• Phasors- example #1

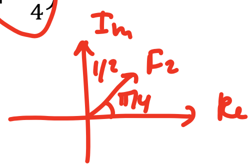
- Express the following co-sinusoidal functions in phasor form:

  - $f_1(t)=1\cos(10t)$

    $F_1 = 1e^{j0} = 1$

  - $f_2(t)=\frac{1}{2}\cos(10t+\frac{\pi}{4})$

    $F_2 = \frac{1}{2}e^{j\frac{\pi}{4}}$

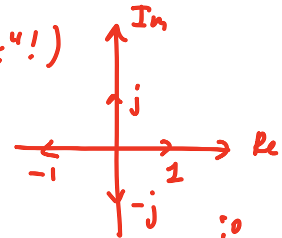


  - $f_3(t)= \sqrt{3}\sin(10t) = \sqrt{3}\cos(10t - \pi/2)$

    $\sin x = \cos(x - \pi/2)$

    $F_3 = \sqrt{3}e^{-j\pi/2} = -j\sqrt{3}$

- Phasors are constants! (no "t"!)
- Don't forget j!
- Convert sin to cos first, then go ahead :)



$e^{j0} = 1$

$e^{j\pi/2} = j$

$e^{-j\pi/2} = -j$

$e^{j\pm\pi} = -1$

- Superposition principle (for phasors)

The weighted superposition

$$f_3(t) = k_1 f_1(t) + k_2 f_2(t),$$

same ω !

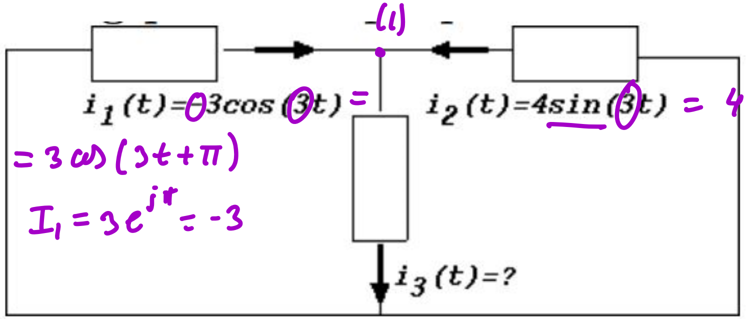
of co-sinusoids $f_1(t) = \text{Re}\left\{\boldsymbol{F}_1 e^{j\omega t}\right\}$ and $f_2(t) = \text{Re}\left\{\boldsymbol{F}_2 e^{j\omega t}\right\}$, with phasors $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ respectively, is also a co-sinusoid with phasor

$$\boxed{\boldsymbol{F}_3 = k_1\boldsymbol{F}_1 + k_2\boldsymbol{F}_2,}$$

$$f_3(t) = Re\{F_3 e^{j\omega t}\} = Re\{(k_1F_1 + k_2F_2)e^{j\omega t}\}$$

• Phasor superposition- example #2

• Consider the circuit below. Obtain $i_3(t)$ as a single co-sinusoid using phasor superposition.

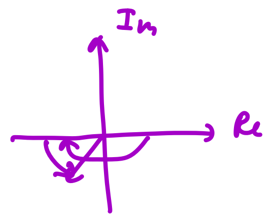


$i_1(t) = -3\cos(3t) = 3\cos(3t+\pi)$

$I_1 = 3e^{j\pi} = -3$

$i_2(t) = 4\sin(3t) = 4\cos(3t-\pi/2)$

$I_2 = 4e^{-j\pi/2} = -j4$



KCL @ (1) : $i_3(t) = i_1(t) + i_2(t)$

$I_3 = I_1 + I_2 = -3 - j4 = Ae^{j\theta} = 5e^{j(-\pi + \tan^{-1}(\frac{4}{3}))}$

$A = \sqrt{(-3)^2 + (-4)^2} = 5$

$\theta = -\pi + \tan^{-1}\left(\frac{-4}{-3}\right)$

$\downarrow$ time

$i_3(t) = 5\cos\left(3t - \pi + \tan^{-1}\left(\frac{4}{3}\right)\right)$ A

- Derivative principle

The derivative

$$g(t) = \frac{d}{dt} f(t)$$

of co-sinusoid $f(t) = \mathrm{Re}\,\{\boldsymbol{F} e^{j\omega t}\}$ is also a co-sinusoid with phasor

$$\boldsymbol{G} = j\omega \boldsymbol{F}$$

n-th derivative:

$\frac{d^n f}{dt^n}$ is a cosinusoid with a phasor $(j\omega)^n F$

$$\frac{df}{dt} = g(t) = \mathrm{Re}\,\{\underbrace{j\omega F}_{G}\, e^{j\omega t}\}$$

- Phasor derivative- example #3

- Consider the co-sinusoid f(t) = $3\cos\left(\overset{\omega}{2}t + \frac{\pi}{4}\right)$

  Determine $\underbrace{f'(t)}$ using phasors

$g(t) = Re\{Ge^{j\omega t}\}$

G ?

$F = 3e^{j\pi/4} = 3\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) =$

$= \frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}}$

G - ?

a) 3

b) $\frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}}$

c) $j\frac{6}{\sqrt{2}} - \frac{6}{\sqrt{2}}$

d) $6 + j6$

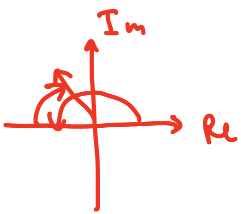
$G = j\omega F = j \cdot 2\left(\frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}}\right) =$

$= j\frac{6}{\sqrt{2}} - \frac{6}{\sqrt{2}} = 6e^{j\frac{3\pi}{4}} \xrightarrow{\text{time}} g(t) = 6\cos\left(2t + \frac{3\pi}{4}\right)$

$j^2 = -1$

$A = \sqrt{\left(-\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2} = 6$

$\theta = \pi + \tan^{-1}(-1) = \pi - \pi/4 = \frac{3\pi}{4}$



- Example #5
  - Determine the steady-state solution to the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = -5\sin(\overset{\omega}{2}t) = 5\cos(2t \underbrace{- \pi/2 + \pi}_{\pi/2})$$

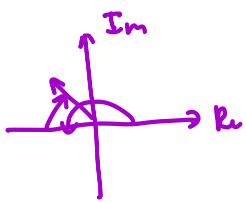
$\downarrow$ phasors

$j^2 = -1$

$$(j\overset{2}{\omega})^2 Y + 2j\omega Y + Y = 5e^{j\pi/2} = 5j$$

$$-4Y + 4jY + Y = j5$$

$$Y(-3 + j4) = j5$$



$$Y = \frac{j5}{-3 + j4} = \frac{5e^{j\pi/2}}{\underbrace{\sqrt{(-3)^2 + 4^2}}_{5}\, e^{j(\pi + \tan^{-1}(\frac{4}{-3}))}}$$

$\Downarrow$

$$y_{ss}(t) = \cos\left(2t + \pi/2 - \pi - \tan^{-1}\left(\frac{4}{-3}\right)\right) =$$

$$= \cos\left(2t - \pi/2 + \tan^{-1}\left(\frac{4}{3}\right)\right)$$