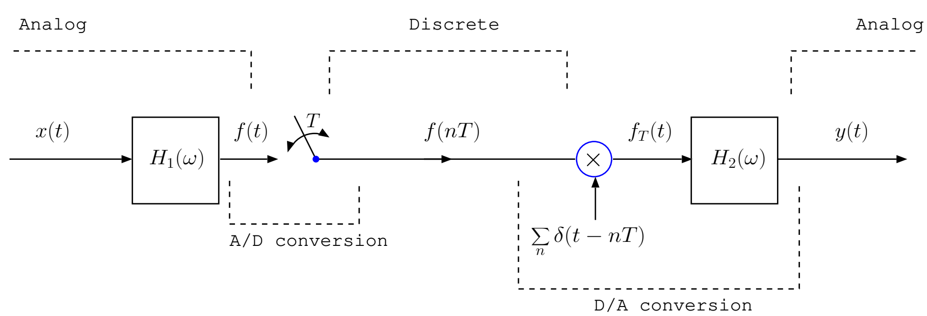


Figure 1: A conceptual system that samples a band-limited continuous-time signal $f(t)$ and reconstructs a continuous-time output $y(t)$. An A/D converter generates the samples $f(nT)$ of its analog input $f(t)$. An ideal D/A converter generates a signal $f_T(t) = \sum_n f(nT)\delta(t - nT)$ from samples $f(nT)$ and low-pass filters $f_T(t)$ with $H_2(\omega)$ to produce an analog $y(t)$. In a practical D/A converter the impulse train $\sum_n \delta(t - nT)$ is replaced by a practical pulse train $\sum_n p(t - nT)$ such that $p(t) * h_2(t)$ is a closed approximation of a delayed $\mathrm{sinc}(\frac{\pi}{T}t)$.

[1]The term "continuous time" is used generically to refer to signals that are functions of a continuous independent variable. Often that variable represents time, but it may instead represent distance, etc. "Discrete time" is used in the same way.

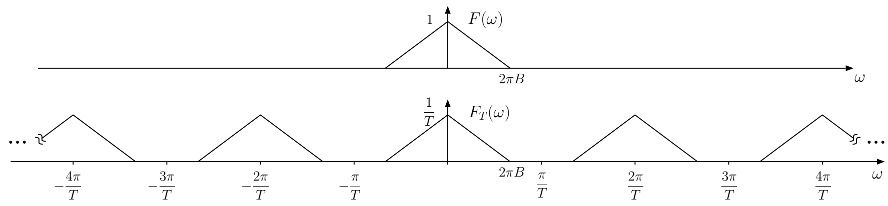


Figure 2: An example comparing the Fourier transforms of signals $f(t)$ and $f_T(t)$ defined in Figure 1. Since for $|\omega| < \frac{\pi}{T}$ the two Fourier transforms have the same shape, low-pass filtering of $f_T(t)$ yields the original analog signal $f(t)$. $F_T(\omega)$ is constructed as a superposition of replicas of $\frac{F(\omega)}{T}$ shifted in ω by all integer multiples of $\frac{2\pi}{T}$ (see item 25 in Table 7.2 in the text).

The following prelab exercises concern the system shown Figure 1. Assume that $T = \frac{1}{44100}$ s (i.e., the sampling frequency is $T^{-1} = 44100$ Hz) and signal $x(t)$ has a Fourier transform $X(\omega)$ shown in Figure 3.

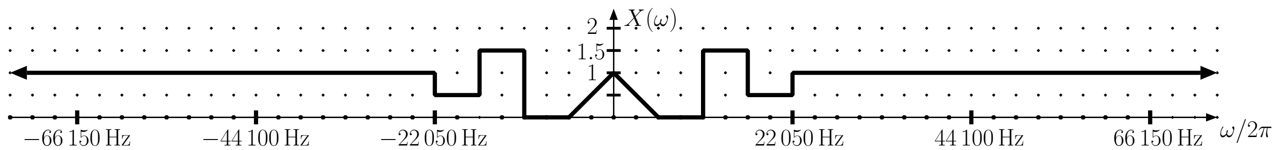


Figure 3: Fourier transform of $x(t)$ for prelab questions.

