

## Lab 5: Sampling, reconstruction, and software radio

Until this point, your study of signals and systems has concerned only the continuous-time case<sup>1</sup>, which dominated the early history of signal processing. About 60 years ago, however, the development of the modern computer generated research interest in digital signal processing (DSP), a type of discrete-time signal processing. Although hardware limitations made most real-time DSP impractical at the time, the continuing maturation of the computer has been matched with a continuing expansion of DSP. Much of that expansion has been into areas previously dominated by continuous-time systems: our telephone network, medical imaging, music recordings, wireless communications, and many more.

You do not need to worry whether the time and effort you have invested in studying continuous-time systems will be wasted because of the growth of DSP — digital systems are practically always hybrids of analog and digital subsystems. Furthermore, many DSP systems are linear and time-invariant, meaning that the same analysis techniques apply, although with some modifications. In this lab, you will explore some of the parallels between continuous-time systems and DSP with a “software radio” designed to the same specifications as the receiver circuit you developed on your protoboard.

## 1 Prelab

Our software radio is typical of many DSP systems in that both the available input and required output are continuous-time signals. The conversion of a continuous-time input signal to a discrete-time signal is called **sampling** (or A/D conversion), and the conversion of a discrete-time signal to a continuous-time output signal is called **reconstruction** (or D/A conversion). As discussed in class, **samples**  $f(nT)$  of a **band-limited** analog signal  $f(t)$  can be used to reconstruct  $f(t)$  exactly when the **sampling interval**  $T$  and **signal bandwidth**  $\Omega = 2\pi B$  satisfy the **Nyquist criterion**  $T < \frac{1}{2B}$ .

This is illustrated by the hypothetical system shown in Figure 1, where the analog signal  $f(t)$  defined at the output stage of a low-pass filter  $H_1(\omega)$  has a bandwidth  $\Omega = 2\pi B$  limited by the bandwidth  $\Omega_1 = 2\pi B_1$  of the filter. A/D converter extracts the samples  $f(nT)$  from  $f(t)$  with a sampling interval of  $T$ , and D/A conversion of samples  $f(nT)$  into an analog signal  $y(t)$  can be envisioned as low-pass filtering of a hypothetical signal  $f_T(t) = \sum_n f(nT)\delta(t - nT)$  using the filter  $H_2(\omega)$ . With an appropriate choice of  $H_2(\omega)$ , the system output  $y(t)$  will be identical to  $f(t)$  in all its details as long as  $T < \frac{1}{2B_1}$ . The reason for that can be easily appreciated after comparing the Fourier transforms  $F(\omega)$  and  $F_T(\omega)$  of signals  $f(t)$  and  $f_T(t)$  with the help of Figure 2.

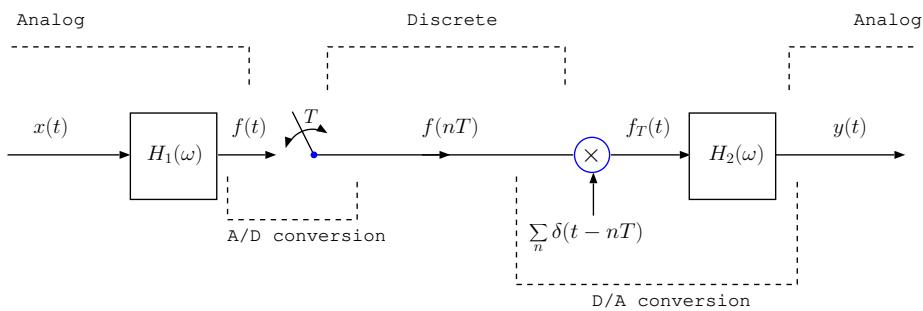


Figure 1: A conceptual system that samples a band-limited continuous-time signal  $f(t)$  and reconstructs a continuous-time output  $y(t)$ . An A/D converter generates the samples  $f(nT)$  of its analog input  $f(t)$ . An ideal D/A converter generates a signal  $f_T(t) = \sum_n f(nT)\delta(t - nT)$  from samples  $f(nT)$  and low-pass filters  $f_T(t)$  with  $H_2(\omega)$  to produce an analog  $y(t)$ . In a practical D/A converter the impulse train  $\sum_n \delta(t - nT)$  is replaced by a practical pulse train  $\sum_n p(t - nT)$  such that  $p(t) * h_2(t)$  is a closed approximation of a delayed sinc( $\frac{\pi}{T}t$ ).

