University of Illinois **Alvarez, Lee, Mironenko, Waldrop**

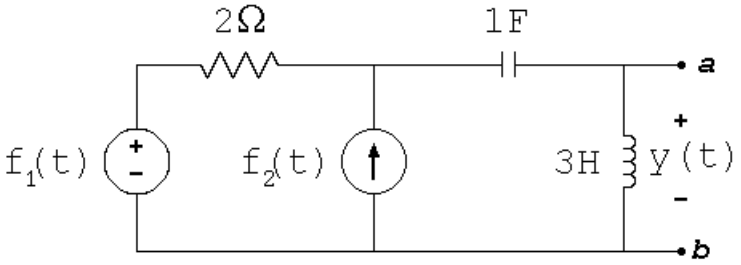
Analog Signal Processing

Thursday, October 22, 8pm

Exam II

- **Exam duration:** The exam is designed for **75 minutes** and you will have an additional 35 minutes to upload your solutions. The proctor will not stop the exam when the 75 minutes are up, so you could potentially keep working on the exam. However, keep in mind that there will be a hard stop (Gradescope will shut down) at 110 minutes and you will **not be allowed** to upload after that, so we strongly advise you to finish your exam with plenty of time to spare. If your submission/upload is not completed within 110 minutes, you will get a **zero** in the exam, unless you start scanning/uploading by the 76th minute.
- **No collaboration allowed:** You are **not allowed** to share or collaborate on this exam and all work should be your own.
- **Closed notes:** : you **cannot** use the textbook, **nor** any notes. However, the Fourier series **tables** (6.1 and 6.3 from the textbook, or 1-2 from the online handout.) **are at** the end of this exam.
- **Calculations:** Calculators and other electronic ways to do calculations, like Wolfram alpha, are **not allowed. Neither** is searching online.
- You will be **penalized** for unauthorized actions.
- **Solving the exam:** you **must** solve the exam in blank sheets of paper. Tablets are **not allowed** for writing.
- **Solution uploads:** Make sure that your scans are legible and that you correctly assign each solution to its question, or you will be **deducted** at least 5% of the corresponding problem part. Instructions for downloading the exam and uploading your pdf solutions to Gradescope have been available on the course website for over a week and you were instructed to try it before the exam to make sure you know how to do it.
- **Proctoring:** You **MUST** remain in the proctoring session until you are finished uploading, even if the proctor says you can leave before that. We were lenient on this for the first exam due to the mixed messages from proctors, but now you are notified that you **cannot** leave the proctoring session before uploading **even if** the proctor says it is OK. Take a screenshot or picture of your successful upload message next to the proctoring window, in case there are issues with timestamps from CBTF vs. Gradescope..
- **Show all your work and simplify** your answers. Answers with no explanation/justification/work will be given **little/no credit**.
- **Box your answers**.
- Answers **should** include units if appropriate.

1. (25 pts) Consider the circuit below, where $f_1(t) = 2\cos\left(\frac{1}{3}t\right)$V and $f_2(t) = 3\sin(t)$A. Determine the steady-state output $y(t)$.

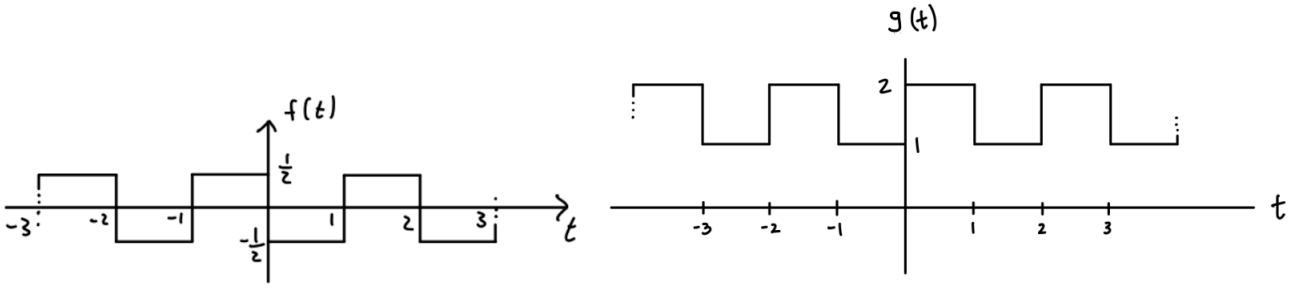


$y(t) =$ _____

2. (25 pts) The two parts of this question are unrelated.

