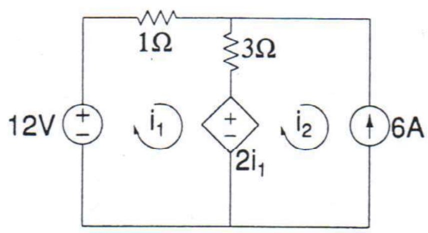
2. (25 pts) The two parts in this problem are unrelated.

(a) Find the loop currents i_1 and i_2 in the following circuit.



$i_1 =$ $-1A$

$i_2 =$ $-6A$

1st loop: $12 - 1 \cdot i_1 - 3(i_1 - i_2) - 2i_1 = 0$

$12 - 6i_1 + 3i_2 = 0$

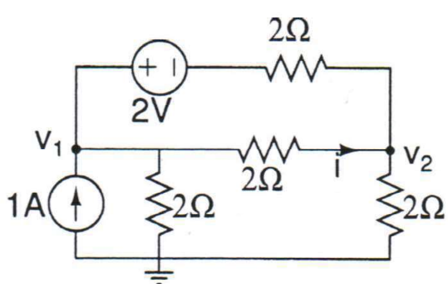
$6i_1 - 3i_2 = 12$

$6i_1 - 3 \cdot -6 = 12$

$6i_1 = 12 - 18$

$i_1 = -\frac{6}{6} = -1A$

(b) Find the node voltages v_1, v_2 and the current i in the following circuit.



$v_1 =$ $8/5\ V$

$v_2 =$ $2/5\ V$

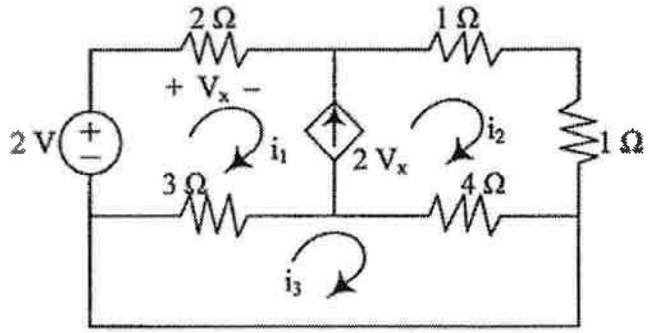
$i =$ $3/5\ A$

1st node: $\frac{V_1}{2} + \frac{V_1 - V_2}{2} + \frac{V_1 - 2 - V_2}{2} = 1 \rightarrow \frac{3V_1}{2} - V_2 = 2$

2nd node: $\frac{V_2}{2} + \frac{V_2 - V_1}{2} + \frac{V_2 - (V_1 - 2)}{2} = 0 \rightarrow \frac{3V_2}{2} - V_1 = -1$

$i = \frac{V_1 - V_2}{2} = \frac{8/5 - 2/5}{2} = \frac{6/5}{2} = 3/5 A$

4. (15 points) For the following circuit:

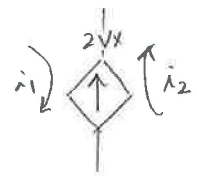


(a) (2 pts) What is V_x in terms of loop current i_1 ?

via Ohm's Law: $V_x = 2i_1$

V_x= $2i_1$

(b) (3 pts) What is loop current i_2 in terms of loop current i_1 ?



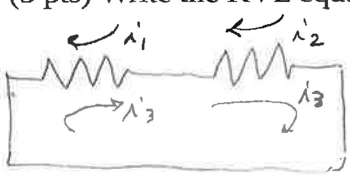
$2V_x = i_2 - i_1$ via part (a)

$= 2(2i_1)$

$\Rightarrow i_2 = 4i_1 + i_1 = 5i_1$

i_2= $5i_1$

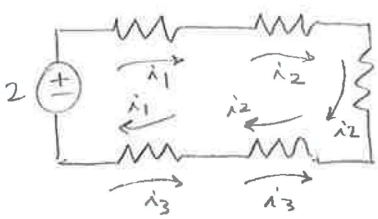
(c) (5 pts) Write the KVL equation for loop 3 in terms of i_1, i_2 and i_3.



equation: $3(i_3 - i_1) + 4(i_3 - i_2) = 0$

(d) (5 pts) Write one additional KVL equation for the circuit in terms of i_1, i_2 and i_3, which makes it possible to solve for these three variables. You are not required to solve the system.

superloop spanning current source:

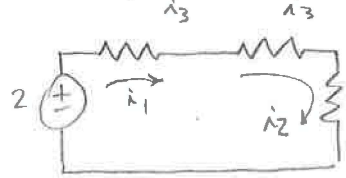


$2 = 2i_1 + i_2 + i_2 + 4(i_2 - i_3) + 3(i_1 - i_3)$

$\boxed{2 = 5i_1 + 6i_2 - 7i_3}$

- or -

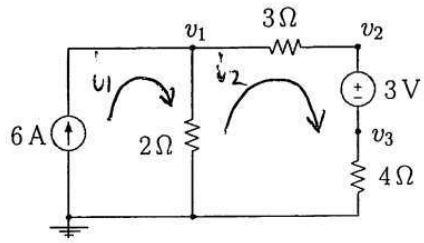
outer current loop:



$\boxed{2 = 2i_1 + 2i_2}$

equation: $2 = 5i_1 + 6i_2 - 7i_3$ - or - $2 = 2i_1 + 2i_2$

(a) (10pt) Find v_1 , v_2 , v_3 .



