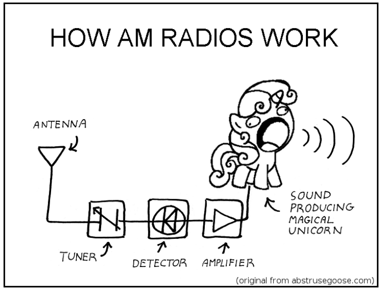
ECE 210 (AL2)

Chapter 8

Modulation and AM Radio



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https://abstrusegoose.com/

Chapter objectives

- Understand modulation
- Understand coherent demodulation of AM signals
- Understand envelope detection of AM signals
- Understand how a superheterodyne receiver with envelope detection works

- Properties of Fourier transform
  - Frequency shift

Recall time shift:

$f(t) \Leftrightarrow F(\omega)$ same sign!

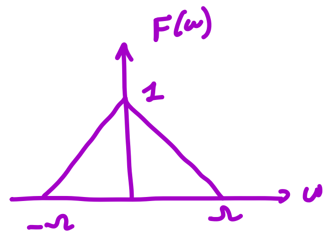
$f(t - t_0) \Leftrightarrow F(\omega)\, e^{j\omega(-t_0)}$

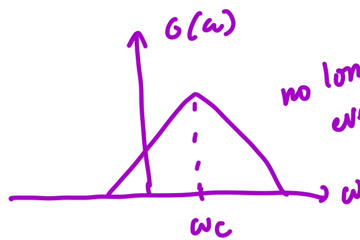
$$f(t) \leftrightarrow F(\omega)$$

opposite sign!

$$g(t) = f(t) e^{j\omega_c t} \leftrightarrow G(\omega) =? \; F(\omega - \omega_c)$$

time-varying, specific freq.





no longer even if $g(t)$ is not real

$g(t) = f(t) e^{j\omega_c t}$

real $f(t)$

$F^*(\omega) = F(-\omega)$

$|F(\omega)|$ even

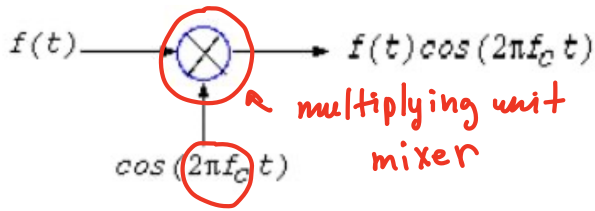
- Properties of Fourier transform-cont

$\omega = 2\pi f$

  - Modulation

$$f(t) \leftrightarrow F(\omega)$$

$$x(t) = f(t)\cos(\omega_c t) \leftrightarrow X(\omega) = ? \quad \frac{1}{2}F(\omega-\omega_c) + \frac{1}{2}F(\omega+\omega_c)$$



multiplying unit

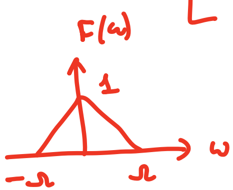
mixer

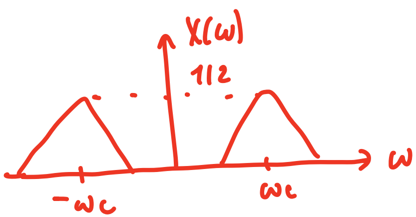
ω_c

$$f(t)\cdot\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right] = \frac{1}{2}f(t)e^{j\omega_c t} + \frac{1}{2}f(t)e^{-j\omega_c t}$$

$$\downarrow \mathcal{F}$$

$$\frac{1}{2}F(\omega-\omega_c) + \frac{1}{2}F(\omega+\omega_c)$$





Multiplication by $\cos(\omega_c t)$ is called modulation.

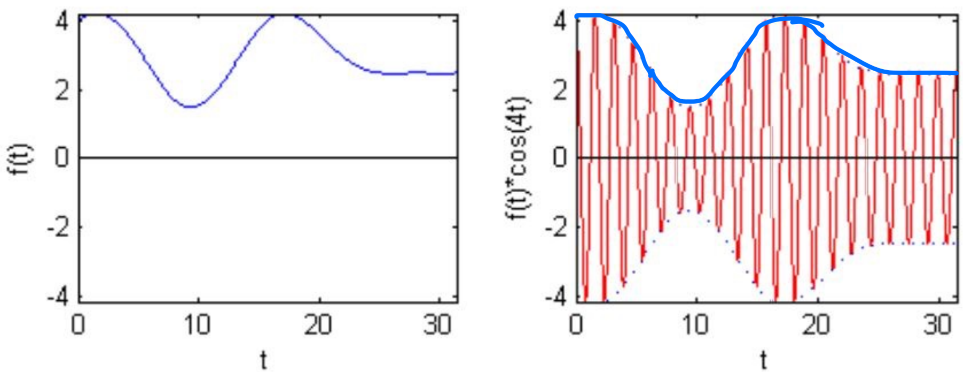
- Properties of Fourier transform-cont
  - Modulation

