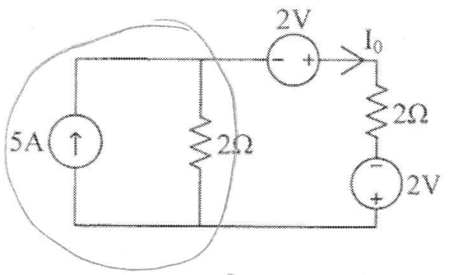
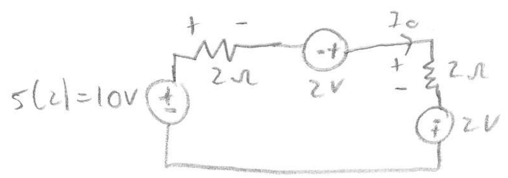
Problem 1 *(12 points)*

(a) In the following circuit find the value of I_0.



Source transformation



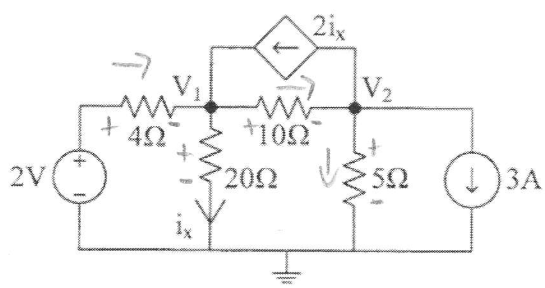
KVL

$10 + 2 + 2 = 2I_0 + 2I_0$

$\Rightarrow I_0 = \frac{10+2+2}{2+2} = \frac{14}{4} = \frac{7}{2}$ A

$I_0 =$ $\frac{7}{2}$ A

(b) In the following circuit, what is the node voltage V_2.



Node-voltage method

① $\frac{2 - V_1}{4} + 2i_x = \frac{V_1 - 0}{20} + \frac{V_1 - V_2}{10}$

② $\frac{V_1 - V_2}{10} = 2i_x + \frac{V_2 - 0}{5} + 3$

$i_x = \frac{V_1 - 0}{20}$

$V_2 =$ -10V

$\frac{1}{2} = V_1\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{4} - \frac{2}{20}\right) + V_2\left(-\frac{1}{10}\right) = V_1 \frac{1+2+5-2}{20} - \frac{1}{10}V_2 = \frac{3}{10}V_1 - \frac{1}{10}V_2$

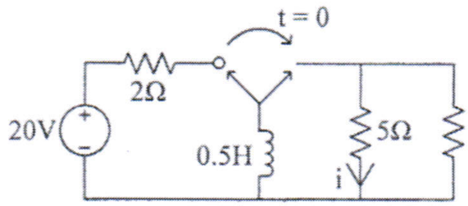
$\Rightarrow V_1 = \left(\frac{1}{2} + \frac{1}{10}V_2\right)\frac{10}{3} = \frac{5 + V_2}{3}$

$V_1\left(\frac{1}{10} - \frac{2}{20}\right) + V_2\left(-\frac{1}{10} - \frac{1}{5}\right) = 3 = \left(\frac{5+V_2}{3}\right)\left(\frac{2-2}{20}\right) + V_2\left(\frac{-1-2}{10}\right) = -\frac{3}{10}V_2$

$\Rightarrow V_2 = 3\left(-\frac{10}{3}\right) = -10$ V

Problem 2 *(13 points)*

(a) The following circuit is at the steady state at t = 0-. Find i(t), t > 0.

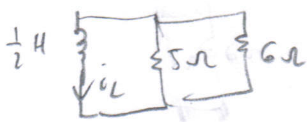


For $t<0$ in steady state

$i(0^-) = 0\ A$

$i_L(0^-) = \frac{20}{2} = 10\ A$

For $t>0$



$i_L(t) = B + Ae^{-t/\tau} = B + Ae^{-t\,60/11} = 0 + 10e^{-60t/11}$

$\tau = \frac{L}{R_{eq}} = \frac{1}{2}\left(\frac{11}{30}\right) = \frac{11}{60}$

$R_{eq} = 5 \| 6 = \frac{5(6)}{5+6} = \frac{30}{11}$

$i_L(\infty) = 0 = B$

$i_L(0^+) = i_L(0^-) = 10 = B + A$

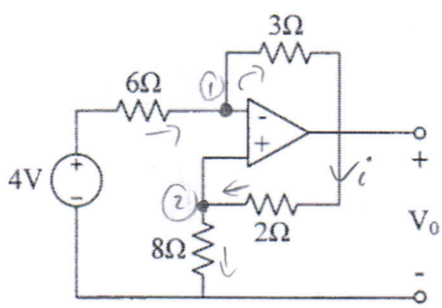
by continuity of i_L

By current division

$i = -i_L \frac{6}{6+5} = -\frac{6}{11} i_L$

i(t) = $-\frac{60}{11} e^{-60t/11}$ A

(b) In the following ideal op-amp circuit, what is the power absorbed by 2Ω resistor?



$V_- = V_+$

KCL @ 1

$\frac{4-V_+}{6} = \frac{V_+ - V_o}{3} \Rightarrow V_o = \left[2 + V_+\left(\frac{-1}{2} - 1\right)\right](-1)$

$= -2 + \frac{3}{2} V_+$

$V_+ = 8i$

$V_o = -2 + \frac{3}{2} 8i = -2 + 12i$

$i = \frac{V_o}{8+2} = \frac{V_o}{10} = \frac{1}{10}(-2 + 12i)$

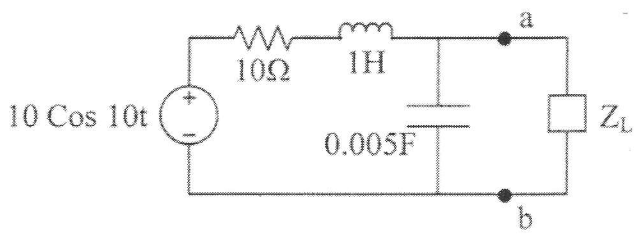
$= -\frac{1}{5} + \frac{6}{5}i \Rightarrow i = \frac{-\frac{1}{5}}{1 - \frac{6}{5}} = -\frac{1}{5}\frac{5}{5-6} = 1\ A$

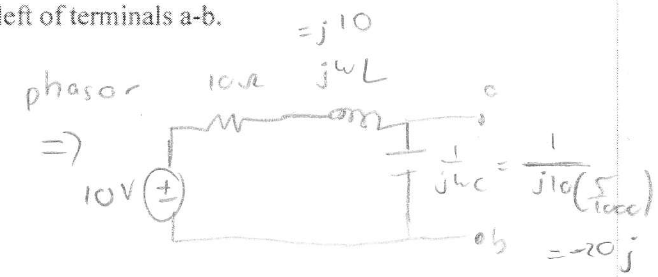
$P_{2\Omega} = Ri^2 = 2(1)^2 = 2W$

$P_{2\Omega}$ = 2W

Problem 3 *(12 points)*

(a) Determine and draw the Thévenin equivalent circuit to the left of terminals a-b.





V_T = open circuit voltage

By voltage division: $V_T = 10 \frac{(-20j)}{-20j + 10 + j10} = \frac{-200j}{10 - 10j} = \frac{-20j}{1-j}\ V = V_T$

Z_T is equivalent impedance after source suppression

$Z_T = (10 + j10) \| (-20j) = \frac{(10 + j10)(-20j)}{10 + j10 - 20j} = \frac{-200j + 200}{10 - 10j} = \frac{200(1-j)}{10(1-j)} =$

$Z_T = 20\,\Omega$

(b) Find the value of Z_L for maximum power transfer to Z_L. And then find the maximum power delivered to Z_L.

