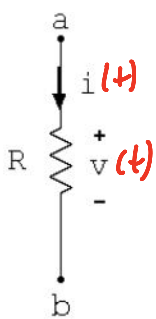
• LTIC circuits

- How to analyze circuits in the s-domain?

- Consider a resistor:



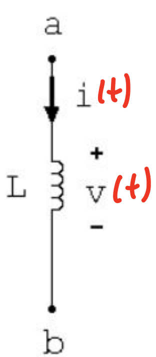
$$v(t) = R \cdot i(t)$$

$$\downarrow \mathcal{L}$$

$$\hat{V} = R \cdot \hat{I}$$

- LTIC circuits-cont

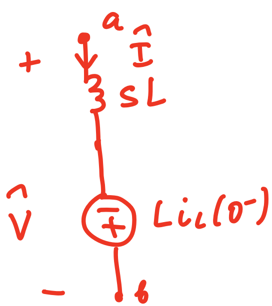
  - Consider an inductor:



$$v(t) = L\frac{di}{dt}$$

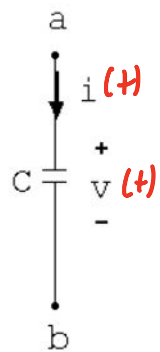
$$\Big\downarrow \mathcal{L}$$

$$\hat{V} = L\left(s\hat{I} - i_L(0^-)\right) = \underbrace{(L\cdot s)}_{\substack{\| \\ Z - \text{s-domain} \\ \text{impedance}}}\cdot\hat{I} - \underbrace{L i_L(0^-)}$$



• LTIC circuits-cont

• Consider a capacitor:

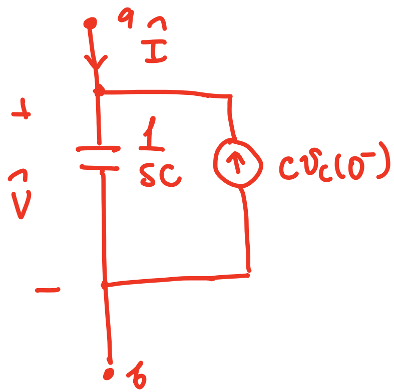


$$i(t) = C\frac{dv}{dt}$$

$$\downarrow \mathcal{L}$$

$$\hat{I} = C\left(s\hat{V} - v_c(0^-)\right) = \underbrace{Cs\hat{V}}_{=\frac{\hat{V}}{Z}} - Cv_c(0^-)$$

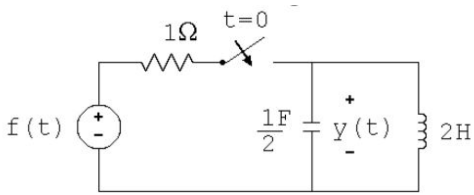
$$\Rightarrow Z = \frac{1}{sC}$$



$$Z = \begin{cases} R & \text{resistor} \\ sL & \text{inductor} \\ \frac{1}{sC} & \text{capacitor} \end{cases}$$

• LTIC circuits in the s-domain - Example # 19

- Consider the following LTIC circuit



where

- $f(t) = e^{3t} V$
- $v_c(0^-) = 1V$
- $i_L(0^-) = 0A$

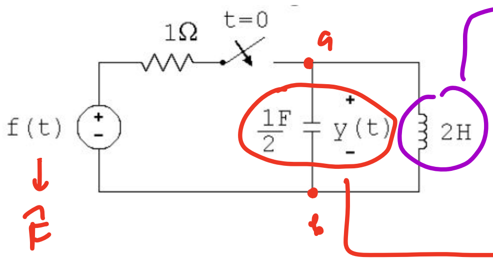
V
- Determine the s-domain equivalent circuit for t > 0.

V
- Determine $\widehat{H}(s)$ and h(t), as well as if the system is BIBO stable.

V
- Determine the characteristic poles and characteristic modes of the system.

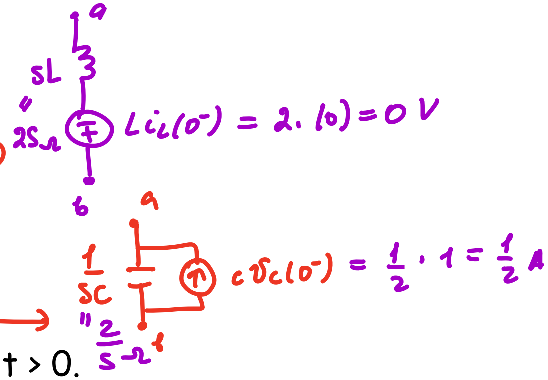
• LTIC circuits in the s-domain - Example # 19-cont

- Consider the following LTIC circuit

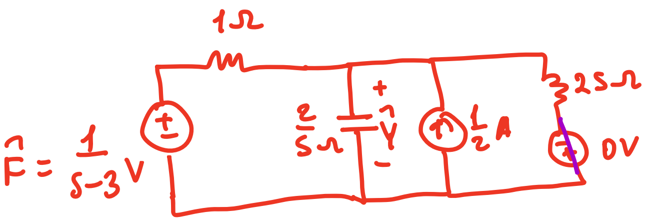


where

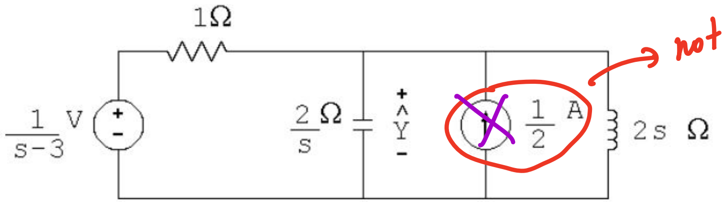
- $f(t) = e^{3t} V$
- $v_c(0^-) = 1V$
- $i_L(0^-) = 0A$



- Determine the s-domain equivalent circuit for t > 0.