<sup>1</sup>The term “continuous time” is used generically to refer to signals that are functions of a continuous independent variable. Often that variable represents time, but it may instead represent distance, etc. “Discrete time” is used in the same way.

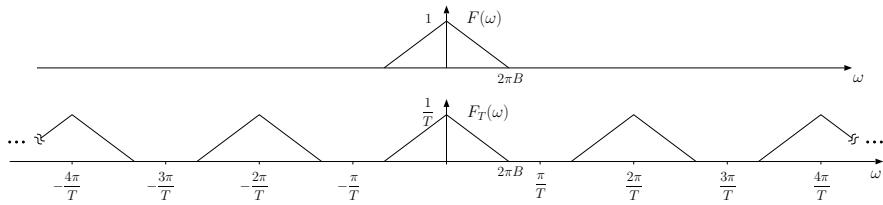


Figure 2: An example comparing the Fourier transforms of signals  $f(t)$  and  $f_T(t)$  defined in Figure 1. Since for  $|\omega| < \frac{\pi}{T}$  the two Fourier transforms have the same shape, low-pass filtering of  $f_T(t)$  yields the original analog signal  $f(t)$ .  $F_T(\omega)$  is constructed as a superposition of replicas of  $\frac{F(\omega)}{T}$  shifted in  $\omega$  by all integer multiples of  $\frac{2\pi}{T}$  (see item 25 in Table 7.2 in the text).

The following prelab exercises concern the system shown Figure 1. Assume that  $T = \frac{1}{44100}$  s (i.e., the sampling frequency is  $T^{-1} = 44100$  Hz) and signal  $x(t)$  has a Fourier transform  $X(\omega)$  shown in Figure 3.

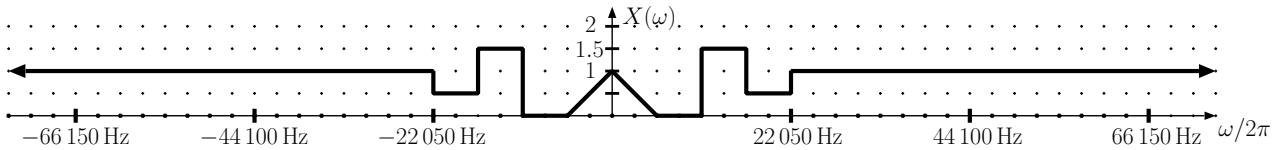


Figure 3: Fourier transform of  $x(t)$  for prelab questions.

