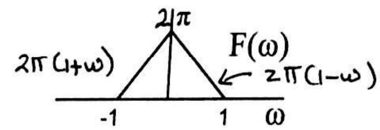
Problem 2

An f(t) signal is given with its Fourier Transform $F(\omega)$.



(a) Find its energy ε_f.

$$\mathcal{E}_f = \frac{1}{2\pi}\int |F(\omega)|^2 d\omega = \frac{2}{2\pi}\int_0^1 (2\pi)^2(1-\omega)^2 = 4\pi\int_0^1 (1-2\omega+\omega^2)\,d\omega = 4\pi/3 \text{ J}$$

$\varepsilon_f =$ $4\pi/3$ J

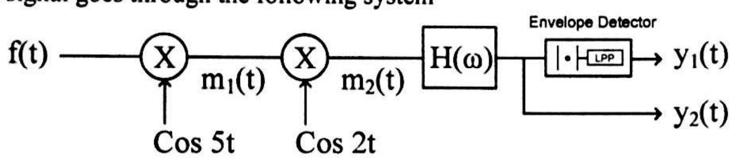
(b) Find its 3dB Bandwidth.

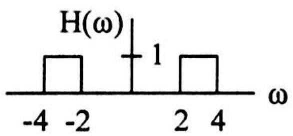
$$\frac{|F(\Omega)|}{|F(0)|} = \frac{1}{\sqrt{2}} = \frac{2\cancel{\pi}(1-\omega)}{2\cancel{\pi}} = \frac{1}{\sqrt{2}}$$

$$1-\omega = \frac{1}{\sqrt{2}} \rightarrow \omega = 1-\frac{1}{\sqrt{2}} \text{ r/sec}$$

$B\omega =$ $\left(1-\frac{1}{\sqrt{2}}\right)$ r/sec

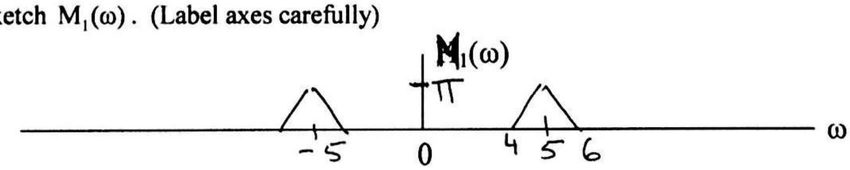
(c) This f(t) signal goes through the following system





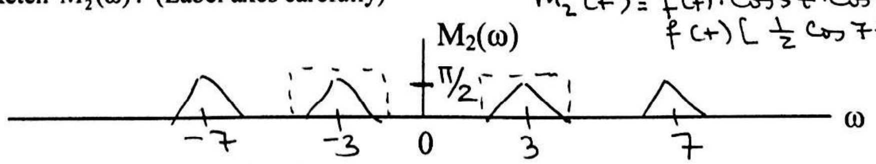
$$m_1(t) = \frac{1}{2}F(\omega-5)+\frac{1}{2}F(\omega+5)$$

i) Sketch $M_1(\omega)$. (Label axes carefully)



ii) Sketch $M_2(\omega)$. (Label axes carefully)

$$m_2(t) = f(t)\cdot \cos 5t \cdot \cos 2t$$
$$f(t)\left[\frac{1}{2}\cos 7t + \frac{1}{2}\cos 3t\right]$$



iii) And, the signals $y_1(t)$ and $y_2(t)$ are:

