

Analog Signal Processing

Thursday, February 25, 7-8.50pm

Exam I — Solutions

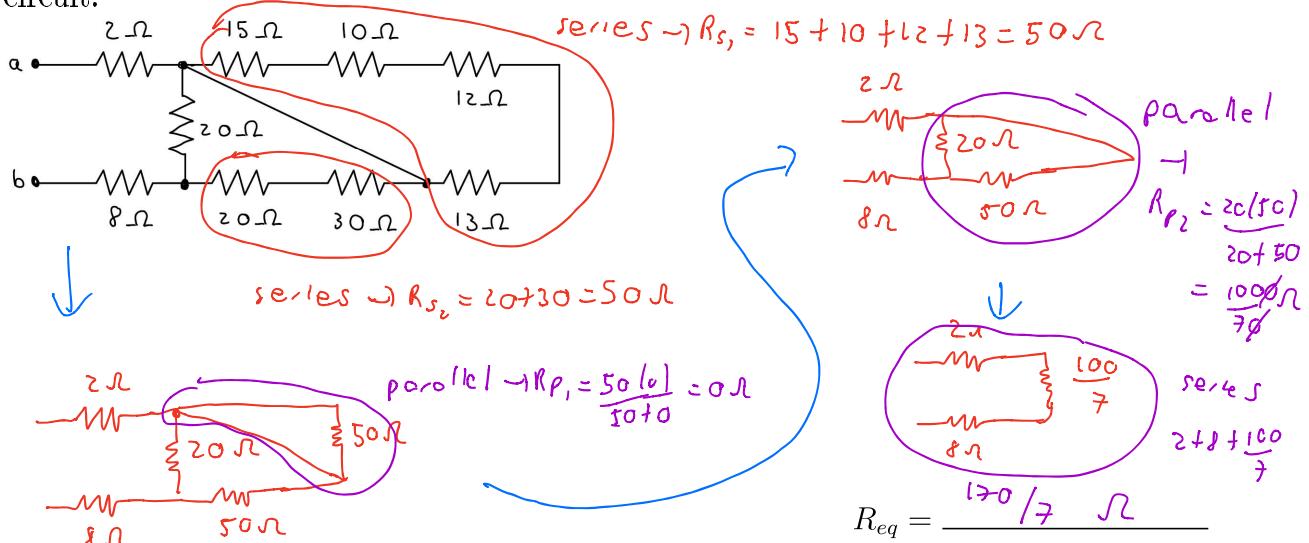
- **Exam duration:** The midterm exams will be **110 minutes** (1 hour and 50 minutes) long, **including** scan/upload time, so we strongly advise you to finish your exam with plenty of time to spare, in case you run into issues scanning/uploading. We will **deduct** 20 points per late minute after those first 110 minutes, and Gradescope will automatically stop accepting exams after those 115 minutes. **No** late exams will be accepted and you will get a **zero** in the exam if it is not uploaded on time (no exceptions).
- **No collaboration allowed:** You are **not allowed** to share or collaborate on this exam and all work should be your own; otherwise, an Academic Integrity report will be filed against you and sanctions will be applied.
- **Closed notes:** you cannot use the textbook, nor any notes; otherwise, an Academic Integrity report will be filed against you and sanctions will be applied.
- **Calculations:** Calculators and other electronic ways to do calculations, like Wolfram alpha, are **not allowed and neither** is searching online; otherwise, an Academic Integrity report will be filed against you and sanctions will be applied.
- **Solving the exam:** you **must** solve the exam in blank sheets of paper. Tablets are **not allowed** for writing and you may not print the exam; otherwise, an Academic Integrity report will be filed against you and sanctions will be applied.
- An Academic Integrity report will be filed against you and sanctions will be applied for unauthorized actions.
- **Solution uploads:** Make sure that your scans or photos are legible and that you correctly assign each solution to its question, or you will be **deducted** at least 5% of the corresponding problem part. - Instructions for uploading your solutions to Gradescope were provided by you, as well as opportunities to practice doing so. You **MUST** remain in the proctoring session until you are finished uploading, even if the proctor says you can leave before that. We will **not accept** your exam if you leave the proctoring session before finishing your submission to Gradescope, and you will get a **zero** in the exam.
- **Proctoring:** This course uses the College of Engineering Computer-Based Testing Facility service CBTF Online for its exams. You must already have reviewed those policies and be aware of them. Any violation/disregard of CBTF policy, like incorrect camera positioning, ignoring proctor's instructions, etc. may result in your exam not being accepted (giving a zero) or in an Academic Integrity report being filed against you with sanctions applied, depending on the situation. If you have any issue during an exam, please inform the proctor immediately. Work with the proctor to resolve the issue at the time before logging off.
- **Show all your work and simplify** your answers. Answers with no explanation/justification/work will be given **little/no credit**.
- **Box your answers.**
- Answers **should** include units if appropriate.

