

ECE 210/211 HWs HW 10

Student XRU7 MZG2

TOTAL POINTS

63.5 / 70

QUESTION 1

1 0 / 0

✓ - 0 pts Correct

QUESTION 2

30 pts

2.1 10 / 10

✓ - 0 pts Correct

2.2 10 / 10

✓ - 0 pts Correct

2.3 8.5 / 10

✓ - 1.5 pts Graph Incorrect, but Exists

QUESTION 3

20 pts

3.1 5 / 5

✓ - 0 pts Correct

3.2 10 / 10

✓ - 0 pts Correct

3.3 5 / 5

✓ - 0 pts Correct

QUESTION 4

20 pts

4.1 10 / 10

✓ - 0 pts Correct

4.2 5 / 10

✓ - 5 pts Incorrect

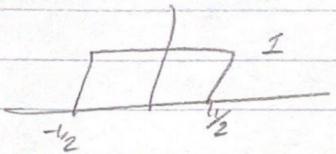
Varenya S Jain, varenya, 655479542

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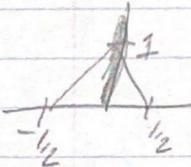
04/01/2022 - ECE 210 - MW #10

1. Varenya Jain

L(a) Unit rect:



Unit diracite:

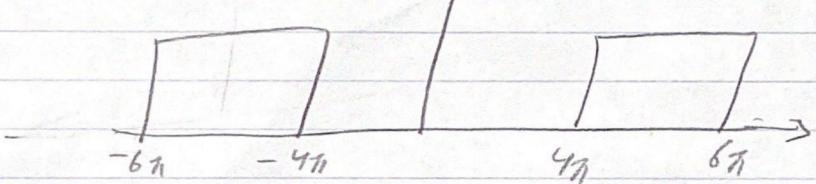


$$H_{\text{ay}} = 3 \text{rect}\left(\frac{w - 5\pi}{2\pi}\right) + 3 \text{rect}\left(\frac{w + 5\pi}{2\pi}\right)$$

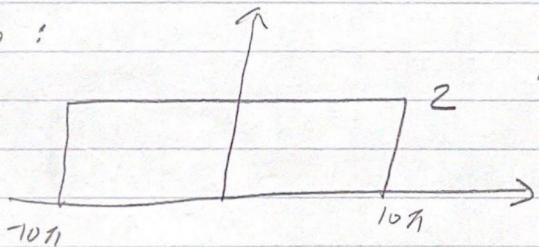


H_{ay}

f_s



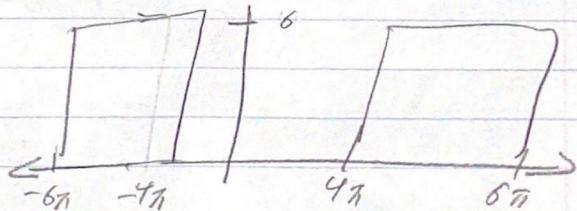
F_{ay} :



$$\checkmark f_{\text{ay}} = 2 \text{rect}\left(\frac{w}{20\pi}\right)$$

$$Y_{\text{ay}} = f_{\text{ay}} \cdot F_{\text{ay}} :$$

\downarrow
 $y_{\text{ay}} \rightarrow y_{\text{ay}}$



$$\hookrightarrow 6 \text{rect}\left(\frac{w - 5\pi}{2\pi}\right) + 6 \text{rect}\left(\frac{w + 5\pi}{2\pi}\right) = Y_{\text{ay}} \rightarrow$$

1 0 / 0

✓ - 0 pts Correct

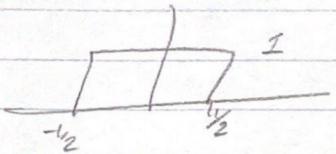
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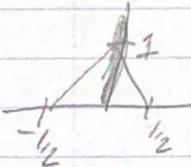
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1. Varenya Jain