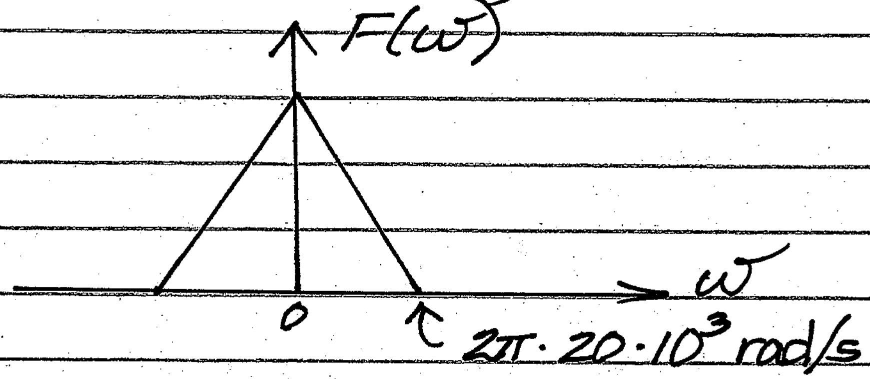
$y_1(t) =$ $\frac{1}{2}f(t) = \frac{1}{2}\,\mathrm{Sinc}^2\frac{t}{2}$

$y_2(t) =$ $\frac{1}{2}f(t)\cdot\cos 3t$

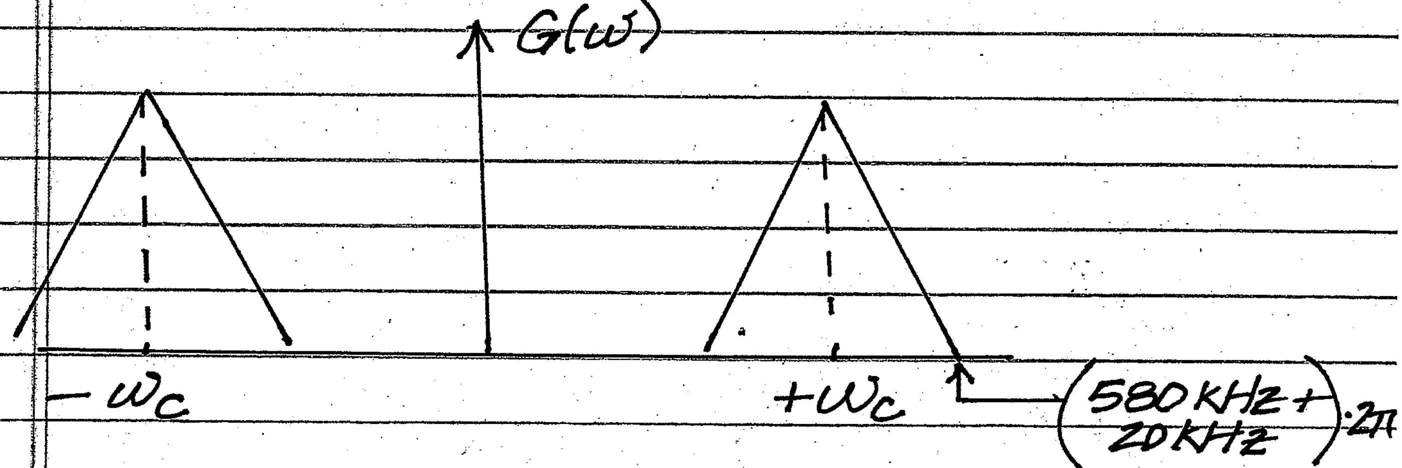
$\frac{1}{2}\,\mathrm{Sinc}^2\frac{t}{2}\cdot\cos 3t$

SOLUTION TO PROBLEM 2
EXAM #3

1. Since $F(\omega)$ may be represented as:



then the largest positive frequency component in $G(\omega)$ is 600 kHz:

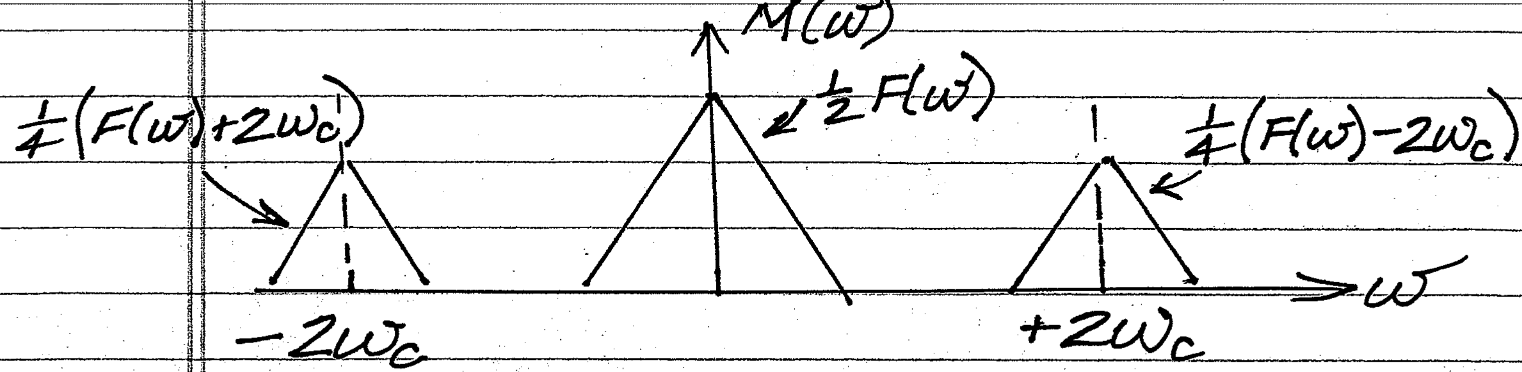


where ω_c is the carrier frequency $580 \cdot 10^3 \cdot 2\pi$ rad/sec or, simply, 580 kHz.

2. Remember that the energy of a signal is given by:

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

and $M(\omega)$ can be drawn as:



Therefore, $\dfrac{W_{Y(\omega)}}{W_{F(\omega)}} = \dfrac{\left|\frac{1}{2}F(\omega)\right|^2}{\left|F(\omega)\right|^2} = \dfrac{1}{4}$ OR 25%

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3. $G_1(\omega) = \frac{1}{2}\left[F(\omega-\omega_c)+F(\omega+\omega_c)\right]e^{-j\omega t_0}$

