ECE 210/211 **Fall 2020**

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Analog Signal Processing

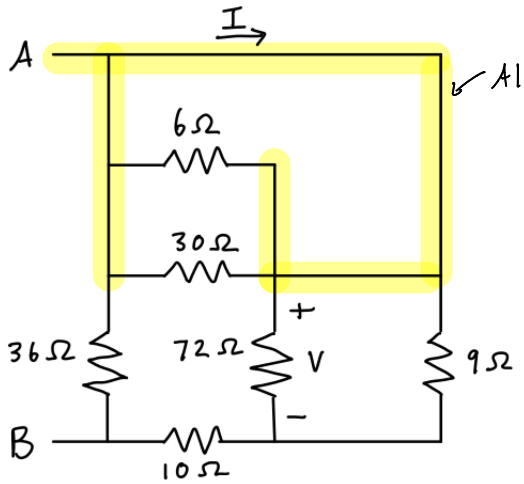
Thursday, September 24, 8pm

Exam I Solutions

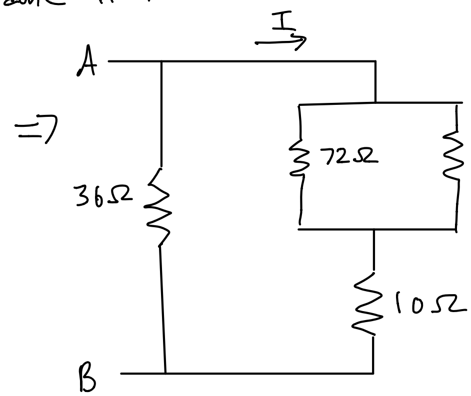
- **Exam duration:** The exam is designed for **75 minutes** and you will have an additional 35 minutes to upload your solutions. The proctor will not stop the exam when the 75 minutes are up, so you could potentially keep working on the exam. However, keep in mind that there will be a **hard stop at 110 minutes** and you will not be allowed to upload after that, so we strongly advise you to finish your exam with plenty of time to spare. If your submission/upload is not completed within the 110 minutes, you will get a **zero** in the exam.
- **No collaboration allowed:** You are **not allowed** to share or collaborate on this exam and all work should be your own.
- **Open textbook:** you can use a hard copy of the textbook. However, you **should not** depend on it, browsing and searching through it will only waste your time. If you know what you are doing, you should not need any aides. Handwritten notes are **not allowed** in paper nor in your tablet nor electronic device.
- **Calculations:** Calculators and other electronic ways to do calculations, like Wolfram alpha, are **not allowed. Neither** is searching online.
- You will be **penalized** for unauthorized actions.
- **PDF of exam:** To solve the exam you may write on the pdf on your tablet, print the pdf and write on it, or write your solutions in blank sheets of paper. If you have paper solutions you will need to scan them at the end of the exam.
- **Solution uploads:** Make sure that your scans or photos are legible and that you correctly assign each solution to its question, or you will be **deducted** at least 5% of the corresponding problem part. Instructions for downloading the exam and uploading your solutions to Gradescope have been available on the course website since the beginning of the semester and you were instructed to try it before the exam to make sure you know how to do it.
- **Show all your work and simplify** your answers. Answers with no explanation/justification/work will be given **little/no credit**.
- **Box your answers**.
- Answers **should** include units if appropriate.

1. (25 pts) Solve the following for the circuit below with $V_{AB} = 12\text{V}$. Assume that the terminals A-B are not open.

· Notice 6Ω, 30Ω resistors are shorted + 4



$\Rightarrow$



$9\Omega \Rightarrow 72\|9 = 8\Omega$

10Ω in series w/ 8Ω $\Rightarrow 18\Omega$

(a) [8 pts] The equivalent resistance, R_{eq}, between A and B

· Notice $\|$, series, $\|$ resistor combos + correct arithmetic + 3

$$R_{eq} = (72\Omega \| 9\Omega + 10\Omega) \| 36\Omega$$

$$= \left(\frac{72 \cdot 9}{72+9}\Omega + 10\Omega\right) \| 36\Omega$$

$$= 18\Omega \| 36\Omega$$

unit +1

$R_{eq} =$ $12\ \Omega$

(b) [8 pts] The voltage, V, across the 72Ω resistor.