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Unit diracite:

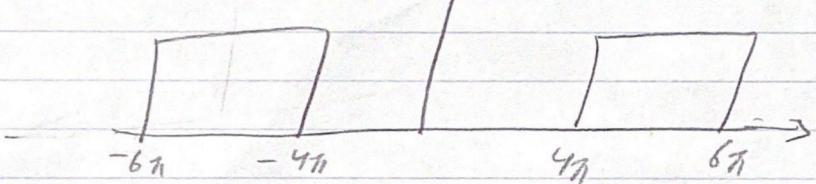


$$H_{\text{ay}} = 3 \text{rect}\left(\frac{w - 5\pi}{2\pi}\right) + 3 \text{rect}\left(\frac{w + 5\pi}{2\pi}\right)$$

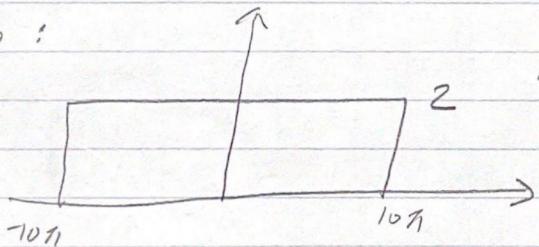


H_{ay}

f_s



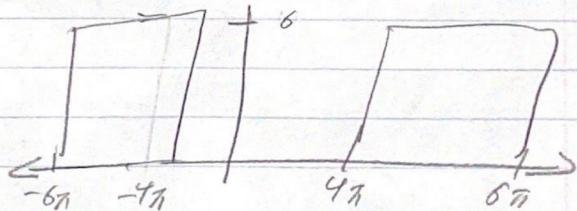
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\downarrow
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$$\hookrightarrow 6 \text{rect}\left(\frac{w - 5\pi}{2\pi}\right) + 6 \text{rect}\left(\frac{w + 5\pi}{2\pi}\right) = Y_{\text{ay}} \rightarrow$$

2

Table 3, # 9: \rightarrow Frequency Shift

$$f(t) \leftrightarrow f(\omega), \quad f(t)e^{j\omega_0 t} \leftrightarrow f(\omega - \omega_0)$$

↓

$$6 \operatorname{rect}\left(\frac{\omega-5\pi}{2\pi}\right) + 6\operatorname{rect}\left(\frac{\omega+5\pi}{2\pi}\right) \rightarrow 6 f(t)e^{-j5\pi t} + 6 f(t)e^{j5\pi t}$$

Table 4 # 8: $\operatorname{sinc}(wt) \leftrightarrow \frac{1}{w} \operatorname{rect}\left(\frac{\omega}{2w}\right)$

$$W = \pi \rightarrow f(t) = \operatorname{sinc}(gt) \quad * \text{ we accounted for the } \pm 5\pi \text{ shift}$$

↓

$$\begin{aligned} y(t) &= 6 \operatorname{sinc}(gt) e^{-j5\pi t} + 6 \operatorname{sinc}(gt) e^{j5\pi t} \\ &= 6 \operatorname{sinc}(gt) \cdot (e^{-j5\pi t} + e^{j5\pi t}) \end{aligned}$$

$$= 6 \operatorname{sinc}(gt) \cdot 2 \cos(5\pi t)$$

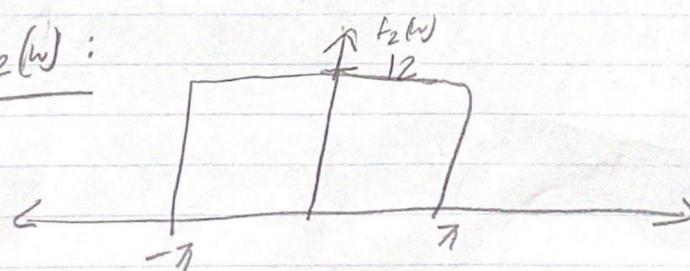
$$\boxed{y(t) = 12 \operatorname{sinc}(gt) \cdot \cos(5\pi t)}$$