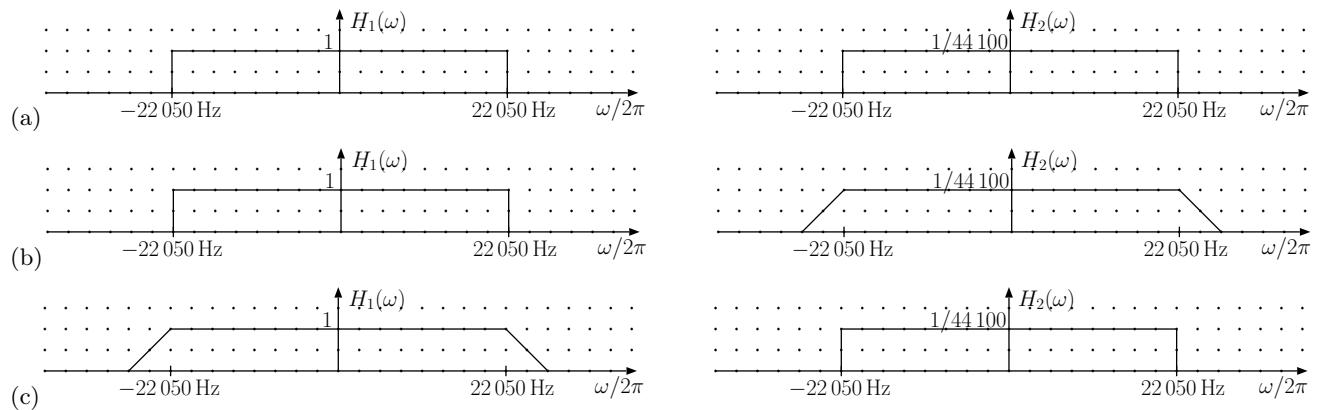
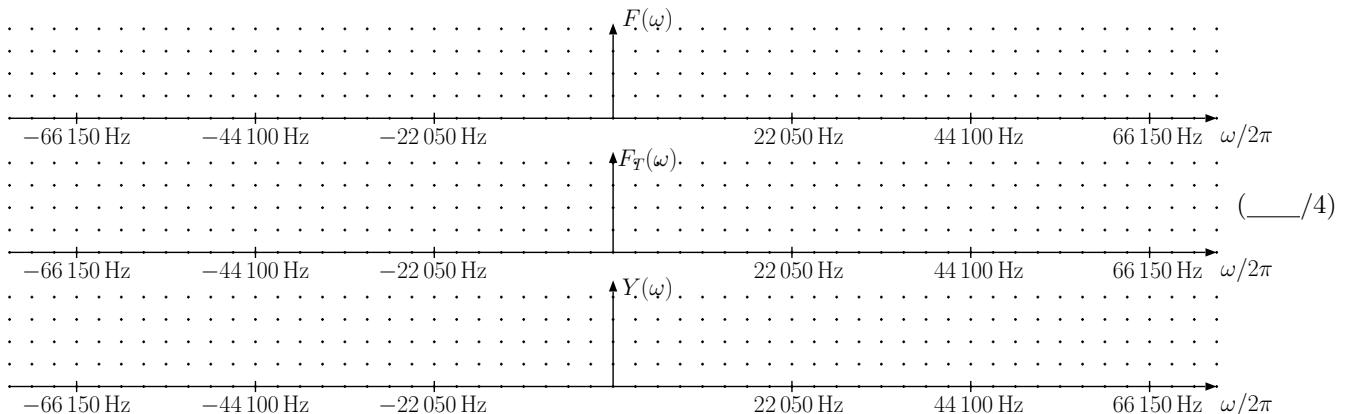


Figure 4: Filter frequency responses for the prelab problems.

1. For frequency responses  $H_1(\omega)$  and  $H_2(\omega)$  in Figure 4a, sketch the Fourier transform of  $f(t)$ ,  $f_T(t) = \sum_n f(nT)\delta(t-nT)$ , and  $y(t)$ . Label the y axis of your plot carefully using appropriate tick marks and tags.



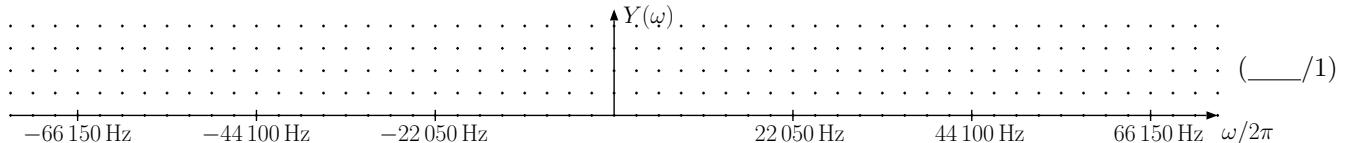
Is  $y(t)$  a perfect reconstruction of  $x(t)$ ? Explain your reasoning.

( \_\_\_/2)

Is  $y(t)$  a perfect reconstruction of  $f(t)$ ? Explain your reasoning.

(\_\_\_\_/2)

2. Now suppose an ideal  $H_1(\omega)$  but a non-ideal  $H_2(\omega)$  given by Figure 4b. The signals  $f(t)$  and  $f_T(t)$  are unchanged, but sketch the Fourier transform of the new  $y(t)$ .