For max power $Z_L = Z_T^* = 20\,\Omega$

$P_a = \frac{1}{2} \left| \frac{V_T}{2} \right|^2 \frac{1}{R_T} = \frac{1}{2} \frac{10^2}{1^2 + 1^2} \frac{1}{20} = \frac{100}{80} = \frac{10}{8} = \frac{5}{4}\ W$

Z_L = $20\,\Omega$

P_{max} = $5/4$ W

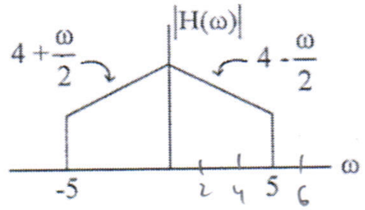
Problem 4 *(13 points)*

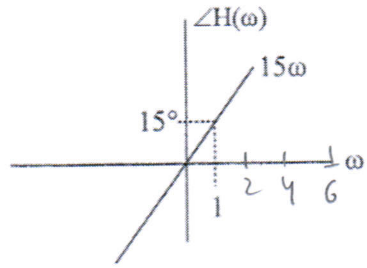
The periodic input signal f(t) is described by

$$f(t) = 3 + 2\,\mathrm{Cos}\,(2t) + \mathrm{Cos}\,(4t - 70°) + 6\,\mathrm{Cos}\,(6t + 45°)$$

$\omega_1 \quad \omega_2 \quad \omega_3$

Determine the output signal y(t) for which the magnitude and phase of transfer function is given by





$|H(0)| = 4$

$|H(2)| = 4 - \frac{2}{2} = 3$

$|H(4)| = 4 - \frac{4}{2} = 2$

$|H(6)| = 0$

$\measuredangle H(0) = 0$

$\measuredangle H(2) = 15(2) = 30$

$\measuredangle H(4) = 15(4) = 60$

$\measuredangle H(6) = 15(6) = 90$

If

$$f(t) = \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \theta_n)$$

then

$$y(t) = \sum_{n=0}^{\infty} A_n |H(\omega_n)| \cos(\omega_n t + \theta_n + \measuredangle H(\omega_n))$$

$$= 3\,H(0) + 2\,|H(2)| \cos(2t + 30°) + |H(4)| \cos(4t - 70° + 60°)$$
$$+ 6\,|H(6)| \cos(6t + 45° + 90°)$$

y(t) = $12 + 6\cos(2t + 30°) + 2\cos(4t - 10°)$

Problem 5 *(12 points)*

An LTI system is given by $H(\omega) = \dfrac{1}{2+j\omega}$. If the input is $f(t) = e^{2t}\,u(-t)$, determine the zero-state response y(t) of this system.

$\Big\downarrow \mathcal{F}$ by #2 in table 4 (7.2)

$$F(\omega) = \frac{1}{2-j\omega}$$

$$\Rightarrow Y(\omega) = F(\omega)H(\omega) = \frac{1}{2-j\omega}\,\frac{1}{2+j\omega} = \frac{1}{4+\omega^2} = \frac{1}{4}\,\frac{2(2)}{2^2+\omega^2}$$

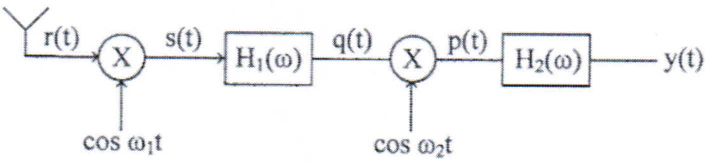
$\Big\downarrow \mathcal{F}^{-1}$ by # 3 in table 4 (7.2)

$$e^{-2|t|}$$

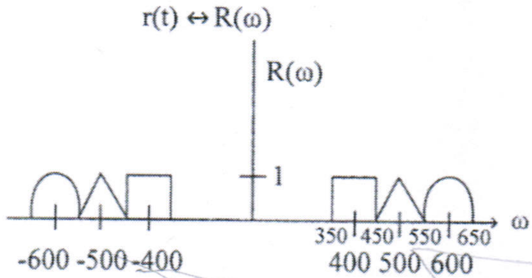
$y_{zs}(t) = \frac{1}{4} e^{-2|t|}$

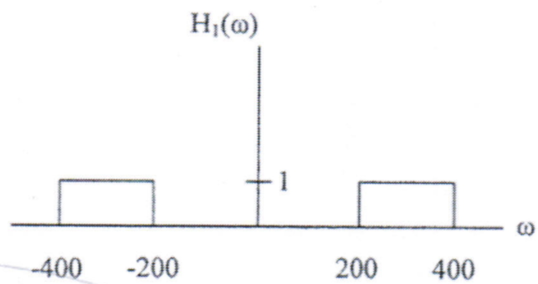
Problem 6 *(13 points)*

Consider the following system

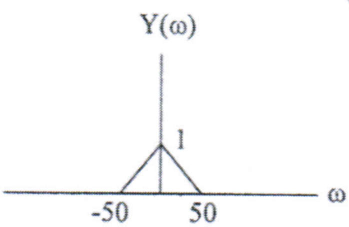


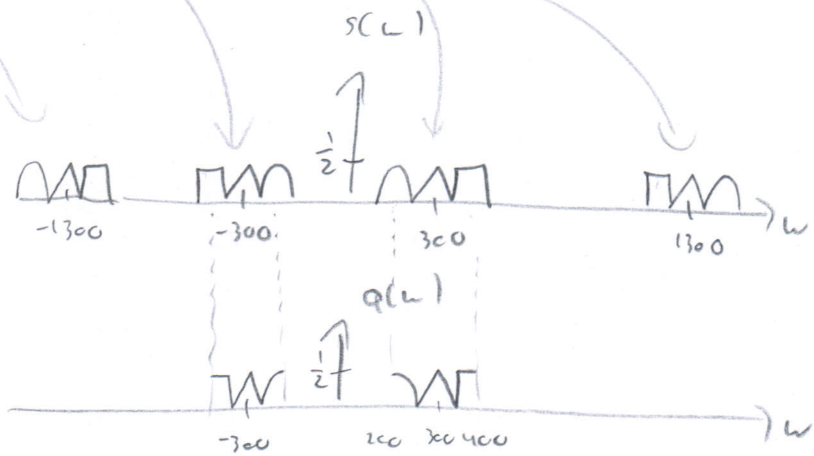
with $\omega_1 = 800$ rad/s





Find ω_2 and $H_2(\omega)$ such that the output of the system is

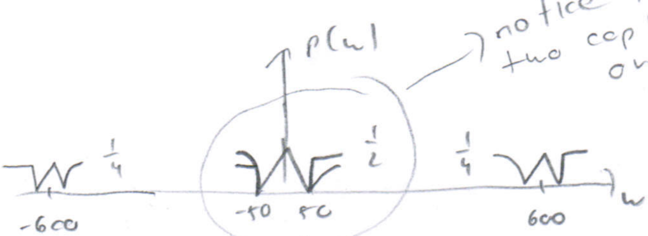




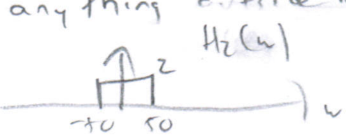
need to move by 300

If $\omega_2 = 300$

notice that two copies overlap



need to remove anything outside here

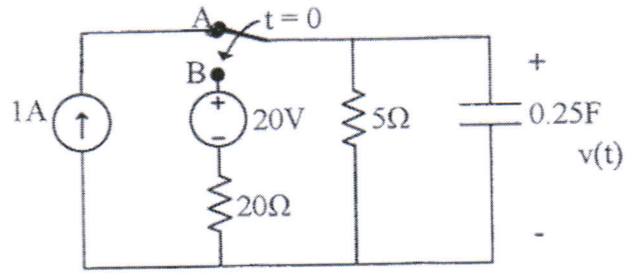


$\omega_2 =$ 300 rad/s

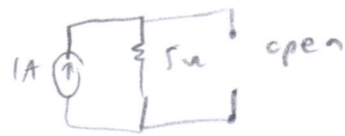
$H_2(\omega) =$ $2\,\mathrm{rect}\left(\frac{\omega}{100}\right)$

Problem 7 *(13 points)*

In the following circuit, the switch has been in the A position for a long time and the circuit is at the steady state. The switch goes to the B position at time t = 0.