b) $y_2(t) = f_2(t) \cos(gt) \equiv y(t) \rightarrow \boxed{\omega_0 = 5\pi}$

$$f_2(t) = 12 \operatorname{sinc}(gt) \leftrightarrow \underline{12 \operatorname{rect}\left(\frac{\omega}{2\pi}\right)} = F_2(\omega)$$

mid rect
freq. scale by 2π

$$\underline{F_2(\omega)}$$



2.1 10 / 10

✓ - 0 pts Correct

2

Table 3, # 9: \rightarrow Frequency Shift

$$f(t) \leftrightarrow f(\omega), \quad f(t)e^{j\omega_0 t} \leftrightarrow f(\omega - \omega_0)$$

↓

$$6 \operatorname{rect}\left(\frac{\omega-5\pi}{2\pi}\right) + 6\operatorname{rect}\left(\frac{\omega+5\pi}{2\pi}\right) \rightarrow 6 f(t)e^{-j5\pi t} + 6 f(t)e^{j5\pi t}$$

Table 4 # 8: $\operatorname{sinc}(wt) \leftrightarrow \frac{1}{w} \operatorname{rect}\left(\frac{\omega}{2w}\right)$

$$W = \pi \rightarrow f(t) = \operatorname{sinc}(gt) \quad * \text{ we accounted for the } \pm 5\pi \text{ shift}$$

↓

$$\begin{aligned} y(t) &= 6 \operatorname{sinc}(gt) e^{-j5\pi t} + 6 \operatorname{sinc}(gt) e^{j5\pi t} \\ &= 6 \operatorname{sinc}(gt) \cdot (e^{-j5\pi t} + e^{j5\pi t}) \end{aligned}$$

$$= 6 \operatorname{sinc}(gt) \cdot 2 \cos(5\pi t)$$

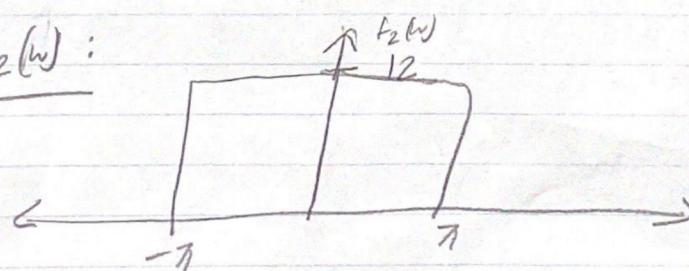
$$\boxed{y(t) = 12 \operatorname{sinc}(gt) \cdot \cos(5\pi t)}$$

b) $y_2(t) = f_2(t) \cos(gt) \equiv y(t) \rightarrow \boxed{\omega_0 = 5\pi}$

$$f_2(t) = 12 \operatorname{sinc}(gt) \leftrightarrow \underline{12 \operatorname{rect}\left(\frac{\omega}{2\pi}\right)} = F_2(\omega)$$

mid rect
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2.2 10 / 10

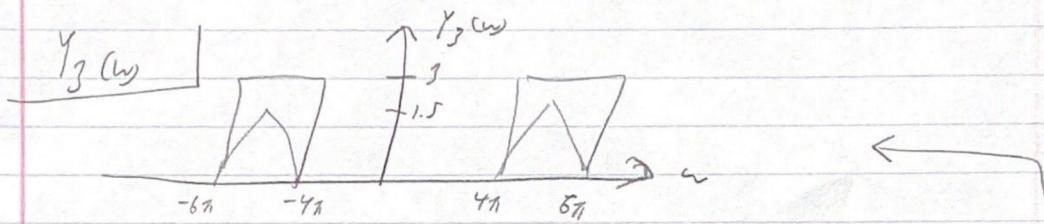
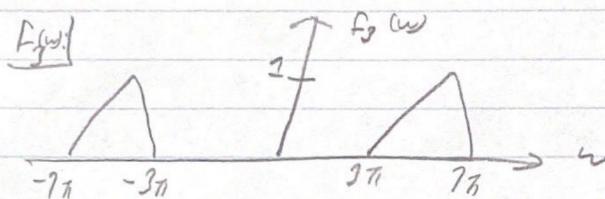
✓ - 0 pts Correct