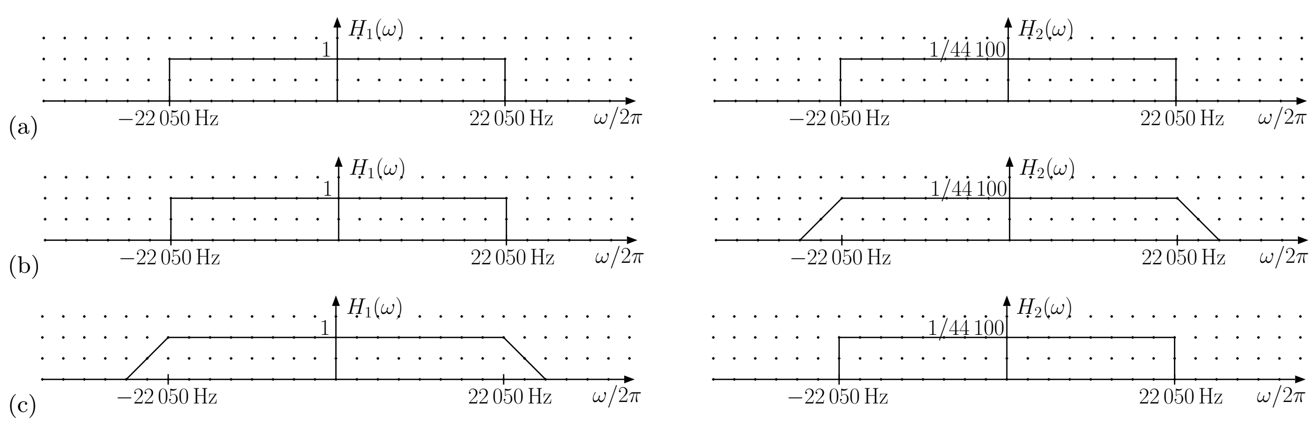
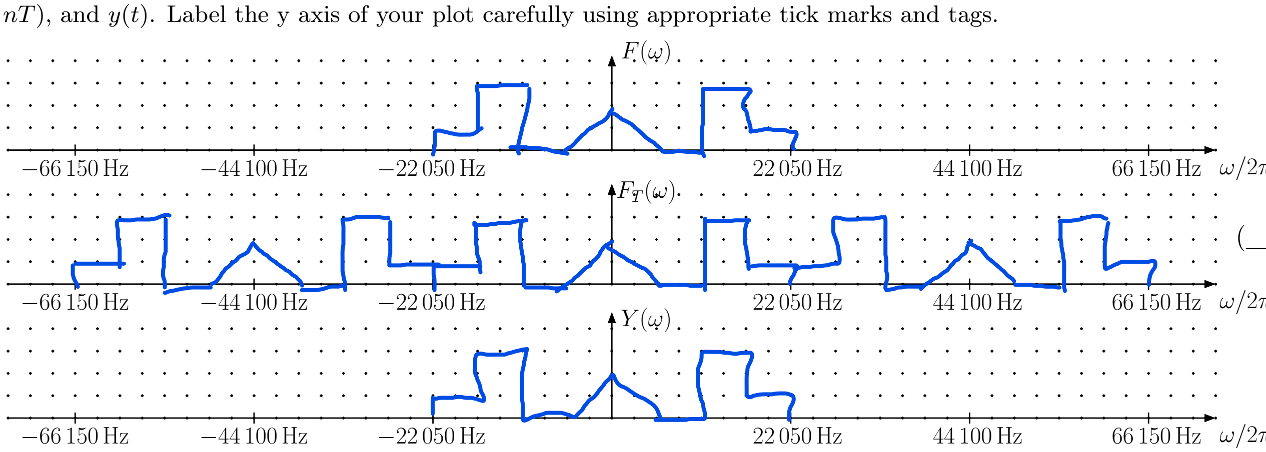


Figure 4: Filter frequency responses for the prelab problems.

