ECE 210/211 HWs HW 9

Student ABB9 CUX4

TOTAL POINTS

32.5 / 40

QUESTION 1

1 0 / 0

✓ **- 0 pts** Correct

QUESTION 2

2 8 / 10

✓ **- 1 pts didn't use rect->sinc formula (not a real value answer)**

✓ **- 1 pts didn't transfer f(t) to a rect function**

QUESTION 3

3 10 / 10

✓ **- 0 pts** Correct

QUESTION 4

4 10 / 10

✓ **- 0 pts** Correct

QUESTION 5

10 pts

5.1 2 / 6

✓ **- 6 pts** $$|F(\omega)|^2 = \frac{A}{2}\Delta(\frac{w+1.5\pi}{\pi}) + A\Delta(\frac{\omega}{2\pi})+\frac{A}{2}\Delta(\frac{w-1.5\pi}{\pi})$$

✓ **+ 3 pts Wrote solution as sum of three triangle functions**

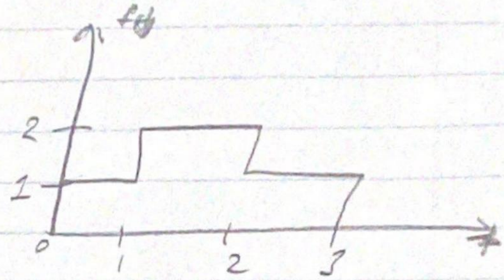
✓ **- 1 pts Answer should be in terms of ω and not t**

5.2 2.5 / 4

✓ **- 1.5 pts Correct Steps, but pass over correct answer**

1. Vareenya Jain

2. $f(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t\leq 1 \\ 2 & 1<t\leq 2 \\ 1 & 2<t\leq 3 \\ 0 & t>3 \end{cases} \rightarrow$



$$\hookrightarrow \int_0^1 f(t)\, e^{-j\omega t}\, dt + \int_1^2 f(t)\, e^{-j\omega t}\, dt + \int_2^3 f(t)\, e^{-j\omega t}\, dt$$

$$\hookrightarrow 1\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^1 + 2\left[\frac{e^{-j\omega t}}{-j\omega}\right]_1^2 + 1\left[\frac{e^{-j\omega t}}{-j\omega}\right]_2^3$$

$$= \frac{e^{-j\omega} - e^{0}}{-j\omega} + 2\left(\frac{e^{-2j\omega} - e^{-j\omega}}{-j\omega}\right) + \frac{e^{-3j\omega} - e^{-2j\omega}}{-j\omega}$$

$$= \frac{e^{0} - e^{-2j\omega} + e^{-j\omega} - e^{-3j\omega}}{j\omega} = \frac{1 + e^{-j\omega} - e^{-2j\omega} - e^{-3j\omega}}{j\omega}$$

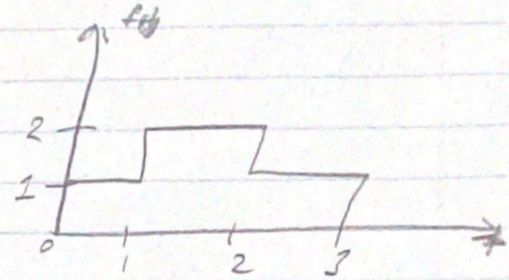
$$\hookrightarrow \boxed{F(\omega) = \frac{1 + e^{-j\omega} - e^{-2j\omega} - e^{-3j\omega}}{j\omega}}$$

1 0 / 0

✓ - 0 pts Correct

1. Varenya Jain

2. $f(t) \begin{cases} 0 & t<0 \\ 1 & 0<t\leq 1 \\ 2 & 1<t\leq 2 \\ 1 & 2<t\leq 3 \\ 0 & t>3 \end{cases} \rightarrow$



$\hookrightarrow \int_0^1 f(t) e^{-j\omega t} dt + \int_1^2 f(t) e^{-j\omega t} dt + \int_2^3 f(t) e^{-j\omega t} dt$

$\hookrightarrow 1\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^1 + 2\left[\frac{e^{-j\omega t}}{-j\omega}\right]_1^2 + 1\left[\frac{e^{-j\omega t}}{-j\omega}\right]_2^3$

$= \frac{e^{-j\omega} - e^0}{-j\omega} + 2\left(\frac{e^{-2j\omega} - e^{-j\omega}}{-j\omega}\right) + \frac{e^{-3j\omega} - e^{-2j\omega}}{-j\omega}$

$= \frac{e^0 - e^{-2j\omega} + e^{-j\omega} - e^{-3j\omega}}{j\omega} = \frac{1 + e^{-j\omega} - e^{-2j\omega} - e^{-3j\omega}}{j\omega}$

$\hookrightarrow \boxed{F(\omega) = \frac{1 + e^{-j\omega} - e^{-2j\omega} - e^{-3j\omega}}{j\omega}}$

