Lecture 31, Monday, March 21, 2022

- Fourier transform:

$$F(\omega) \; = \; \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \; \in \; \mathbb{C}$$

  – There should be no t variable left after integration.

  – $F(\omega)$ is the projection of $f(t)$ onto $e^{j\omega t}$

- Inverse Fourier transform:

$$f(t) \; = \; \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

  – There should be no ω variable left after integration.

  – $f(t)$ can be represented in terms of time-varying complex exponentials

- There are different definitions of these transforms, but we will use these ones.

- These transforms form a unique transform pair:

$$f(t) \; \leftrightarrow \; F(\omega)$$

- If $f(t)$ is absolutely integrable (A.I.), then its Fourier transform exists.

$$\int_{-\infty}^{\infty} |f(t)| dt \; < \; \infty \quad \rightarrow \quad \text{absolutely integrable (A.I.)}$$

- Some functions that are not A.I. do have a Fourier transform.

continued on next page....

Lecture 31, continued from previous page...

- LTI system response with Fourier transforms:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t}d\omega$$

$$F(\omega) \longrightarrow \boxed{\text{LTI}} \longrightarrow Y(\omega) = H(\omega)F(\omega)$$

$F(\omega)$ acts like the phasor representation of $f(t)$ at frequency ω

- Well-known Fourier transform pair:

$$rect(t) \leftrightarrow sinc\left(\frac{\omega}{2}\right)$$

where

$$rect(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \text{else.} \end{cases} \qquad \text{and} \qquad sinc(x) = \frac{sin(x)}{x}$$