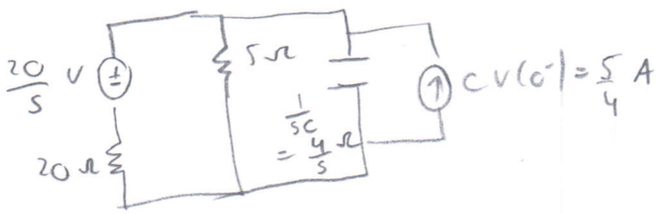


For $t<0$



$\Rightarrow v(0^-) = 5(1) = 5V$

(a) Draw the equivalent circuit in the s-domain, t > 0.

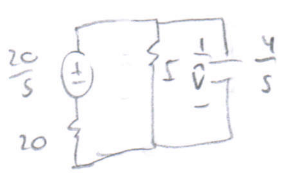


(b) Obtain $\hat{V}(s)$, t > 0.

Superposition

• $\frac{20}{s}$ V only : $5 \| \frac{4}{s} = \frac{5\left(\frac{4}{s}\right)}{5+\frac{4}{s}} = \frac{20}{5s+4}$

By voltage division $\hat{V} = \frac{20}{s}\cdot\frac{\frac{20}{5s+4}}{\frac{20}{5s+4}+20} = \frac{(\cancel{20})^{2}(5s+4)}{s(\cancel{5s+4})(20+20(5s+4))}$

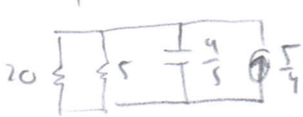


$= \frac{20}{s(5+5s)} = \frac{4}{s(1+s)}$

$\hat{V}(s) = \boxed{\frac{4+5s}{s(s+1)}}$ ✓

• $\frac{5}{4}$A only : $5 \| 20 = \frac{5(20)}{5+20} = \frac{100}{25} = 4$

By current division $\hat{I}_c = \frac{5}{4}\cdot\frac{4}{4+\frac{4}{s}} = \frac{20s}{4(4s+4)} = \frac{5s}{4(s+1)}$



$\hat{V} = \frac{4}{s}\hat{I}_c = \frac{4}{\cancel{s}}\cdot\frac{5\cancel{s}}{4(s+1)} = \frac{5}{s+1}$

$\Rightarrow \hat{V} =$ the terms : $\frac{4}{s(1+s)} + \frac{5}{s+1} = \frac{4+5s}{s(s+1)}$

Problem 7 *(continued)*

(c) Obtain v(t), for t > 0s.

By PFE

$$\hat{V} = \frac{4+5s}{s(s+1)} = \frac{A_0}{s} + \frac{A_1}{s+1}$$

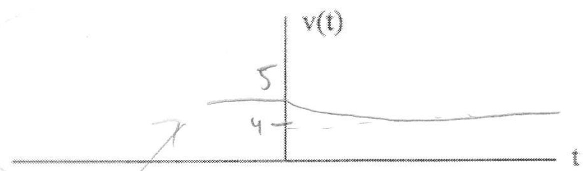
$$= \frac{\frac{4+5(0)}{(0+1)}}{s} + \frac{\frac{4+5(-1)}{-1}}{s+1} = \frac{4}{s} + \frac{1}{s+1}$$

$$\mathcal{L}^{-1} \Downarrow$$

$$4u(t) + e^{-t}u(t)$$

v(t) = $4u(t) + e^{-t}u(t)$ V

(d) Plot v(t) for t > -1s.



Notice that we had initial condition of 5V in steady state, which the inverse Laplace would not tell us (it only tells us what happens for t>0)

Problem 8 *(12 points)*

(a) Consider the following differential equation describing an LTI system.

$$\frac{dy}{dt} + 3y(t) = f(t) \Rightarrow s\hat{Y} - y(0^-) + 3\hat{Y} = \hat{F}$$

$$\Rightarrow \hat{Y} = \frac{\hat{F} + y(0^-)}{s+3}$$

i) If $f(t) = u(t)$ and $y(0\text{-}) = 1$, find $y_{ZI}(t)$, $y_{ZS}(t)$ and $y(t)$.

$\hat{F}(s) = \frac{1}{s}$

$$\hat{Y}_{ZI} = \frac{y(0^-)}{s+3} = \frac{1}{s+3} \xrightarrow{\mathcal{L}^{-1}} e^{-3t}u(t)$$

$$\hat{Y}_{ZS} = \frac{\hat{F}}{s+3} = \frac{1}{s(s+3)} = \frac{A_0}{s} + \frac{A_1}{s+3}$$

$$= \frac{\frac{1}{0+3}}{s} + \frac{\frac{1}{-3}}{s+3} = \frac{\frac{1}{3}}{s} - \frac{\frac{1}{3}}{s+3}$$

$$\mathcal{L}^{-1} \downarrow \quad \frac{1}{3}u(t) - \frac{1}{3}e^{-3t}u(t)$$

$y_{ZI}(t) =$ $e^{-3t}u(t)$

$y_{ZS}(t) =$ $\frac{1}{3}u(t) - \frac{1}{3}e^{-3t}u(t)$

$y(t) =$ $\frac{1}{3}u(t) + \frac{2}{3}e^{-3t}u(t)$

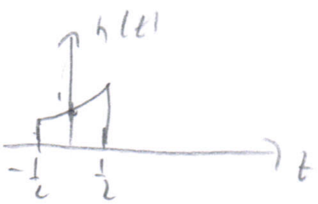
$y_{ZI} + y_{ZS}$

ii) Find the impulse response h(t) of this system.

$$\hat{H}(s) = \frac{\hat{Y}_{ZS}(s)}{\hat{F}(s)} = \frac{1}{s+3} \xrightarrow{\mathcal{L}^{-1}} e^{-3t}u(t)$$

$h(t) =$ $e^{-3t}u(t)$

(b) If impulse response h(t) is given as $h(t) = e^{3t}\,\text{rect}(t)$ for a system, is this system BIBO stable or not? Causal or not? (You have to justify your answer, no justification will not get any credit)



BIBO Stable ✓ yes ___ no

Causal ___ yes ✓ no

$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ ✓ $\Rightarrow$ BIBO

$h(t) \neq 0$ for some $t < 0$ $\Rightarrow$ not causal

By voltage division

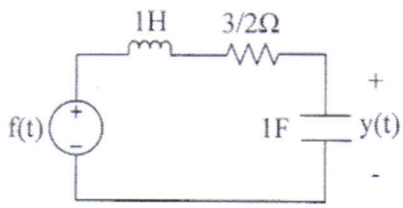
$$\Rightarrow Y = F \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + j\omega + \frac{3}{2}} = F \frac{2}{2(1-\omega^2) + 3j\omega}$$

$$= H(\omega)$$

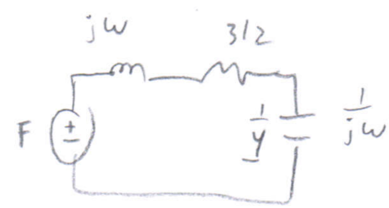
Problem 3 *(12 points)*

For f(t) = 3 + 2cos(t) + sin(2t)

and the following circuit



$\rightarrow$ phasor



(a) Is f(t) periodic? If so, what is the period?

Yes $\left.\begin{array}{l}\omega_1 = 0 \\ \omega_2 = 1 \\ \omega_3 = 2\end{array}\right\}$ ratios are rational numbers

$\omega_0 = 1$ rad/s

$\Rightarrow T = \frac{2\pi}{\omega_0} = 2\pi$ s

Periodic: ☑ YES ☐ NO

T = 2π sec

(b) What are the trigonometric fourier series coefficients of f(t)?

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right)$$

$\Rightarrow a_0 = 6$

$a_1 = 2$

$b_2 = 1$

all others are zero

(c) Find y(t), the output of the circuit above, for the given f(t).