where $\omega_c = 2\pi \cdot 580 \cdot 10^3\ s^{-1}$

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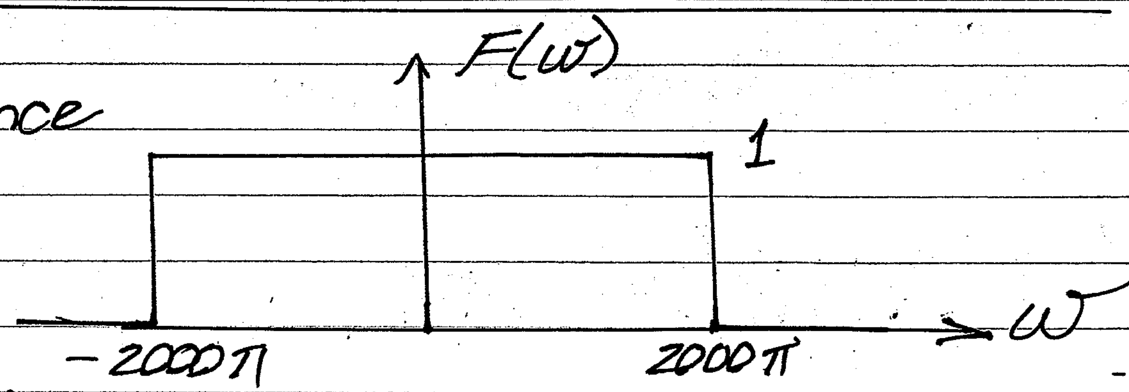
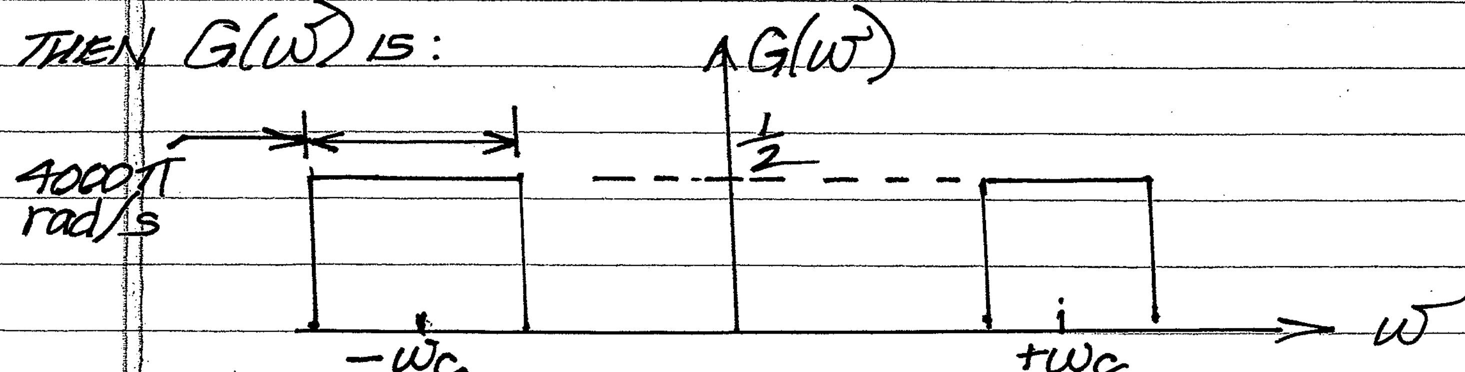
4. Since



THEN $G(\omega)$ IS:



5. The Local Oscillator frequency (ω_{LO}) for a superheterodyne receiver is defined as:
generally

$$\omega_{LO} - \omega_{IF} = \omega_C$$

OR $\boxed{f_{LO} - f_{IF} = f_C}$

So $\boxed{f_{LO} = 1035 \text{ kHz} = 1.035 \text{ MHz}}$

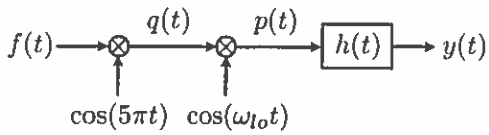
if $\omega_{IF} = 2\pi \cdot 455 \cdot 10^3$, $f_{IF} = 455 \text{ kHz}$

and $\omega_C = 2\pi \cdot 580 \cdot 10^3 s^{-1}$, $f_C = 580 \text{ kHz}$

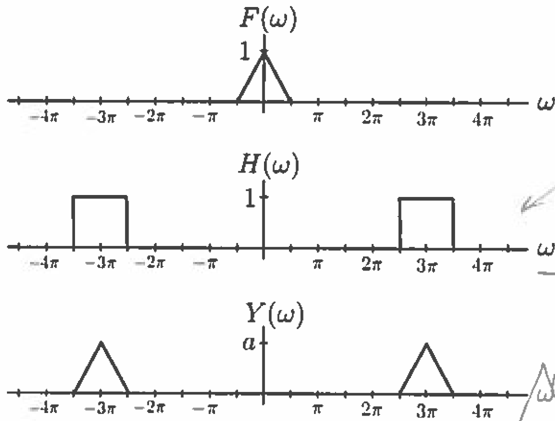
BUT $\boxed{f_{LO} \equiv f_C - f_{IF} = 125 \text{ kHz}}$