Voltage divider

$$(V_{9\Omega} =) V_{72\Omega} = V_{AB} \cdot \frac{\overset{=8}{72\|9}}{\underset{=18}{72\|9 + 10}} = 12 \cdot \frac{4}{9} = \frac{48}{9} V \ (= 5.33V)$$

$$= \frac{16}{3} V$$

Write voltage divider formula + 3
Plug in correct values + 2
Correct arithmetic + 2

unit + 1

$V =$ $\frac{16}{3}$ V

(c) [9 pts] The current, I,at the top.

$$I = \frac{V_{AB}}{R_{eq,\text{ right branch}}}$$

Ohms Law + 4

$$= \frac{12}{72\|9 + 10} = \frac{12V}{18\Omega} = \frac{2}{3} A$$

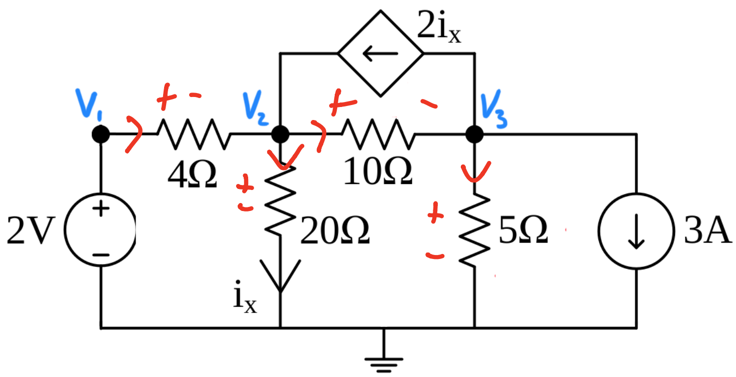
unit + 1

$I =$ $\frac{2}{3}$ A

Correct values + 2
Correct arithmetic + 2

2. (25 pts) The two parts of this question are unrelated.

(a) [18 pts] Consider the circuit below. Use the node-voltage method to obtain a set of three equations, in terms of the node voltages V_1, V_2 and V_3, that can be used to determine the node voltages. Then, determine the values of the node voltages. Note: equations not derived using the node-voltage method will receive no credit.



Equations: $\underline{1}\,V_1 + \underline{0}\,V_2 + \underline{0}\,V_3 = \underline{2}$

$\underline{5}\,V_1 + \underline{-6}\,V_2 + \underline{2}\,V_3 = \underline{0}$

$\underline{0}\,V_1 + \underline{0}\,V_2 + \underline{3}\,V_3 = \underline{-30}$

Values: $V_1 = \underline{2\ V}$

$V_2 = \underline{-\frac{5}{3}\ V}$

$V_3 = \underline{-10\ V}$

From voltage source: $2 = V_1 - 0$

$\rightarrow V_1 = 2$ eq ①

KCL @ ② $\frac{V_1 - V_2}{4} + 2i_x = \frac{V_2 - 0}{20} + \frac{V_2 - V_3}{10}$

From definition $i_x = \frac{V_2 - 0}{20} = \frac{V_2}{20}$

$\rightarrow \left(\frac{V_1 - V_2}{4} + 2\frac{V_2}{20} = \frac{V_2}{20} + \frac{V_2 - V_3}{10}\right) \times 20$

eq ②

$5V_1 - 5V_2 + 2V_2 = V_2 + 2V_2 - 2V_3 \rightarrow 5V_1 - 6V_2 + 2V_3 = 0$

$\downarrow$ substitute ① & ③

$5(2) - 6V_2 + 2(-10) = 0$

$\rightarrow V_2 = \frac{-10}{6} = -\frac{5}{3}\ V$

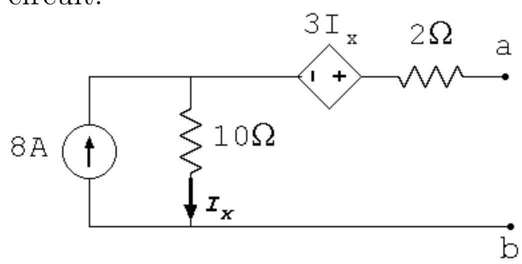
KCL @ ③ $\frac{V_2 - V_3}{10} = 2i_x + \frac{V_3 - 0}{5} + 3$