2 8 / 10

✓ **- 1 pts** didn't use rect->sinc formula (not a real value answer)

✓ **- 1 pts** didn't transfer f(t) to a rect function

3. $f(t) = e^{-t^2/2\sigma^2} \Longleftrightarrow f(\omega) = \sigma\sqrt{2\pi}\cdot e^{-\sigma^2\omega^2/2}$

$\hookrightarrow \quad f(t) = \frac{3}{\sqrt{2\pi}} e^{-4.5t^2} \Longleftrightarrow F(\omega) = ?$

$\downarrow$

$f(t) = \frac{3}{\sqrt{2\pi}} \cdot e^{-4.5t^2} \rightarrow \frac{1}{2\sigma^2} = 4.5$

$\hookrightarrow \sigma = \sqrt{\frac{1}{9}} = \frac{1}{3}$

$\downarrow$

$F(\omega) = \frac{3}{\sqrt{2\pi}} \cdot (\sigma\sqrt{2\pi}) \cdot (e^{-\frac{\sigma^2\omega^2}{2}})$

$\downarrow$

$\boxed{f(\omega) = e^{\frac{-\omega^2}{18}}}$

3 **10 / 10**

✓ **- 0 pts** Correct

4. $f(t) = \begin{cases} A\sin\left(\frac{2\pi}{T} t\right) & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$

$\hookrightarrow \int_{-T/2}^{T/2} \left(A\sin(\omega_0 t) \cdot e^{-j\omega t}\right) dt = F(\omega)$

$\hookrightarrow \sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j} \rightarrow \sin(\omega_0 t) = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$\hookrightarrow F(\omega) = \int_{-T/2}^{T/2} A\left(\dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right)\left(e^{-j\omega t}\right) dt$

$= \dfrac{A}{2j} \int_{-T/2}^{T/2} \left(e^{jt\left(\frac{2\pi}{T} - \omega\right)} - e^{-jt\left(\frac{2\pi}{T} - \omega\right)}\right) dt$

$$\boxed{\hookrightarrow \frac{Aj}{\frac{2\pi}{T} + \omega} \sin\left(\pi + \frac{\omega T}{2}\right) - \frac{Aj}{\frac{2\pi}{T} - \omega} \sin\left(\pi - \frac{\omega T}{2}\right) = F(\omega)}$$

$\dfrac{A}{2j}\left[\dfrac{e^{j\left(\pi - \frac{\omega T}{2}\right)}}{\frac{2\pi}{T} - \omega} + \dfrac{e^{-j\left(\pi + \omega T/2\right)}}{\frac{2\pi}{T} + \omega} - \dfrac{e^{-j\left(\pi - \frac{\omega T}{2}\right)}}{\frac{2\pi}{T} - \omega} - \dfrac{e^{j\left(\pi + \frac{\omega T}{2}\right)}}{\frac{2\pi}{T} + \omega}\right]$

$Aj\left[\dfrac{-1}{\frac{2\pi}{T} - \omega} \sin\left(\pi - \frac{\omega T}{2}\right) \quad \dfrac{+1}{\frac{2\pi}{T} + \omega} \sin\left(\pi + \frac{\omega T}{2}\right)\right]$

* $f(t) \Leftrightarrow F(\omega)$

$e^{-at} u(t) \Leftrightarrow \dfrac{1}{a + j\omega}, a > 0$

4 **10 / 10**

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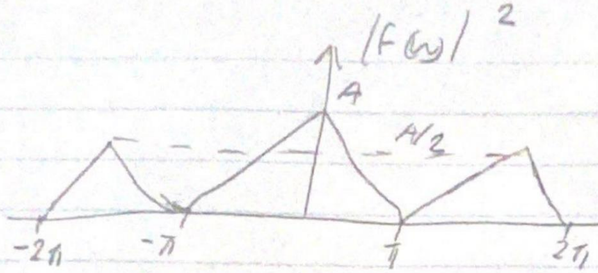
5.



a) $|F(\omega)|^2 = A \cdot \Delta\left(\frac{t}{2\pi}\right) + \frac{A}{2} \cdot \Delta\left(\frac{t}{\pi} + \frac{3\pi}{2}\right) + \frac{A}{2} \cdot \Delta\left(\frac{t}{\pi} - \frac{3\pi}{2}\right)$

$$\boxed{\hookrightarrow |F(\omega)|^2 = A\left(\Delta\left(\frac{t}{2\pi}\right) + \frac{1}{2} \cdot \Delta\left(\frac{t}{\pi} + \frac{3\pi}{2}\right) + \frac{1}{2} \cdot \Delta\left(\frac{t}{\pi} - \frac{3\pi}{2}\right)\right)}$$

↳ sum of 3 rectangle functions ; (p. 231 txtbk) ?

b) 95% bandwidth of this signal?

