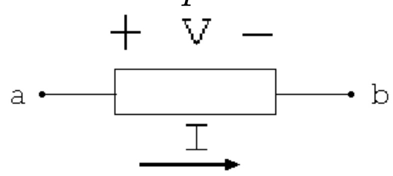
Lecture 3, Friday, January 21, 2022

- ECE 210 students: remember to order your lab kit!

- *Absorbed power*: $P = VI$

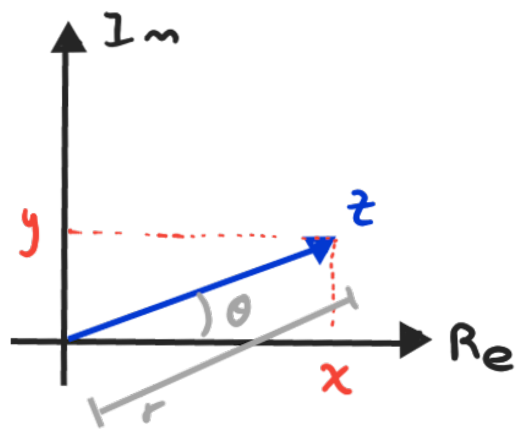


$$P = \frac{dW}{dt} = \frac{dW}{dq}\frac{dq}{dt} = VI \begin{cases} > 0 \text{ absorbs} \\ = 0 \\ < 0 \text{ injects} \end{cases}$$

- *Energy conservation*: sum of absorbed power across all elements in a circuit equals zero:

$$\sum P = 0$$

- Complex numbers
  - A *complex number* $\mathbf{Z}$ is a two-dimensional vector



  * In rectangular form $\mathbf{Z} = \mathbf{x} + \mathbf{jy}$

$$\mathbf{j} = \sqrt{\mathbf{-1}}$$
$$\mathbf{x} = \mathbf{Re}\{\mathbf{Z}\} \in \mathbb{R}$$
$$\mathbf{y} = \mathbf{Im}\{\mathbf{Z}\} \in \mathbb{R}$$

  * In exponential form $\mathbf{Z} = \mathbf{r}\mathbf{e}^{\mathbf{j}\theta}$

$$\mathbf{r} = |\mathbf{Z}|$$
$$\theta = \angle\mathbf{Z} \;\in [-\pi, \pi]$$

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– To convert between exponential and rectangular forms:

$$\mathbf{Re}\{\mathbf{Z}\} = \mathbf{r}\cos(\theta) \qquad \mathbf{Im}\{\mathbf{Z}\} = \mathbf{r}\sin(\theta)$$

$$\mathbf{r} = \sqrt{\mathbf{x^2} + \mathbf{y^2}} \qquad \theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & x > 0 \\ \pi + \tan^{-1}\left(\frac{y}{x}\right) & x < 0, y > 0 \\ -\pi + \tan^{-1}\left(\frac{y}{x}\right) & x < 0, y < 0 \end{cases}$$

– Special cases:

$$\mathbf{e^{j0} = 1} \qquad \mathbf{e^{\pm j\pi} = -1} \qquad \mathbf{e^{\pm j\pi/2} = \pm j}$$

– Euler's identity: $\mathbf{e^{j\theta}} = \cos(\theta) + \mathbf{j}\sin(\theta)$

$$\Rightarrow \qquad \cos(\theta) = \frac{\mathbf{e^{j\theta} + e^{-j\theta}}}{\mathbf{2}} \qquad \sin(\theta) = \frac{\mathbf{e^{j\theta} - e^{-j\theta}}}{\mathbf{2j}}$$

– Operations:

Addition/subtraction: $\mathbf{Z_1 \pm Z_2} = \mathbf{(a_1 + jb_1) \pm (a_2 + jb_2)} = \mathbf{(a_1 \pm a_2) + j(b_1 \pm b_2)}$

Multiplication: $\mathbf{Z_1 Z_2} = \left(\mathbf{r_1 e^{j\theta_1}}\right)\left(\mathbf{r_2 e^{j\theta_2}}\right) = \mathbf{(r_1 r_2)\, e^{j(\theta_1 + \theta_2)}}$

Division: $\dfrac{\mathbf{Z_1}}{\mathbf{Z_2}} = \dfrac{\mathbf{r_1 e^{j\theta_1}}}{\mathbf{r_2 e^{j\theta_2}}} = \left(\dfrac{\mathbf{r_1}}{\mathbf{r_2}}\right)\mathbf{e^{j(\theta_1 - \theta_2)}}$

Powers: $\mathbf{Z_1^n} = \left(\mathbf{r_1 e^{j\theta_1}}\right)^{\mathbf{n}} = \mathbf{r_1^n e^{jn\theta_1}}$

Complex conjugate: $\mathbf{Z_1^*} = \left(\mathbf{r_1 e^{j\theta_1}}\right)^* = \mathbf{r_1 e^{-j\theta_1}} = \mathbf{(a_1 + jb_1)}^* = \mathbf{a_1 - jb_1}$