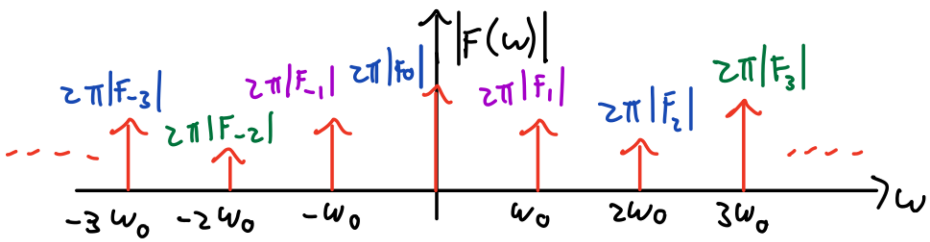
## Lecture 43, Wednesday, April 13, 2022

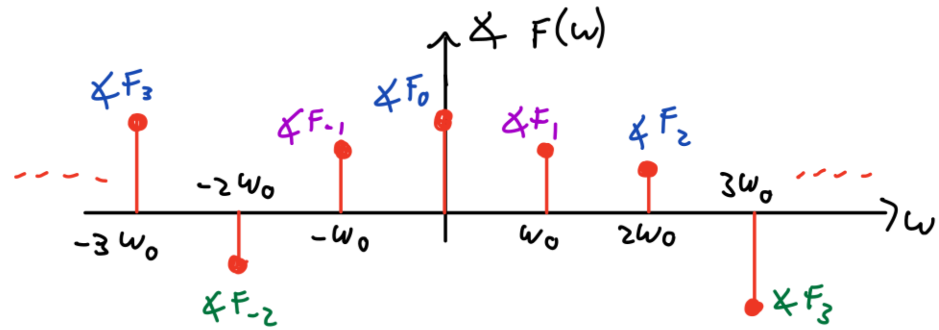
- Multiplication in time corresponds to convolution in frequency:

$$f(t)h(t) \longleftrightarrow \frac{1}{2\pi}F(\omega) * H(\omega)$$

- Fourier transform representation of periodic signals:

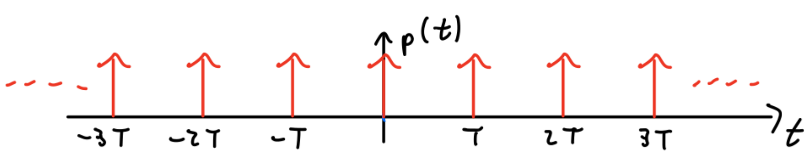
$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \longleftrightarrow F(\omega) = \sum_{n=-\infty}^{\infty} F_n 2\pi\delta(\omega - n\omega_0)$$

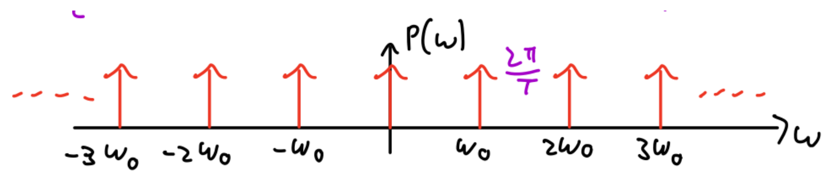



- Impulse train, $p(t)$,

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \longleftrightarrow P(\omega) = \sum_{n=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - n\omega_0)$$

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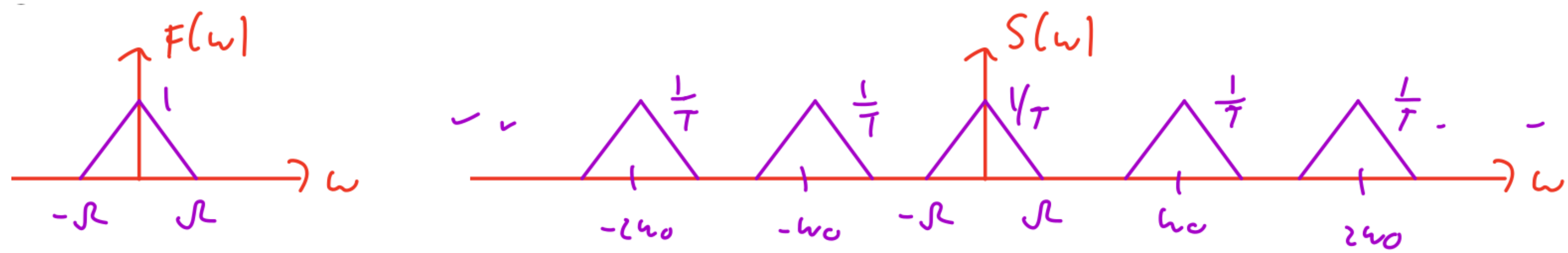
**Lecture 43, continued from previous page...**

- Sampling and reconstruction, assuming $f(t)$ has bandwidth $\Omega$:
  - Sample $f(t)$ by multiplying, $f(t)$ with and impulse train:

$$s(t) = f(t)p(t) \longleftrightarrow S(\omega) = \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\omega_0)$$




Sampling $f(t)$ creates an infinite number of copies of $F(\omega)$, centered at multiples of $\omega_0 = \frac{2\pi}{T}$ and scaled by $\frac{1}{T}$

  - Reconstruct by low-pass filtering with $H(\omega) = T \text{ rect}\left(\frac{\omega}{2\Omega}\right)$

$$Y(\omega) = S(\omega)\left(T\text{rect}\left(\frac{\omega}{2\Omega}\right)\right) = F(\omega) \longleftrightarrow y(t) = \left(\sum_{n=-\infty}^{\infty} f(nT)\delta(t - nT)\right) * \text{sinc}\left(\frac{\pi t}{T}\right)$$

$$y(t) = \sum_{n=-\infty}^{\infty} f(nT)\text{sinc}\left(\frac{\pi}{T}(t - nT)\right) = f(t), \quad \text{reconstruction formula}$$

    * $f(t)$ is reconstructed via an infinite sum of sinc functions centered at multiples of $T$