ECE 210/211

University of Illinois

Spring 2020

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Analog Signal Processing

Thursday, February 20, 8:45-10pm

Exam I

Full Name (First Last): (all capital letters)	Solutions

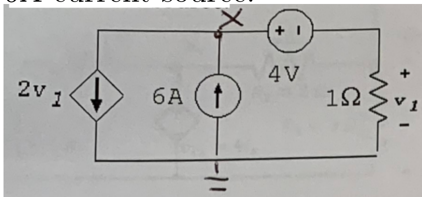
UIN:			netID:	

Course: (circle one)		ECE210	ECE211	
Section to return exam: (circle one)	10AM	11AM	1PM	2PM

	DO NOT write in these spaces.
Clearly PRINT your name in CAPITAL LETTERS.	Problem 1 (25 points):________
This is a closed book and closed notes exam.	Problem 2 (20 points):________
Calculators are not allowed.	Problem 3 (30 points):________
To get full credit, please SHOW all your work and simplify your answers.	Problem 4 (25 points):________
Write your final answers in the spaces provided.	Total: (100 points):________
All answers should INCLUDE UNITS whenever appropriate.	
The exam is printed **double-sided**.	

1. (25 pts) The two parts of this problem are unrelated.

(a) [10 pts] For the circuit below, determine v_1 and the power absorbed or delivered by the 6A current source.



$\frac{x-4}{1} - 6 + 2v_1 = 0$

$x = v_1 + 4$ $\qquad$ $x = 6V$

$v_1 + 4 - 4 - 6 + 2v_1$ $\qquad$ $P = -6 \cdot 6$

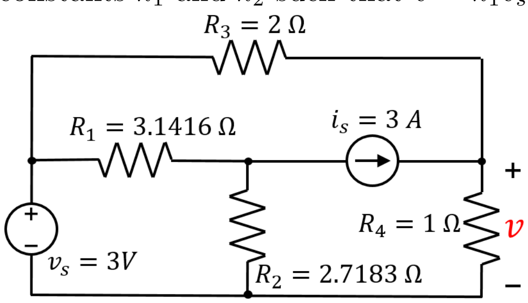
$3v_1 = 6$ $\qquad$ $v_1 = 2V$

$v_1 =$ 2V

$P_{6A} =$ −36 W

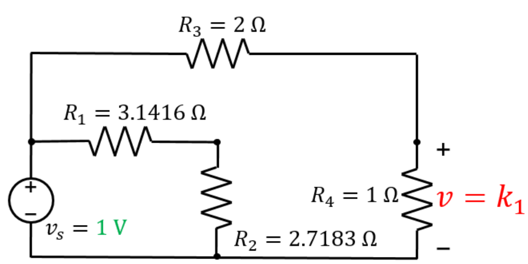
____ absorbed ✓ delivered

(b) [15 pts] Consider the circuit below. Use the superposition method to determine the constants k_1 and k_2 such that $v = k_1 v_s + k_2 i_s$.

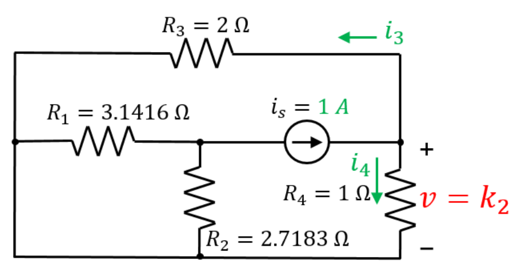


Solution: Start from $v = k_1 v_s + k_2 i_s$

1) Let $v_s = 1$ V, $i_s = 0$ A. The original circuit becomes sub-circuit a, where $v = k_1$.
   According to voltage division, $v = v_s \frac{R_4}{R_4+R_3} = 1 \times \frac{1}{1+2} = \frac{1}{3} \Rightarrow k_1 = \frac{1}{3}$.
2) Let $v_s = 0$ V, $i_s = 1$ A. The original circuit becomes sub-circuit b, where $v = k_2$.
   According to current division (because $R_3||R_4$), $i_4 = i_s \frac{R_3}{R_4+R_3} = 1 \times \frac{2}{1+2} = \frac{2}{3}$ A.
   $v = i_4 R_4 = \frac{2}{3}$ V $\Rightarrow k_2 = \frac{2}{3}\ \Omega$