$$f(t) \leftrightarrow F(\omega)$$

It's carrying f(t) via its amplitude

carrier

$$x(t) = f(t)\cos(\omega_c t) \leftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$



f(t) is modulating the amplitude of $\cos(\omega_c t)$ → amplitude modulation (AM)

- Modulation
  - Why modulate?

1. Antenna length

   Signal wavelength: $\lambda = \frac{c}{f_c}$

   Antenna length for efficient transmission: $L > \frac{\lambda}{4} = \frac{c}{4f_c}$

   Audio bandwidth: ≈ 15KHz ⇒ L > 5Km
   AM radio: (WILL) 580KHz ⇒ L > 130m
   FM radio: 100MHz ⇒ L > 75cm
   Satellite: 10GHz ⇒ L > 7.5mm

2. Available bandwidth
   Can't all transmit at baseband
   Transmitters are assigned frequency bands

- ## Coherent demodulation of AM signals
    - How to demodulate?

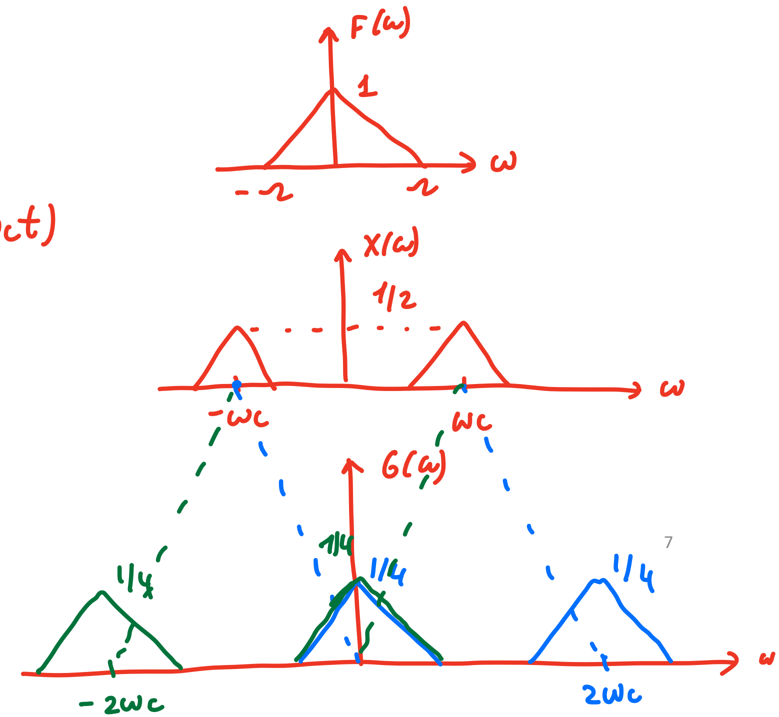
$$\underbrace{x(t) = f(t)\cos(\omega_c t)}_{} \longleftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$