1. (1 pt) Sign acknowledging you will abide by this course's policies, CBTF policies and the University's Academic Integrity policies or face sanctions for not doing so. If your solutions upload does not include your signature, your exam will NOT be graded, resulting in a zero.

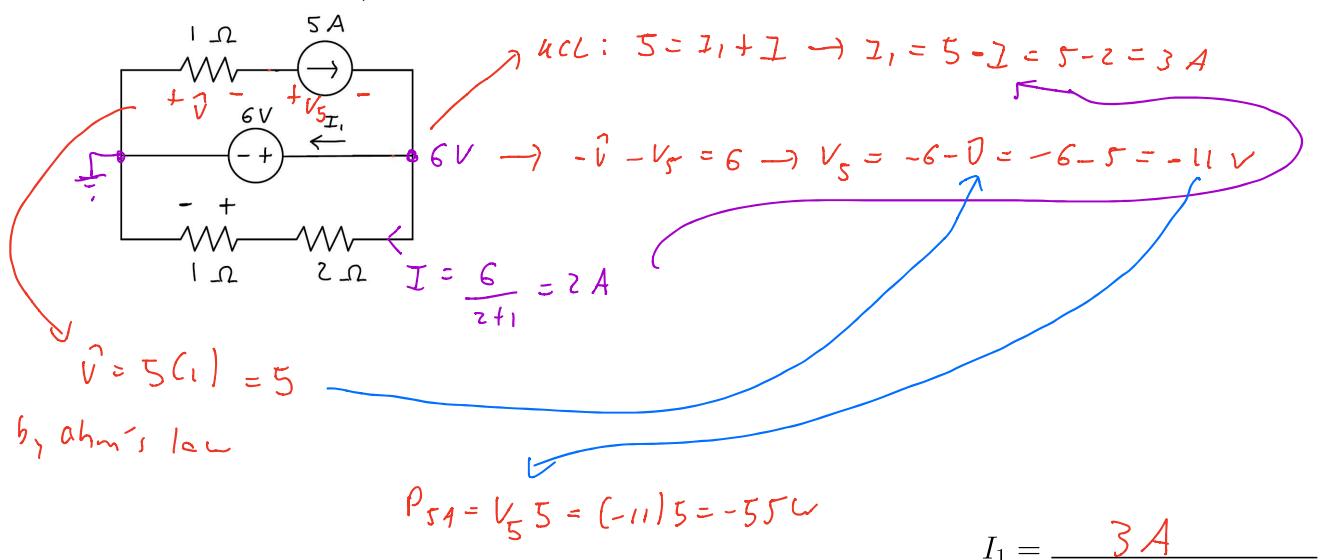
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2. (25 pts) The three parts of this problem are unrelated.

- (a) [8 pts] Determine the equivalent resistance, R_{eq} , between terminals a-b in the following circuit.