$\left(\frac{V_2 - V_3}{10} = \frac{2V_2}{20} + \frac{V_3}{5} + 3\right) \times 10$

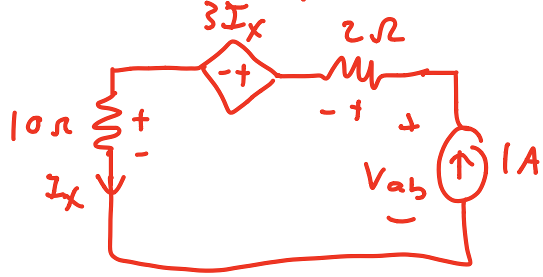
$\rightarrow V_3 = -10\ V$

$\rightarrow V_2 - V_3 = V_2 + 2V_3 + 30 \rightarrow -30 = 3V_3$ eq ③

(b) [07 pts] Determine Thevenin's resistance, R_T, between terminals a-b in the following circuit.



Use test signal



Do KVL : $-10 I_x - 3I_x - 2 I_x + V_{ab} = 0$

$\rightarrow V_{ab} = 15 I_x$

$I_x = 1$

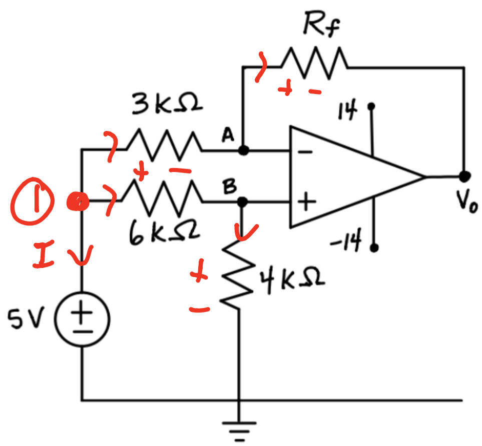
$= 15(1)$

$= 15V$

$\rightarrow R_T = \frac{V_{ab}}{1A} = 15\,\Omega$

$R_T =$ ________________

3. (25 pts) Consider the following op-amp circuit, in which the op amp is powered by a DC voltage biasing source of 14 V that shares a common ground with the source (note that the bias network is not shown in the figure). You can assume that the op-amp is not saturated and it operates in ideal op-amp conditions.



(a) [4 pts] Assuming $R_f = 12k\Omega$, find V_B.

KCL @ B

$$\frac{5-V_B}{6\cancel{K}} = \frac{V_B - 0}{4\cancel{K}}$$

$$20 - 4V_B = 6V_B \rightarrow \frac{20}{10} = V_B = 2V$$

$V_B =$ 2V

(b) [5 pts] Assuming again that $R_f = 12k\Omega$, find V_0.

KCL @ A

$$\frac{5-V_A}{3\cancel{K}} = \frac{V_A - V_o}{12\cancel{K}}$$

$$60 - 12V_A = 3V_A - 3V_o$$

$$\rightarrow 3V_o = 15V_A - 60 = 15V_B - 60 = 15(2) - 60 = -30\text{ V}$$

$$\rightarrow V_o = \frac{-30}{3} = -10$$

$V_0 =$ −10 V

(c) [8 pts] Assuming again that $R_f = 12k\Omega$, determine the absorbed power at the 5V source and indicate if it is injecting or dissipating power? Justify your answer by drawing the direction of positive current through all four resistors.

KCL @ 1

$$I + \frac{5-V_A}{3K} + \frac{5-V_B}{6K} = 0$$

$$\rightarrow I\,6K = 0 - 2V_A + 5 - V_B$$

$$I = \frac{-10 + 2(2) - 5 + 2}{6K} = \frac{-9}{6K} = \frac{-3}{2}\text{mA}$$

$$\rightarrow P = VI = 5\left(\frac{-3}{2}m\right)$$

$P_{5V} =$ $\frac{-15}{2}$ mW

< 0 ↓ injecting

(d) [8 pts] What is the maximum possible value for R_f that would keep the op-amp operating in the linear (non-saturating) regime?