3

$$2c \quad H(\omega) = 3 \left[\text{rect} \left(\frac{\omega - 5\pi}{2\pi} \right) + \text{rect} \left(\frac{\omega + 5\pi}{2\pi} \right) \right]$$

$$f_g(\omega) = \Delta \left(\frac{\omega - 5\pi}{4\pi} \right) + \Delta \left(\frac{\omega + 5\pi}{4\pi} \right)$$

$$Y_g(\omega) = H(\omega) \cdot f_g(\omega) \rightarrow \mathcal{X}^{-1} \rightarrow y_g(t) = ?$$



$$\begin{aligned} Y_g(\omega) &= 3 \text{rect} \left(\frac{\omega - 5\pi}{2\pi} \right) + 3 \text{rect} \left(\frac{\omega + 5\pi}{2\pi} \right) + 1.5 \Delta \left(\frac{\omega - 5\pi}{2\pi} \right) + 1.5 \Delta \left(\frac{\omega + 5\pi}{2\pi} \right) \\ &= 3 \cdot f_0 \cdot (e^{j\omega\pi/2} - e^{-j\omega\pi/2}) + 1.5 \cdot f_0 \cdot (e^{j\omega\pi/2} - e^{-j\omega\pi/2}) \\ &= 3 \sin(\omega\pi/2) (e^{j\omega\pi/2} + e^{-j\omega\pi/2}) = \frac{3}{2} (\sqrt{2} \sin(\frac{\omega\pi}{2})) e^{j\omega\pi/2} + \frac{3}{2} (\sqrt{2} \sin(\frac{\omega\pi}{2})) e^{-j\omega\pi/2} \end{aligned}$$

$$y_g(t) = 3 \sin(\pi t) \cdot 2 \cos(5\pi t) + \frac{3}{2} \sin^2(\frac{\pi t}{2}) \cdot \cos(5\pi t) = 3 \cos(\pi t) \cdot (\sin(\pi t) + \frac{1}{2} \cos(5\pi t))$$

$$y_g(t) = 3 \cos(\pi t) \cdot (\sin(\pi t) + \frac{1}{2} \cos(5\pi t))$$

2.3 8.5 / 10

✓ - 1.5 pts Graph Incorrect, but Exists

(A)

3. a) $f(t) = \sin(10^5 t)$, A.M. w/ car($\omega_0 t$), $\omega_c = 10^6 \text{ rad/s}$

$$X(\omega) = f(t) \cdot \cos(\omega_0 t) \Leftrightarrow X(\omega) = \frac{1}{2} (F_{\omega-\omega_0} + F_{\omega+\omega_0})$$

$$f(t) \cdot \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) \Leftrightarrow X(\omega), \quad \boxed{\text{Table #3, #10}}$$

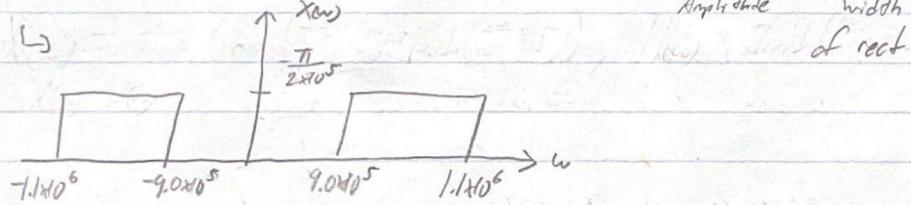
$$H(\omega) = 2\text{rect}\left(\frac{\omega+1.05\omega_c}{10^5}\right) + 2\text{rect}\left(\frac{\omega-1.05\omega_c}{10^5}\right)$$