$H(0) = 1$

$H(1) = \frac{2}{3j} = \frac{2}{3} e^{-j\pi/2}$

$H(2) = \frac{2}{-6+6j} = \frac{2}{6\sqrt{2}} e^{-j3\pi/4} = \frac{1}{3\sqrt{2}} e^{-j3\pi/4}$

y(t) = $3 + \frac{4}{3}\cos(t - \frac{\pi}{2}) + \frac{1}{3\sqrt{2}} \sin(2t - \frac{3\pi}{4})$ ✓

4

$f(t) = 3 + 2\cos(t) + \sin(2t)$

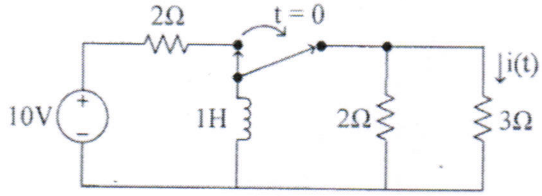
$\downarrow$

$y(t) = 3H(0) + 2|H(1)|\cos(t + \angle H(1)) + |H(2)| \sin(2t + \angle H(2))$

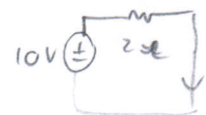
$= 3 + \frac{4}{3}\cos(t - \pi/2) + \frac{1}{3\sqrt{2}} \sin(2t - 3\pi/4)$

Problem 1 *(12 points)*

(a) The following circuit is at the steady state at t = 0⁻. Find i(t), t > 0.



For $t<0$ in steady state



$i_L = \frac{10}{2} = 5A = i_L(0^-)$

For $t>0$

$i_L(t) = B + Ae^{-t/\tau}$

$\tau = \frac{L}{R_{eq}} = \frac{1}{6/5} = \frac{5}{6}$

$R_{eq} = 2\|3 = \frac{2(3)}{2+3} = \frac{6}{5}$

$i_L(\infty) = B = 0$

$i_L(0^+) = i_L(0^-) = 5 = B + A$

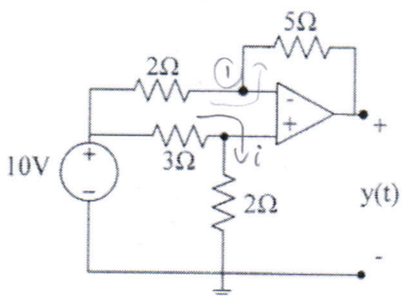
$\Rightarrow i_L(t) = 5e^{-6t/5}$

By current division

$i = -i_L \frac{2}{2+3} = -\frac{2}{5} 5 e^{-6t/5}$

i(t) = $-2e^{-6t/5}$ A

(b) In the following circuit, find the output y(t).



$i = \frac{10}{3+2} = \frac{10}{5} = 2A$

$V_+ = 2i = 2(2) = 4V = V_-$

KCL @ ①

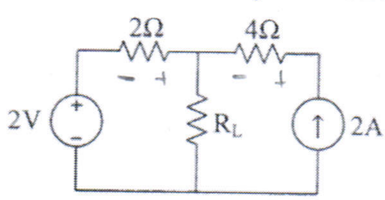
$\frac{10 - V_+}{2} = \frac{V_+ - y}{5}$

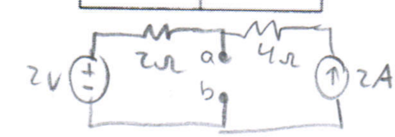
$\Rightarrow y = \left[\left(5 - \frac{V_+}{2}\right)5 - V_+\right](-1) = \left[25 - V_+\left(\frac{5}{2}+1\right)\right](-1)$

$= \left(25 - \frac{7}{2}V_+\right)(-1) = \left(25 - \frac{7}{2}(4)\right)(-1) = (25-14)(-1)$

y(t) = $-11V$

(c) What is the maximum power absorbed by R_L.





V_T is open circuit voltage

$V_T = 2 + 2(+2) = 2 + 4 = 6V$

R_T is R_{eq} after source suppression

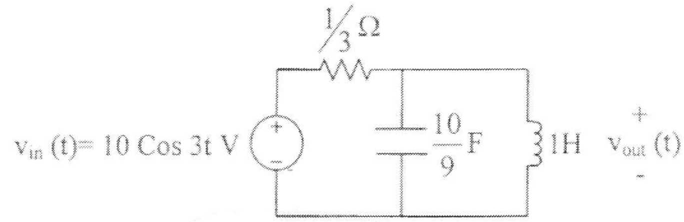
$R_T = 2\Omega$

$P_a = \left(\frac{V_T}{2}\right)^2 \frac{1}{R_T} = \left(\frac{6}{2}\right)^2 \frac{1}{2} = \frac{9}{2}W$

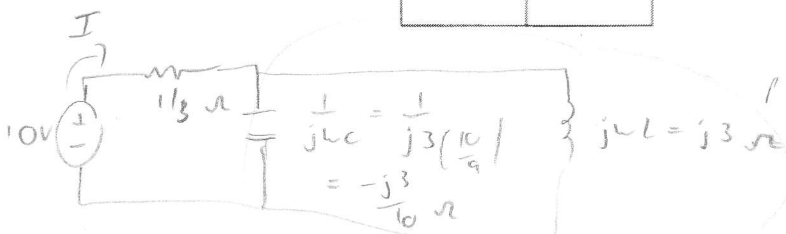
P_{max} = $\frac{9}{2}$ W

Problem 2 *(12 points)*

(a) Determine $v_{out}(t)$.



phasor



$$\frac{-j\frac{3}{10}(j3)}{-j\frac{3}{10} + j3} = \frac{9}{10}\frac{10}{-j3+j30} = \frac{9}{j27} = \frac{-j}{3}$$

By voltage division

$$\tilde{V} = 10\frac{\left(\frac{-j}{3}\right)}{\frac{-j}{3}+\frac{1}{3}} = \frac{-j10}{3}\frac{3}{-j+1} = \frac{-j10}{1-j} = \frac{10}{\sqrt{1^2+1^2}}e^{j(-\frac{\pi}{2}-(-\frac{\pi}{4}))}$$

$$= \frac{10}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$v_{out}(t) = \frac{10}{\sqrt{2}}\cos\left(3t - \frac{\pi}{4}\right)$ V

(b) Determine the average power supplied by the input source $v_{in}(t)$.

In phasor, $I = \frac{10}{\frac{1}{3}-\frac{j}{3}} = \frac{30}{1-j} = \frac{30}{\sqrt{2}}e^{+j\pi/4}$

because of SRS convention

$$P = \frac{1}{2}\mathrm{Re}\{VI^*\} = \frac{1}{2}\mathrm{Re}\left\{10\left(-\frac{30}{\sqrt{2}}e^{j\pi/4}\right)^*\right\} = \frac{1}{2}(10)\left(\frac{-30}{\sqrt{2}}\right)\mathrm{Re}\left\{e^{-j\pi/4}\right\}$$

$$= \frac{-150}{\sqrt{2}}\cos\left(-\frac{\pi}{4}\right) = \frac{-150}{\sqrt{2}}\frac{1}{\sqrt{2}} = -75\ \mathrm{W}$$

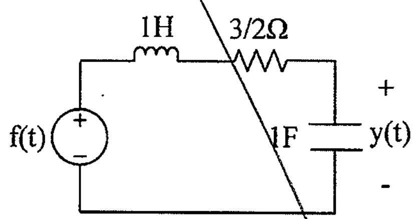
absorbed

Answer = +75 W supplied

Problem 3 *(12 points)*

For $f(t) = 3 + 2\cos(t) + \sin(2t)$

and the following circuit



REPEATED
PROBLEM

(a) Is f(t) periodic? If so, what is the period?

Periodic: ☐ YES ☐ NO

T = ________________ sec

(b) What are the trigonometric fourier series coefficients of f(t)?