(a) A periodic signal $f(t)$ is given by

$$f(t) = \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{j}{n\pi} e^{jn\pi t}$$



i. [15 pts] Determine G_n, the exponential Fourier series coefficients of $g(t)$, for all n.

$G_n =$ _____

ii. [5 pts] Circle which of the following statements is true regarding the Fourier series expansion in trigonometric form of the function $g(t)$ shown in part (a). Briefly explain your choice.

A. $a_m = 0$ and $b_m \neq 0$ for all m

B. $a_m \neq 0$ and $b_m = 0$ for all m

C. $a_m \neq 0$ and $b_m \neq 0$ for all m

(b) [5 pts] A periodic signal $x(t)$ is given by

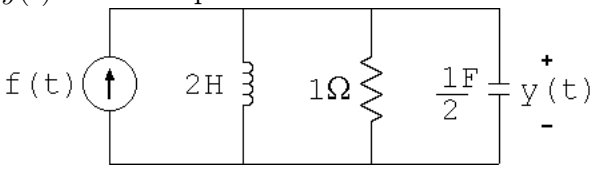
$$x(t) = e^{j\frac{\pi}{3}} e^{-j\frac{\pi}{5}t} + je^{-j\frac{\pi}{4}t} + e^{-j\frac{\pi}{2}t} + e^{-j\frac{\pi}{3}} e^{j\frac{\pi}{5}t} - je^{j\frac{\pi}{4}t} + e^{j\frac{\pi}{2}t}$$

What is the exponential Fourier series coefficient X_4?

$X_4 =$ _____

3. (25 pts) The two parts of this question are unrelated.

(a) Consider the circuit below, where the current source $f(t)$ is the input and the voltage $y(t)$ is the output.



i. [10 pts] Determine the frequency response of the system.

$H(\omega) =$ _____

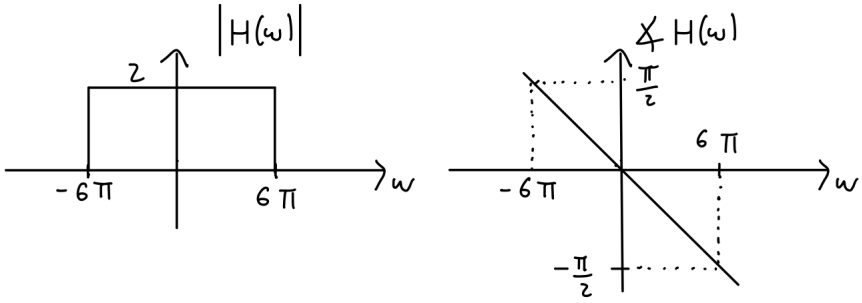
ii. [5 pts] Determine if this is a low-pass filter, a band-pass filter, a high-pass filter or none of them.

Explain: _____

(b) [10 pts] The input signal to the ideal low-pass filter with a frequency response $H(\omega)$ is

$$f(t) = \frac{1}{4} + \sum_{n=-\infty, n\ odd}^{\infty} \frac{1}{n\pi} e^{jn\pi t}.$$

The amplitude response $|H(\omega)|$ and phase response $\angle H(\omega)$ are plotted below:



Determine the steady-state output $y(t)$.

$y(t) =$ _____

4. (25 pts) The two parts of this problem are unrelated.

(a) Given the initial conditions $y = 0$ and $\frac{dy}{dt} = 6$, determine the general form zero-input solution of the second order ODE
$2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 16y(t) = 0$

i. [10 pts] Determine the zero-input solution of the second order ODE:

$y_{ZI}(t) =$ _____

ii. [3 pts] Is the zero–input response transient? **Explain.**

Yes/No

iii. [3 pts] Is the zero-input response dissipative? **Explain.**

Yes/No

(b) (9 pts) The input-output relation for a system with the input f(t) is given as

$$y(t) = 2f(t) + f^2(t) + f^3(t).$$

Determine the total harmonic distortion (THD) of the system response to a pure cosine input $f(t) = \cos(\omega_0 t)$.

$THD =$ _____

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K f(t) \leftrightarrow K F_n$
2	Addition	$f(t) \leftrightarrow F_n$, $g(t) \leftrightarrow G_n$, ...	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

Table 2: Fourier series properties