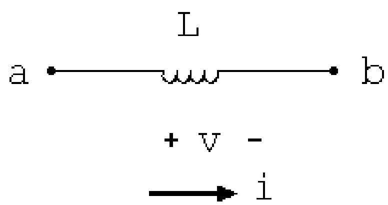
Lecture 11, Monday, February 7, 2022

- Differentiators and integrators
  - *Inductor:*

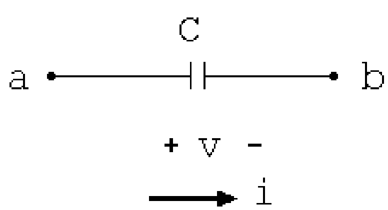


$* \; v(t) = L\dfrac{di}{dt} \quad \rightarrow \quad i(t) \;=\; \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v(s)ds$

$*$ current in inductor is continuous for practical sources

$\rightarrow \; i(t) \;=\; i(t^-) \;=\; i(t^+)$

  - *Capacitor:*



$* \; i(t) = C\dfrac{dv}{dt} \quad \rightarrow \quad v(t) \;=\; \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(s)ds$

$*$ voltage in capacitor is continuous for practical sources

$\rightarrow \; v(t) \;=\; v(t^-) \;=\; v(t^+)$

- *Zero-state* linearity: in a zero-state linear system, a weighted sum of inputs produces a similarly weighted sum of corresponding *zero-state* outputs, consistent with superposition:

$$\mathbf{f_1(t)} \;\rightarrow\; \boxed{\text{zero-state linear}} \;\rightarrow\; \mathbf{y_{ZS,1}(t)}, \;\text{and}$$

$$\mathbf{f_2(t)} \;\rightarrow\; \boxed{\text{zero-state linear}} \;\rightarrow\; \mathbf{y_{ZS,2}(t)}, \;\text{then}$$

$$\mathbf{f_3(t) = k_1 f_1(t) + k_2 f_2(t)} \rightarrow \boxed{\text{zero-state linear}} \rightarrow \mathbf{y_{ZS,3}(t) = k_1 y_{ZS,1}(t) + k_2 y_{ZS,2}(t)}$$

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  - Basically a linear combination of inputs gives the same linear combination of their individual outputs
  - Sometimes you can determine by inspection: if you plot y as a function of f, is it a line through the origin?

- *Zero-input linearity* : in a zero-input linear system, a weighted sum of initial states produces a similarly weighted sum of corresponding *zero-input* outputs, consistent with the superposition principle.

- *Linearity*: a system is linear if it is zero-state linear, zero-input linear and the output is the sum of the zero-state response (output if you force the initial state to be zero) and the zero-input response (output if you force the input to be zero).

$$\mathbf{y(t)} \;=\; \mathbf{y_{ZS}(t)} \;+\; \mathbf{y_{ZI}(t)}$$