(c) Find y(t), the output of the circuit above, for the given f(t).

y(t) = ________________

Problem 4 *(12 points)*

Let f(t) be the periodic function with period T =4 given by

$$f(t)=\begin{cases} 2t & 0\le t\le 2 \\ 0 & 2<t<4 \end{cases}$$

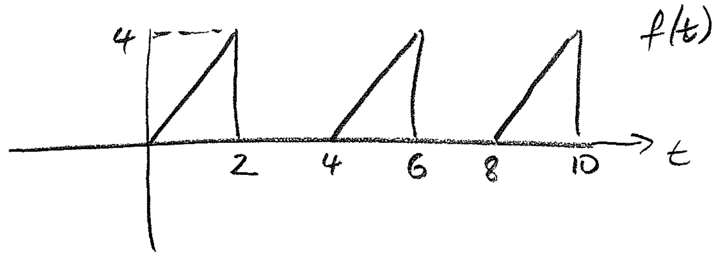
The following coefficients of its exponential Fourier series are known:

$F_{-1}=\dfrac{j2\pi-4}{\pi^2}$

$F_{-2}=\dfrac{-j}{\pi}$

$F_{-3}=\dfrac{j6\pi-4}{9\pi^2}$

$F_{-4}=\dfrac{-j}{2\pi}$



(a) Find the exponential coefficients F_0, F_1, F_2, F_3, and F_4.

$$F_0 = \text{DC value} = \frac{1}{4}\int_0^4 f(t)\,dt = \frac{\frac{1}{2}(2)(4)}{4} = 1$$

$F_n = F_{-n}^*$ since $f(t)$ is real

$F_0 =$ 1

$F_1 =$ $(-j2\pi-4)/\pi^2$

$F_2 =$ j/π

$F_3 =$ $(-j6\pi-4)/(9\pi^2)$

$F_4 =$ $j/2\pi$

(b) Find the compact Fourier coefficients c_0, c_1, c_2, c_3, c_4 and θ_1, θ_2, θ_3, θ_4.

$C_n = 2|F_n| \qquad \theta_n = \angle F_n$

Note that θ_1 and θ_3 are in 3rd quadrant, so they are $-\pi+\tan^{-1}(\cdots)$

$c_0 =$ $2F_0 = 2$

$c_1 =$ $2\sqrt{4\pi^2+16}/\pi^2$ $\quad \theta_1 =$ $-\pi+\tan^{-1}(2\pi/4)$

$c_2 =$ $2/\pi$ $\quad \theta_2 =$ $\pi/2$

$c_3 =$ $2\sqrt{36\pi^2+16}/(9\pi^2)$ $\quad \theta_3 =$ $-\pi+\tan^{-1}(6\pi/4)$

$c_4 =$ $1/\pi$ $\quad \theta_4 =$ $\pi/2$

(c) The signal f(t) is the input to an LTI system with frequency response $H(\omega) = \text{sinc}(\omega)$. Obtain the compact Fourier series coefficients of the output $\hat{c}_0,\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ and $\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,\hat{\theta}_4$.

$T=4 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi/2$

$H(0) = \text{sinc}(0) = 1$

$H(\omega_0) = \text{sinc}(\pi/2) > 0$

$H(2\omega_0) = \text{sinc}(\pi) = 0$

$H(3\omega_0) = \text{sinc}(3\pi/2) < 0$

$H(4\omega_0) = \text{sinc}(2\pi) = 0$

$\hat{c}_0 =$ $H(0)c_0 = 2$

$\hat{c}_1 =$ $\text{sinc}(\pi/2)\,c_1$ $\quad \hat{\theta}_1 =$ $-\pi+\tan^{-1}(2\pi/4)$

$\hat{c}_2 =$ 0 $\quad \hat{\theta}_2 =$ not applicable

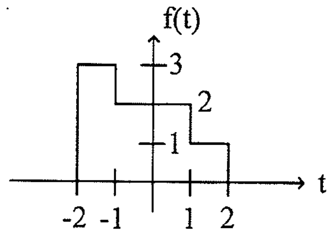
$\hat{c}_3 =$ $-\text{sinc}(3\pi/2)\,c_3$ $\quad \hat{\theta}_3 =$ $\tan^{-1}(6\pi/4)$

$\hat{c}_4 =$ 0 $\quad \hat{\theta}_4 =$ not applicable

<u>Note</u>: $|H(3\omega_0)| = -\text{sinc}(3\pi/2) \quad \angle H(3\omega_0) = \pi$

Problem 5 *(13 points)*

(a) f(t) is given as in the figure.



i) Find $F(\omega)$.

$$f(t) = 3\,\mathrm{rect}(t+1.5) + 2\,\mathrm{rect}(t/2) + \mathrm{rect}(t-1.5)$$

$$F(\omega) = 3\,\mathrm{sinc}\left(\frac{\omega}{2}\right)e^{j\omega(1.5)} + 4\,\mathrm{sinc}(\omega) + \mathrm{sinc}\left(\frac{\omega}{2}\right)e^{-j\omega(1.5)}$$

$F(\omega) =$ ______

ii) Let $g(t) = \sqrt{f(t)}$. Find the energy of g(t).

$$\mathcal{E} = \int_{-\infty}^{\infty} |g(t)|^2\,dt = \int_{-\infty}^{\infty} f(t)\,dt = \text{Area under } f(t)$$

$$= 1\times 3 + 2\times 2 + 1\times 1$$

$$= 8 \text{ J}$$

$\varepsilon =$ 8 J

(b) Let $x(t) = 2e^{-t}\,\mathrm{rect}\left(\frac{t}{2}\right)$.

i) Find $X(\omega)$.

$$X(\omega) = \int_{-\infty}^{\infty} 2e^{-t}\,\mathrm{rect}(t/2)\,e^{-j\omega t}\,dt = 2\int_{-1}^{1} e^{-(1+j\omega)t}\,dt = 2\left.\frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_{-1}^{1}$$

$$= 2\left(\frac{e^{-(1+j\omega)}}{-(1+j\omega)} - \frac{e^{(1+j\omega)}}{-(1+j\omega)}\right) = \frac{2}{1+j\omega}\left(e^{1+j\omega} - e^{-(1+j\omega)}\right)$$

$X(\omega) =$ ______

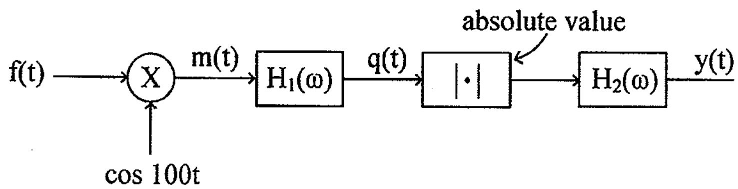
ii) Find the energy of x(t).

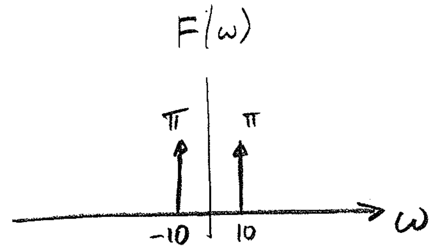
$$\mathcal{E} = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-1}^{1} 4e^{-2t}\,dt = 4\left.\frac{e^{-2t}}{-2}\right|_{-1}^{1} = -2\left(e^{-2} - e^{2}\right) = 2\left(e^{2} - e^{-2}\right) \text{ J}$$

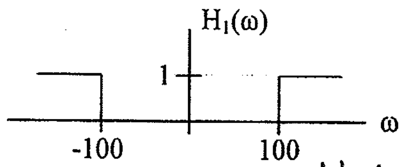
$\varepsilon =$ $2(e^{2} - e^{-2})$ J

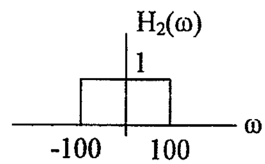
Problem 6 *(13 points)*

(a) The signal $f(t) = \cos(10t)$ is input to the following system.