Loop method

① $i_1 = 6$

② $2(i_2 - i_1)$ (in terms of voltage drops) $+ 3 i_2 + 3 + 4 i_2 = 0$

$i_2\ (2+3+4) = 2 \cdot i_1 - 3 = 12 - 3 = 9$

$i_2 = \frac{9}{9} = 1$

So

$v_3 = 4 \cdot i_2 = 4$

$v_2 = v_3 + 3 = 7$

$v_1 = v_2 + 3 i_3 = 7 + 3 = 10$

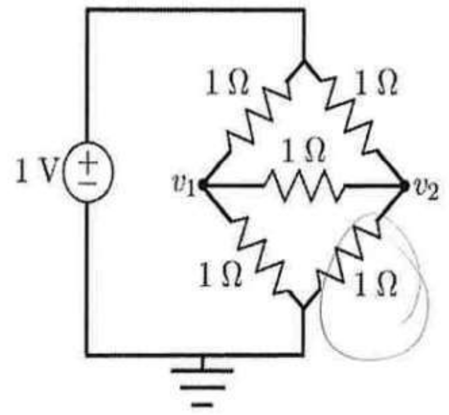
Node method

@ v_1 : $6 = \frac{v_1}{2} + \frac{v_1 - v_2}{3}$: $5v_1 - 2v_2 = 36$

@ v_2 : $\frac{v_1 - v_2}{3} = \frac{v_3}{4}$: $4v_1 - 7v_2 = -9$

@ v_3 : $v_3 = v_2 - 3$

3. (10 pts) Write, *but do not solve*, two node equations in order to solve for v_1 and v_2. Fill in your final answer in the space provided, but be sure to show your work.

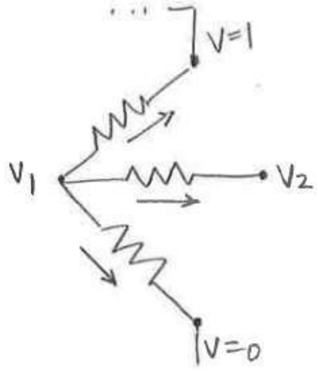


Answer:

(3)v_1 + (−1)v_2 = (1)

(−1)v_1 + (3)v_2 = (1)

@ V_1 :

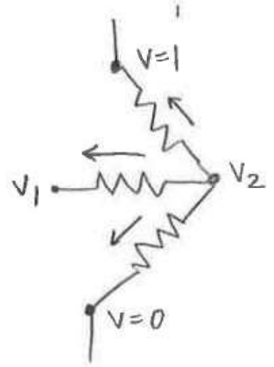


$$\frac{(V_1-1)}{1}+\frac{(V_1-V_2)}{1}+\frac{(V_1-0)}{1}=0$$

so $V_1-1+V_1-V_2+V_1=0$

$\Rightarrow 3V_1-V_2=1$

@ V_2 :

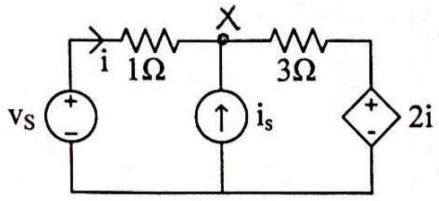


$$\frac{(V_2-1)}{1}+\frac{(V_2-V_1)}{1}+\frac{(V_2-0)}{1}=0$$

so $V_2-1+V_2-V_1+V_2=0$

$\Rightarrow 3V_2-V_1=1$

(a) The current i in the following circuit can be expressed as $i = K_1 v_S + K_2 i_S$. Find the values of K_1 and K_2 if v_S=12V, i_S= 6A.