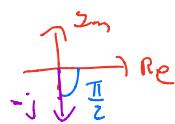
- (b) [9 pts] Consider the circuit below. Determine the current, I_1 and the absorbed power in the current source, P_{5A} .



- (c) [8 pts] Determine the magnitude and phase of the complex number $Z = \frac{2e^{j\frac{\pi}{3}} - 2e^{-j\frac{\pi}{3}}}{2e^{j\pi} + 2e^{-j\pi}}$.

$$Z = \frac{4j \sin(\frac{\pi}{3})}{4 \cos(\pi)} = j \frac{\sqrt{3}}{2} e^{-j\pi} = -j \frac{\sqrt{3}}{2}$$

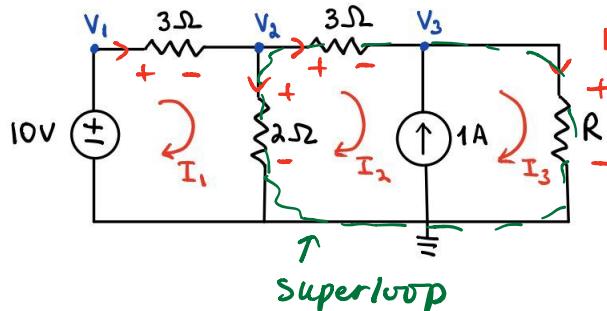
Diagram: A right triangle with hypotenuse $\sqrt{3}$ and angle $\frac{\pi}{3}$ from the negative real axis.



$$|Z| = \sqrt{3}/2$$

$$\angle Z = -\pi/2 \text{ rad}$$

3. (25 pts) Consider the circuit below,



From voltage source: $V_1 - 0 = 10V \Rightarrow V_1 = 10V$

$$\text{KCL @ (2)}: \frac{V_1 - V_2}{3} = \frac{V_2}{2} + \frac{V_2 - V_3}{3}$$

$$\frac{1}{3}V_1 + \underbrace{\left(-\frac{1}{3} - \frac{1}{2} - \frac{1}{3}\right)V_2}_{-\frac{7}{6}} + \frac{1}{3}V_3 = 0$$

- (a) [10 pts] Use the node-voltage method to obtain a set of three equations, in terms of the node voltages V_1 , V_2 , V_3 and R , that can be used to determine the node voltages. DO NOT SOLVE. Note: equations not derived using the node-voltage method will receive no credit.

KCL @ (3):

$$\frac{V_2 - V_3}{3} + 1 = \frac{V_3}{R}$$

$$\frac{1}{3}V_2 + \left(-\frac{1}{3} - \frac{1}{R}\right)V_3 = -1$$

$$\frac{1}{3}V_1 + 0V_2 + 0V_3 = 10$$

$$\frac{1}{3}V_1 + -\frac{7}{6}V_2 + \frac{1}{3}V_3 = 0$$

$$0V_1 + \frac{1}{3}V_2 + \frac{-1}{3}V_3 = -1$$

- (b) [10 pts] Use the loop-current method to obtain a set of three equations, in terms of the loop currents I_1 , I_2 , I_3 and R , that can be used to determine the loop currents. DO NOT SOLVE. Note: equations not derived using the loop-current method will receive no credit.

From current source:

$$I_3 - I_2 = 1$$

KVL @ loop 1:

$$-10 + 3I_1 + 2(I_1 - I_2) = 0$$

$$\text{KVL @ superloop: } -2(I_1 - I_2) + 3I_2 + RI_3 = 0$$

$$0I_1 + -1I_2 + 1I_3 = 1$$

$$5I_1 + -2I_2 + 0I_3 = 10$$

$$-2I_1 + 5I_2 + RI_3 = 0$$

- (c) [5 pts] It is known that if $R = 1\Omega$ then $I_1 = \frac{29}{13}A$, $I_2 = \frac{15}{26}A$ and $I_3 = \frac{41}{26}A$. Determine the corresponding node voltages V_1 , V_2 , V_3 .