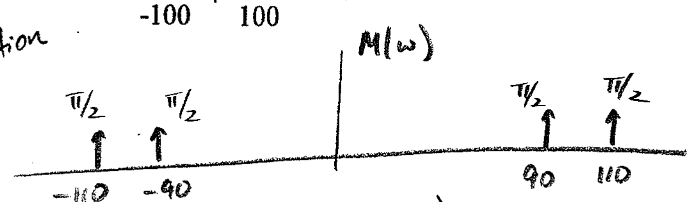




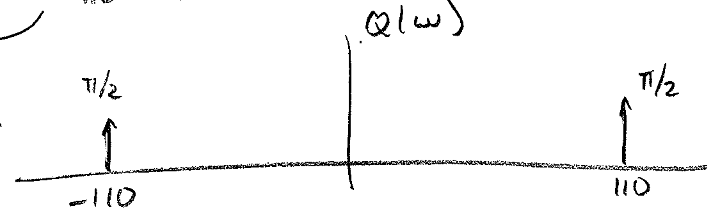


$m(t) = \cos(10t)\cos(100t)$ modulation

$= \frac{1}{2}\cos(90t) + \frac{1}{2}\cos(110t)$

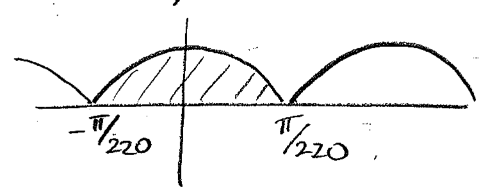


$q(t) = \frac{1}{2}\cos(110t)$



$$|q(t)| = \frac{1}{2}|\cos(110t)| = \frac{1}{2}\left(\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\,220t + \theta_n)\right)$$

where $c_0 = 2\left(\text{DC value of } |\cos(110t)|\right)$



$$= 2\,\frac{110}{\pi}\int_{-\pi/220}^{\pi/220} \cos(110t)\,dt$$

$$= 2\,\frac{\cancel{110}}{\pi}\,\frac{\sin(110t)}{\cancel{110}}\Bigg|_{-\pi/220}^{\pi/220}$$

$$= \frac{2}{\pi}\left(\sin(\pi/2) - \sin(-\pi/2)\right) = \frac{4}{\pi}$$

$$y(t) = \frac{1}{2}\,\frac{c_0}{2} = \frac{1}{\pi}$$

Find:

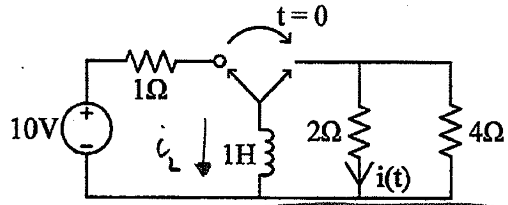
$m(t) =$ $\frac{1}{2}\cos(90t) + \frac{1}{2}\cos(110t)$

$q(t) =$ $\frac{1}{2}\cos(110t)$

$y(t) =$ $\frac{1}{\pi}$

Problem 1

(a) The following circuit is in the steady state at $t = 0^-$. Find i(t), t > 0.



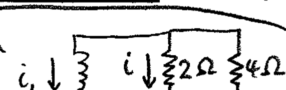
$i_L(0^-) = 10\text{ A}$ $\quad i_L(\infty) = 0$ $\quad \tau = \frac{L}{R_{\|}} = \frac{1}{8/6} = 0.75$

$i_L(t) = 10e^{-t/0.75}$

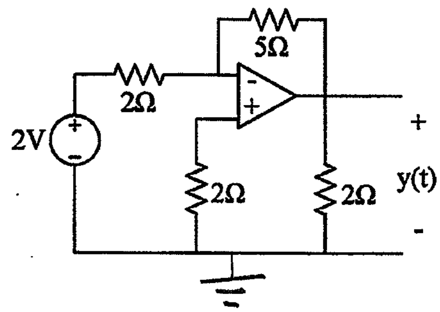
$i(t) = -\frac{4}{6}\, i_L(t) = -\frac{20}{3} e^{-t/0.75}$

i(t) = ______________

By current division



(b) Find the output y(t) in the following ideal op-amp circuit.



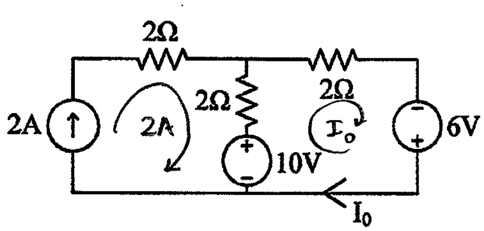
$V_+ = 0$ since current into ⊕ is 0

$\Rightarrow V_- = 0$

KCL @ ⊖: $\frac{2 - V_-}{2} = \frac{V_- - y(t)}{5} \Rightarrow y(t) = -5\text{ V}$

y(t) = -5 V

(c) Find I_0 in the following circuit.



KVL around loop I_o

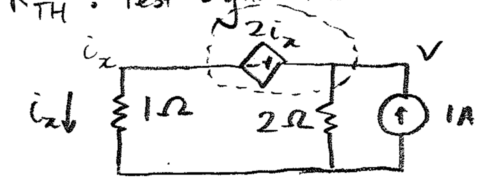
$6 + 10 = 2(I_o - 2) + 2I_o$

$16 = 4I_o - 4$

$I_o = 5\text{ A}$

I_0 = 5 A

R_{TH} : Test signal method



$V = i_x(1) + 2i_x = 3i_x$

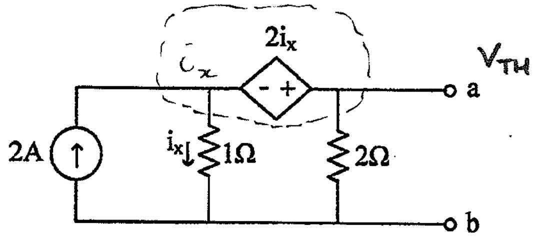
KCL around supernode

$i_x + \frac{3i_x}{2} = 1 \Rightarrow i_x = 2/5$

$\Rightarrow V = 6/5 \Rightarrow R_{TH} = 6/5\ \Omega$

Problem 2

(a) For the network shown below, determine the Thevenin voltage, Thevenin resistance, and Norton current between nodes a and b.



$V_{TH} =$ $12/5$ V

$R_{TH} =$ $6/5\ \Omega$

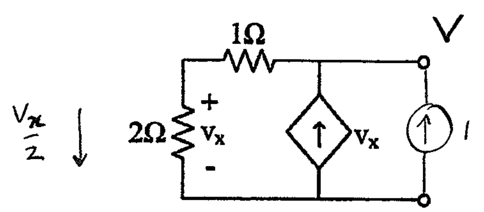
$I_N =$ $V_{TH}/R_{TH} = 2$ A

$V_{TH} = i_x(1) + 2i_x = 3i_x$

KCL around supernode

$2 = i_x + \frac{3i_x}{2} \Rightarrow i_x = 4/5 \Rightarrow V_{TH} = 12/5$ V

(b) Determine the Thevenin Resistance for the network shown below.



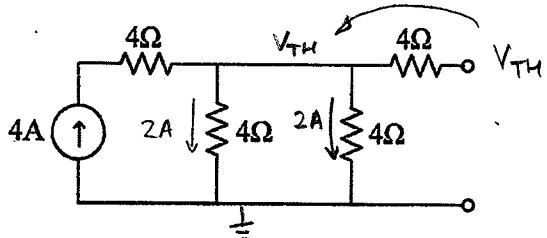
Test signal method

$R_{TH} =$ $-3\ \Omega$

KCL @ V : $\frac{V_x}{2} = V_x + 1 \Rightarrow V_x = -2$

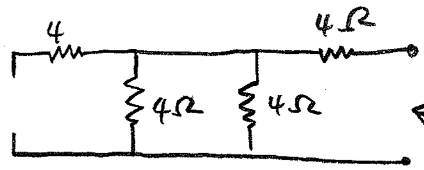
$V = \left(\frac{V_x}{2}\right)(1+2) = -3 \Rightarrow R_{TH} = -3\ \Omega$

(c) Determine the maximum available power for the network shown below.