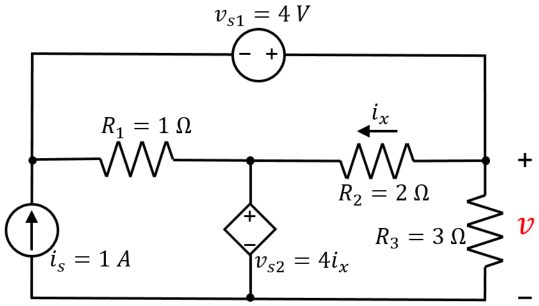


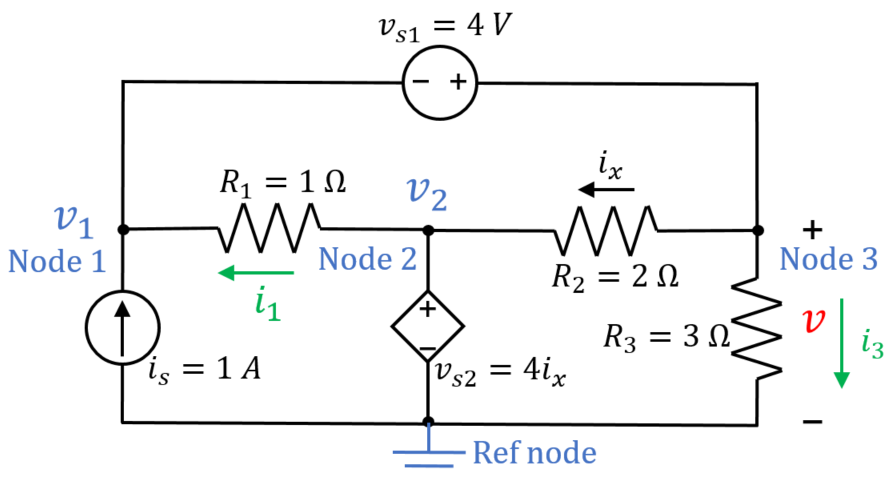
Sub-circuit a



Sub-circuit b

2. (20 pts) Consider the circuit below. Use the node-voltage method to determine v.





Solution: 1) Choose the reference node, declare node voltage variables, and assign current directions to the resistors.

2) According to $v_{s1} = 4$ V, we obtain $v_1 = v - 4$.

3) According to $v_{s2} = 4i_x$, we obtain $v_2 = 4i_x \Rightarrow v_2 = 4 \times \frac{v - v_2}{2} \Rightarrow v_2 = \frac{2}{3}v$

4) Nodes 1 and 3 form a super node. Applying KCL at this super node yields

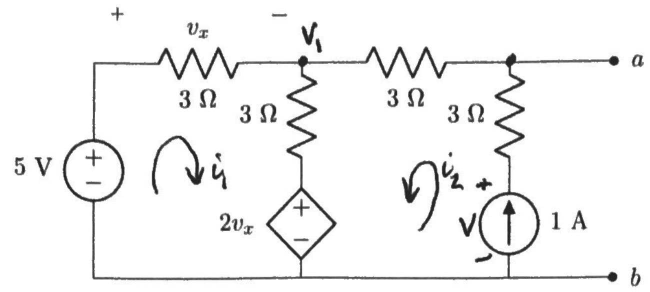
$$i_s + i_1 = i_x + i_3 \Rightarrow 1 + \frac{v_2 - v_1}{1} = \frac{v_2}{4} + \frac{v}{3}$$

$$\Rightarrow 1 + \frac{2}{3}v - v + 4 = \frac{v}{6} + \frac{v}{3} \Rightarrow \boxed{v = 6\text{V}}$$

$v =$ ____________________

3. (30 pts) The two parts of this problem are unrelated.

(a) [20] Consider the circuit below.



i. [10 pts] Determine the Thevenin equivalent voltage, V_T, between nodes a and b. .