$$\frac{V_2}{2} = I_1 - I_2 \Rightarrow V_2 = 2\left(\frac{29}{13} - \frac{15}{26}\right) = \frac{43}{13}V$$

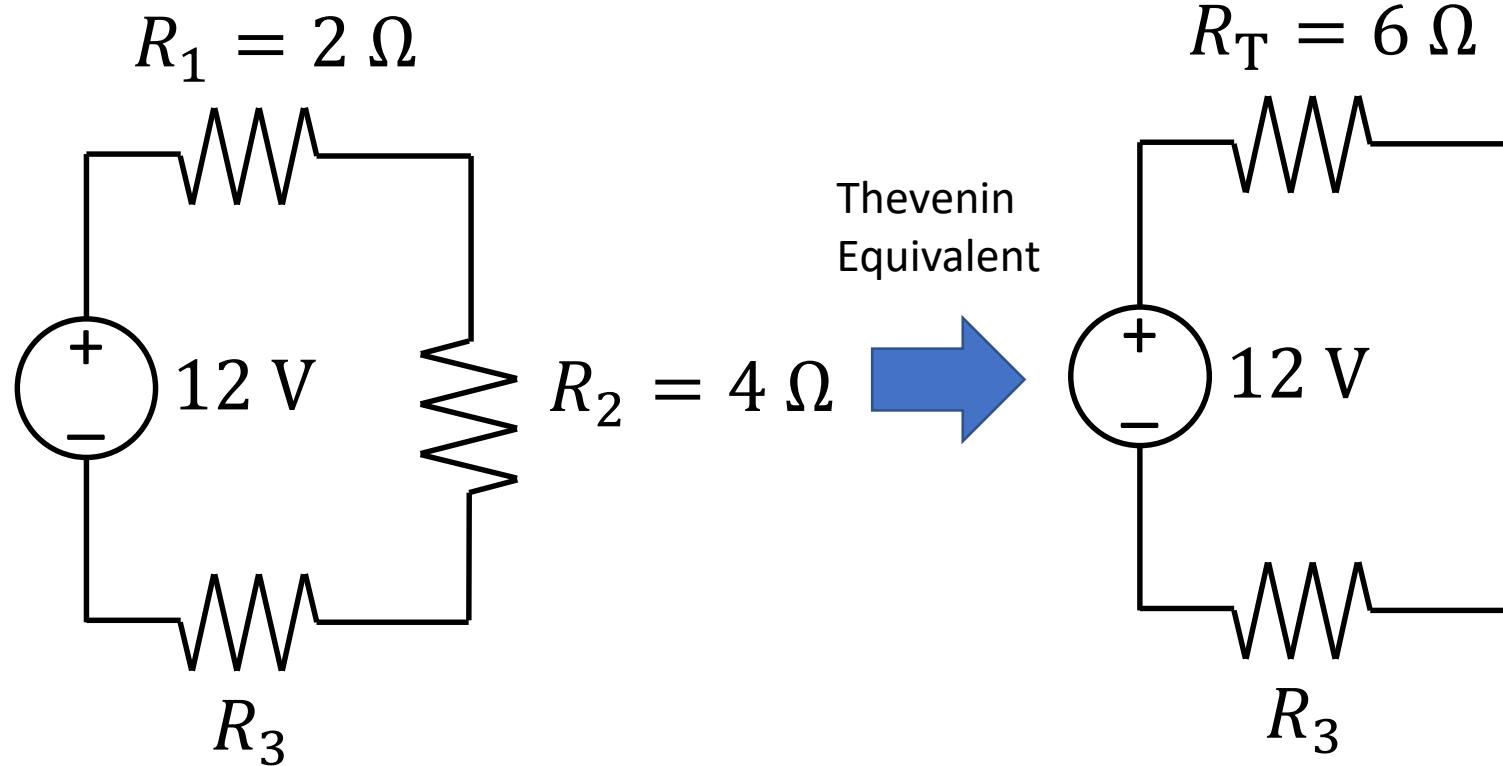
$$\frac{V_3}{R} = I_3 \Rightarrow V_3 = 1I_3 = \frac{41}{26}V$$

