ECE 210 (AL2)

Chapter 10

Impulse Response, Stability, Causality, and LTIC Systems

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Chapter objectives

- Understand the meaning of an LTI system's impulse response and its relation to the frequency response
- Understand and test for BIBO stability
- Understand and test for causality of systems and signals

- Convolution and impulse response

  - Recall that

$$F(\omega) \rightarrow \boxed{\text{LTI with } H(\omega)} \rightarrow Y(\omega) = F(\omega)H(\omega)$$

  - What is $F(\omega)$ or $H(\omega)$ doesn't exist?

$$f(t) \rightarrow \boxed{\text{LTI with } h(t)} \rightarrow y(t) = f(t) * h(t)$$

  - Q: How to get h(t) if we do not know it?
    - Recall that

$$\delta(t) \rightarrow \boxed{\text{LTI with } h(t)} \rightarrow y(t) = \delta(t) * h(t) = h(t)$$

h(t) is the impulse response

- Convolution and unit-step response

  - Consider

$$u(t) \rightarrow \boxed{\text{LTI with } h(t)} \rightarrow y(t) = u(t) * h(t)$$

y(t) is the *unit-step* response

$$\frac{dy(t)}{dt} = \left(\frac{d}{dt} u(t)\right) * h(t) =$$

$$= \delta(t) * h(t) = h(t)$$

  - Q: How to get h(t) from y(t) ?

$$y(t) = f(t) * h(t)$$

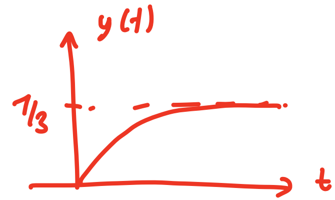
$$\frac{dy(t)}{dt} = \frac{d}{dt} f(t) * h(t) = f(t) * \frac{d}{dt} h(t)$$

$$\boxed{\frac{dy(t)}{dt} = h(t)}$$

- ## Unit-step response - Example # 1

  - Let the unit-step response of an LTI system be

$$y(t) = \frac{1}{3}(1 - e^{-3t})u(t)$$



  - Determine h(t) :

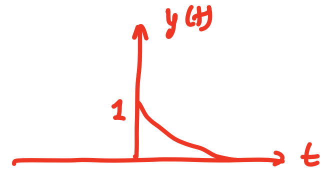
$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = \frac{1}{3}(1-e^{-3t})u(t)$$

@ t=0

$$h(t) = \frac{dy(t)}{dt} = \frac{1}{3}\Big(\underbrace{(1-e^{-3t})\delta(t)}_{\text{sampling}} + 3e^{-3t}u(t)\Big) =$$

$$= \frac{1}{3}\Big(\underbrace{(\underbrace{1-e^{-3(0)}}_{0})\delta(t)}_{0} + 3e^{-3t}u(t)\Big) = e^{-3t}u(t)$$

- Unit-step response - Example # 2



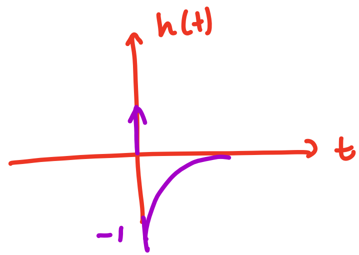
  - Let the unit-step response of an LTI system be

$$y(t) = e^{-t}u(t)$$

  - Determine h(t) :

$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = e^{-t}u(t)$$

$$h(t) = \frac{dy(t)}{dt} = \underbrace{e^{-t}\delta(t)} - e^{-t}u(t) = \underbrace{e^{-0}}_{1}\delta(t) - e^{-t}u(t)$$



- Bounded input-bounded output (BIBO) stability
  - Consider

general

$f(t) \rightarrow$ system $\rightarrow y(t)$

  - In a bounded input-bounded output stable system

bounded $\rightarrow$ system $\rightarrow$ bounded

for any bounded f(t) $|f(t)| \le C_1 \Rightarrow |y(t)| \le C_2$

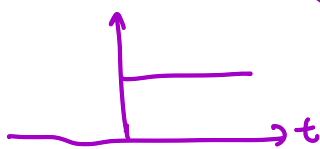
Note: BIBO doesn't care what happens to unbounded inputs

$u(t) \longrightarrow \boxed{h(t) = u(t)} \longrightarrow y(t) = u(t) * u(t) =$

$= t\,u(t)$

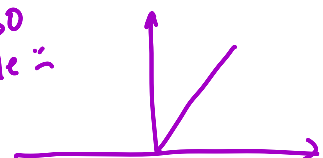
LTI

$\Downarrow$ system is not BIBO stable



bounded

$|u(t)| \le 1$



unbounded

- BIBO stability and LTI systems
  - If the system is LTI, then it is BIBO if and only if its impulse response is absolutely integrable.

$$\text{BIBO stable} \iff \int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

Assuming h(t) is a function. If not, use original definition

check the system, not the input!

Note: do not check boundness of h(t), just A.I.!

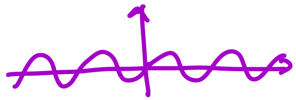
- BIBO stability and LTI systems - Example # 3

  - Determine which of the following impulse responses correspond to BIBO stable systems:

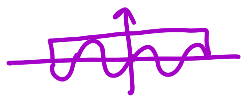
Check $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

X not BIBO stable

1. $h(t) = \sin(\omega_0 t)$



2. $h(t) = \sin(\omega_0 t) rect(t)$



V ☺ BIBO stable

3. $h(t) = \cos(\omega_0 t)$u(t)

4. $h(t) = 2u(t-1)$

5. $h(t) = \delta(t-1)$