$$S_{\omega} = X(\omega) \cdot H(\omega)$$

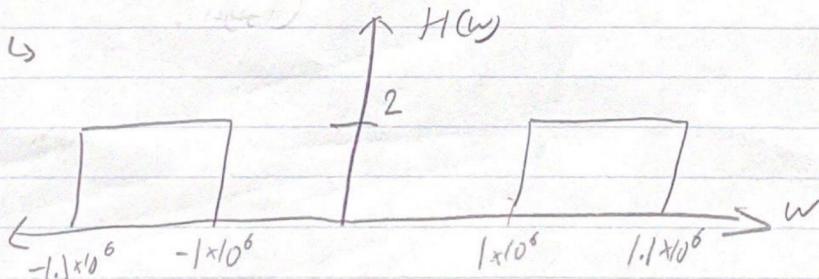
Table 4, #8: $\sin(wt) \Leftrightarrow \frac{\pi}{w} \text{rect}\left(\frac{w}{2w}\right)$

$$W = 10^5 \rightarrow \frac{\pi}{10^5} \text{rect}\left(\frac{w}{2 \cdot 10^5}\right) = F(\omega) \quad \text{shift}$$

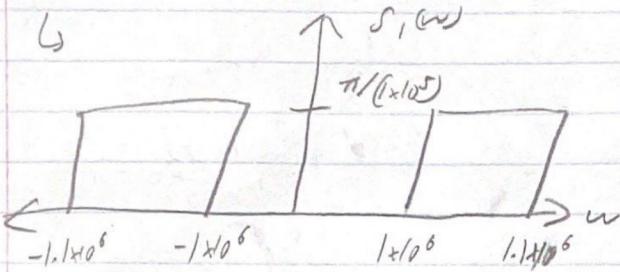
$$X(\omega) = \frac{1}{2} [F_{\omega-\omega_c} + F_{\omega+\omega_c}] = \frac{\pi}{2 \cdot 10^5} [\text{rect}\left(\frac{\omega-\omega_c}{2 \cdot 10^5}\right) + \text{rect}\left(\frac{\omega+\omega_c}{2 \cdot 10^5}\right)]$$



$$\rightarrow H(\omega) = 2\text{rect}\left(\frac{\omega-1.05\omega_c}{10^5}\right) + 2\text{rect}\left(\frac{\omega+1.05\omega_c}{10^5}\right)$$



2b) $S_{c\omega} = X_{c\omega} \cdot H_{c\omega} = \text{overlap of graphs}$



$$\hookrightarrow S_{c\omega} = \frac{\pi}{10^5} \text{rect}\left(\frac{w - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{w + (1.05 \times 10^6)}{10^5}\right)$$

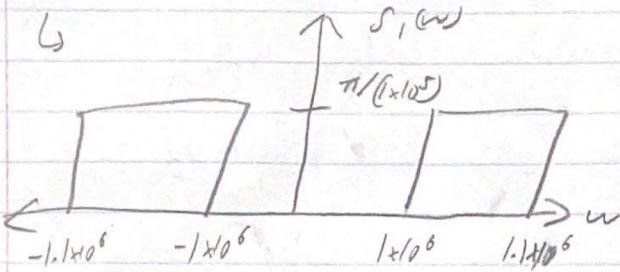
3b) To recover $f(t)$ from $S_{c\omega}$, we need to mix $S_{c\omega}$ with a delayed carrier, pass this mix through a low-pass filter, feed this through an amplifier with a gain of 2 in order to retain the original signal's amplitude. This results in the system returning the original $f(t)$.