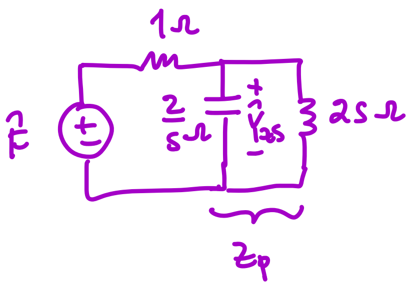


• LTIC circuits in the s-domain - Example # 19-cont



not an input. It comes from initial state.

- Determine $\hat{H}(s)$, as well as if the system is BIBO stable.



$$\hat F \to \boxed{\hat H} \to \hat Y_{zs} = \hat F \cdot \hat H$$

Solve for $\hat Y_{zs}$:

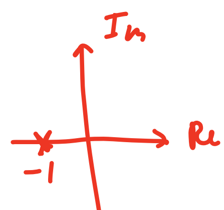
Voltage division:

$$\hat Y_{zs} = \hat F\left(\frac{Z_p}{Z_p+1}\right) =$$

$$Z_p = \frac{\frac{2}{s}(2s)}{\frac{2}{s}+2s} = \frac{2s}{s^2+1}$$

$$= \hat F\left(\frac{\frac{2s}{s^2+1}}{\frac{2s}{s^2+1}+1}\right) = \underbrace{\hat F\left(\frac{2s}{(s+1)^2}\right)}_{\hat H}$$

poles @ $s=-1,-1$



LHP ⇒ BIBO stable

- LTIC circuits in the s-domain - Example # 19-cont

$te^{pt}u(t) \leftrightarrow \frac{1}{(s-p)^2}$

$$\hat{H}(s) = \frac{2s}{(s+1)^2} = \frac{A_1}{s+1} + \frac{A_2}{(s+1)^2}$$

- Determine the characteristic poles and characteristic modes of the system.

$\hat{Y} = \hat{F} \cdot \hat{H} + \frac{I.C.}{\text{char. polyn.}}$

charact. poles : $-1, -1$

charact. modes : e^{-t}, te^{-t}

1 ind + 1 cap $\Rightarrow$ 2^{nd} order ODE

$\Downarrow$

ch. polyn. has a degree of 2

$\Downarrow$

2 ch. poles

• LTIC circuits in the s-domain - Example # 19-cont

$$\widehat{Y_{zs}}(s) = \frac{2s\hat{F}}{(s+1)^2}$$

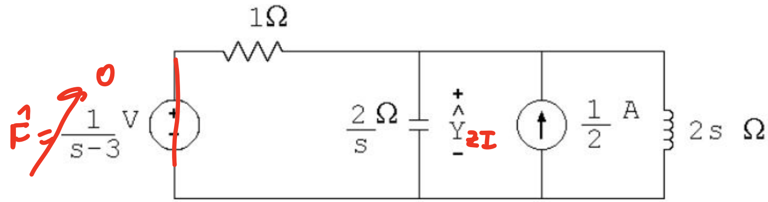
- Recall that $f(t) = e^{3t} \rightarrow \hat{F}(s) = \frac{1}{s-3}$
- Determine $y_{zs}(t)$

$$\hat{Y}_{zs} = \frac{2s}{(s+1)^2} \cdot \frac{1}{(s-3)} = \frac{A_0}{(s-3)} + \frac{A_1}{s+1} + \frac{A_2}{(s+1)^2} = \frac{\frac{6}{16}}{s-3} + \frac{-\frac{6}{16}}{s+1} + \frac{\frac{1}{2}}{(s+1)^2}$$

$$\downarrow \mathcal{L}^{-1}$$

$$y_{zs}(t) = \frac{3}{8} e^{3t} u(t) - \frac{3}{8} e^{-t} u(t) + \frac{1}{2} t e^{-t} u(t) \ V$$

• LTIC circuits in the s-domain - Example # 19-cont



- Determine $y_{ZI}(t)$

$$\frac{1}{Z_p} = \frac{1}{1} + \frac{1}{\frac{2}{s}} + \frac{1}{2s} \Rightarrow Z_p = \frac{2s}{(s+1)^2}$$

$$\hat{Y}_{ZI} = Z_p \cdot \frac{1}{2} = \frac{s}{(s+1)^2}$$

$$y_{ZI}(t) = e^{-t}u(t) - te^{-t}u(t) \quad V$$

- s-domain analysis of LTIC systems - Example # 19-cont
  - Recall that

$$y_{zs}(t) = -\frac{3}{8}e^{-t}u(t) + \frac{1}{2}te^{-t}u(t) + \frac{3}{8}e^{3t}u(t)$$

$$y_{zI}(t) = (1-t)e^{-t}u(t)$$

  - Determine y(t) $= y_{zs}(t) + y_{zI}(t) = \frac{5}{8}e^{-t}u(t) - \frac{1}{2}te^{-t}u(t) + \frac{3}{8}e^{3t}u(t)$ ✓