- Example #20:

- Consider the first-order ODE:

$$\frac{dy}{dt} + y = e^{-2t}, \quad \text{with } y(0^+) = 1$$

a, f(t), p

- Determine y_{ZS}, y_{ZI} and y(t). for $t>0$

$f(t) = e^{-2t}$ $(p \neq a) \Rightarrow$

$y_p(t) = Be^{-2t}$

$y_h(t) = Ae^{-at} =$

$= Ae^{-t}$

$$\boxed{y(t) = Be^{-2t} + Ae^{-t}}$$

$t>0$

① Substitute y_p into ODE:

$$\frac{dy_p}{dt} + y_p = e^{-2t}$$

$$\frac{d}{dt}(Be^{-2t}) + Be^{-2t} = e^{-2t}$$

$$-2Be^{-2t} + Be^{-2t} = e^{-2t}$$

$$-Be^{-2t} = e^{-2t}$$

$$\boxed{B = -1}$$

$$y(t) = -e^{-2t} + Ae^{-t}$$

- Example #20-cont:

$$y(t) = -e^{-2t} + Ae^{-t}$$

$$y(t) = y_{ZS}(t) + y_{ZI}(t)$$

- Consider the first-order ODE:

$$\frac{dy}{dt} + y = e^{-2t}, \quad \text{with } y(0^+) = 1$$

- Determine y_{ZS}, y_{ZI} and y(t).

② Use I.C. to find A:

$$y(0^+) = -e^{-2(0)} + Ae^{-(0)} = -1 + A \Rightarrow A = y(0^+) + 1$$

$$y(t) = -e^{-2t} + (y(0^+) + 1)e^{-t} = \underbrace{-e^{-2t} + e^{-t}}_{ZS} + \underbrace{y(0^+)e^{-t}}_{ZI}$$

$$y_{ZS}(t) = -e^{-2t} + e^{-t}$$

$$+ \quad y_{ZI}(t) = y(0^+)e^{-t} = e^{-t}$$

$$y(t) = -e^{-2t} + e^{-t} + e^{-t} = -e^{-2t} + 2e^{-t} \qquad t > 0$$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$H\cos(t+\psi) = H(\cos t \cos\psi - \sin t \sin\psi) =$

$= \underbrace{H\cos\psi}_{B} \cos t \underbrace{- H\sin\psi}_{D} \sin t$

$y_h(t) = Ae^{-t}$

- Example #21:

- Consider the first-order ODE:

$y_p(t) = \underbrace{H\cos(t+\psi)}_{} =$

$$\frac{dy}{dt} + 1y = \overset{\text{" } f(t)}{\cos(t)}, \text{ with } y(0^+) = 1$$

$= B\cos t + D\sin t$

$\omega = 1$ rad/s

- Determine y_{ZS}, y_{ZI} and y(t).

① Substitute y_p into ODE:

$$\frac{dy_p}{dt} + y_p = \cos t$$

$$\frac{d}{dt}\left(B\cos t + D\sin t\right) + B\cos t + D\sin t = \cos t$$

$$-B\sin t + D\cos t + B\cos t + D\sin t = \cos t$$

$$\underbrace{(-B+D)}_{0}\sin t + \underbrace{(D+B)}_{1}\cos t = 0\sin t + 1\cos t$$

$\Downarrow$

$D = B = \frac{1}{2}$

$y_p = \frac{1}{2}\cos t + \frac{1}{2}\sin t$

$y(t) = y_p(t) + y_h(t) = \frac{1}{2}\cos t + \frac{1}{2}\sin t + Ae^{-t}$

- Example #21-cont:

- Consider the first-order ODE:

$$\frac{dy}{dt} + y = \cos(t), \text{ with } y(0^+) = 1$$

- Determine y_{ZS}, y_{ZI} and y(t).

2 Use I.C. to find A:

$$y(0^+) = \frac{1}{2}\underbrace{\cos(0)}_{1} + \frac{1}{2}\underbrace{\sin(0)}_{0} + A = \frac{1}{2} + A \Rightarrow A = y(0^+) - \frac{1}{2}$$

$$y(t) = \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + \left(y(0^+) - \frac{1}{2}\right)e^{-t}$$

$$y_{zs}(t) \; (y(0^+) = 0) = \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) - \frac{1}{2}e^{-t}$$

$$+ \quad y_{zI}(t) = y(0^+)\,e^{-t} = e^{-t}$$

$$y(t) = \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) - \frac{1}{2}e^{-t} + e^{-t} \quad t > 0$$

- Example #22:
- Consider the particular solution:

$$y_p(t) = \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) = H\cos(t+\psi),$$

- Determine H and ψ

$= H\cos\psi\cos t - H\sin\psi\sin t$

B D

$\Downarrow$

$H\cos\psi = \frac{1}{2}$

$-H\sin\psi = \frac{1}{2}$

Get ψ:

$\frac{\cancel{H}\sin\psi}{\cancel{H}\cos\psi} = \frac{-1/2}{1/2} \Rightarrow \tan\psi = -1 \Rightarrow$

$\psi = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$

Get H:

$H\cos\psi = \frac{1}{2}$

$H\cos(-\pi/4) = \frac{1}{2}$

$H = \frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$

$y_p(t) = \frac{\sqrt{2}}{2}\cos(t - \pi/4) =$

$= -\frac{\sqrt{2}}{2}\cos(t + \frac{3\pi}{4})$

- Transient vs steady-state response

- *Transient response*, $y_{tr}(t)$ is such that

$$y_{tr}(t) \overset{t \to \infty}{\longrightarrow} 0$$

- *Steady-state response*, $y_{ss}(t)$ is what is left after $y_{tr}(t) \longrightarrow 0$

$$y(t) = y_{tr}(t) + y_{ss}(t)$$

- Recall

$t \to \infty$ $y_h \to 0$ y_{tr}

$y_p \to y_{ss}$

$$y(t) = Ae^{-at} + B$$

y_p $t \to \infty$ 0 y_{tr}

$$y(t) = Ae^{-at} + Be^{-pt}$$

$\to y_p$

$t \to \infty$ $\to 0$ y_{tr}

$$y(t) = Ae^{-at} + H\cos(\omega t + \psi)$$

y_{tr} $t \to \infty$ $\to 0$

$\to y_{ss}$