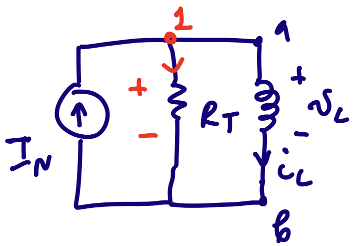
- What about inductors?



KCL @(1): $I_N = \frac{v_L}{R_T} + i_L = $

$$= \frac{L}{R_T}\frac{di_L}{dt} + i_L \quad \Big| \times \frac{R_T}{L}$$

$$\boxed{v_L = L\frac{di_L}{dt}}$$

⇓

in DC steady-state inductor acts like short

$$\boxed{\frac{di_L}{dt} + \underbrace{\frac{R_T}{L}}_{a} i_L = \frac{R_T}{L} I_N}$$

for $t>0$ $\frac{1}{\tau} \Rightarrow \boxed{\tau = \frac{L}{R_T}}$

first order ODE with const. coeff. and const. input

⇓

solution should be in this form:

$$i_L(t) = B + A e^{-at} = B + A e^{-t/\tau} \qquad a = \frac{1}{\tau}$$

$$i_L(\infty) = B$$

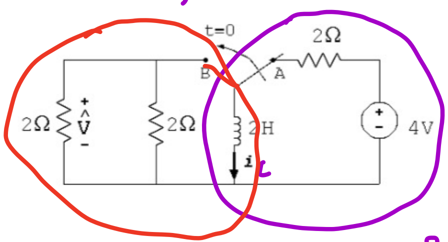
$$i_L(0^+) = i_L(0^-) = B + A$$

• Example #16:

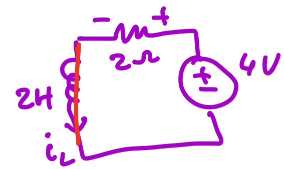
- Consider the circuit below.
- Assume the switch has been in position A for a long time and it switches to position B at time t = 0
- Determine $i_{ZS}(t)$, and $i_{ZI}(t)$

$i_L(t)$ for $t>0$

in DC steady-state ⇒ inductor is a short

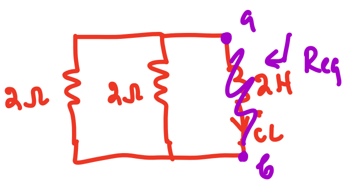


$t<0$



$i_L(0^-) = \frac{4}{2} = 2\text{ A}$

$t>0$



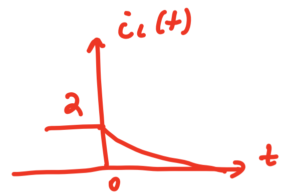
$i_L(t) = \overset{?}{B} + \overset{?}{A}e^{-t/\tau ?}$

$i_L(\infty) = B = 0$ (no source)

$i_L(0^+) = i_L(0^-) = \cancel{B} + A$

Find τ: $R_T = \frac{2\cdot 2}{2+2} = 1\Omega$

$\tau = \frac{L}{R_T} = \frac{2}{1} = 2\text{s}$



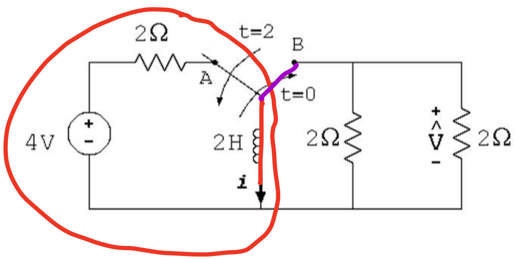
$i_L(t) = i_L(0^-)\,e^{-t/2}$ A

$i_{ZS}(t) = 0$ (force $i_L(0^-)$ to be 0)

$i_{ZI}(t) = i_L(0^-)\,e^{-t/2} = 2e^{-t/2}$ A

• Example #17:

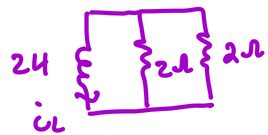
- Consider the circuit below.
- Assume the switch has been in position A for a long time and it switches to position B at time t = 0
- Determine and sketch $i_L(t)$



$t<0$

$i_L(0^-) = \frac{4}{2} = 2A$

$0<t<2$



Find τ:

$\tau = \frac{L}{R_T} = \frac{2}{1} = 2s$

$R_T = \frac{2\cdot 2}{2+2} = 1\Omega$

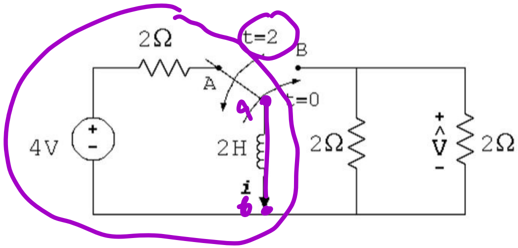
$i_L(t) = B + Ae^{-t/\tau}$

$i_L(\infty) = B = 0$

$i_L(0^+) = i_L(0^-) = A = 2$

$i_L(t) = 2e^{-t/2}$ A

- Example #17-cont:



Find $\hat{\tau}$:

$\hat{\tau} = \frac{L}{R_T} = \frac{2}{2} = 1s$

$R_T = 2\Omega$

$\underline{t>2}$

$i_L(t) = \hat{B} + \hat{A} e^{-t/\hat{\tau}}$

$i_L(\infty) = \hat{B} = \frac{4}{2} = 2A$

? $i_L(2^+) = i_L(2^-) = \hat{B} + \hat{A} e^{-2/\hat{\tau}} =$

$= 2 + \hat{A} e^{-2/\hat{\tau}} = 2 + \hat{A} e^{-2} = 2e^{-1}$

$\Downarrow$

$\hat{A} = (2e^{-1} - 2)e^{2}$

$i_L(t) = 2 + (2e^{-1} - 2) e^{2} \cdot e^{-t} =$

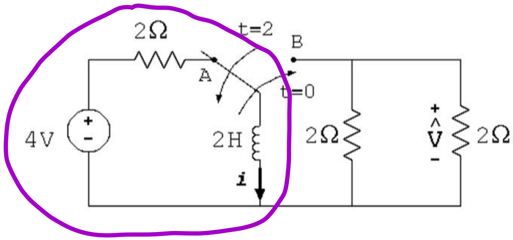
$= 2 + 2(e^{-1} - 1) e^{-(t-2)}$ A

$\underline{0<t<2}$

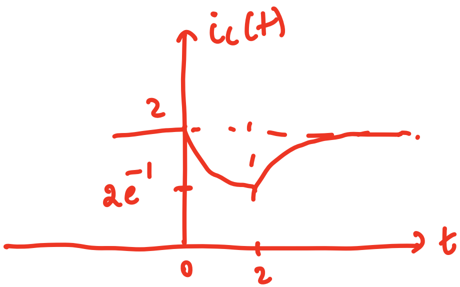
$i_L(t) = 2e^{-t/2} \Rightarrow$

$i_L(2^-) = 2e^{-2/2} = 2e^{-1}$

• Example #17-cont:



$$i_L(t) = \begin{cases} 2\,A & \underline{t<0} \\ 2e^{-t/2}\,A & \underline{0<t<2} \\ \underline{2+2(e^{-1}-1)\,e^{-(t-2)}\,A} & \underline{t>2} \end{cases}$$



Alternative:

t>2

Reference to t=2

$$i_L(t) = \hat{B} + \hat{A}\,e^{-(t-2)} \qquad \hat{\tau} = 1$$

$$i_L(\infty) = \hat{B} = \frac{4}{2} = 2 \quad \text{same}$$

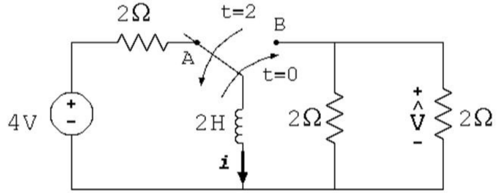
$$i_L(2^+) = \hat{B} + \hat{A} =$$

$$= 2 + \hat{A} = 2e^{-1}$$

$$\hat{A} = 2e^{-1} - 2 = 2(e^{-1}-1) \text{ - different}$$

$$\underline{i_L(t) = 2 + 2(e^{-1}-1)\,e^{-(t-2)}\,A}$$

- Example #17-cont:
- Determine and sketch $i_L(t)$



- Solution to first-order ODE with time-varying sources

- Recall that if the input is constant, i.e. f(t) = K,

$$\underbrace{K}_{\text{constant}} = \frac{dy}{dt} + ay,$$

then $$y(t) = y_p(t) + y_h(t) = \underbrace{B}_{y_p} + \underbrace{Ae^{-at}}_{y_h}$$

y_h solves $0 = \frac{dy_h}{dt} + ay_h$

← input has no effect on hom. solution

- If the input is not constant, then

$$f(t) = \frac{dy}{dt} + ay,$$

is still the first-order ODE with constant coefficients

  - The homogeneous solution is the same no matter what the input is:

$$y_h(t) = Ae^{-at}$$

  - However, the particular solution changes depending on the input

• Solution to first-order ODE with time-varying sources-cont.

$y_h = Ae^{-at}$

$$y_p(t) = \begin{cases} B & \text{if } f(t) = K \text{ - constant} \\ Be^{-pt} & \text{if } f(t) = Ke^{-pt}, \ a \neq p \\ Bte^{-at} & \text{if } f(t) = Ke^{-at} \quad p=a \\ H\cos(\omega t + \psi) & \text{if } f(t) = K\cos(\omega t + \theta) \end{cases}$$

same!