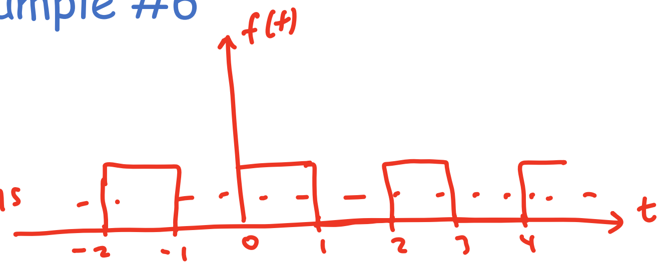
- Fourier series coefficients-example #6

  - Let $f(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & t \in [1,2) \end{cases}$

  have period $T = 2s$. $\Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$ rad/s



  - Determine its exponential Fourier series of f(t)

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \quad \omega_0 = \pi \text{ rad/s}$$

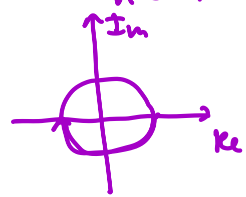
$$F_n = \frac{1}{T}\int_T f(t)\, e^{-jn\pi t}\, dt = \frac{1}{2}\left[\int_0^1 (1)\, e^{-jn\pi t}\, dt + \int_1^2 \underbrace{(0)\, e^{-jn\pi t}}_{0}\, dt\right] =$$

$$= \frac{1}{2}\, \frac{e^{-jn\pi t}}{(-jn\pi)}\Bigg|_0^1 = -\frac{1}{2jn\pi}\left(e^{-jn\pi} - 1\right)$$

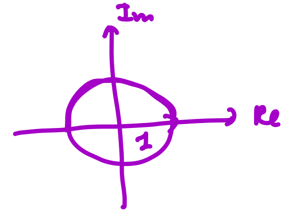
$e^{-jn\pi} = 1$ if n even, $= -1$ if n odd

$n \neq 0$

n odd



n even



$$F_0 = \frac{1}{T}\int_T f(t)\, e^{-j0\pi t}\, dt = \frac{1}{2}\int_T f(t)\, dt = \frac{1}{2} \leftarrow \text{average of } f(t)$$

DC term

- Fourier series coefficients-example #6-cont

- Let $f(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & t \in [1,2) \end{cases}$

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

have period $T = 2s$. , $\omega_0 = \pi$ rad/s

- Determine its exponential Fourier series of f(t)

$$F_n = \begin{cases} -\frac{1}{jn2\pi}(1-1) = 0 & n \text{ even}, \; n \neq 0 \\ -\frac{1}{jn2\pi}(-1-1) = \frac{1}{jn\pi} & n \text{ odd} \\ \frac{1}{2} & n = 0 \end{cases}$$

$$\boxed{f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{1}{jn\pi} e^{jn\pi t}}$$

– Exp. F.S. of f(t)

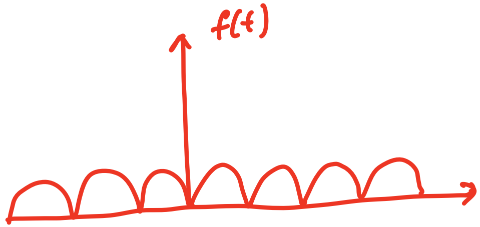
- Fourier series coefficients-cont
  - F_0 is the DC component (DC term) (when $F_0 = 0$, $f(t)$ has zero mean)
  - F_1 corresponds to the fundamental frequency term
  - F_n corresponds to the n-th harmonic

- Existence of Fourier series representation

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_{-1} e^{-j\omega_0 t} + \dots$$

① $f(t) = |\sin(t)|$



②

• Existence of Fourier series representation-cont

• For the existence of the Fourier series representation of a periodic signal f(t), we need:

✓ Absolute integrability of f(t) over period T

$$\int_T |f(t)|dt < \infty$$ has to be finite

✓ Plottability

over one period
- finite number of maxima and minima
- finite number of finite discontinuities (series converges to midpoint at discontinuity)

Dirichlet conditions for F.S.

- Fourier series forms

Table 6.1.

- Exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

- Trigonometric Fourier series:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t))$$

$a_0 = 2F_0$ $\quad a_n = F_n + F_{-n}$ $\quad b_n = j(F_n - F_{-n})$

$F_0 = \dfrac{a_0}{2}$ $\quad F_n = \dfrac{a_n - jb_n}{2}$ $\quad F_{-n} = \dfrac{a_n + jb_n}{2}$

Hermitian property

If $f(t)$ is real:

$F_n = F_{-n}^*$

$|F_n| = |F_{-n}|$

$\angle F_n = -\angle F_{-n}$

- Compact Fourier series, for real-valued functions only:

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$c_0 = 2F_0$ $\quad c_n = 2|F_n|$ $\quad \theta_n = \angle F_n$

$F_0 = \dfrac{c_0}{2}$ $\quad F_n = \dfrac{c_n}{2} e^{j\theta_n} = F_{-n}^*$

- Fourier series forms-cont

$$f(t) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n\ odd}}^{\infty} \frac{1}{jn\pi} e^{jn\pi t} \ominus \rightarrow \text{trig. F.S. of } f(t)$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$\ominus\ \frac{1}{2} + \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \left(\frac{1}{jn\pi} e^{jn\pi t} + \frac{1}{j(-n)\pi} e^{j(-n)\pi t} \right) =$$

$$= \frac{1}{2} + \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{1}{n\pi} \left(2 \frac{e^{jn\pi t} - e^{-jn\pi t}}{2j} \right) = \frac{1}{2} + \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{2}{n\pi} \sin(n\pi t)$$

$\sin(n\pi t)$

$$a_n = \begin{cases} 1 & n=0 \\ 0 & \text{all others} \end{cases}$$

$$b_n = \begin{cases} 0 & n\ even \\ \frac{2}{n\pi} & n\ odd \end{cases}$$

Exp. FS $\rightarrow$ compact F.S.

$$F_n = \begin{cases} \frac{1}{2} & n=0 \\ \frac{1}{jn\pi} & n\ odd \\ 0 & else \end{cases}$$

$C_0 = 2F_0 = 1$

$$C_n = 2|F_n| = 2\left|\frac{1}{jn\pi}\right| = \begin{cases} \frac{2}{n\pi} & n\ odd \\ 0 & n\ even,\ n \neq 0 \end{cases}$$

$\frac{1}{jn\pi} = \frac{-j}{n\pi}$

$$\theta_n = \angle F_n = \begin{cases} -\pi/2 & n\ odd \\ 0 & else \end{cases}$$

$$f(t) = \frac{1}{2} + \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{2}{n\pi} \cos(n\pi t - \pi/2)$$