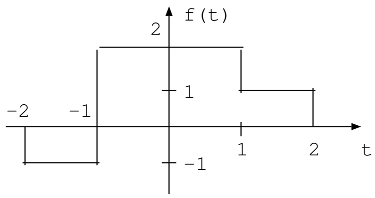
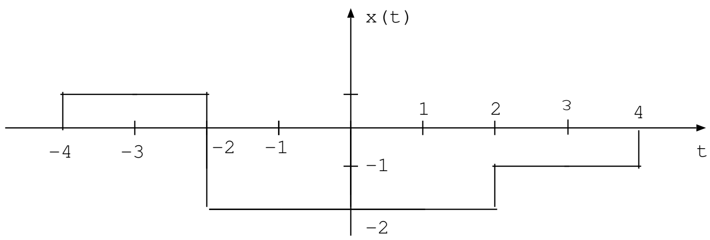
1. (10 pts) Let $f(t)$, plotted below, have Fourier transform $F(\omega)$.

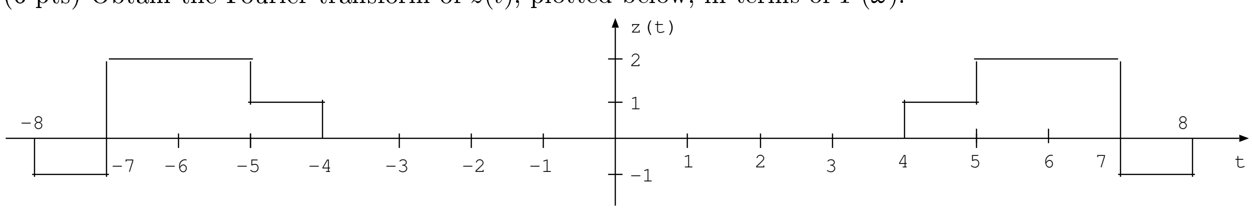


(a) (4 pts) Obtain the Fourier transform of $x(t)$, plotted below, in terms of $F(\omega)$.



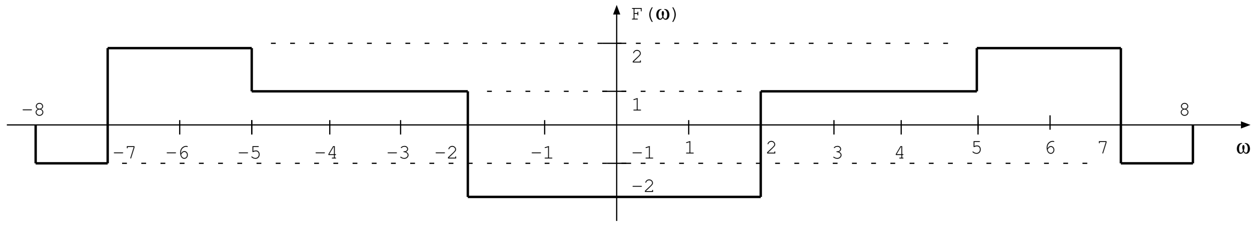
$X(\omega) =$ ____.

(b) (6 pts) Obtain the Fourier transform of $z(t)$, plotted below, in terms of $F(\omega)$.



$Z(\omega) =$ ____.

2. (15 pts) Let $f(t)$ have Fourier transform $F(\omega)$, plotted below.



(a) (5 pts) Obtain the 90% energy bandwidth, $\Omega_{90\%}$, of f.

$\Omega_{90\%} =$ _______________ .

(b) (10 pts) Let $f(t)$ be the input to an LTI system with frequency response $H(\omega) = rect(\frac{\omega}{4})e^{-j\frac{\pi}{6}\omega}$. Obtain the zero-state output $y_{zs(t)}$.

$y_{zs(t)}$ ____ .

problem 1 cont'd

(c) Let $f(t) = e^{-t}u(t - 2)$. Obtain its Fourier transform $F(\omega)$, evaluated at $\omega = 1$.

$F(1) =$ ____________________

(d) The function f(t) has Fourier transform $F(\omega) = 3\pi\, e^{-3\omega}u(\omega)$. Indicate (circle) which of the following functions corresponds to the function f(t), and explain why (no points if no valid reason).