3c) A Fourier transform is an even function - a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single-Sideband Modulation, aptly named because one sideband (the 2 sidebands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.1 5 / 5

✓ - 0 pts Correct

2b) $S_{c\omega} = X_{c\omega} \cdot H_{c\omega} = \text{overlap of graphs}$



$$\hookrightarrow S_{c\omega} = \frac{\pi}{10^5} \text{rect}\left(\frac{w - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{w + (1.05 \times 10^6)}{10^5}\right)$$

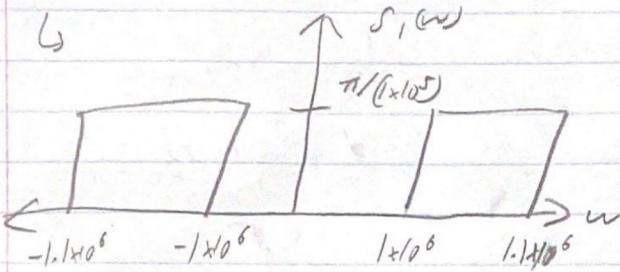
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3c) A Fourier transform is an even function - a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single-Sideband Modulation, aptly named because one sideband (the 2 sidebands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.2 10 / 10

✓ - 0 pts Correct

2b) $S_{c\omega} = X_{c\omega} \cdot H_{c\omega} = \text{overlap of graphs}$



$$\hookrightarrow S_{c\omega} = \frac{\pi}{10^5} \text{rect}\left(\frac{w - (1.05 \times 10^6)}{10^5}\right) + \frac{\pi}{10^5} \text{rect}\left(\frac{w + (1.05 \times 10^6)}{10^5}\right)$$

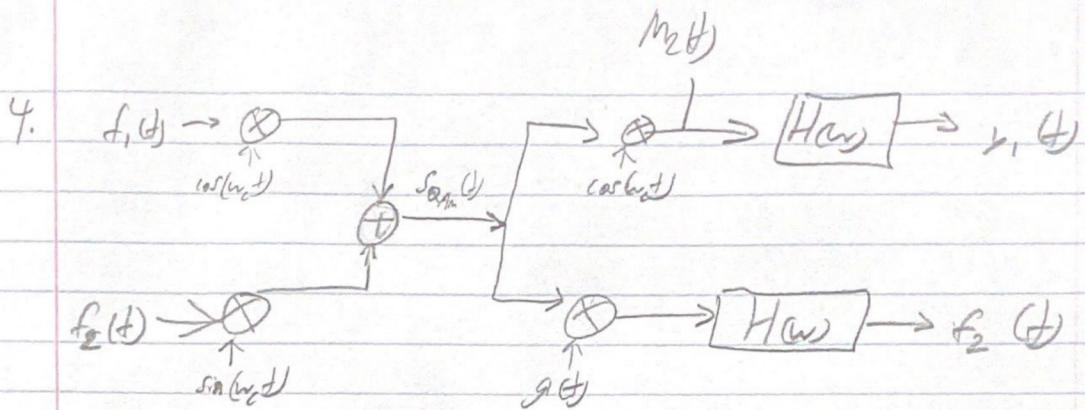
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3c) A Fourier transform is an even function - a rect function in this case. Removing some components of the ordinary AM signal improves the transmission efficiency. The carrier - a steady state signal which itself carries no information - only provides only a reference in this demodulation process. This gets removed in Single-Sideband Modulation, aptly named because one sideband (the 2 sidebands are identical and carry the same information), streamlining the signal of unnecessary bandwidth occupation while allowing the carrier to be re-introduced in the receiver.