1. For frequency responses $H_1(\omega)$ and $H_2(\omega)$ in Figure 4a, sketch the Fourier transform of $f(t)$, $f_T(t) = \sum_n f(nT)\delta(t-nT)$, and $y(t)$. Label the y axis of your plot carefully using appropriate tick marks and tags.



(____/4)

Is $y(t)$ a perfect reconstruction of $x(t)$? Explain your reasoning.

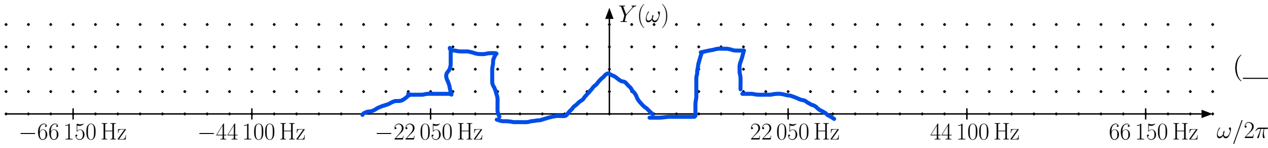
(____/2)

This is an imperfect reconstruction of x(t). Since X(w) does not equal Y(w), they are not exactly equal when transformed back into time-domain.

Is $y(t)$ a perfect reconstruction of $f(t)$? Explain your reasoning. (____/2)

y(t) is a perfect reconstruction of f(t). Since their frequency spectrums are identical, they are equal when transformed back.

2. Now suppose an ideal $H_1(\omega)$ but a non-ideal $H_2(\omega)$ given by Figure 4b. The signals $f(t)$ and $f_T(t)$ are unchanged, but sketch the Fourier transform of the new $y(t)$.



(____/1)

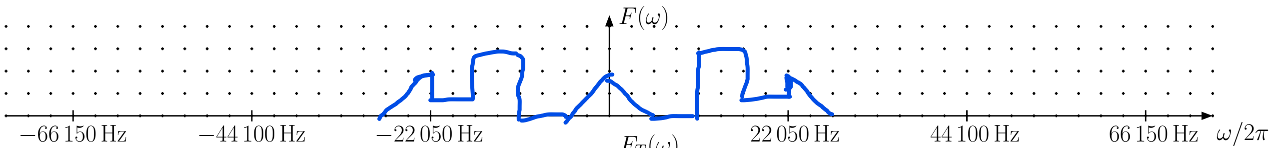
Is $y(t)$ a perfect reconstruction of $f(t)$? Explain your reasoning. (____/2)

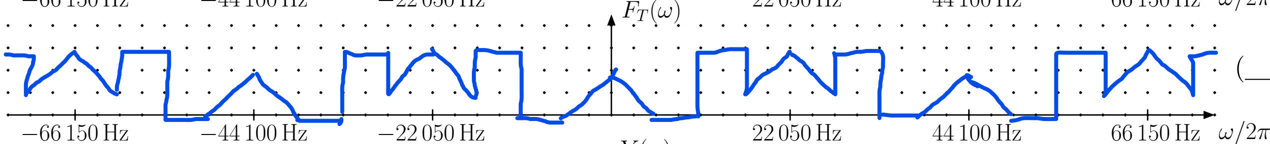
No, y(t) is not a perfect reconstruction of f(t). There will be additional frequencies not present in the original f(t). Therefore when the signals are transformed back, they will not match.

