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ZS linear ?, T.I?, Causal?

[?] 2. a) $y(t) = f(t-t_0) + f(t+t_0)$, $t_0 = 3s$

i) $\boxed{\text{linear}}$ ("$f(t)$" has a degree of 1) ?

? ii) $\boxed{\text{Time invariant}}$ - introducing a delay affects all terms, in the same way that delaying the output would mean a delay in all terms)

iii) $\boxed{\text{Non-causal}}$ - the same "t" is not present in all terms?

b) $y(t) = f(t) * u(t)$

i) $\boxed{\text{Linear}}$ - "$f(t)$" has a degree of 1 ?

ii) $\boxed{\text{Time Invariant}}$ - introducing a delay affects all terms similarly ?

iii) Causal - the same "t" is present in all terms ?

c) $y(t) = \int_{-\infty}^{t/2} f(\tau)\,d\tau$

i) Linear ii) Not Time Invariant iii) Non - Causal

d) $y(t) = f(t-2) * f(t) + \int_{-\infty}^{t-2} f(\tau)\,d\tau$

i) Linear - unique input/outputs

output shift reflects input shift → ii) Time Invariant

only depends on past outputs → iii) Causal

e) $y(t) \Leftrightarrow Y(\omega)$, $f(t) \Leftrightarrow F(\omega)$,
where $Y(\omega) = F(\omega)e^{j\omega t_0}$, i) Linear ii) Time Invariant iii) Not causal

unique inputs/outputs
output shift reflects input shifts
dependent on input values

②

3. Impulse responses $h(t)$ for LTI systems w/ the following unit-step responses

a) $g(t) = K u(t-t_0)$, where $t_0 > 0$ and K is a real-valued constant

unit-step response

$$y(t) = f(t) * h(t) \quad , \quad \frac{dy(t)}{dt} = \frac{d}{dt} f(t) * h(t) = f(t) * \frac{d}{dt} u(t)$$

$$\frac{dy(t)}{dt} = \left(\frac{d}{dt} u(t)\right) * h(t) = \delta(t) * h(t) = h(t)$$

$$\hookrightarrow \frac{dy(t)}{dt} = h(t)$$

$$\hookrightarrow \frac{d}{dt} g(t) = \frac{d}{dt}(K u(t-t_0)) = K\left(\frac{d}{dt} u(t-t_0)\right) = K \cdot \delta(t-t_0)$$

$$\boxed{\hookrightarrow h(t) = K \delta(t-t_0)}$$

b) $g(t) = t^2 u(t) - t^2 u(t-1)$

$$\frac{d}{dt}(g(t)) = \frac{d}{dt}[t^2 u(t) - t^2 u(t-1)] = \frac{d}{dt}[t^2 u(t)] - \frac{d}{dt}[t^2 \cdot u(t-1)]$$

$$h(t) = [2t u(t) + t^2 \delta(t)] - [2t \cdot u(t-1) + t^2 \delta(t-1)]$$

$\hookrightarrow$ sampling $\rightarrow f(t)\delta(t) = f(0)\delta(t) \rightarrow f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$

$\hookrightarrow t^2 \delta(t) \rightarrow \delta(t)$ @ $t=0 \Rightarrow (0)^2 \delta(t) = 0$

$\hookrightarrow t^2 \delta(t-1) \rightarrow \delta(t)$ @ $t-1=0 \rightarrow t=1 \Rightarrow (1)^2 \delta(t-1) = \delta(t-1)$