3.3 5 / 5

✓ - 0 pts Correct

6



$f_1(t)$ and $f_2(t)$ have 100% bandwidth $\rightarrow R_{100\%} = 40\pi \times 10^3$ rad/s

$$\Rightarrow w_c = 1 \times 10^6 \text{ rad/s}, H_{av} = 2 \operatorname{rect}\left(\frac{w}{20\pi \times 10^3}\right)$$

$$M_1(t) = f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)$$

$$\begin{aligned} M_2(t) &= \cos(w_c t) \cdot (f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)) \\ &= f_1(t) \cdot \cos^2(w_c t) + f_2(t) \cdot \cos(w_c t) \sin(w_c t) \\ &= f_1(t) \cdot \left(1 + 2 \cos(w_c t)\right) + f_2(t) \cdot \left(2 \cdot \sin(w_c t)\right) \end{aligned}$$

$$\hookrightarrow 2w_c = 2 \times 10^6 \Rightarrow R_{100\%}, \text{ Amplitude of } H_{av} = 2$$

$$f_1(t) \cdot 2 = f_1(t) \rightarrow \boxed{y_1(t) = f_1(t)}$$

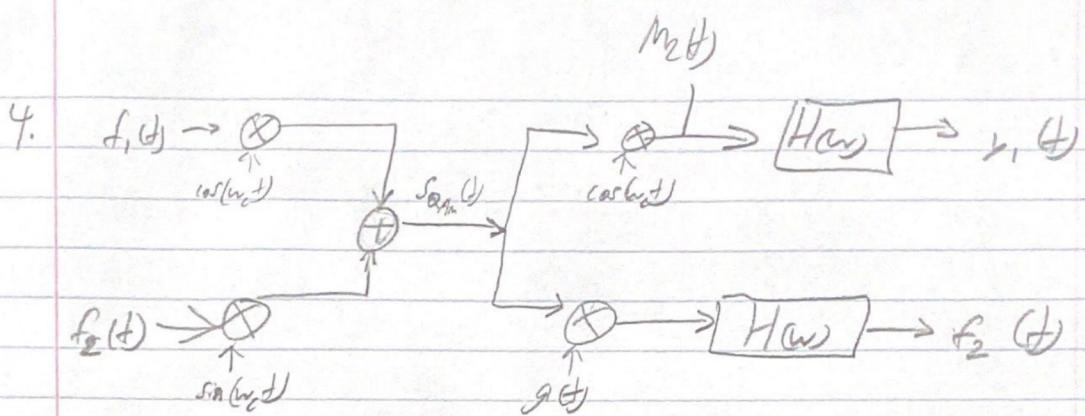
$$\hookrightarrow M_1(t) = f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)$$

$$\hookrightarrow M_2(t) = [f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)] \cdot g(t)$$

4.1 10 / 10

✓ - 0 pts Correct

6



$f_1(t)$ and $f_2(t)$ have 100% bandwidth $\rightarrow R_{100\%} = 40\pi \times 10^3$ rad/s

$$\Rightarrow w_c = 1 \times 10^6 \text{ rad/s}, H_{av} = 2 \operatorname{rect}\left(\frac{w}{20\pi \times 10^3}\right)$$

$$M_1(t) = f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)$$

$$\begin{aligned} M_2(t) &= \cos(w_c t) \cdot (f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)) \\ &= f_1(t) \cdot \cos^2(w_c t) + f_2(t) \cdot \cos(w_c t) \sin(w_c t) \\ &= f_1(t) \cdot \left(1 + 2 \cos(w_c t)\right) + f_2(t) \cdot \left(2 \cdot \sin(w_c t)\right) \end{aligned}$$

$$\hookrightarrow 2w_c = 2 \times 10^6 \Rightarrow R_{100\%}, \text{ Amplitude of } H_{av} = 2$$

$$f_1(t) \cdot 2 = f_1(t) \rightarrow \boxed{y_1(t) = f_1(t)}$$

$$\hookrightarrow M_1(t) = f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)$$

$$\hookrightarrow M_2(t) = [f_1(t) \cdot \cos(w_c t) + f_2(t) \cdot \sin(w_c t)] \cdot g(t)$$

4.2 5 / 10

✓ - 5 pts Incorrect