

Inverse Laplace transform

In mathematics, the **inverse Laplace transform** of a function $F(s)$ is the piecewise-continuous and exponentially-restricted real function $f(t)$ which has the property:

$$\mathcal{L}\{f\}(s) = \mathcal{L}\{f(t)\}(s) = F(s),$$

where \mathcal{L} denotes the Laplace transform.

It can be proven that, if a function $F(s)$ has the inverse Laplace transform $f(t)$, then $f(t)$ is uniquely determined (considering functions which differ from each other only on a point set having Lebesgue measure zero as the same). This result was first proven by Mathias Lerch in 1903 and is known as Lerch's theorem.^{[1][2]}

The Laplace transform and the inverse Laplace transform together have a number of properties that make them useful for analysing linear dynamical systems.

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Mellin's inverse formula

An integral formula for the inverse Laplace transform, called the *Mellin's inverse formula*, the *Bromwich integral*, or the *Fourier–Mellin integral*, is given by the line integral:

$$f(t) = \mathcal{L}^{-1}\{F(s)\}(t) = \frac{1}{2\pi i} \lim_{T \rightarrow \infty} \int_{\gamma-iT}^{\gamma+iT} e^{st} F(s) \, ds$$

where the integration is done along the vertical line $\operatorname{Re}(s) = \gamma$ in the complex plane such that γ is greater than the real part of all singularities of $F(s)$ and $F(s)$ is bounded on the line, for example if the contour path is in the region of convergence. If all singularities are in the left half-plane, or $F(s)$ is an entire function , then γ can be set to zero and the above inverse integral formula becomes identical to the inverse Fourier transform.

In practice, computing the complex integral can be done by using the Cauchy residue theorem.

Post's inversion formula

Post's inversion formula for Laplace transforms, named after Emil Post,^[3] is a simple-looking but usually impractical formula for evaluating an inverse Laplace transform.

The statement of the formula is as follows: Let $f(t)$ be a continuous function on the interval $[0, \infty)$ of exponential order, i.e.

$$\sup_{t>0} \frac{f(t)}{e^{bt}} < \infty$$

for some real number b . Then for all $s > b$, the Laplace transform for $f(t)$ exists and is infinitely differentiable with respect to s . Furthermore, if $F(s)$ is the Laplace transform of $f(t)$, then the inverse Laplace transform of $F(s)$ is given by

$$f(t) = \mathcal{L}^{-1}\{F\}(t) = \lim_{k \rightarrow \infty} \frac{(-1)^k}{k!} \left(\frac{k}{t}\right)^{k+1} F^{(k)}\left(\frac{k}{t}\right)$$

for $t > 0$, where $F^{(k)}$ is the k -th derivative of F with respect to s .

As can be seen from the formula, the need to evaluate derivatives of arbitrarily high orders renders this formula impractical for most purposes.

With the advent of powerful personal computers, the main efforts to use this formula have come from dealing with approximations or asymptotic analysis of the Inverse Laplace transform, using the Grunwald–Letnikov differintegral to evaluate the derivatives.

Post's inversion has attracted interest due to the improvement in computational science and the fact that it is not necessary to know where the poles of $F(s)$ lie, which make it possible to calculate the asymptotic behaviour for big x using inverse Mellin transforms for several arithmetical functions related to the Riemann hypothesis.

Software tools

- InverseLaplaceTransform (<http://reference.wolfram.com/mathematica/ref/InverseLaplaceTransform.html>) performs symbolic inverse transforms in Mathematica
- Numerical Inversion of Laplace Transform with Multiple Precision Using the Complex Domain (<http://library.wolfram.com/infocenter/MathSource/5026/>) in Mathematica gives numerical solutions^[4]
- ilaplace (<http://www.mathworks.co.uk/help/symbolic/ilaplace.html>) performs symbolic inverse transforms in MATLAB
- Numerical Inversion of Laplace Transforms in Matlab (<http://www.mathworks.co.uk/matlabcentral/fileexchange/32824-numerical-inversion-of-laplace-transforms-in-matlab>)
- Numerical Inversion of Laplace Transforms based on concentrated matrix-exponential functions (<https://www.mathworks.com/matlabcentral/fileexchange/71511-a-cme-based-numerical-inverse-laplace-transformation-method>) in Matlab

See also

- Inverse Fourier transform

- [Poisson summation formula](#)

References

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2. Lerch, M. (1903). "Sur un point de la théorie des fonctions génératrices d'Abel" (https://doi.org/10.1007%2FBF02421315). *Acta Mathematica*. **27**: 339. doi:10.1007/BF02421315 (https://doi.org/10.1007%2FBF02421315).
3. Post, Emil L. (1930). "Generalized differentiation" (https://doi.org/10.1090%2FS0002-9947-1930-1501560-X). *Transactions of the American Mathematical Society*. **32** (4): 723–723. doi:10.1090/S0002-9947-1930-1501560-X (https://doi.org/10.1090%2FS0002-9947-1930-1501560-X). ISSN 0002-9947 (https://www.worldcat.org/issn/0002-9947).
4. Abate, J.; Valkó, P. P. (2004). "Multi-precision Laplace transform inversion". *International Journal for Numerical Methods in Engineering*. **60** (5): 979. doi:10.1002/nme.995 (https://doi.org/10.1002%2Fnme.995).

Further reading

- Davies, B. J. (2002), *Integral transforms and their applications* (3rd ed.), Berlin, New York: Springer-Verlag, ISBN 978-0-387-95314-4
- Manzhirrov, A. V.; Polyanin, Andrei D. (1998), *Handbook of integral equations*, London: CRC Press, ISBN 978-0-8493-2876-3
- Boas, Mary (1983), *Mathematical Methods in the physical sciences* (https://archive.org/details/mathematicalmeth00boas/page/662), John Wiley & Sons, p. 662 (https://archive.org/details/mathematicalmeth00boas/page/662), ISBN 0-471-04409-1 (p. 662 or search Index for "Bromwich Integral", a nice explanation showing the connection to the Fourier transform)
- Widder, D. V. (1946), *The Laplace Transform*, Princeton University Press
- Elementary inversion of the Laplace transform (http://www.rose-hulman.edu/~bryan/invlap.pdf). Bryan, Kurt. Accessed June 14, 2006.

External links

- [Tables of Integral Transforms](http://eqworld.ipmnet.ru/en/auxiliary/aux-inttrans.htm) (http://eqworld.ipmnet.ru/en/auxiliary/aux-inttrans.htm) at EqWorld: The World of Mathematical Equations.

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