Is it still possible to have a perfect reconstruction of $f(t)$ based on the signal $y(t)$? Explain your reasoning. (____/2)

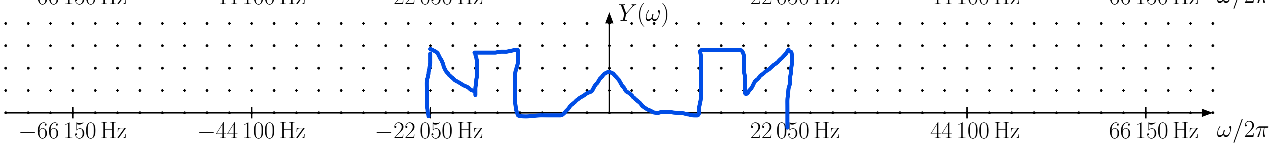
Yes, it is possible to have a perfecct reconstruction of f(t) based on the signal y(t). If we pass y(t) through an idea lowpass filter such as rect(w/44100), the signal will be recovered.

3. Now suppose a non-ideal $H_1(\omega)$ and an ideal $H_2(\omega)$ given by Figure 4c. Sketch the Fourier transform of $f(t)$, $f_T(t)$, and $y(t)$.





(____/4)



Is $y(t)$ a perfect reconstruction of $f(t)$? Explain your reasoning. (____/2)

No, y(t) is not a perfect reconstruction of x(t). Since X(w) does not equal Y(w), then when they are transformed back, there will be noticeable differences.

Is it still possible to have a perfect reconstruction of $f(t)$ based on the signal $y(t)$? Explain your reasoning. (____/2)

No, it is not still possible to have a perfect reconstruction of f(t) based on the signal of y(t). Since the aliasing changed the higher frequencies of Y(w) which were preserved, the signal will not be clean when transformed.

4. Discuss the role of the filter $H_1(\omega)$ in the system examined above. In what way does it impact the system output? (____/2)

H1(w) filters out the higher frequencies present in X(w). This system implementation results in a clean output such that the original input signal can be completely recovered.

5. Discuss the role of the filter $H_2(\omega)$ in the system examined above. In what way does it impact the system output? (____/2)

H2(w) is used to filter out the symmetric copies of Ft(w). This implementation also allows the original signal to be recovered.

2 Laboratory exercise

Lab 5 is a lab based on Jupyter notebook which gives you step-by-step access to python code.