$$V_1 = \frac{10V}{\frac{43}{13}V}$$

$$V_2 = \frac{\frac{43}{13}V}{\frac{43}{13}V}$$

$$V_3 = \frac{\frac{41}{26}V}{\frac{41}{26}V}$$

Regular exam 4(a)



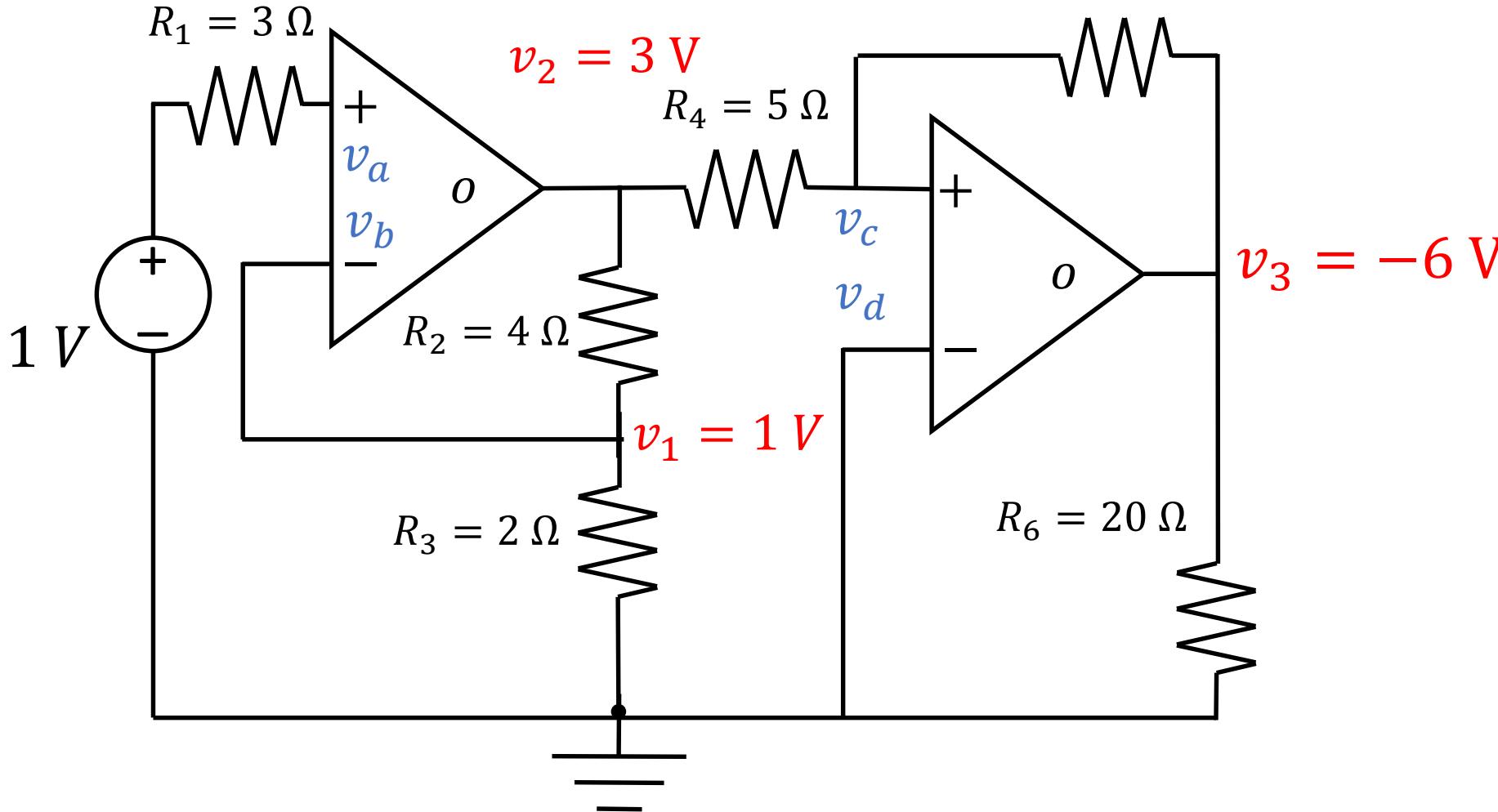
The power of R_3 is maximum
when $R_3 = R_T = 6 \Omega$ (matched load)

