Problem 1

(a) Compute the phasors associated with three signals below.

$f_1(t) = 2\sin\left(t + \frac{\pi}{6}\right)$
$= 2\cos\left(t + \frac{\pi}{6} - \frac{\pi}{2}\right)$

$F_1 =$ $2e^{-j60^\circ} = 2e^{-j\pi/3} = 2\angle -60^\circ$ (2)

$f_2(t) = -2\cos t$

$F_2 =$ $2e^{j180^\circ} = 2e^{\pm j\pi} = 2\angle \pm 180^\circ$ (2)

$f_3(t) = \sin 2t + \cos 2t$

$F = \frac{1}{j} + 1 = 1 - j = \sqrt{2}\angle -45^\circ$

$F_3 =$ $\sqrt{2}\,e^{-j45^\circ} = \sqrt{2}\,e^{-j\pi/4} = \sqrt{2}\angle -45^\circ$ (2)

Problem 2

(a) Convert each of the following time domain signals into phasor.

(i) $f_1(t) = 5\sin\left(3t - \frac{\pi}{6}\right) = 5\cos\left(3t - \frac{\pi}{6} - \frac{\pi}{2}\right) = 5\cos\left(3t - \frac{4\pi}{6}\right)$ $F_1 =$ $5\angle -\frac{2\pi}{3}\text{ rad} = 5\angle -120^\circ$

(ii) $f_2(t) = \cos t + \sin t = \cos t + \cos\left(t - \frac{\pi}{2}\right)$

$F_2 = 1 - j = \sqrt{2}\angle -45^\circ$

(b) Convert each of the following phasors into a time domain signal. Assume $\omega = 1$ rad/sec.

(i) $F_1 = 2 + 2j = 2\sqrt{2}\angle 45^\circ$

$f_1(t) = 2\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$

(ii) $F_2 = \dfrac{e^{j\pi/4}}{e^{-j\pi/2} + e^{j\pi/4}\left(1 + e^{j\pi/4}\right)} = \dfrac{e^{j\pi/4}}{e^{-j\pi/2} + e^{+j\pi/4} + e^{j\pi/2}}$

$f_2(t) = \cos(t)$

$= \dfrac{e^{j\pi/4}}{2\cos\left(\frac{\pi}{2}\right) + e^{+j\pi/4}} = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

Problem 3

(a) Find the cosine function f(t) with frequency $\omega = 10$ rad/sec corresponding the following phasors.

pnasors.

(i) $F = (1+j)^3$ $= \left(\sqrt{2}\,e^{j45^\circ}\right)^3 = 2\sqrt{2}\,e^{j135^\circ}$

$f(t) =$ $2\sqrt{2}\cos(10t + 135^\circ)$

(ii) $F = e^{j\pi/4} + e^{-j\pi/4}(1 + \sqrt{2}\,e^{-j\pi/4})$

$f(t) =$ $2\cos(10t - \pi/4)$

$= \frac{1+j}{\sqrt{2}} + \frac{1-j}{\sqrt{2}} + \sqrt{2}\,e^{-j\pi/2} = \sqrt{2} - j\sqrt{2}$

$= 2e^{-j\pi/4}$

(b) Convert each of the following time domain signals into phasor form F.

i) $f_1(t) = \text{Cost} - \text{Sint}$

ii) $f_2(t) = -2 \text{ Sin } 5t$

iii) $f_3(t) = -3 \text{ Cos}\left(10t - \frac{\pi}{6}\right)$

$F_1 = \sqrt{2}\angle 45^\circ$

$F_2 = 2\angle 90^\circ$

$F_3 = 3\angle 5\pi/6$

Problem 4

(a) If possible, convert the following signals into phasor form. If not possible, indicate why not.

i) $f_1(t) = \sqrt{3}\sin(3t + \frac{\pi}{2}) = \sqrt{3}\cos(3t + \frac{\pi}{2} - \frac{\pi}{2})$

$= \sqrt{3}\cos(3t)$

$\rightarrow \sqrt{3}\, e^{j0} = \sqrt{3}$

$F_1 = \sqrt{3}$

ii) $f_2(t) = \cos(2t) + \sqrt{3}\sin(2t + \frac{\pi}{2})$

$= \cos(2t) + \sqrt{3}\cos(2t + \frac{\pi}{2} - \frac{\pi}{2})$

$= \cos(2t) + \sqrt{3}\cos(2t)$

$= (1 + \sqrt{3})\cos(2t)$

$\rightarrow (1+\sqrt{3})\, e^{j0} = 1 + \sqrt{3}$

$F_2 = 1 + \sqrt{3}$

iii) $f_3(t) = \sqrt{3}\cos(t + \frac{\pi}{2}) + \sin(2t + \frac{\pi}{2})$

$\omega = 1 \frac{rad}{s}$ $\omega = 2 \frac{rad}{s}$

different frequencies so no possible phasor representation.

$F_3 =$ not possible because $\omega_1 \neq \omega_2$