2.1 Jupyter Notebook instruction

1. Go to `https://mybinder.org/`

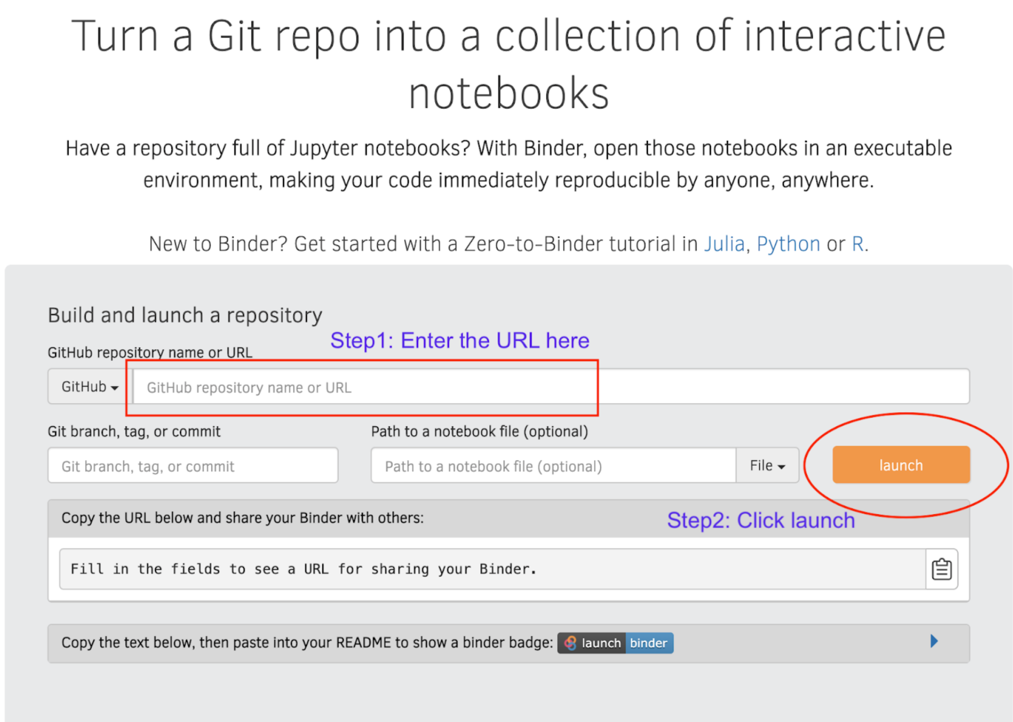


Figure 5: Instruction for using Jupyter Notebook in binder

2. You will see the webpage looks like Figure 5. Enter URL https://github.com/Krasus7/ECE-210-Lab-5 in the rectangle box,then click the icon “launch”.

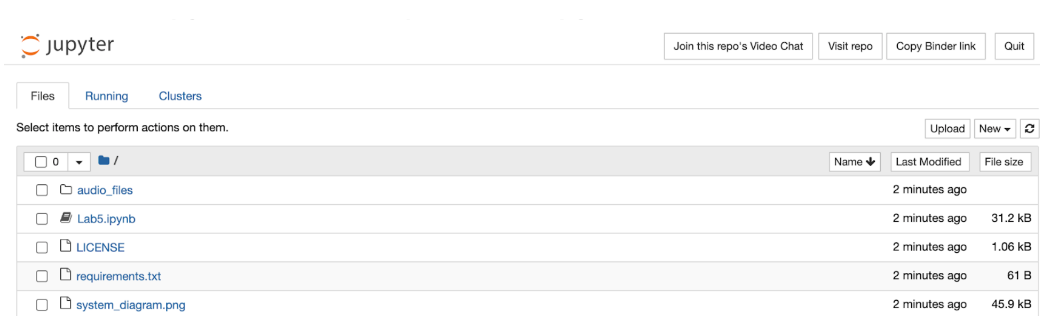


Figure 6: Instruction for opening terminal in Jupyter Notebook

3. The launching might take several minutes, then in the next page, click "Lab5.ipynb", this will open the Jupyter notebook.

4. Binder will automatically shut down user sessions that have more than 10 minutes of inactivity (if you leave your window open, this will be counted as "activity"). Otherwise it will last up to 12 hours. If you want to save your work, you may download "Lab5.ipynb" in the page shown in Figure 6. (Check the box next to the file and download option will appear)

2.2 Answer sheet

There are instructions and questions in the notebook you just opened, however, the final submission file should be this lab report only. Please fill your answers for the notebook in the following blanks.

1.1 Describe what you see in time domain of the sampled input signal (top right plot) (____/2)

In time domain, the graph appears to be a pure sinusoid. The wave amplitude is about 0.2 and has a period of about 0.55 ms.

1.2 Describe what you see in frequency domain of the ouput (bottom right plot) (____/2)

In the frequency domain, the peak frequency appears to be around 2kHz.

1.3 Does the output $y(nT)$ resemble the input signal $f(t)$? If not, then explain the difference. (____/2)

The output of y(nT) resembles in the input signal f(t). When the frequency is larger, it is difficult to discern the original signal from the sampled signal.

1.4 Describe what happens to the output signal in the time and frequency domain. (____/2)

The output signal approximately matches the input until the 20.5 kHz. The output frequency increases until almost 22.05 kHz, at which point it starts to decrease.

1.5 At 24 kHz, does the output $y(nT)$ resemble the input signal $f(t)$ or do we have an aliased component ? (____/2)

At 24 kHz, the output of y(nT) does not resemble the input signal f(t) because there is aliasing. This aliasing is due to the fact that the signal variation is under the Nyquist frequency of 44.1 kHz.

1.6 What is the significance of the frequency 22.05 kHz ? Remember the sampling rate is 44100 Hz. (____/4)

At 22.05 kHz, the signal frequency is the same as the Nyquist frequency for this system. Any frequencies larger that this will result in aliasing and an imperfect reconstruction.