$\downarrow x(t)$

$\cos(\omega_c t) \rightarrow \otimes$

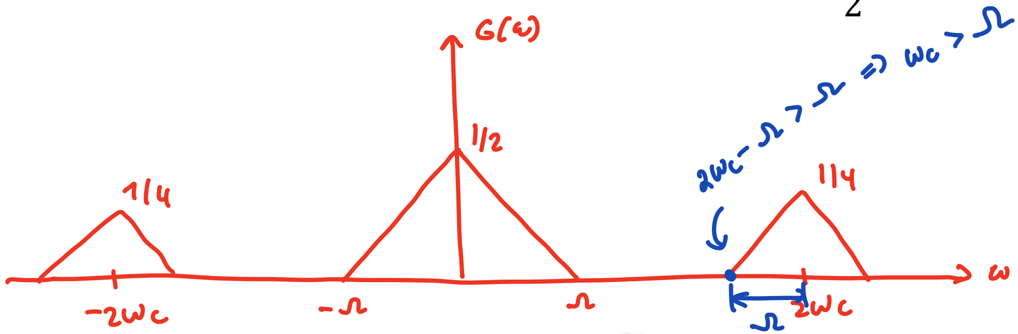
$\downarrow$

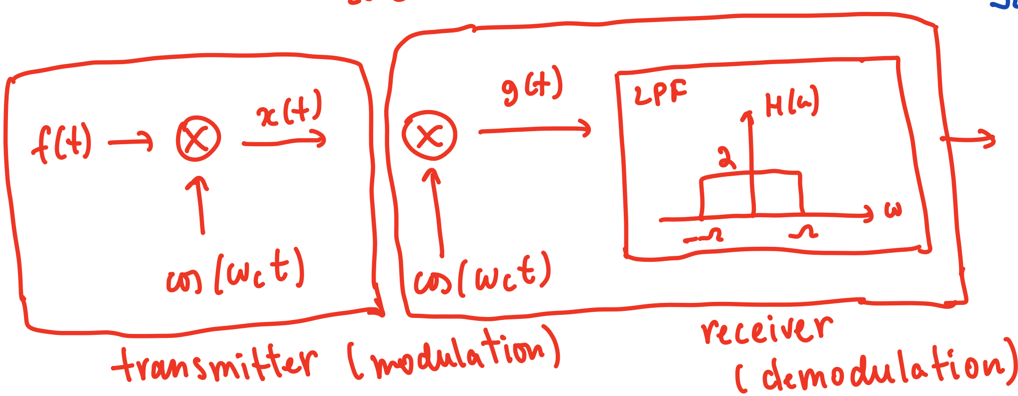
$g(t) = x(t)\cos(\omega_c t)$

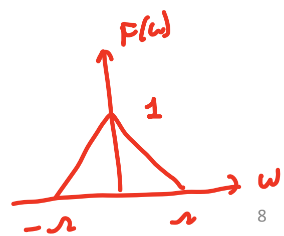


- Coherent demodulation of AM signals-cont
  - How to demodulate?

$$x(t) = f(t)\cos(\omega_c t) \Leftrightarrow X(\omega) = \frac{F(\omega-\omega_c)+F(\omega+\omega_c)}{2}$$







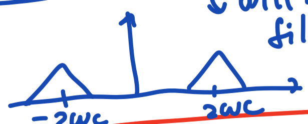
- Coherent demodulation of AM signals-cont
  - What if the channel is not ideal, e.g. there is a time delay?

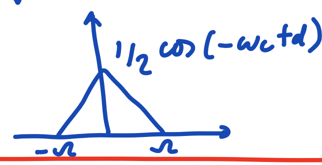
$$\cos x \cos y = \frac{\cos(x-y)+\cos(x+y)}{2}$$

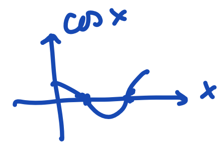
$$g(t) = f(t-t_d)\cos(\omega_c(t-t_d))\cos(\omega_c t) =$$

$$= f(t-t_d)\left[\frac{\cos(2\omega_c t - \omega_c t_d) + \cos(-\omega_c t_d)}{2}\right] =$$

$$= \underbrace{\frac{1}{2} f(t-t_d)\cos(2\omega_c t - \omega_c t_d)}_{\text{will be filtered by low-pass filter}} + \underbrace{\frac{1}{2} f(t-t_d)\overbrace{\cos(-\omega_c t_d)}}$$







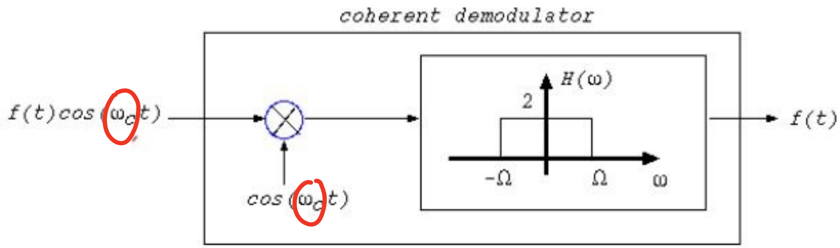
$$g(t) = f(t)\cos(\omega_c t)\cdot\cos(\omega_c t) = f(t)\cos^2(\omega_c t) = f(t)\left[\frac{1+\cos(2\omega_c t)}{2}\right] =$$

$$= \frac{1}{2} f(t) + \overbrace{\frac{1}{2} f(t)\cos(2\omega_c t)}$$

$$\downarrow \mathcal{F}$$

$$\frac{1}{2}F(\omega) + \frac{1}{2}\left[\frac{F(\omega-2\omega_c)+F(\omega+2\omega_c)}{2}\right]$$

- Coherent demodulation of AM signals-cont



- Needs same phase at modulator and at demodulator.