$P_a =$ $8/3$ W

$V_{TH} = 2 \times 4 = 8$ V

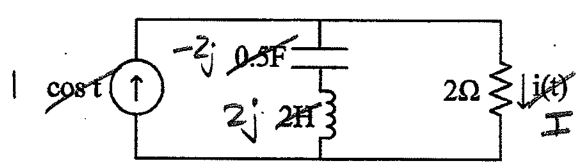


$\leftarrow R_{TH} = 4 + (4 \| 4) = 6\ \Omega$

$$P_a = \frac{V_{TH}^2}{4R_{TH}} = \frac{64}{24} = 8/3\ \text{W}$$

Problem 3

(a) Use the phasor method to determine the steady-state current i(t).



$\omega = 1$

$$\frac{1}{j\omega C} = \frac{1}{j(0.5)} = -2j$$

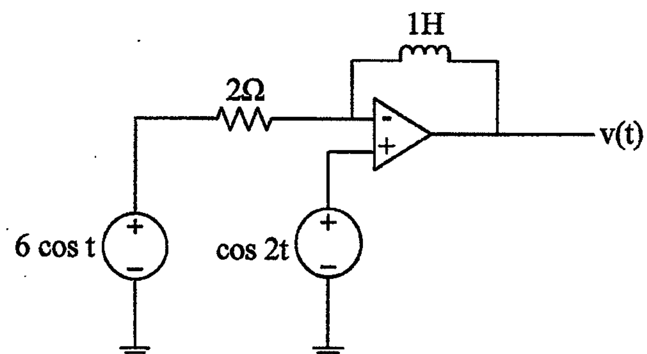
$$j\omega L = 2j$$

Current division :

$$I = 1\,\frac{-2j+2j}{-2j+2j+2} = 0$$

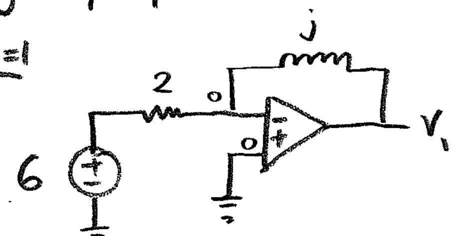
i(t) = 0 A

(b) Use the phasor method to find the steady-state voltage v(t). Assume an ideal op-amp.



By superposition

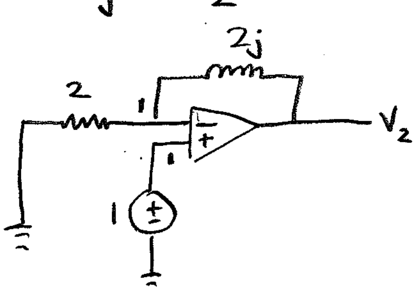
$\underline{\omega = 1}$



v(t) = $3\cos(t - \pi/2) + \sqrt{2}\cos(2t + \pi/4)$

KCL @ ⊖ $\quad \dfrac{V_1 - 0}{j} = \dfrac{0-6}{2} \Rightarrow V_1 = -3j \Rightarrow |V_1| = 3,\ \angle V_1 = -\pi/2 \Rightarrow v_1(t) = 3\cos(t - \pi/2)$

$\underline{\omega = 2}$



KCL @ ⊖ $\quad \dfrac{V_2 - 1}{2j} = \dfrac{1-0}{2} \Rightarrow V_2 = 1 + j$

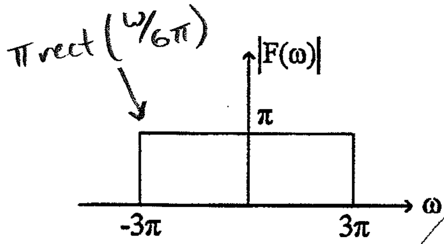
$\Rightarrow |V_2| = \sqrt{2},\ \angle V_2 = \pi/4$

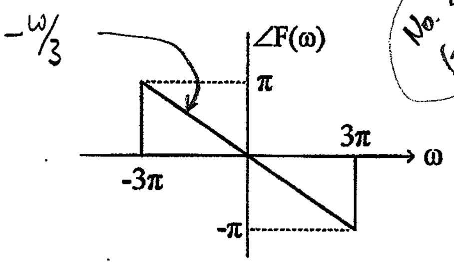
$\Rightarrow v_2(t) = \sqrt{2}\cos(2t + \pi/4)$

$$v(t) = v_1(t) + v_2(t) = 3\cos(t - \pi/2) + \sqrt{2}\cos(2t + \pi/4)$$

Problem 4

(a) Express f(t), the inverse Fourier transform of F(ω), as a real function.





$$F(\omega) = \underbrace{\pi\, rect\left(\frac{\omega}{6\pi}\right)}_{\updownarrow} \underbrace{e^{-j\omega\frac{1}{3}}}_{\text{shift in time by } t_0 = 1/3}$$

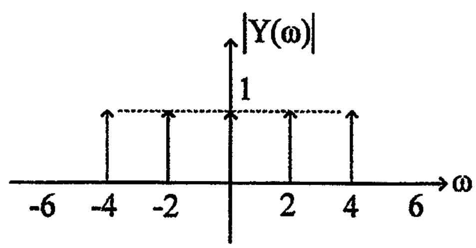
No. 8 from Table 4 (7.2) with $W = 3\pi$

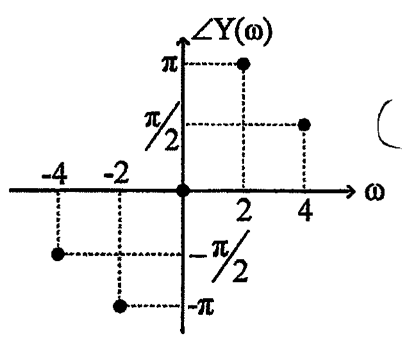
$$3\pi\, sinc(3\pi t)$$

$$f(t) = 3\pi\, sinc\left(3\pi\left(t - \tfrac{1}{3}\right)\right)$$

f(t) = $3\pi\, sinc(3\pi t - \pi)$

(b) Express y(t), the inverse Fourier transform of Y(ω), as a real function.





$$Y(\omega) = \delta(\omega+4)e^{-j\pi/2} + \delta(\omega+2)e^{-j\pi} + \delta(\omega) + \delta(\omega-2)e^{j\pi} + \delta(\omega-4)e^{j\pi/2}$$

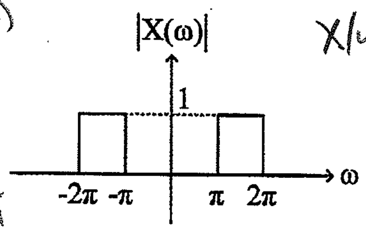
$$= \delta(\omega) - \left(\delta(\omega-2) + \delta(\omega+2)\right) - j\left(\delta(\omega+4) - \delta(\omega-4)\right)$$

$$y(t) = \frac{1}{2\pi} - \frac{1}{\pi}\cos(2t) - \frac{1}{\pi}\sin(4t)$$

(No. 15, 18, 19 from Table 4 (7.2))

y(t) = $\dfrac{1 - 2\cos(2t) - 2\sin(4t)}{2\pi}$

(c) Express x(t), the inverse Fourier transform of X(ω), as a real function.

