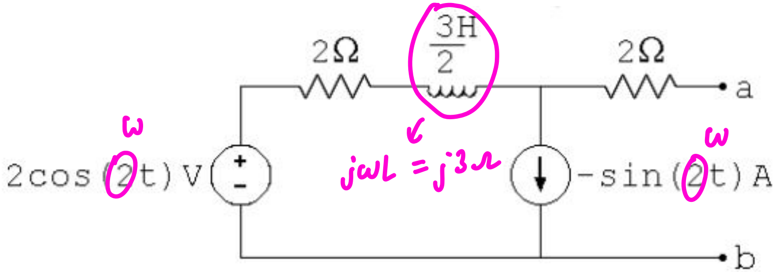
- Example #9
- Determine the average absorbed power in each element:

$p = \frac{1}{2}\,\mathrm{Re}\{V I^*\}$

$P_R = \frac{1}{2} R |I|^2$

$I \cdot I^* = |I|^2$

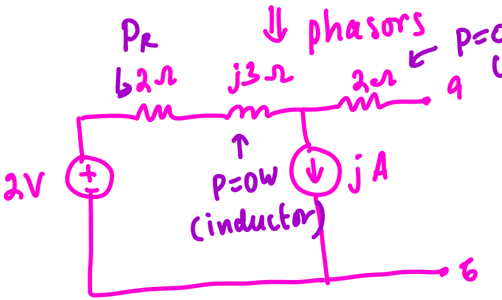


$= \cos(2t \underbrace{-\pi/2 + \pi}_{\pi/2})$

$\downarrow$

$1 e^{j\pi/2} = j\,A$

⇓ phasors



$P_R = \frac{1}{2} R |I|^2 = \frac{1}{2} \cdot R |j|^2 =$

$= \frac{1}{2} \cdot R = 1W$

$P_{2V} = \frac{1}{2}\,\mathrm{Re}\{V I^*\} = \frac{1}{2}\,\mathrm{Re}\{2 \cdot (-j)^*\} =$

$= \frac{1}{2}\,\underbrace{\mathrm{Re}\{2j\}}_{0} = 0W$

By energy conservation:

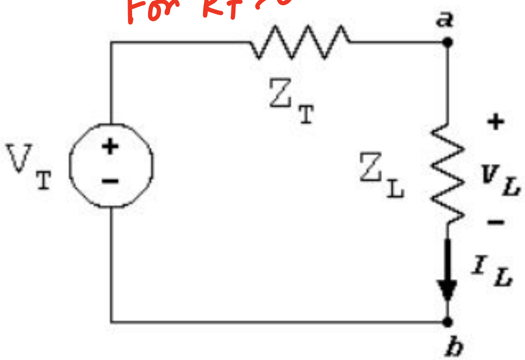
$\Sigma P = 0$

$P_{jA} = -1W$

get magnitude $0 + j \to \sqrt{0^2 + 1^2} = 1$

- Available power

For $R_T > 0$



$$P_L = \frac{1}{2} Re\{V_L \cdot I_L^*\} = \frac{1}{2} Re\left\{ \frac{V_T Z_L}{(Z_L + Z_T)} \cdot \frac{V_T^*}{(Z_T + Z_L)^*} \right\} =$$

$$I_L = \frac{V_L}{Z_L} = \frac{V_T}{Z_T + Z_L}$$

$$V_L = V_T \left(\frac{Z_L}{Z_T + Z_L} \right)$$

$$= \frac{1}{2} Re\left\{ \frac{|V_T|^2 \cdot Z_L}{|Z_L + Z_T|^2} \right\} =$$

$$= \frac{1}{2} \frac{|V_T|^2 \cdot R_L}{|Z_L + Z_T|^2} =$$

$$= \frac{1}{2} \frac{|V_T|^2 \cdot R_L}{|(R_L + R_T) + j\underbrace{(X_L + X_T)}|^2}$$

$$Z_T = R_T + jX_T$$

$$Z_L = R_L + jX_L$$

$$\boxed{Z_L = R_T - jX_T = Z_T^*} \leftarrow \begin{cases} X_L = -X_T \\ R_L = R_T \end{cases}$$

matched load

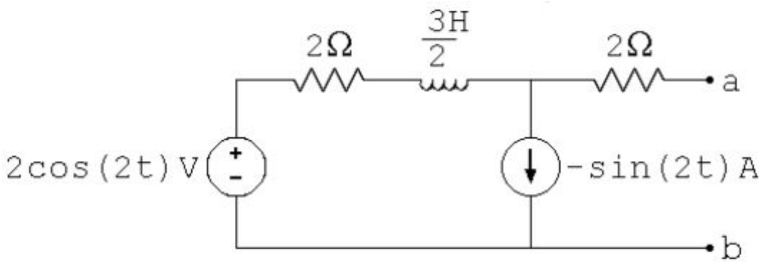
$$P_a = \frac{|V_T|^2}{8R_T} = \frac{1}{2} \cdot \frac{|V_T|^2}{4R_T}$$

take magnitude of V_T

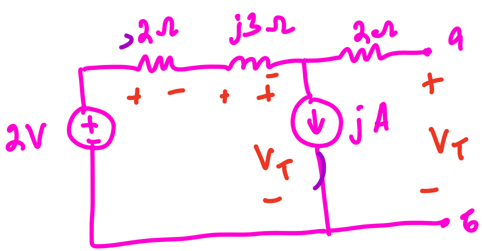
from phasors

only real part of Z_T

- Example #9-cont
- Determine the available average absorbed power:



↓ phasors



$$P_a = \frac{1}{2}\cdot\frac{|V_T|^2}{4R_T} = \frac{1}{8}\,\frac{\left(\sqrt{5^2+(-2)^2}\right)^2}{4} = \frac{29}{32}\,W \text{ only if}$$

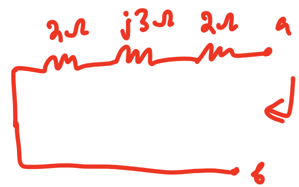
$$Z_L = Z_T^* = 4-j3\,\Omega$$

Get V_T:

KVL: $-2 + 2j + j(j3) + V_T = 0$

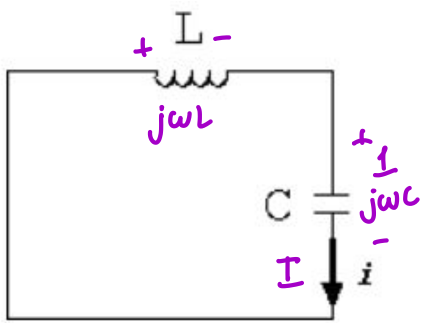
$V_T = 5-j2$ V

Get Z_T:



$Z_T = 4+j3\,\Omega$

- Resonance
  - Recall the circuit below with $v_c(0^-)$ = 1V, $i_L(0^-)$ = 0A and $v_c(t) = A\cos(t + \theta)$



KVL: $V_L + V_c = 0$

$$j\omega L \cdot I + \frac{1}{j\omega C} \cdot I = 0$$

$$I\underbrace{\left(j\omega L - \frac{j}{\omega C}\right)}_{0} = 0$$

can be satisfied by any I as long as

$$j\omega L - \frac{j}{\omega C} = 0$$

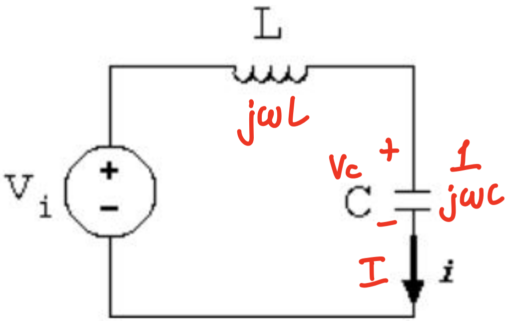
$$\omega L = \frac{1}{\omega C}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

resonant frequency

- Resonance: possible existence of steady-state co-sinusoidal oscillations in a source-free circuit.

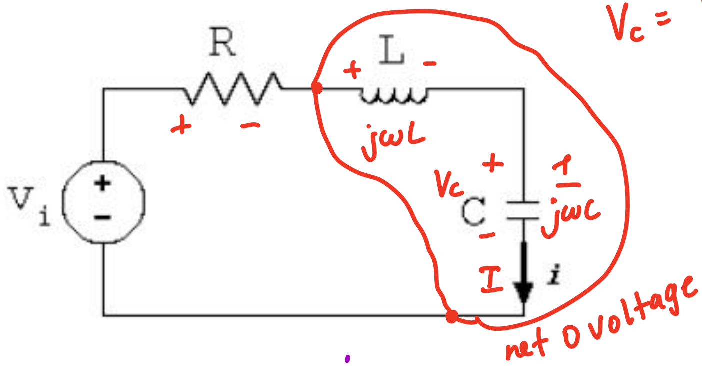
- Resonance-cont
  - What if we add a source?



$$V_c = V_i\left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+j\omega L}\right) = \frac{V_i}{\underbrace{-\omega^2 LC+1}_{=0}} \to V_c \to \infty$$ ☹

if $\omega = \frac{1}{\sqrt{LC}}$

- Resonance-cont
  - What if we add a resistor too?



$$V_c = V_i \frac{\frac{1}{j\omega C}}{\left(\frac{1}{j\omega C}+j\omega L+R\right)} = \frac{V_i}{\underbrace{(1-\omega^2 LC)}_{=0 \text{ if } \omega = \frac{1}{\sqrt{LC}}}+j\omega RC}$$

$$Z_s = R + \underbrace{j\omega L - \frac{j}{\omega C}}_{=0 \text{ If } \omega = \frac{1}{\sqrt{LC}}}$$

L and C act like a short, but they still have voltage, just opposite in signs to each other
$\Rightarrow$ max current achieved!