KCL: $\frac{x-2i}{3} - i_s + \frac{x-v_s}{1} = 0 \rightarrow$

$\frac{v_s - x}{1} = i \rightarrow \quad v_s - i = x$

$\frac{v_s - i - 2i}{3} - i_s + \cancel{v_s} - i - \cancel{v_s} = 0$

$\frac{v_s}{3} - i - i_s - i = 0$

$\frac{v_s}{3} - i_s = 2i \rightarrow \quad i = \frac{1}{6} v_s - \frac{1}{2} i_s$

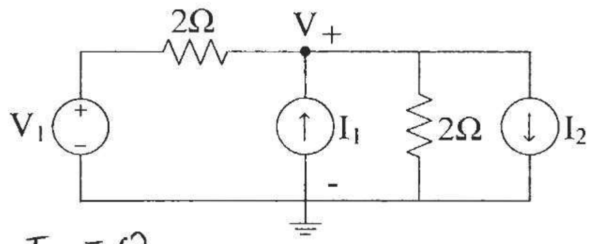
$K_1 =$ 1/6

$K_2 =$ $-\frac{1}{2}$

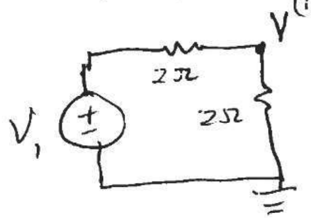
(a) In the following circuit, using the principle of linearity and superposition, we can write the node voltage V as

$$V = AV_1 + BI_1 + CI_2$$

Use source suppression to find the constants A, B, and C.



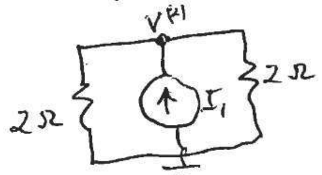
$I_1 = I_2 = 0$



$V^{(1)} = V_1/2$

so $A = 1/2$

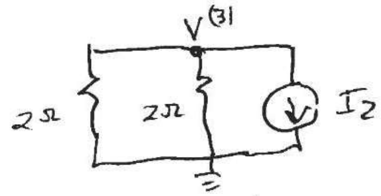
$V_1 = I_2 = 0$



$V^{(2)} = (I_1/2)\, 2\Omega = I_1$

so $B = 1\Omega$

$V_1 = I_1 = 0$



$V^{(3)} = (-I_2/2)\, 2\Omega = -I_2$

so $C = -1\Omega$

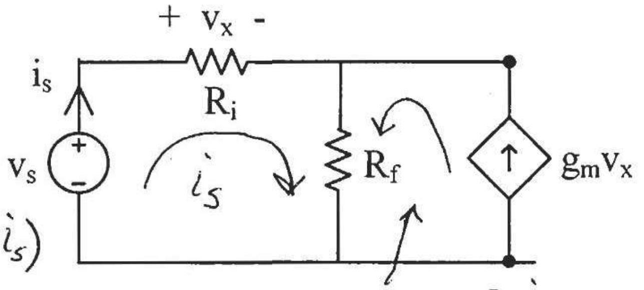
A = $1/2$

B = 1Ω

C = -1Ω

(a) Use the loop method to find the current i_s. Assume that v_s, R_i, R_f, and g_m are known.

(/10)



$g_m v_x = g_m R_i i_s$

KVL: $-v_s + R_i i_s + R_f(i_s + g_m R_i i_s) = 0$

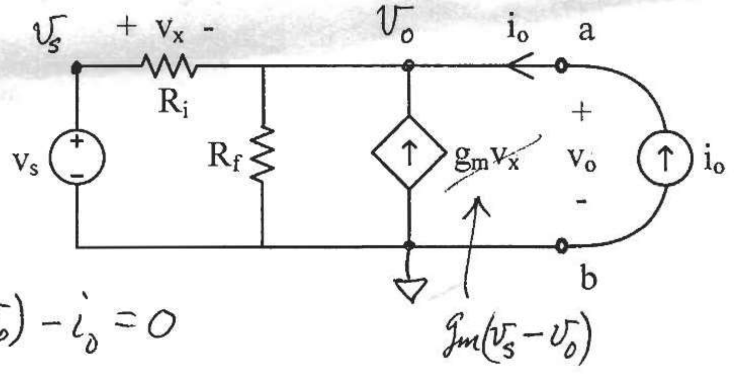
$$v_s = (R_i + R_f + g_m R_i R_f)\, i_s$$

$$i_s = \frac{v_s}{R_i + R_f + g_m R_i R_f}$$

Note: equivalent input resistance

(15)

(b) Use the node method to find v_o as a function of the independent sources v_s and i_o. At terminals a-b what is the equivalent resistance and the open circuit voltage.



$$\frac{V_o - V_s}{R_i} + \frac{V_o}{R_f} - g_m(V_s - V_o) - i_o = 0$$

becomes

$$\left(\frac{1}{R_i} + \frac{1}{R_f} + g_m\right)V_o = i_o + \frac{V_s}{R_i} + g_m V_s$$

so

$$V_o = \underbrace{\frac{1}{\frac{1}{R_i} + \frac{1}{R_f} + g_m}}_{R_T} \cdot i_o + \underbrace{\left(\frac{1 + g_m R_i}{1 + \frac{R_i}{R_f} + g_m R_i}\right) V_s}_{V_T}$$