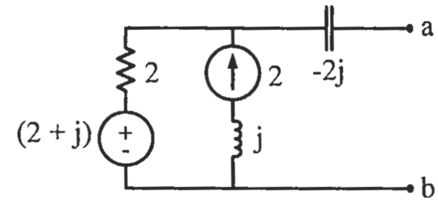
Problem 1 (6 *points each*)

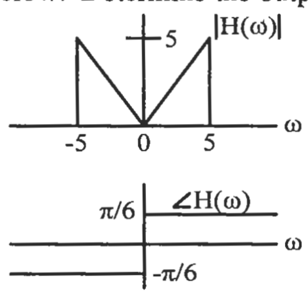
(**a**) Determine the impedance Z_L of a load that is matched to the following circuit at terminals a and b, and determine the net power absorbed by the matched load.



$Z_L =$ ______________

$P =$ ______________

(**b**) The signal $f(t) = 3 + \cos\left(2t + \frac{\pi}{6}\right) + 5\cos\left(6t + \frac{\pi}{3}\right)$ is the input to an LTI system described by the frequency response shown below. Determine the output.



$y(t) =$ ______________

(**c**) For an LTI system if the input $f(t) = \cos 2t$, the steady state response is $y(t) = \sqrt{2}\cos\left(2t + \frac{\pi}{4}\right)$. For this system if $f(t) = \sqrt{2}\cos\left(2t + \frac{\pi}{4}\right)$, what is the steady state response $y(t)$?

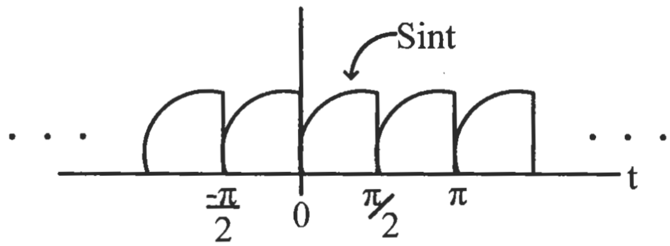
$y(t) =$ ______________

(**d**) A signal $f(t)$ is given as $f(t) = e^{j\frac{\pi}{4}} e^{-j\frac{2}{3}t} - je^{-j\frac{t}{2}} + e^{-j\frac{t}{3}} + e^{j\frac{t}{3}} + je^{j\frac{t}{2}} + e^{-j\frac{\pi}{4}} e^{j\frac{2}{3}t}$. What is the exponential Fourier series coefficient F_3?

$F_3 =$ ______________

Problem 4 *(26 points)*

Consider the periodic waveform $f(t)$



(a) What is the fundamental frequency ω_o?

$\omega_o =$ ______________

(b) Find the exponential Fourier series coefficient F_n. (Simplify your expression.)

$F_n =$ ______________

(c) Exponential Fourier series for a periodic signal is given as

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{0.504}{1+j4n} e^{jn2t}$$

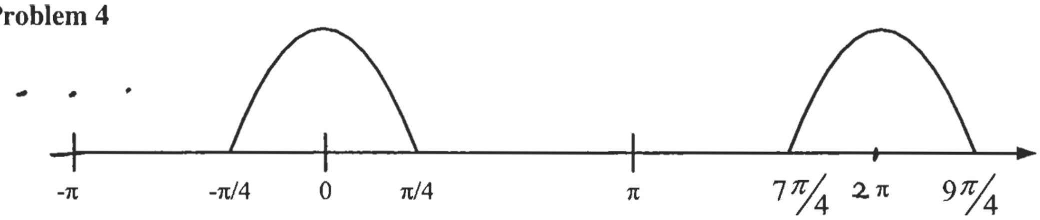
Suppose $f(t)$ is input to a system having the frequency response

$$H(\omega) = \begin{cases} e^{-j\omega/6}, & |\omega| \leq 3 \\ 0, & else \end{cases}$$

What is the output signal $y(t)$ in real form?

$y(t) =$ ______________

Problem 4



The periodic signal shown above is given by $f(t) = \begin{cases} \cos(2t), & -\pi/4 < t < \pi/4 \\ 0, & \pi/4 < t < 7\pi/4 \end{cases}$

(a) What is the period T and the fundamental frequency ω_o ?

$T =$ __________, $\omega_o =$ __________

(b) Is the complex Fourier coefficient F_n
(circle the correct answer)

totally real, totally imaginary, or both parts non-zero?

(c) Write the integral equation for F_n (leave in integral form):

(d) Compute F_0 :

$F_0 =$ ________________

(e) Compute F_1 :

errata: $\left(F_n = \begin{cases} 0, & n=0 \\ 1/n, & n>0 \end{cases} \right)$

$F_1 =$ ________________

(f) Given input Fourier coefficient $(F_n = \cancel{1/n \text{ for}} \text{ all } n)$, system response $H(\omega) = \dfrac{1}{1+j\omega}$, and fundamental frequency $\omega_o = 2$, write the equation for the output Fourier coefficient Y_n in **polar form**.

4. (25 pts) The two parts in this problem are unreleated.

(a) Consider the following periodic function

$$f(t) = cos^2(\pi t) + sin(2\pi t + \frac{\pi}{2})$$

i. What is the fundamental frequency ω_0 and period T of $f(t)$?

$\omega_0 =$ ________________

$T =$ ________________

ii. $f(t)$ can be expressed as an exponential Fourier series, where $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$, what is F_n for $n = 0, 1, -1, 2, -2$?

$F_0 =$ ________________

$F_1 =$ ________________

$F_{-1} =$ ________________

$F_2 =$ ________________

$F_{-2} =$ ________________

(b) The periodic function $f(t) = |sin(t)|$ can be expressed in Fourier series form as

$$f(t) = |sin(t)| = \sum_{n=-\infty}^{\infty} \frac{2}{\pi} \frac{1}{1-4n^2} e^{jn2t}$$

Let $f(t)$ be input to an LTI system having frequency response

$$H(\omega) = \frac{j\omega}{1+j\omega}$$

Then the output $y(t)$ is also periodic and can be expressed in Fourier Series form as

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{jn2t}$$

i. Write the expression for the $n = 2$ coefficient, Y_2. You do not have to simplify your answer.

. Y_2 =______________________________.

ii. Determine whether the following statements are true or false. Briefly justify your answer.
TRUE / FALSE: The DC component of the output from this system is zero regardless of the input.

TRUE / FALSE: This system acts as a band pass filter.

TRUE / FALSE: $y(t)$ is real-valued when $f(t)$ is real valued..

TRUE /FALSE: $H(-\omega) = H(\omega)^*$ ONLY when f(t) is real-valued.