ECE 210 (AL2) - ECE 211 (E)

Chapter 5

Frequency Response $H(\omega)$ of LTI Systems

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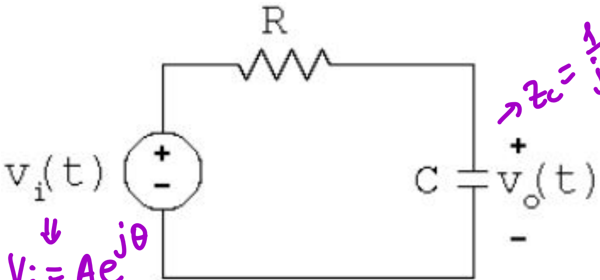
Chapter objectives

- Understand the meaning and application of an LTI system's frequency response
- Be able to obtain the frequency response of an LTI system
- Know the properties of the frequency response of an LTI system
- Use the frequency response of an LTI system obtain the system's response to co-sinusoidal inputs
- Use the frequency response of an LTI system obtain the system's response to multifrequency co-sinusoidal inputs

• Example #1



$\Downarrow$

$V_i = Ae^{j\theta}$

$\to Z_c = \frac{1}{j\omega c} = \frac{1}{j2c}\,\Omega$

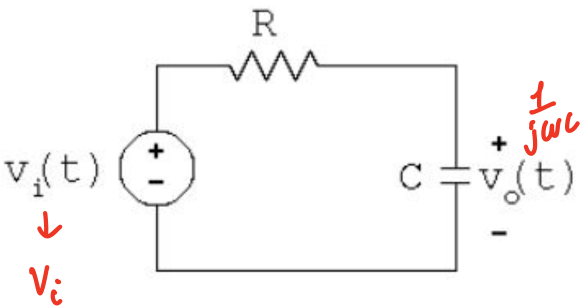
- Let $v_i(t) = A\cos(2t + \theta)$
- Determine $v_{o_{ss}}(t)$

$$V_o = V_i \frac{\frac{1}{j2C}}{R + \frac{1}{j2C}} = V_i \frac{1}{j2RC+1} = \frac{|V_i|\, e^{j\angle V_i}}{\sqrt{1^2 + (2RC)^2}\, e^{j\tan^{-1}\left(\frac{2RC}{1}\right)}}$$

$$V_o = \frac{\overbrace{|V_i|}^{A}}{\sqrt{1 + 4R^2C^2}}\, e^{j\left(\underbrace{\angle V_i}_{\theta} - \tan^{-1}(2RC)\right)}$$

$$v_o(t) = \frac{A}{\sqrt{1 + 4R^2C^2}} \cos\left(2t + \theta - \tan^{-1}(2RC)\right) \text{ V}$$

- Example #1



$$V_o = \frac{V_i \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_i \frac{1}{j\omega RC + 1} = V_o$$

$H(\omega)$ frequency response

- What if we change the frequency of the input?

$$V_o = \frac{|V_i|}{\sqrt{1+\omega^2 R^2 C^2}} e^{j(\measuredangle V_i - \tan^{-1}(\omega RC))}$$

$$v_{o_{ss}}(t) = |V_i| \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\omega t + \measuredangle V_i - \tan^{-1}(\omega RC))\ V$$

amplitude response $|H(\omega)|$

phase response $\measuredangle H(\omega)$

$$H(\omega) = |H(\omega)| e^{j\measuredangle H(\omega)}$$

- Frequency response of LTI systems

output phasor

$$V_o = V_i H(\omega)$$

input phasor

- Frequency response $H(\omega)$

$$V_i = |V_i| e^{j(\angle V_i)}$$

$$V_o = |V_i||H(\omega)| e^{j(\angle V_i + \angle H(\omega))}$$

real signal $\rightarrow \boxed{LTI} \rightarrow$ real signal

$|V_i|$ $\cancel{H(\omega)}$ $\cos(\ldots\ )$

$$v_i(t) = |V_i| \cos(\omega t + \angle V_i)$$

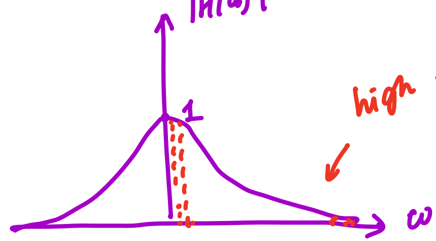
$$v_o(t) = |V_i||H(\omega)| \cos(\omega t + \angle V_i + \angle H(\omega))$$

- Frequency response of LTI systems-cont

$$v_o(t) = |V_i||H(\omega)|\cos(\omega t + \angle V_i + \angle H(\omega))$$

- Amplitude response $|H(\omega)|$ $= \dfrac{1}{\sqrt{1+\omega^2R^2C^2}}$

$H(\omega) = \dfrac{1}{1+j\omega RC}$



high frequencies are very attenuated

low frequencies pass through almost untouched

$\Downarrow$ low-pass filter

- Phase response $\angle H(\omega)$

$\dfrac{e^{j0}}{e^{j\tan^{-1}(\omega RC)}}$

$\angle H(\omega) = -\tan^{-1}(\omega RC)$