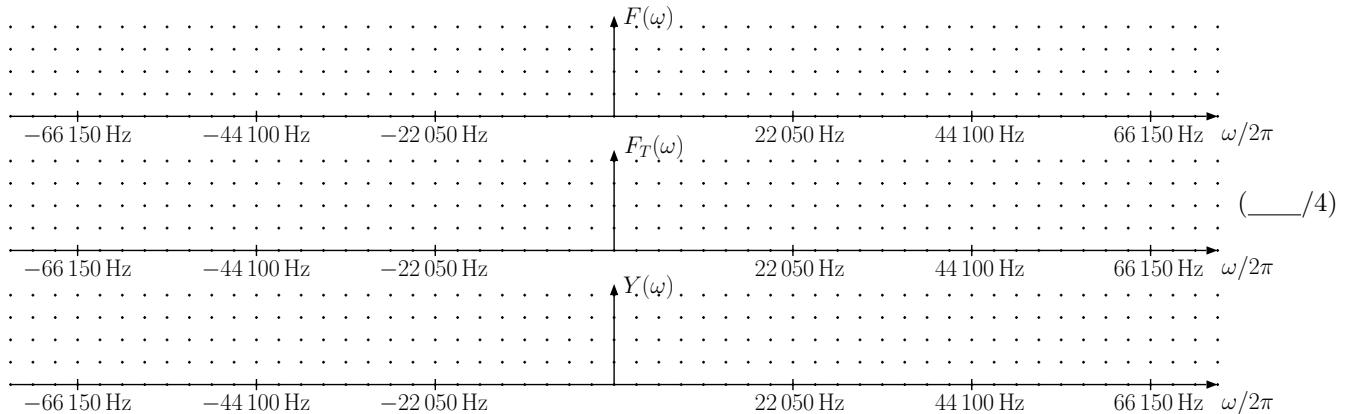
Is  $y(t)$  a perfect reconstruction of  $f(t)$ ? Explain your reasoning.

(\_\_\_\_/2)

Is it still possible to have a perfect reconstruction of  $f(t)$  based on the signal  $y(t)$ ? Explain your reasoning.

(\_\_\_\_/2)

3. Now suppose a non-ideal  $H_1(\omega)$  and an ideal  $H_2(\omega)$  given by Figure 4c. Sketch the Fourier transform of  $f(t)$ ,  $f_T(t)$ , and  $y(t)$ .



Is  $y(t)$  a perfect reconstruction of  $f(t)$ ? Explain your reasoning.

(\_\_\_\_/2)

Is it still possible to have a perfect reconstruction of  $f(t)$  based on the signal  $y(t)$ ? Explain your reasoning.

(\_\_\_\_/2)

4. Discuss the role of the filter  $H_1(\omega)$  in the system examined above. In what way does it impact the system output?

(\_\_\_\_/2)

5. Discuss the role of the filter  $H_2(\omega)$  in the system examined above. In what way does it impact the system output?

(\_\_\_\_/2)

## 2 Laboratory exercise

Lab 5 is a lab based on Jupyter notebook which gives you step-by-step access to python code.

### 2.1 Jupyter Notebook instruction

1. Go to <https://mybinder.org/>

### Turn a Git repo into a collection of interactive notebooks

Have a repository full of Jupyter notebooks? With Binder, open those notebooks in an executable environment, making your code immediately reproducible by anyone, anywhere.

New to Binder? Get started with a Zero-to-Binder tutorial in [Julia](#), [Python](#) or [R](#).

Build and launch a repository

**Step1: Enter the URL here**

GitHub repository name or URL

Git branch, tag, or commit

Path to a notebook file (optional)  File

Copy the URL below and share your Binder with others: **Step2: Click launch**

Fill in the fields to see a URL for sharing your Binder.

Copy the text below, then paste into your README to show a binder badge:  [launch binder](#)