1.7 Finally increase the input frequency from 24 to 30 kHz. What do you observe ? Explain what is happening. (____/4)

When increasing the input frequency from 24 to 30 kHz, we observe that multiple different sinusoids appear to be present in our sampled signal. This is due to us not sampling fast enough for the given input frequency.
This method results in a large amount of aliasing in the output.

2.1 What is the frequency of the output $y(t)$? 7.5 kHz (____/2)

2.2 Explain why do you have such output, when the input is a 1.5 kHz square wave. (Hint: Having in mind the analysis of the perodic square wave done in lab 3, recall that the input signal should consist on decreasing odd harmonics of the fundamental frequency.) (____/4)

While the square wave has a fundamental frequency of 1.5 kHz, it is also comprised of higher frequencies wherein the respective amplitudes decay rapidly, but the band pass filter allows them within its band and to eclipse the other frequencies.
This is similar to the analysis of the periodic square wave from lab 3 where the odd harmonics of the fundamental frequency makeup the square wave signal. This is why there is a single peak around the 7.5 kHz mark.

2.3 Explain why the second largest frequency component is around 5 kHz. (____/4)

The second largest frequency component is around 5 kHz due to aliasing. The large frequencies of the 13kHz square wave progressively gets larger than 22.05 kHz.
This results in the frequency of the sampled output being lower than that of the input, causing a peak around 5 kHz.

3.1 What does the strong impulse at $f = 13$ kHz represent in this AM signal. (____/2)

The strong impulse at f = 13 kHz is the carrier signal in this AM signal.

3.2 At what frequency locations are the other two impulses surrounding the 13 kHz impulse, and what do they represent ? (____/4)

The two other impulses surrounding the 13 kHz impulse are located at 12120 Hz and 13880 Hz.
This variation of 13kHz +/- 880 Hz represents the local oscillator frequencies.

3.3 Compare the frequency response of the digital band pass filter with the frequency response of the analog band pass filter built in lab 3, which one is considered a better filter and why ? (____/4)

Comparing the frequency responses of the digital and analog band pass filters, the digital BPF is better because it allows for more precise attenuation of input signals and preserves a higher percentage of the frequencies passed withing the band.
It can be described as close to a shifted and scaled rect function.

3.4 In the time domain, what type of signal is that. Describe it. And in the frequency domain, can you find the message signal ? (____/2)

In the time domain, the output signal is a sine function with a frequency of 880 Hz with a DC term to maintain the positive attribute.
In frequency domain, the signal is located at the peak around the 880Hz mark.

3.5 Overall, what does the envelope detector or the coherent detector accomplish ? (____/2)

The envelope detector/coherent detector recovers the original signal and shifts is down to counteract the affect of the DC offset.

4.1 In the spectrum of input, what is the frequency of the peak and what does it represent ? (____/2)

In the spectrum of the input, the peak in the frequency domain represents the carrier frequency around 14kHz.

4.2 Approximately what is the bandwidth of the output signal ? Does it align with the frequency response of the band pass filter ? (____/2)

The band spans approximately 9kHz to 18kHz, aka a bandwidth 9kHz, very similar to that of the digital bandpass filter.

4.3 Did you successfully demodulate the AM signal ? How was it compare to the one you did with circuit ? Explain the advantage of digital filter. (____/4)

Compared to the circuit lab, this AM signal was successfully demodulated. The advantage of the digital BPF is that clear sound can be heard over the static signal. The result is closer to the ideal demodulation than the physical implementation.

Demo : Please have your Jupyter notebook running before your demo, demo questions will be related to the notebook.

The End

Congratulations on completing the ECE 210 lab! Over these five labs you have learned and applied the most important principles of continuous-time signals and systems and explored their parallels in discrete-time signals and systems. Advanced coursework in ECE will require you to apply these principles again and again. You are well prepared!