Lecture 17, Tuesday, February 15, 2022

- The response in the first order ODE with constant coefficients can also be written as

$$y(t) \;=\; y_{tr}(t) + y_{ss}(t)$$

  - The *transient response*, $y_{tr}(t)$ is such that

$$y_{tr}(t) \;\stackrel{t\to\infty}{\longrightarrow}\; 0$$

  - The *steady-state response*, $y_{ss}(t)$ is what remains after $y_{tr}(t) \longrightarrow 0$.

    Steady-state does not mean constant, it means is does not go to zero as $t \to \infty$.

- A *dissipative system* has a transient zero-input response:
  - The energy stored is eventually dissipated.
  - The steady-state response to a cosinusoidal input applied at $\mathbf{t} = -\infty$ will be a cosinusoidal independent of the initial state:

$$y(t) \;=\; Ae^{-at} + \underbrace{H\cos(\omega t + \psi)}_{y_{ss}(t)}$$

    Basically, if we wait long enough, we just see a cosinusoidal.

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- n-th order ODE with constant coefficients:

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = f(t)$$

  n is the number of energy storage elements: $L,\ C$

  - Solution is:

$$y(t) \;=\; y_p(t) + y_h(t)$$

  - $y_p(t)$ matches the input, like in 1st order ODE case
  - $y_h(t)$ changes, in most cases, to:

$$y_h(t) \;=\; A_1 e^{-p_1 t} \;+\; A_2 e^{-p_2 t} \;+\; \ldots \;+\; A_n e^{-p_n t}$$

    * the constants, A_i are used to match the n initial conditions
    * the p_i are related to the coefficients of the ODE
  - We will see later how to solve this type of equations.
  - Higher order systems might no longer be dissipative

