Lecture 14, Wednesday, February 9, 2022

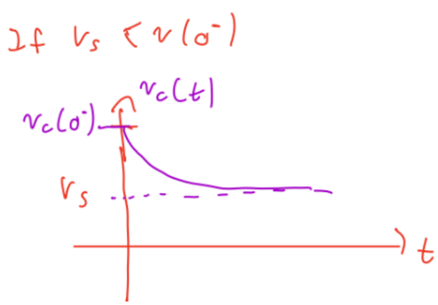
- First order ODE with constant coefficients and constant input:
  - Solution for $t > 0$:

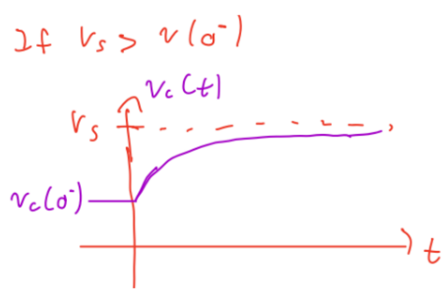
$$y(t) = \underbrace{\frac{K}{a}}_{= y_p(t)} + \underbrace{\left(y(0^+) - \frac{K}{a}\right) e^{-at}}_{= y_h(t)}$$

  - $y_p(t)$ is the particular solution, specific to constant inputs
  - $y_h(t)$ is the homogeneous solution
    - satisfies the ODE $= \; 0$
    - is needed to match the initial state of the output
  - solution increases/decreases exponentially from $y(0^+)$ to $\frac{K}{a}$
- For the RC circuit, $K = \frac{V_s}{RC}$, $a = \frac{1}{RC}$ and $y(0^+) = v_c(0^+) = v_c(0) = v_c(0^-)$

$$v_c(t) = V_s + \left(v_c(0^-) - V_s\right) e^{-\frac{t}{RC}}$$

  - solution increases/decreases exponentially from $v_c(0^-)$ to V_S





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- $RC = \tau$ is called the time-constant
- The larger the value of τ, the slower the exponential term decays
- A simple way to solve for $v_c(t)$ for $t > 0$ is to consider
  - $v_c(t) \;=\; B \;+\; Ae^{-\frac{t}{\tau}}$
  - $v_c(\infty) = V_T \;=\; B$
  - $v_c(0^+) \;=\; v_c(0^-) \;=\; B + A$
  - $\tau \;=\; R_T C$

- In a general first order ODE with constant coefficients and constant input

$$y(t) \;=\; y_{ZS}(t) \;+\; y_{ZI}(t) \;=\; \underbrace{\frac{K}{a}\left(1 - e^{-at}\right)}_{= y_{ZS}} \;+\; \underbrace{y\left(0^+\right)e^{-at}}_{= y_{ZI}}$$

  - Therefore, the system is linear