Need $-14 < V_o < 14$

KCL @ A

$$\frac{5-2}{3K} = \frac{2-V_o}{R_f}$$

$$R_f(5-2) = 6K - 3kV_o$$

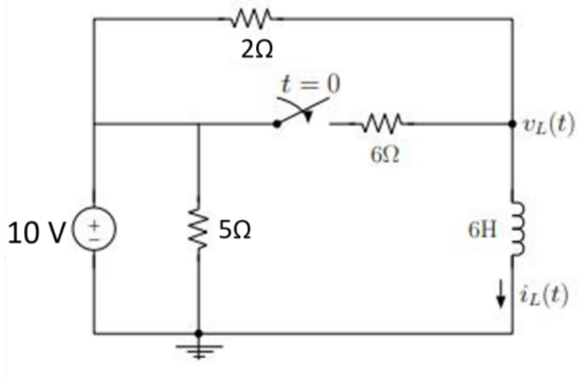
$$\frac{-3R_f + 6K}{3K} = V_o = -R_f m + 2$$

$$\rightarrow -14 < -R_f m + 2 < 14$$

$$R_f < \frac{2+14}{m} = 16K$$

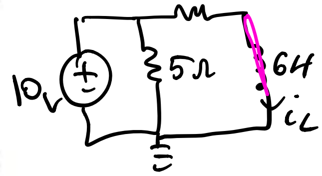
$R_f =$ 16K Ω

4. (25 pts) Consider the circuit shown below. Assume the switch has been open for a long time and it closes at t=0.



$t<0$

$v_L(0^-) = 0V$ (inductor acts as short in the DC steady-state)



$i_L(0^-) = \frac{10}{2} = 5A$

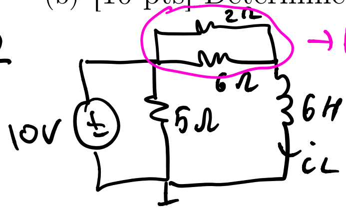
(a) [5 pts] Find $v_L(0^-)$ and $i_L(0^-)$.

$v_L(0^-) =$ 0V

$i_L(0^-) =$ 5A

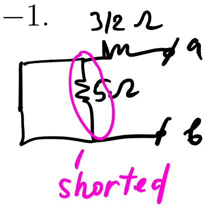
(b) [10 pts] Determine $i_L(t)$ for $t > 0$ and **sketch** it for $t > -1$.

$t>0$



$\rightarrow R_p = \frac{6\cdot 2}{6+2} = \frac{3}{2}\Omega$

Find R_T:



$R_T = \frac{3}{2}\Omega$

$i_L(\infty) = \frac{10}{3/2} = \frac{20}{3}A = B$

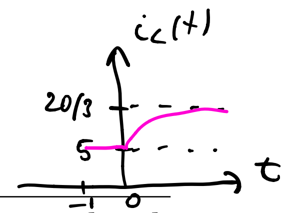
$i_L(0^+) = i_L(0^-) = B + A =$

$= \frac{20}{3} + A = 5 \Rightarrow A = 5 - \frac{20}{3} = -\frac{5}{3}$

$i_L(t) = B + Ae^{-t/\tau}$

$\tau = \frac{L}{R_T}$

$i_L(t) =$ $\frac{20}{3} - \frac{5}{3}e^{-t/4}$ Amps



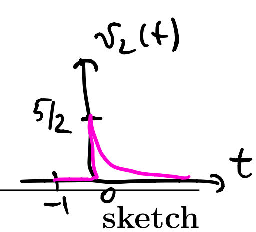
sketch

(c) [10 pts] Find $v_L(t)$ for $t > 0$ and **sketch** it for $t > -1$.

$v_L(t) = L\frac{di}{dt} = 6\left(\frac{5}{3\cdot 4}e^{-t/4}\right) = \frac{5}{2}e^{-t/4}$ V

$v_L(0^+) = \frac{5}{2}$ V

$v_L(t) =$ $\frac{5}{2}e^{-t/4}$ V



sketch