$\hookrightarrow \frac{W}{2} \cdot 2\pi = \frac{A}{2} \cdot \pi + \frac{\pi}{2} \cdot \frac{A}{2} = A\pi\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4} \cdot A\pi$

$0.95\left(\frac{W}{2} \cdot 2\pi\right) = \int_0^{\Omega_{95}} |F(\omega)|^2 d\omega$

$0.95 \cdot \frac{3}{4} A\pi = \int_0^{3\pi/2} |F(\omega)|^2 d\omega + \int_0^x \left(\frac{A}{2} - \frac{A}{\pi} \cdot \omega\right) d\omega \rightarrow \Omega_{95} = x + \frac{3\pi}{2}$

$0.95 \cdot \frac{3}{4} A\pi = \frac{A}{2} \cdot \pi + \frac{A}{2} \cdot \frac{\pi}{4} + \left[\frac{A}{2} \omega - \frac{A}{2\pi} \cdot \omega^2\right]_0^x$

$0.95 \cdot \frac{3}{4} A\pi = \frac{A}{2} \cdot \pi + \frac{A}{2} \cdot \frac{\pi}{4} + \left(\frac{A}{2}(\Omega_{95}) - \frac{A}{2\pi}(\Omega_{95})^2\right)$

$\frac{1}{2\pi} \cdot (\Omega_{95})^2 - \frac{1}{2}(\Omega_{95}) + \frac{\pi}{2} + \frac{\pi}{8} \quad \frac{3}{4} \cdot 0.95 \cdot \pi = 0$

$\Omega_{95} = \left(\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4\left(\frac{1}{2\pi}\right)\left(\frac{5}{8} - 3 \cdot 0.95/\pi\right)}\right) \frac{1}{\pi}$

$= \pi\left(\frac{1}{2} \pm \left(\frac{1}{4} + \frac{157}{100}\pi^2\right)\right)$

$$\boxed{\hookrightarrow \Omega_{95} \approx 5.988}$$

5.1 2 / 6

✓ **- 6 pts** $$|F(\omega)|^2 = \frac{A}{2}\Delta(\frac{w+1.5\pi}{\pi}) + A\Delta(\frac{\omega}{2\pi})+\frac{A}{2}\Delta(\frac{w-1.5\pi}{\pi})$$

✓ **+ 3 pts** Wrote solution as sum of three triangle functions

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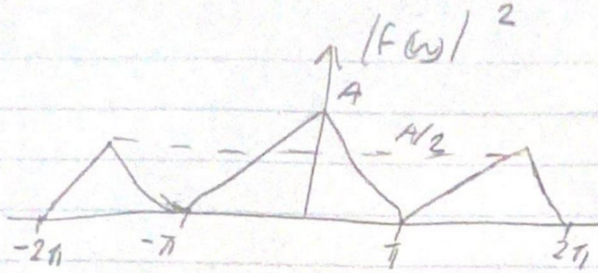

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$\hookrightarrow$ sum of 3 rectangle functions ; (p. 231 txtbk) ?

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$\hookrightarrow \frac{W}{2} \cdot 2\pi = \frac{A}{2} \cdot \pi + \frac{\pi}{2} \cdot \frac{A}{2} = A\pi\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4} \cdot A\pi$

$0.95\left(\frac{W}{2} \cdot 2\pi\right) = \int_0^{\Omega_{95}} |F(\omega)|^2 d\omega$

$0.95 \cdot \frac{3}{4} A\pi = \int_0^{3\pi/2} |F(\omega)|^2 d\omega + \int_0^{x}\left(\frac{A}{2} - \frac{A}{\pi} \cdot \omega\right) d\omega \rightarrow \Omega_{95} = x + \frac{3\pi}{2}$

$0.95 \cdot \frac{3}{4} A\pi = \frac{A}{2} \cdot \pi + \frac{A}{2} \cdot \frac{\pi}{4} + \left[\frac{A}{2} \omega - \frac{A}{2\pi} \cdot \omega^2\right]_0^x$

$0.95 \cdot \frac{3}{4} A\pi = \frac{A}{2} \cdot \pi + \frac{A}{2} \cdot \frac{\pi}{4} + \left(\frac{A}{2}(\Omega_{95}) - \frac{A}{2\pi}(\Omega_{95})^2\right)$

$\frac{1}{2\pi} \cdot (\Omega_{95})^2 - \frac{1}{2}(\Omega_{95}) + \frac{\pi}{2} + \frac{\pi}{8} - \frac{3}{4} \cdot 0.95 \cdot \pi = 0$

$\Omega_{95} = \left(\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4\left(\frac{1}{2\pi}\right)\left(\frac{5}{8} - 3 \cdot 0.95/\pi\right)}\right) \frac{1}{\pi}$

$= \pi\left(\frac{1}{2} \pm \left(\frac{1}{4} + \frac{157}{100}\pi^2\right)\right)$

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5.2 **2.5 / 4**

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