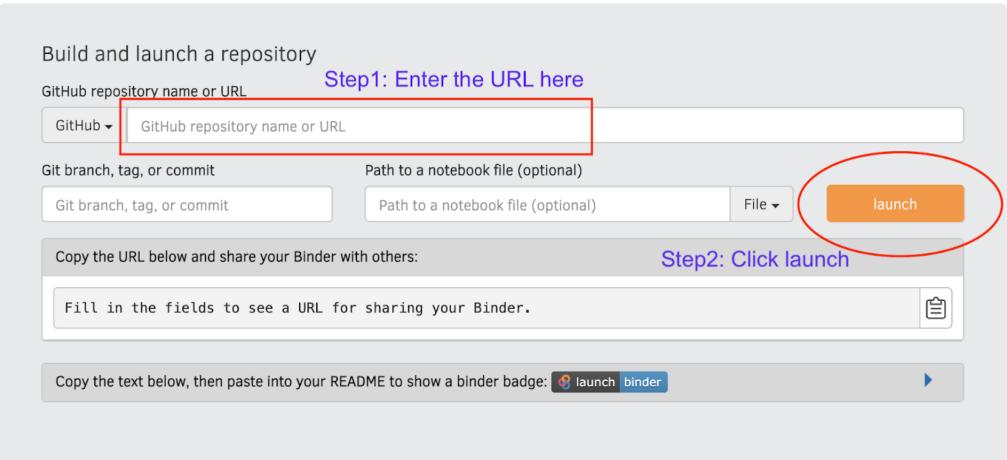


Figure 5: Instruction for using Jupyter Notebook in binder

2. You will see the webpage looks like Figure 5. Enter URL <https://github.com/Krasus7/ECE-210-Lab-5> in the rectangle box, then click the icon “launch”.



Figure 6: Instruction for opening terminal in Jupyter Notebook

3. The launching might take several minutes, then in the next page, click “Lab5.ipynb”, this will open the Jupyter notebook.
4. Binder will automatically shut down user sessions that have more than 10 minutes of inactivity (if you leave your window open, this will be counted as “activity”). Otherwise it will last up to 12 hours. If you want to save your work, you may download “Lab5.ipynb” in the page shown in Figure 6. (Check the box next to the file and download option will appear)

## 2.2 Answer sheet

There are instructions and questions in the notebook you just opened, however, the final submission file should be this lab report only. Please fill your answers for the notebook in the following blanks.

1.1 Describe what you see in time domain of the sampled input signal (top right plot)

(\_\_\_\_/2)

1.2 Describe what you see in frequency domain of the ouput (bottom right plot)

(\_\_\_\_/2)

1.3 Does the output  $y(nT)$  resemble the input signal  $f(t)$  ? If not, then explain the difference.

(\_\_\_\_/2)

1.4 Describe what happens to the output signal in the time and frequency domain.

(\_\_\_\_/2)

1.5 At 24 kHz, does the output  $y(nT)$  resemble the input signal  $f(t)$  or do we have an aliased component ?

(\_\_\_\_/2)

1.6 What is the significance of the frequency 22.05 kHz ? Remember the sampling rate is 44100 Hz.

(\_\_\_\_/4)

1.7 Finally increase the input frequency from 24 to 30 kHz. What do you observe ? Explain what is happening.

(\_\_\_\_/4)

2.1 What is the frequency of the output  $y(t)$ ?  (\_\_\_\_/2)

2.2 Explain why do you have such output, when the input is a 1.5 kHz square wave. (Hint: Having in mind the analysis of the periodic square wave done in lab 3, recall that the input signal should consist of decreasing odd harmonics of the fundamental frequency.)  (\_\_\_\_/4)

2.3 Explain why the second largest frequency component is around 5 kHz.  (\_\_\_\_/4)

3.1 What does the strong impulse at  $f = 13$  kHz represent in this AM signal.  (\_\_\_\_/2)

3.2 At what frequency locations are the other two impulses surrounding the 13 kHz impulse, and what do they represent ?  (\_\_\_\_/4)

3.3 Compare the frequency response of the digital band pass filter with the frequency response of the analog band pass filter built in lab 3, which one is considered a better filter and why ?  (\_\_\_\_/4)

3.4 In the time domain, what type of signal is that. Describe it. And in the frequency domain, can you find the message signal ?  (\_\_\_\_/2)

3.5 Overall, what does the envelope detector or the coherent detector accomplish ?  (\_\_\_\_/2)

4.1 In the spectrum of input, what is the frequency of the peak and what does it represent ?

(\_\_\_\_/2)

4.2 Approximately what is the bandwidth of the output signal ? Does it align with the frequency response of the band pass filter ?

(\_\_\_\_/2)

4.3 Did you successfully demodulate the AM signal ? How was it compare to the one you did with circuit ? Explain the advantage of digital filter.

(\_\_\_\_/4)

**Demo : Please have your Jupyter notebook running before your demo, demo questions will be related to the notebook.**

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## The End

Congratulations on completing the ECE 210 lab! Over these five labs you have learned and applied the most important principles of continuous-time signals and systems and explored their parallels in discrete-time signals and systems. Advanced coursework in ECE will require you to apply these principles again and again. You are well prepared!