$i_2 = 1\,A$

$V_x = 3i_1 \Rightarrow i_1 = \frac{V_x}{3}$

$5 = V_x + 3(i_1 + i_2) + 2V_x$

$5 = 3V_x + 3i_1 + 3i_2$

$5 = 4V_x + 3(1)$

$2 = 4V_x$

$V_x = \frac{1}{2}V$

$i_1 = \frac{1}{6}A$

$V_1 = 5 - V_x$

$V_1 = 4.5\,V$

$1 = \frac{V_T - V_1}{3}$

$3 = V_T - 4.5$

$\boxed{V_T = 7.5\,V}$

OR

$V = 3i_2 + 3i_2 + 3(i_1 + i_2) + 2V_x$

$V = 3(1) + 3(1) + 3(\frac{7}{6}) + 1$

$V = 7 + \frac{7}{2} = \frac{21}{2}$

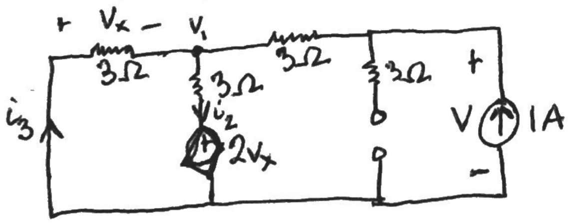
$V_T = V - 3i_2$

$V_T = \frac{21}{2} - 3$

$\boxed{V_T = \frac{15}{2}V}$

$V_T =$ 7.5 V

ii. [10 pts] Determine the Thevenin equivalent resistance, R_T, between nodes a and b.



$\frac{V - V_1}{3} = 1A$

$\frac{V_1 - 2V_x}{3} = i_2$

$\frac{V_x}{3} = i_3$

$\frac{V - \frac{3}{4}}{3} = 1$

$1 + \frac{V_x}{3} = \frac{V_1 - 2V_x}{3}$

$3 + V_x = V_1 - 2V_x$

$3 + 3V_x = V_1$

$3 + 3V_x = -V_x$

$3 = -4V_x$

$V_x = -\frac{3}{4}V$

$V_1 = 0 - V_x$

$V_1 = -V_x$

$V_1 = \frac{3}{4}V$

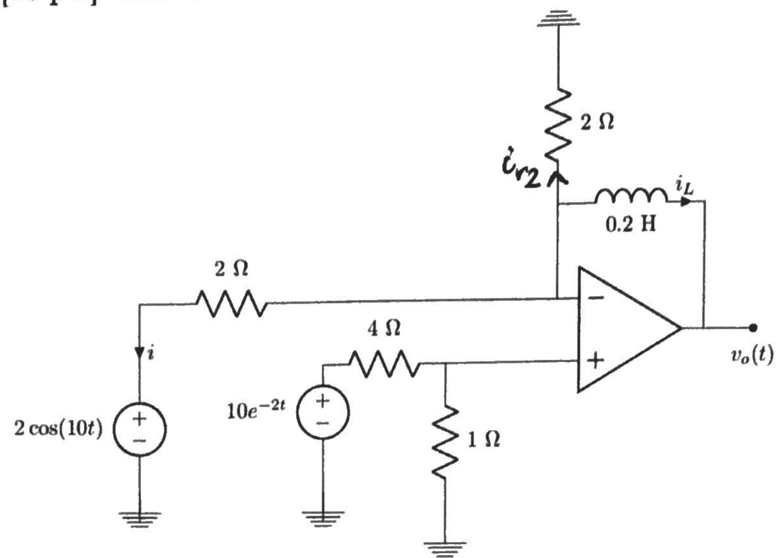
$V - \frac{3}{4} = 3$

$V = \frac{15}{4}V$

$R_T = \frac{15/4}{1} \Rightarrow \boxed{R_T = \frac{15}{4}\Omega}$

$R_T =$ [redacted] $\frac{15}{4}\Omega$

(b) [10 pts] Consider the circuit below.



i. [07 pts] Determine the current through the inductor, i_L.