(a) $f(t) = \frac{3}{2}\frac{1}{3 - t}$

(b) $f(t) = \frac{3}{2}\frac{1}{3 - jt}$

(c) $f(t) = \frac{3}{2}(e^{3t} + e^{-3t})$

(d) $f(t) = \frac{3}{2}(e^{j3t} + e^{-j3t})$

(e) $f(t) = \frac{3}{2j}(e^{j3t} + e^{-j3t})$

Why?

(b) Obtain the Fourier transform of

i) $f(t) = e^{-t}u(t - 1)$

$F(\omega) =$ ____________________

ii) $g(t) = \dfrac{1}{1 + j(t-1)} + \dfrac{1}{1 - j(t-1)}$

(Hint: Do not combine the fractions)

$G(\omega) =$ ____________________

Problem 1

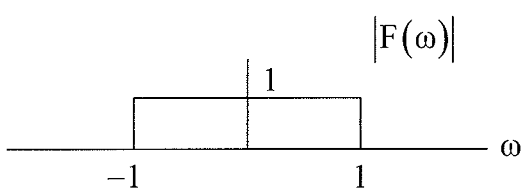
a) Find the Fourier transform $F(\omega)$ and the energy of the following signal $f(t)$.

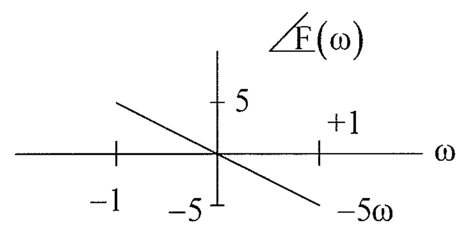
$$f(t)=\left[u(t-1)-u(t-3)\right]e^{-t}$$

$F(\omega)=$ ______________________________ (6 points)

$\varepsilon=$ ______________________________ (6 points)

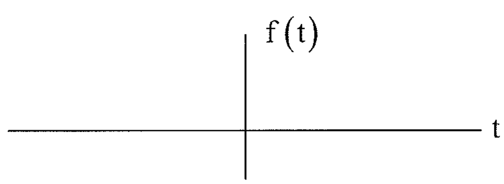
b) Fourier transform of a signal $f(t)$ is given as





i) Find the inverse Fourier transform $f(t)$. Simplify your answer. Sketch $f(t)$, label axis carefully.

$f(t)=$ ______________________________ (7 points)



ii) Find the 90% energy bandwidth of $f(t)$.

BW = ______________________________ (6 points)

1. (25 pts) Given the Fourier transform pair $f(t) = e^{-t}u(t)$ and $F(\omega) = \frac{1}{1+j\omega}$, obtain the Fourier transforms of the following functions:

   (a) $g(t) = \frac{d^2 f}{dt^2}$. Obtain $G(\omega) =$ ______________________________

   (b) $h(t) = e^{-t}u(t-2)$. Obtain $H(\omega) =$ ______________________________

   (c) $x(t) = e^{-t} rect\left(\frac{t}{4}\right)$. Obtain $X(\omega) =$ ______________________________

   (d) $y(t) = \frac{1}{1+j(t+1)}$. Obtain $Y(\omega) =$ ______________________________

6. (?? pts) The three parts of this problem are unrelated.

(a) Determine the Fourier transform of $f(t) = \frac{1}{1-j2t}$.

$F(\omega)=$ ________________________

(b) Determine the inverse Fourier transform of $G(\omega) = \pi e^{-\frac{\omega}{2}} u(\omega)$.

$f(t)=$ ________________________

(c) Given that $x(t)$ has the Fourier transform $X(\omega)$, express the Fourier transforms of the signals listed below in terms of $X(\omega)$.

i. $x_1(t) = x(1-t) + x(-1-t) \quad \rightarrow \quad X_1(\omega)=$ ________________________

ii. $x_2(t) = x(3t-6) \quad \rightarrow \quad X_2(\omega)=$ ________________________

iii. $x_3(t) = \frac{d^2}{dt^2}x(t-1) \quad \rightarrow \quad X_3(\omega)=$ ________________________