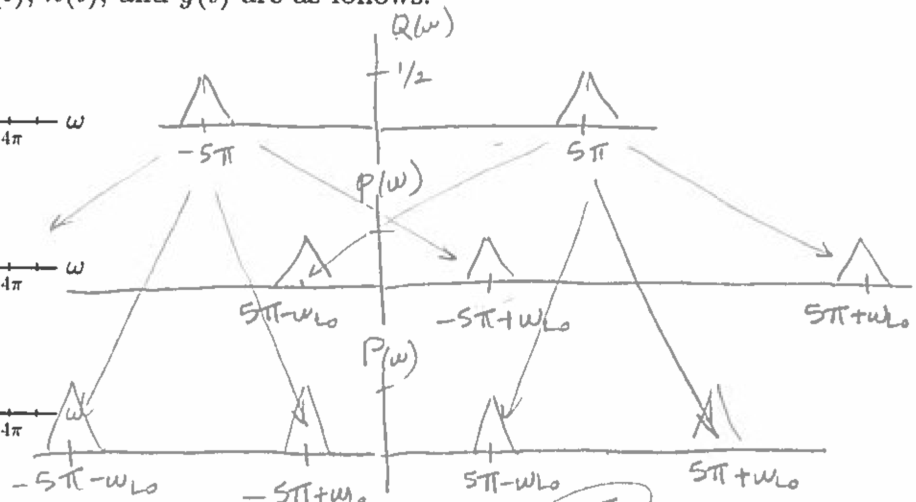
is also acceptable.

3. (20 pts) Consider the system



where the Fourier transforms of $f(t)$, $h(t)$, and $y(t)$ are as follows:





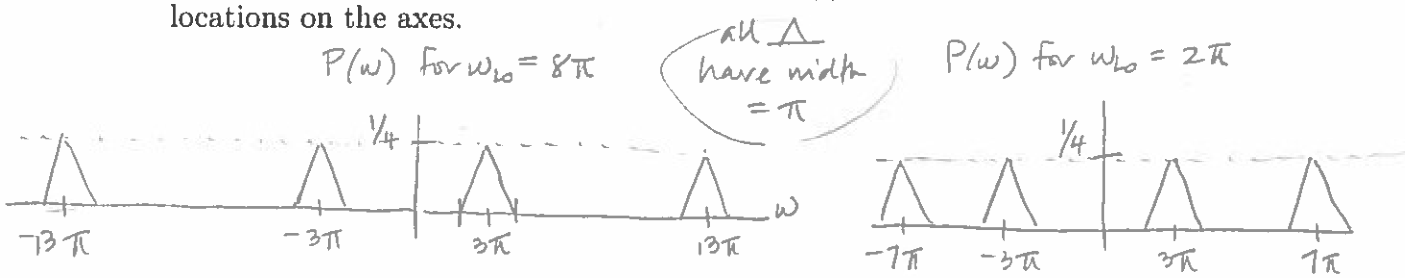
(a) (4 pts) What is ω_{lo}? $\omega_{lo} =$ 2π -or- 8π

need $3\pi = -5\pi + \omega_{Lo}$ (top plot) -or- $3\pi = 5\pi - \omega_{Lo}$

(b) (4 pts) Is your answer to part (a) unique? If so, briefly explain. If not, give another possible value for ω_{lo}.

No. There are two possible values, either $\omega_{Lo} = 2\pi$ or $\omega_{Lo} = 8\pi$.

(c) (4 pts) Plot $P(\omega)$, the Fourier transform of $p(t)$, below. Be sure to mark all relevant locations on the axes.



(d) (4 pts) What is $Y(\omega)$ in terms of $F(\omega)$? You do not need to write the expression for $F(\omega)$ explicitly but your answer should specify the maximum value of $Y(\omega)$, which is denoted as a in the figure.

$Y(\omega) =$ $\frac{1}{4}\left(F(\omega - 3\pi) + F(\omega + 3\pi)\right)$

ampl $= \frac{1}{4}$ since $F(\omega)$ went through two modulations (each scaled by $\frac{1}{2}$)

(e) (4 pts) What is $y(t)$ in terms of $f(t)$? $y(t) =$ $\frac{1}{2} f(t) \cos(3\pi t)$