$V_+ = \left(\frac{1}{4+1}\right)10e^{-2t} \Rightarrow V_+ = \left(\frac{1}{5}\right)10e^{-2t} \Rightarrow V_+ = 2e^{-2t} = V_-$

$i = \frac{2e^{-2t} - 2\cos(10t)}{2} \Rightarrow i = e^{-2t} - \cos(10t)$

$i_{r2} = \frac{2e^{-2t} - 0}{2} \Rightarrow i_{r2} = e^{-2t}$

$i_L + i_{r2} + i = 0$

$i_L = -i - i_{r2} \Rightarrow i_L = \cos(10t) - e^{-2t} - e^{-2t}$

$\boxed{i_L = \cos(10t) - 2e^{-2t}}$

$i_L =$ $\cos(10t) - 2e^{-2t}$

ii. [03 pts] Determine the output voltage, v_o.

$V_L = V_- - V_0$ $\qquad V_L = L\frac{di_L}{dt}$

$V_0 = V_- - V_L$

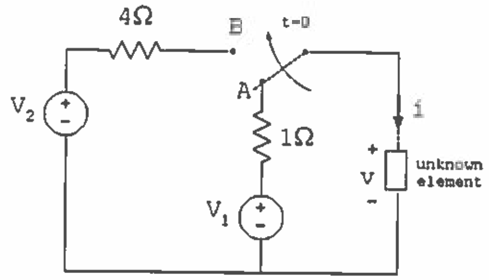
$V_0 = 2e^{-2t} - 0.2(-10\sin(10t) + 4e^{-2t})$

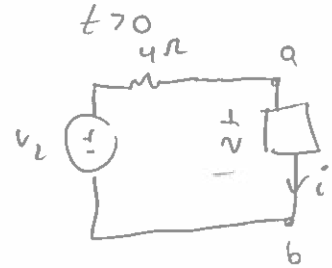
$V_0 = 2e^{-2t} + 2\sin(10t) - 0.8e^{-2t}$

$\boxed{V_0 = 2\sin(10t) + 1.2e^{-2t}}$

$v_o =$ $2\sin(10t) + 1.2e^{-2t}$

4. (25 pts) Consider the circuit below. The switch is originally in position A it switches over to position B at time $t = 0$.





$\Rightarrow R_T = 4\Omega$

(a) [03 pts] If it is known that that $v(0^-) = 5$V, $i(0^-) = 5$A, $v(0^+) = 4$V and $i(0^+) = 4$A, is the unknown element a resistor, an inductor or a capacitor? Explain why.

✓ resistor ___ inductor ___ capacitor

In capacitor $v_c(0^-) = v_c(0^+)$
In inductor $i_L(0^-) = i_L(0^+)$

Explain: In resistor, no such restrictions

(b) [06 pts] If the unknown element is an inductor, and it is known that $v(1) = -20e^{-2}$V, determine the value of the inductance, L.

$i_L(t) = B + Ae^{-t/\tau}$

$v_L(t) = L\frac{di}{dt} = LA\left(-\frac{1}{\tau}\right)e^{-t/\tau}$

$v_L(1) = -\frac{LA}{\tau}e^{-1/\tau} = -20e^{-2}$

$\Rightarrow \frac{1}{\tau} = 2 \Rightarrow \frac{1}{2} = \tau = \frac{L}{R_T} = \frac{L}{4}$ $\Rightarrow L = 2$

$L =$ 2H

(c) [16 pts] Assume the unknown element is a 0.01F capacitor and that the switch was in position A for a long time. Determine the constants a, K_1, K_2 and K_3, such that $v(t) = K_1 + K_2e^{-at} + K_3t$, for $t > 0$. You may leave your answers in terms of V_1 and V_2.

$v_c(t) = B + Ae^{-t/\tau} \Rightarrow K_3 = 0$

$v_c(\infty) = B = V_2$ (see top)
$= K_1$

$v_c(0^-) = v_c(0^+) = B + A = V_2 + A = V_1$ (see top)

$\Rightarrow A = V_1 - V_2 = K_2$

$\tau = R_T C = 4(0.01) = \frac{4}{100} = \frac{1}{25}$

$a = \frac{1}{\tau} \Rightarrow a = 25$

$K_1 =$ V_2

$K_2 =$ $V_1 - V_2$

$K_3 =$ 0

$a =$ 25