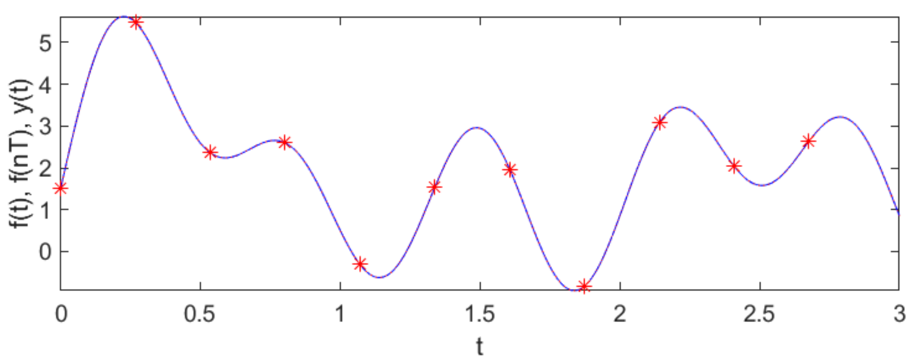
Lecture 44, Friday, April 15, 2022

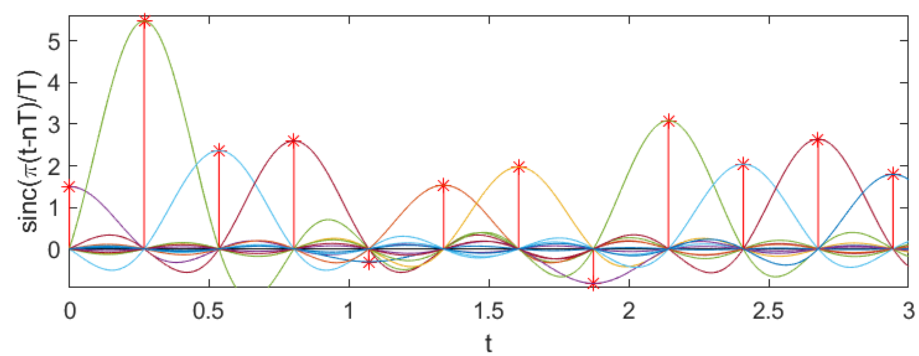
- Sampling and the reconstruction formula

  – Reconstruction formula:

$$y(t) \;=\; \sum_{n=-\infty}^{\infty} f(nT)\text{sinc}\left(\frac{\pi}{T}(t-nT)\right) \;=\; f(t), \qquad \text{reconstruction formula}$$

  * $f(t)$ is reconstructed via an infinite sum of sinc functions centered at multiples of T
  * At each sinc peak, all other sincs have a zero crossing.





  – **Note:** The sampling bandwidth, Ω, is the largest frequency ω such that $F(\omega) \neq 0$.

  For bandpass signals, that is very different than the *energy bandwidth,* discussed in earlier chapters, which would ignore the zero portions of the spectrum around zero.

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– Recall

$$s(t) = f(t)p(t) \longleftrightarrow \; S(\omega) = \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\omega_0)$$

- ∗ If $\omega_0 < 2\Omega$, the frequency copies overlap, creating *aliasing*

  It is called *undersampling,* and signal cannot be usually reconstructed
- ∗ If $\omega_0 = 2\Omega$, the frequency copies touch each other right at the boundary

  It is sampled at *Nyquist sampling frequency*
- ∗ If $\omega_0 > 2\Omega$, the frequency copies are spaced out and there are gaps in-between

  It is called *oversampling,* and signal can be reconstructed

- Impulse response

  – Recall $h(t)$ is called the *impulse response,* because it is the response to an impulse:

$$\delta(t) \longrightarrow \boxed{\text{h(t)}} \longrightarrow y_{ZS}(t) = \delta(t) * h(t) = h(t)$$

  – The *unit-step response,* is the LTI system response to a unit-step:

$$u(t) \longrightarrow \boxed{\text{h(t)}} \longrightarrow y_{ZS}(t) = u(t) * h(t)$$

  ∗ By properties of convolution

$$h(t) = \frac{d}{dt} y_{ZS}(t) \quad \text{only if} \;\; y_{ZS}(t) \text{ is the unit-step response}$$

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- Bounded input-bounded output stability (BIBO) stability
  - In a bounded input-bounded output (BIBO) stable system

    $$\text{bounded input } f(t) \longrightarrow \boxed{\text{system}} \longrightarrow \text{ bounded output } y(t)$$

    for **all** bounded $f(t)$
  - Bounded $f(t)$ means $|f(t)| \leq C$ for some real-valued constant C and all t
  - BIBO may apply to any type of system, but if the system is LTI with impulse response $h(t)$, then

    $$\text{BIBO} \leftrightarrow h(t) \text{ is absolutely integrable} : \int_{-\infty}^{\infty} |h(t)| dt < \infty$$

  - Boundedness of $h(t)$ does not imply BIBO, only absolute integrability does