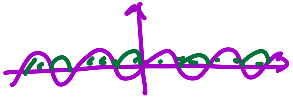
- BIBO stability and LTI systems - Example # 3
  - Determine which of the following impulse responses correspond to BIBO stable systems:

Check $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

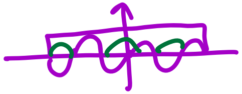
X not BIBO stable

1. $h(t) = \sin(\omega_0 t)$



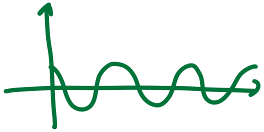
2. $h(t) = \sin(\omega_0 t) rect(t)$

V ☺ BIBO stable



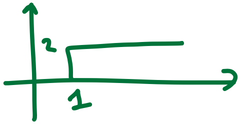
3. $h(t) = \cos(\omega_0 t)$u(t)

X BIBO stable



4. $h(t) = 2u(t-1)$

X BIBO stable



5. $h(t) = \delta(t-1)$

V BIBO stable

$$f(t) \rightarrow \overset{\delta(t-1)}{\boxed{h(t)}} \rightarrow y(t) = f(t) * \delta(t-1) = f(t-1)$$

$$|f(t)| \le C_1 \Rightarrow |f(t-1)| \le C_1$$

• Causality and LTIC systems

- Consider

$$f(t) \rightarrow \overset{\text{general}}{\boxed{\text{system}}} \rightarrow y(t)$$

- If the output y(t) does not depend on the future of the input f(t), then the system is *causal.*

This has to be true for any input f(t)

- If the output y(t) does depend on the future of the input f(t), then the system is *non-causal* (unrealizable).

- Causality and LTI systems
  - If the system is LTI, then it is causal if and only if its impulse response has the following property:

causal $\Longleftrightarrow$ $h(t) = 0$ for $t<0$

Note: only if $h(t)$ is a function. If not, use original definition.

$$y(t) = \int_{-\infty}^{\infty} h(\tau) f(t-\tau)\, d\tau$$

t_1

$$y(t_1) = \int_{-\infty}^{\infty} \underbrace{h(\tau)}_{\substack{= \\ 0 \\ \text{for} \\ \tau<0}} f(t_1-\tau)\, d\tau$$

$\tau>0$: past values of the input

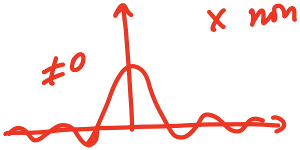
$\tau=0$: present values of the input

$\tau<0$: future values of the input

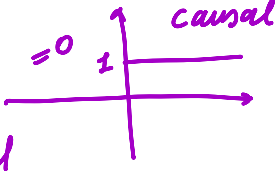
Causality and LTI systems - Example # 4

- Determine which of the following impulse responses correspond to causal systems:

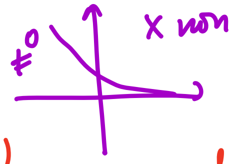
1. $h(t) = \text{sinc}(\omega_0 t)$
2. $h(t) = u(t)$
3. $h(t) = e^{-t}$
4. $h(t) = e^{-t}u(t-1)$
5. $h(t) = \delta(t)$



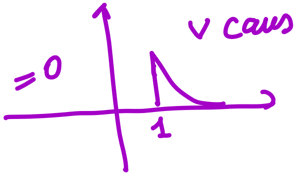
x non-causal



causal ✓



x non-causal



✓ causal

$y(t) = f(t-1)$ ↑ past ⇒ causal

$$f(t) \rightarrow \boxed{\delta(t)} \rightarrow y(t) = f(t) * \delta(t) = f(t)$$

$\delta(t-1)$

$\delta(t+1)$

same t, present ⇒ causal

$y(t) = f(t+1)$

↑ future ⇒ non-causal

Causality

- An LTIC system is
  - Linear
  - Time-invariant
  - Causal

- A signal f(t) is causal if it could be an LTIC impulse response

$f(t) = 0 \quad t<0$

$$\left.\begin{array}{l}\delta(t)\\ u(t-1)\end{array}\right\rangle \text{causal signals}$$

$$\left.\begin{array}{l}u(t+1)\\ \delta(t+2)\end{array}\right\rangle \text{non-causal signals}$$

• LTIC systems - Example # 5

- For each of the following systems, determine if they are $\overset{\text{Z-S}}{\text{linear}}$, time-invariant, BIBO stable and/or causal:

1. $y(t) = \overset{A}{\sin(t+3)}\, f(t)$ ← Yes ✓.

① Z-S linear :

$y(t) = \sin(t+3)\, f(t)$ ↙ f only changes this t

② T.I :

$f(t) \rightarrow \square \rightarrow y(t)$

y changes both ts ⇒ not the same shift ⇒ not T.I. ✗

$f_1(t) = f(t-t_d) \rightarrow \square \rightarrow y_1(t) = y(t-t_d)$

③ BIBO : $y(t) = \underbrace{\sin(t+3)}_{|\sin(t+3)| \le 1}\, \underbrace{f(t)}_{|f(t)| \le C}$ ⇒ $|y(t)| \le C$ ⇒ BIBO stable ☺ ✓

④ Causal : $y(t) = \sin(t+3)\, f(t)$ same t, present ⇒ causal ✓

• LTIC systems - Example # 5-cont

- For each of the following systems, determine if they are linear, time-invariant, BIBO stable and/or causal:

2. $y(t) = f((t-1)^2)$

① Z-S linear : ✓

② T.I : $y(t) = f((t-1)^2)$

$$f(t) \rightarrow \boxed{\ } \rightarrow y(t) = f((t-1)^2)$$

$$f_1(t) \rightarrow \boxed{\ } \rightarrow y_1(t) \overset{?}{=} y(t-t_d)$$

$$'' \quad f(t-t_d)$$

$$'' \quad f_1((t-1)^2) = f((t-1)^2 - t_d)$$

$$\neq$$

$$y(t-t_d) = f((t-t_d-1)^2)$$

not T.I. ×

③ BIBO :

$y(t) = f((t-1)^2)$

↑

amplitude is not affected ⇒ BIBO stable ✓

④ Causal :

$\overset{-3}{y(t)} = f((\overset{-3}{t}-1)^2)$

$y(-3) = f(16)$

$16 > -3$ ⇒ future of $f(t)$ ⇒ non-causal ×