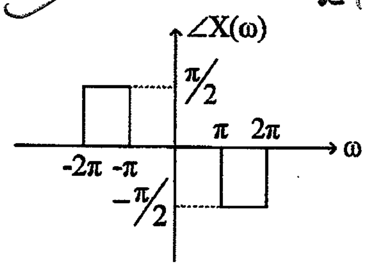


No. 8 in Table 4 (7.2) with $W = \pi/2$ and freq shift property of FT

$$X(\omega) = \text{rect}\left(\frac{\omega + \frac{3}{2}\pi}{\pi}\right) e^{j\pi/2} + \text{rect}\left(\frac{\omega - \frac{3}{2}\pi}{\pi}\right) e^{-j\pi/2}$$

$$= j\left(\text{rect}\left(\frac{\omega + \frac{3}{2}\pi}{\pi}\right) - \text{rect}\left(\frac{\omega - \frac{3}{2}\pi}{\pi}\right)\right)$$

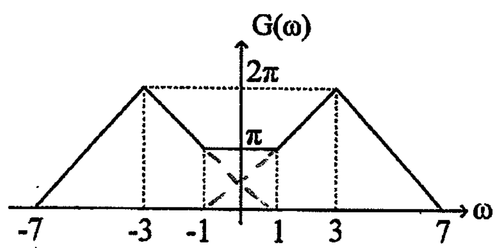
$$x(t) = j\left(\frac{1}{2}\text{sinc}\left(\frac{\pi}{2}t\right) e^{-j\frac{3}{2}\pi t} - \frac{1}{2}\text{sinc}\left(\frac{\pi}{2}t\right) e^{j\frac{3}{2}\pi t}\right)$$



$$= \text{sinc}\left(\frac{\pi}{2}t\right) \frac{1}{2j}\left(e^{j\frac{3}{2}\pi t} - e^{-j\frac{3}{2}\pi t}\right)$$

x(t) = $\underline{\text{sinc}\left(\frac{\pi}{2}t\right) \sin\left(\frac{3}{2}\pi t\right)}$

(d) Express g(t), the inverse Fourier transform of G(ω), as a real function.



g(t) = $\underline{8\,\text{sinc}^2(2t) \cos(3t)}$

$$G(\omega) = 2\pi\, \Delta\left(\frac{\omega+3}{8}\right) + 2\pi\, \Delta\left(\frac{\omega-3}{8}\right)$$

No. 10 in Table 4 (7.2) with $W=4$

$$= \underbrace{2\,\Delta\left(\frac{\omega}{8}\right)}_{\updownarrow\ \frac{4}{\pi}\text{sinc}^2(2t)} * \underbrace{\left(\pi\delta(\omega+3) + \pi\delta(\omega-3)\right)}_{\updownarrow\ \cos(3t)}$$

Using freq. convolution property of F.T.

$$g(t) = 2\pi\left(\frac{4}{\pi}\text{sinc}^2(2t)\right)\left(\cos(3t)\right) = 8\,\text{sinc}^2(2t)\cos(3t)$$

Problem 5

Let $f(t) = \dfrac{1}{1+t^2}$ and $y(t) = \dfrac{1}{1+(t-1)^2}$.

(a) Consider the LTI system $f(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$. Obtain the Fourier transform $Y(\omega)$.

From #4 in Table 4 (7.2), $F(\omega) = \pi e^{-|\omega|}$

Since $y(t) = f(t-1)$, $Y(\omega) = F(\omega) e^{-j\omega 1}$ (time shift property)

$Y(\omega) =$ $\underline{\pi e^{-|\omega|} e^{-j\omega}}$

(b) Let h(t) be the impulse response form part (a) and consider the LTI system

$h(t+1) \rightarrow \boxed{\dfrac{1}{1+(t+1)^2}} \rightarrow \hat{y}(t)$. Obtain $\hat{y}(t)$.

Since $y(t) = f(t-1) = f(t) * \delta(t-1)$, $h(t) = \delta(t-1)$.

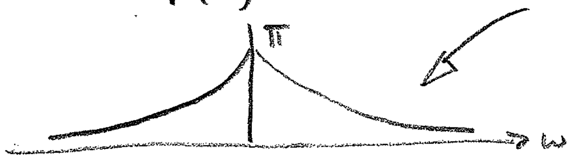
So, $h(t+1) = \delta(t)$;

therefore $\hat{y}(t) = \delta(t) * \dfrac{1}{1+(t+1)^2} = \dfrac{1}{1+(t+1)^2}$

$\hat{y}(t) =$ $\underline{\dfrac{1}{1+(t+1)^2}}$

(c) Can f(t) be sampled without aliasing? If so, which is the minimum required sampling rate ω_0 (rad/s). If not, why?

$F(\omega) = \pi e^{-|\omega|}$



$F(\omega)$ is not bandlimited, so it cannot be sampled without aliasing.

(d) Suppose that before sampling at 10 rad/s, f(t) goes through an ideal low-pass filter LPF. For which range of values of the filter's bandwidth (BW) will the sampling not produce aliasing?

We require sampling rate $\omega_s > 2(BW)$

So, $BW < \dfrac{\omega_s}{2} = \dfrac{10 \text{ rad/s}}{2} = 5 \text{ rad/s}$

$\boxed{BW < 5 \text{ rad/s}}$

Problem 6

(a) An LTI system with input f(t) and output y(t) is described by the ODE

$$\frac{d^2y}{dt^2} + 2y(t) = f(t)$$

Determine the frequency response $H(\omega) = \dfrac{Y(\omega)}{F(\omega)}$.

$$(j\omega)^2 Y(\omega) + 2Y(\omega) = F(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{F(\omega)} = \frac{1}{2-\omega^2}$$

$H(\omega) =$ $\dfrac{1}{2-\omega^2}$

(b) A bandpass filter has frequency response $H(\omega) = \dfrac{j\omega}{1-\omega^2+j\omega}$. $\quad H(1) = \dfrac{j}{1-1^2+j} = 1 = 1e^{j0}$

Determine the filter's output, y(t), for input f(t) = 3cos t.

$$y(t) = 3\,|H(1)|\cos(t + \angle H(t))$$

$$= 3 \times 1 \times \cos(t+0)$$

$$= 3\cos t$$

$y(t) =$ $3\cos(t)$

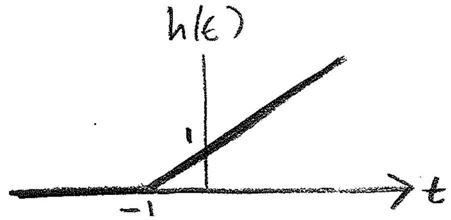
Determine the steady-state response to a DC input f(t) = 10.

$y_{ss}(t) = H(0)\,f(t)$ since DC is at $\omega = 0$

$$= \frac{0}{1-0^2+j0}\,f(t)$$

$$= 0$$

$y_{ss}(t) =$ 0



Problem 7 Let h(t) = (t + 1) u (t + 1) be the impulse response of an LTI system.

(a) Obtain the Laplace transform $\hat{H}(s)$, as well as its poles, zeros, and region of convergence, ROC.

$$\hat{H}(s) = \int_{0^-}^{\infty} (t+1)u(t+1)e^{-st}\,dt = \int_{0^-}^{\infty} (t+1)u(t)e^{-st}\,dt$$ because integral starts at 0^-

$$= \mathcal{L}\{t\,u(t) + u(t)\} = \frac{1}{s^2} + \frac{1}{s} = \frac{s+1}{s^2}$$

See also Example 11.9 (p. 375) in textbook

$\hat{H}(s)$ = $\frac{(s+1)}{s^2}$

poles = 0 (double pole)

zeros = -1, $\pm\infty$

ROC = $\{Re(s) > 0\}$

(b) Is the system causal and/or BIBO stable? Why or why not? (no points if no valid reason)

Noncausal because $h(-\frac{1}{2}) = \frac{1}{2} \neq 0$

Not BIBO stable because $\int_{-\infty}^{\infty} |h(t)|\,dt \to \infty$

(see graph)

Causal: Yes ______ No ✓

Also not BIBO stable because poles (at 0) are not in LHP

BIBO Stable: Yes ______ No ✓

(c) Obtain the Laplace transform of $g(t) = \frac{d^2}{dt^2}h(t)$.

$$\hat{G}(s) = s^2\hat{H}(s) - s\,h(0^-) - h'(0^-)$$

From the graph of $h(t)$ above,

$h(0^-) = 1$

$h'(0^-) = 1$

$\hat{G}(s)$ = 0

So $\hat{G}(s) = s^2\,\frac{s+1}{s^2} - s - 1 = 0$

Alternative: $\frac{d}{dt}h(t) = u(t+1)$, so $g(t) = \frac{d^2}{dt^2}h(t) = \delta(t+1)$

$\hat{G}(s) = 0$ since $g(t) = 0$ for all $t > 0$