$$\text{Max power} = \frac{V_T^2}{4R_T} = \frac{12^2}{4 \times 6} W = 6 W$$

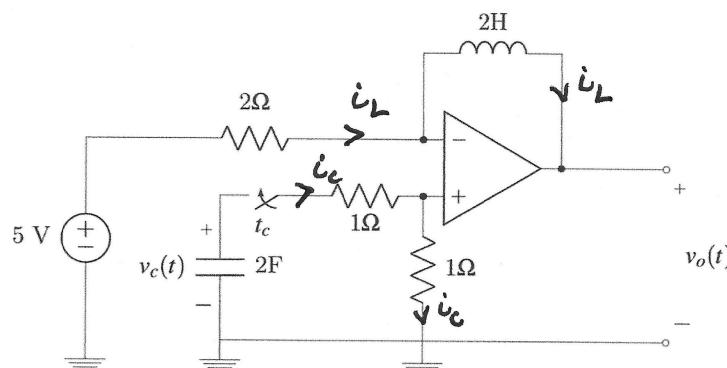
Regular exam 4(b)

$$v_1 = v_b = v_a = 1 \text{ V} \quad v_2 = v_1(R_2 + R_3)/R_3 = 3 \text{ V}$$

$$v_c = v_d = 0 \quad \frac{v_2 - v_c}{R_4} = \frac{v_c - v_3}{R_5} \Rightarrow v_3 = -6 \text{ V}$$



5. (24 pts) In the circuit given below, the switch is closed at time $t_c = 0$. Find the output voltage from the linear op-amp $v_o(t)$ for $t > 0$ if the initial voltage on the capacitor $v_c(t = 0) = 5V$.



$$V_+ = V_- \quad i_+ = i_- = 0$$

Voltage divider: $V_+ = \left(\frac{1}{1+1}\right)V_c(t)$
 $V_+ = \frac{1}{2}V_c(t)$

$$i_C = \frac{V_c(t)}{2} = -2 \frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = -\frac{1}{4}V_c(t)$$

$$V_c(t) = Ae^{-t/4}$$

$$V_c(t=0) = 5V$$

$$5 = A$$

$$V_c(t) = 5e^{-t/4}$$

$$V_+ = \frac{5}{2}e^{-t/4} = V_-$$

$$i_L = \frac{5 - V_-}{2} = \frac{5}{2} - \frac{1}{2}\left(\frac{5}{2}e^{-t/4}\right)$$

$$i_L = \frac{5}{2} - \frac{5}{4}e^{-t/4}$$

$$V_L = V_- - V_o = 2 \frac{di_L}{dt}$$

$$\frac{5}{2}e^{-t/4} - V_o = 2\left(\frac{5}{16}e^{-t/4}\right)$$

$$\frac{5}{2}e^{-t/4} - \frac{5}{8}e^{-t/4} = V_o$$

$$\left(\frac{20}{8} - \frac{5}{8}\right)e^{-t/4} = V_o$$

$$V_o = \frac{15}{8}e^{-t/4}$$

$$v_o(t) = \frac{\frac{15}{8}e^{-t/4}}{V}$$