$$\boxed{\hookrightarrow h(t) = 2t u(t) - 2t u(t-1) - \delta(t-1)}$$

3c) $g(t) = (3 - e^{-2t})\,u(t-4)$

$\frac{d}{dt}g(t) = h(t) = (3 - e^{-2t})\,\delta(t-4) + u(t-4)(2e^{-2t})$

$t - 4 = 0 \rightarrow t = 4$

$h(t) = (3 - e^{-2(4)})\,\delta(t-4) + u(t-4)\,2e^{-2t}$

$\boxed{\hookrightarrow h(t) = 2u(t-4)e^{-2t} + (3 - e^{-8})\,\delta(t-4)}$

4. Laplace transform $\hat{F}(s)$, poles of $\hat{F}(s)$, ROC of $\hat{F}(s)$

a) $f(t) = 3u(t) - 2u(t-3)$

$\geq$ Table 7.8) $u(t) \Leftrightarrow \frac{1}{s}$

i) $\hat{F}(s) = \frac{3}{s} - \frac{2e^{-3s}}{s}$

ii) $\hat{F}(s)$ undefined at $s = 0$

iii) Region of Convergence at $\sigma > 0$

b) $f(t) = te^{3(t-2)}u(t)$

Table 7.3) $te^{pt}u(t) \Leftrightarrow \frac{1}{(s-p)^2}$

$= te^{(3t-6)}u(t) = \frac{t \cdot e^{3t}u(t)}{e^6}$

$= \frac{1}{e^6}\left(te^{3t}u(t)\right)$

$\hat{F}(s) = \frac{1}{e^6}\left(\frac{1}{(s-3)^2}\right)$

i) $\hat{F}(s) = \frac{1}{e^6}\left(\frac{1}{(s-3)^2}\right)$

ii) pole at $s = 3$, iii ROC: $\sigma > 3$

④

4c) $f(t) = (t-2)e^{-2t} + \delta(t)$

$= te^{-2t} - 2e^{-2t} + \delta(t) \rightarrow \hat{F}(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$

$= \hat{F}(s) = \int_{0^-}^{\infty} te^{-t(2+s)}\, dt - 2\int_{0}^{\infty} e^{-t(2+s)} + 1$

$$= \frac{-te^{-t(2+s)}}{(2+s)} + \int_{0^-}^{\infty}\left(\frac{e^{-t(2+s)}}{2+s} + \frac{2e^{-t(2+s)}}{2+s}\right) dt$$

$$= \left[\frac{-te^{-2(2+s)}}{(2+s)} - \frac{e^{-t(2+s)}}{(2+s)^2} + \frac{2e^{-t(2+s)}}{(2+s)}\right]_{0^-}^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-te^{-2(s+2)}}{(2+s)} - \frac{e^{-t(2+s)}}{(2+s)^2} + \frac{2e^{-t(2+s)}}{(2+s)} + \frac{1}{(2+s)^2} - \frac{2}{(2+s)}\right) + 1$$

$\hookrightarrow$ i) $\hat{F}(s) = \dfrac{1}{(2+s)^2} - \dfrac{2}{(2+s)} + 1$

ii) Pole at $s = -2$, iii) ROC @ $\sigma > -2$

4d) $f(t) = e^{4t}\cos(t)\,u(t+2)$

$\downarrow$

$$\hat{F}(s) = \frac{(s-4)}{(s-4)^2 + 1}$$

$$= \frac{s-4}{s^2 - 8s + 16 + 1} = \frac{s-4}{s^2 - 8s + 17}$$

$\hookrightarrow s^2 - 8s + 17 = 0$

$$\frac{8 \pm \sqrt{64 - 68}}{2} = \frac{8 \pm 2j}{2}$$

$= 4 \pm j$

i) $\hat{F}(s) = \dfrac{s-4}{s^2 - 8s + 17}$

ii) Poles at $s = 4 \pm j$

iii) ROC: $\sigma > 4$

5. Is the LTIC system BIBO stable

a) $\hat{H}(s) = \frac{s^4+1}{(s+3)(s+2)}$ → poles at $s = -3, 2$

$\hookrightarrow \lim_{s\to\infty} \hat{H}(s) = \frac{4s^3}{2s} = \frac{12s^2}{2} = 6s^2 = \underline{\infty}$

This LTIC system is not BIBO stable because there is a pole at $s=\infty$

b) $\hat{H}(s) = \frac{1}{s^2+25}$ → $s^2+25=0$ → pole at $s = \pm 5$

This LTIC system is BIBO unstable since there is a pole to the right of the plane

c) $\hat{H}(s) = \frac{(s^2+4s+6)}{(s+1+j6)(s+1-j6)}$ → $s = -1-j6, -1+j6$

This LTIC system is BIBO stable since there are poles at negative sigma values.

d) $\hat{H}(s) = \frac{2(s-3)}{(s^2-9)} = \frac{2s(s-3)}{(s-3)(s+3)} = \frac{2}{s+3}$ → pole at $s=-3$

This system is BIBO stable since there are poles at negative sigma values.