## Lecture 46, Tuesday, April 19, 2022

- We define the Laplace transform of $f(t)$ as

$$\hat{F}(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt$$

  - The exponent $s = \sigma + j\omega$, is a complex number, and so is $\hat{F}(s)$.

- The region of convergence (ROC) of the Laplace transform is the region in the complex plane where the integral converges.

- Notice that if $f(t)$ is causal and $\hat{F}$ is well-defined at $s = j\omega$, then by letting $\sigma = 0$

$$\hat{F}(j\omega) = \int_{0^-}^{\infty} f(t)e^{-j\omega t}dt \;=\; F(\omega),$$

  - The Fourier transform of $f(t)$ can be obtained by looking at its Laplace transform over the $j\omega$-axis. Therefore, the ROC must include it.

- In particular, if $h(t)$ is the impulse response of an LTIC system:

$$\hat{H}(s) = \int_{0^-}^{\infty} h(t)e^{-st}dt \quad \text{is the } \textit{transfer function} \text{ of the LTIC system}$$

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$$f(t) \longrightarrow \boxed{\text{LTIC with } h(t) \leftrightarrow \hat{H}(s)} \longrightarrow y_{ZS}(t) = f(t) * h(t)$$

$$\hat{F}(s) \longrightarrow \boxed{\text{LTIC with } h(t) \leftrightarrow \hat{H}(s)} \longrightarrow \hat{Y}_{ZS}(s) = \hat{F}(s)\hat{H}(s)$$

$$\Rightarrow \ \hat{H}(s) = \frac{\hat{Y}_{ZS}(s)}{\hat{F}(s)}$$

- In an LTIC system

$$e^{st} \longrightarrow \boxed{\hat{H}(s)} \longrightarrow y(t) \ = \ e^{st}\hat{H}(s)$$

- A *pole* of the Laplace transform $\hat{F}(s)$ is a location on the $s$-plane where $|\hat{F}(s)| \to \infty$
- Hidden poles are poles at $\pm\infty$
- The ROC of the Laplace transform is the region in the $s$-plane to the right of the rightmost pole (not including $\infty$.)
- If there is a pole at $s = \infty$, then the Laplace transform has a term proportional to $s$ (or increasing with $s$.)

 