- Exponential Fourier series

  - If f(t) is periodic with fundamental frequency ω_0, then we can express it as

most of periodic signals (there are exceptions)

$$f(t) = \underbrace{\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}}$$

F_n is complex BUT independent of time.

will change depending on the function

  - Distinct functions with the same ω_0 will have distinct sets of F_n

sin(t) and cos(t)

⇓

same ω_0, but will have different F.S. coefficients

  - How to get the coefficients F_n?

- Exponential Fourier series-example #4
  - Let f(t) = $\cos(t)$ $\swarrow$ $\omega_0 = 1$ rad/s
  - Determine its exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} = \underset{\shortparallel\, 0}{F_0} + F_1 e^{jt} + F_{-1} e^{-jt} + F_2 e^{j2t} + F_{-2} e^{-j2t} + \ldots$$

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2} = \frac{1}{2} e^{jt} + \frac{1}{2} e^{-jt}$$

Exp. FS for $f(t) = \frac{1}{2} e^{jt} + \frac{1}{2} e^{-jt}$

$F_0 = 0$

$F_1 = \frac{1}{2}$ $\quad F_{-1} = \frac{1}{2}$

$F_2 = F_{-2} = \ldots = 0$

$F_n = 0$ if $n \neq \pm 1$

- Exponential Fourier series-example #5

  - Let $f(t) = 1 + 2\sin(t) + \sin^2(\frac{5}{4}t)$ ← DC offset
  - Determine its exponential Fourier series

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$f(t) = 1 + 2\sin(t) + \frac{1}{2} - \frac{1}{2}\cos(\frac{5}{2}t) = \frac{3}{2} + 2\sin(t) - \frac{1}{2}\cos(\frac{5}{2}t) =$

$\omega_0 = GCD(1, \frac{5}{2}) = \frac{1}{2}$ rad/s

$= \frac{3}{2} + 2\left(\frac{e^{jt} - e^{-jt}}{2j}\right) - \frac{1}{2}\left(\frac{e^{j\frac{5}{2}t} + e^{-j\frac{5}{2}t}}{2}\right) =$

$\underbrace{= \frac{3}{2} + \frac{1}{j}e^{jt} - \frac{1}{j}e^{-jt} - \frac{1}{4}e^{j\frac{5}{2}t} - \frac{1}{4}e^{-j\frac{5}{2}t}}$ $= f(t)$ ← Exp. F.S. of f(t)

$F_0 = \frac{3}{2}$ $\quad F_5 = -\frac{1}{4}$

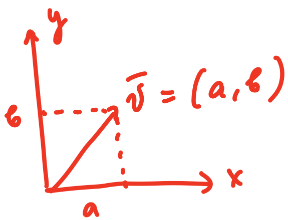
$F_2 = \frac{1}{j}$ $\quad F_{-5} = -\frac{1}{4}$

$F_{-2} = -\frac{1}{j}$ $\quad F_n = 0$ for all others

$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$ ($\omega_0 = \frac{1}{2}$ rad/s) $= F_0 + \underbrace{F_1}_{0} e^{j\frac{1}{2}t} + \underbrace{F_{-1}}_{0} e^{-j\frac{1}{2}t} + F_2 e^{jt} + F_{-2}e^{-jt} +$

$+ F_3 e^{j\frac{3}{2}t} + F_{-3}e^{-j\frac{3}{2}t} + F_4 e^{j2t} + F_{-4}e^{-j2t} + F_5 e^{j\frac{5}{2}t} + F_{-5}e^{-j\frac{5}{2}t} + \ldots$

- Fourier series coefficients
  - If f(t) consists of co-sinusoids or complex exponentials, just match coefficients.
  - If not, but f(t) is periodic, will use $\{e^{jn\omega_0 t}\}_{n=-\infty}^{\infty}$ as a basis to project onto.



a is a projection of $\vec{v}$ onto $\vec{x}$

b is a projection of $\vec{v}$ onto $\vec{y}$

$$F_n = \langle f(t), \underbrace{e^{jn\omega_0 t}} \rangle = \frac{1}{T}\int_T f(t)e^{-jn\omega_0 t}dt$$

0 to T or

-T/2 to T/2

- Fourier series coefficients-cont

$$\omega_0 = \frac{2\pi}{T}$$

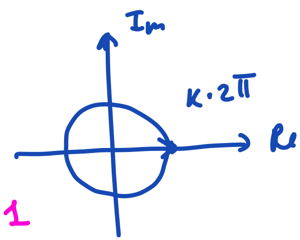
- $\{e^{jn\omega_0 t}\}$ is an orthonormal basis:

$$\langle e^{jn\omega_0 t}, e^{jm\omega_0 t}\rangle = \begin{cases} 1 & n = m \\ 0 & \text{else} \end{cases}$$

$$\langle e^{jn\omega_0 t}, e^{jm\omega_0 t}\rangle = \frac{1}{T}\int_T e^{jn\omega_0 t}\cdot e^{-jm\omega_0 t}\,dt = \frac{1}{T}\int_T e^{j(n-m)\omega_0 t}\,dt =$$

$$= \frac{1}{T}\left.\frac{e^{j(n-m)\omega_0 t}}{j(n-m)\omega_0}\right|_0^T = \frac{1}{T}\frac{\left(e^{j(n-m)\underbrace{\omega_0 T}_{=2\pi}} - 1\right)}{j(n-m)\omega_0} = \frac{1}{T}\left(\frac{\underbrace{e^{j(n-m)2\pi}}_{=1} - 1}{j(n-m)\omega_0}\right) = 0$$

if $n \neq m$



if $\underline{n=m}$

$$\langle e^{jn\omega_0 t}, e^{jm\omega_0 t}\rangle = \frac{1}{T}\int_0^T e^{jn\omega_0 t}\cdot e^{-jn\omega_0 t}\,dt = \frac{1}{T}\int_0^T dt = 1$$