ECE210 Laboratory
PRE-LAB#3

Net ID: _____ ,
Student Name:

Lab section:

Spring 2022

Lab 3: Frequency Response and Fourier Series

In this lab you will build an active bandpass filter circuit with two capacitors and an op-amp, and examine the response of the circuit to periodic inputs over a range of frequencies. The same circuit will be used in Lab 4 in your AM radio receiver system as an intermediate frequency (IF) filter, but in this lab our main focus will be on the frequency response $H(\omega)$ of the filter circuit and the Fourier series of its periodic input and output signals. In particular we want to examine and gain experience about the response of linear time-invariant circuits to periodic inputs.

1 Prelab

1. Determine the compact-form Fourier series of the periodic square wave signal, $f(t)$ shown in Figure 1, with a period T and amplitude A. That is, find c_n and θ_n such that

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n), \text{ where } \omega_o = \frac{2\pi}{T}.$$

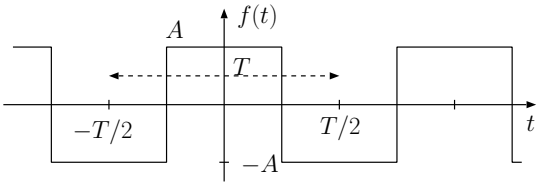


Figure 1: Square wave signal for prelab.

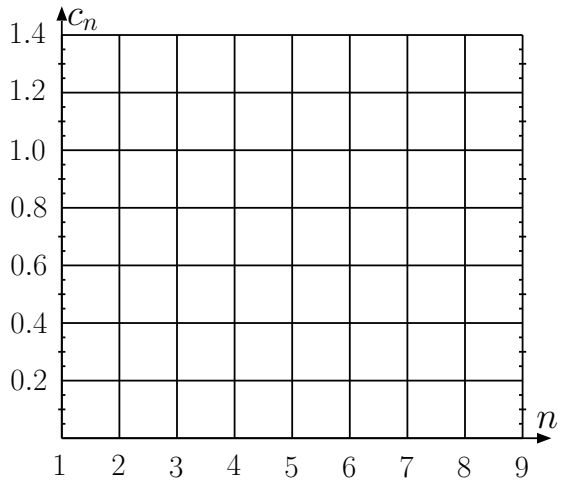
Notice $\frac{C_0}{2} = 0$. How could you have determined that without any calculation? (____/2)

Show your work.(____/3) (____/11)

$F_n =$ _____ (____/2)

$c_n =$ _____ (____/2)

$\theta_n =$ _____ (____/2)



With $A = 1$, plot c_n over n ($n \in [1, 9]$) (____/2)

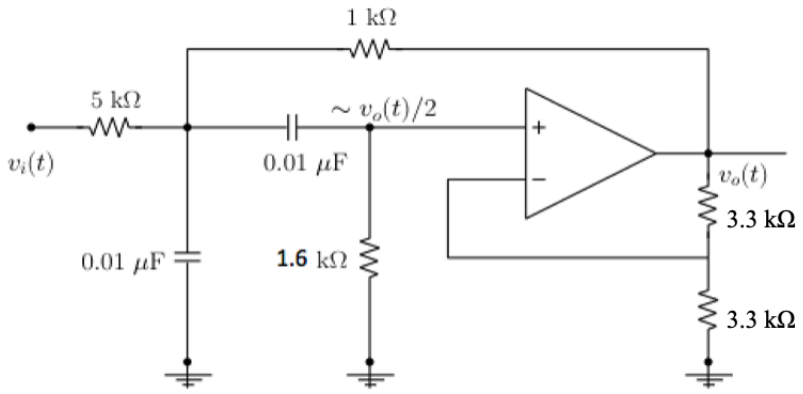


Figure 2: Circuit for analysis in prelab and lab.

2. Consider the circuit in Figure 2 where $v_i(t)$ is a co-sinusoidal input with some radian frequency ω.

   (a) What is the phasor gain $\frac{V_o}{V_i}$ in the circuit as $\omega \to 0$? (Hint: How does one model a capacitor at DC — open or short?)

   Show your work

   (____/3)

   (b) What is the gain $\frac{V_o}{V_i}$ as $\omega \to \infty$? (Hint: think of capacitor behavior in $\omega \to \infty$ limit)

   Show your work

   (____/3)

   (c) In view of the answers to part (a) and (b), and the fact that the circuit is 2nd order (it contains two energy storage elements), try to guess what kind of a filter the system frequency response $H(\omega) \equiv \frac{V_o}{V_i}$ implements — lowpass, highpass, or bandpass? The amplitude response $|H(\omega)|$ of the circuit will be measured in the lab.

   Give your answer and explain your reasoning.

   (____/2)

3. Decibels (dB) is a unit of measurement widely used in science and engineering to compare power or intensity quantities. A decibel (dB) is one-tenth of a Bel (B), which is the name given to $\log_{10}\left(\frac{P_1}{P_0}\right)$, where $\log_{10}$ is the base 10 logarithm, and $\frac{P_1}{P_0}$ is the ratio of two power quantities. The formula for calculating decibels is : $10\log_{10}\left(\frac{P_1}{P_0}\right)$. and for comparing voltages we can use: $20\log_{10}\left(\frac{V_1}{V_0}\right)$, which is derived from $10\log_{10}\left(\frac{V_1^2/R}{V_0^2/R}\right)$. Complete the following table of useful ratios: (use accuracy of 1 decimal in dB row)

P_1/P_0	1000	100	10	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
V_1/V_0	$\sqrt{1000}$	10	$\sqrt{10}$			$\sqrt{2}$						$\frac{1}{\sqrt{1000}}$
Decibels (dB)	30	20				3						-30

(____/2)

4. In the case of the Fourier analysis, the oscilloscope compares the signals with a reference of $V_{\text{rms}} = 1\,\text{V}$. Recall that a sine wave with rms (root mean square) amplitude of $V_{\text{rms}} = 1\,\text{V}$ corresponds to a sine wave with a peak amplitude of $\sqrt{2}\,\text{V}$, which is to say a peak-to-peak amplitude of $2\sqrt{2}\,\text{V}$. To specify that the reference is $V_{\text{rms}} = 1\,\text{V}$, the decibel symbol is modified with the suffix "V" becoming "dBV". Convert the voltages in the table below to dBV units (use accuracy of 1 decimal).

v	$V_{\text{rms}} =$ 1 V	$V_{\text{rms}} =$ 2 V	$V_{\text{rms}} =$ $\sqrt{10}$ V	$V_{\text{rms}} =$ $\sqrt{2}$ V	$V_{\text{rms}} =$ 1/10 V	$2\sqrt{2}$ Vpeak-to-peak	$4\sqrt{5}$ Vpeak-to-peak	2 Vpeak-to-peak	$\frac{2\sqrt{2}}{\sqrt{10}}$ Vpeak-to-peak
v (dBV)	0.0								

(____/2)