ECE 210 Final Exam Review: Chapters 1-11

Major Topics: Node and Loop Analysis, Op-amps, Superposition, Thevenin and Norton Equivalency, RLC circuit analysis, Phasors, Frequency Response, Fourier Series, Fourier Transform, Modulation, Sampling, Bandwidth, Energy, Convolution, Linearity, Time-Invariance, Stability, Causality, Laplace Transform, Laplace-domain circuit analysis

Stability and Causality

a) $h(t) = e^{-3t}u(t)$

Causal because it is zero for all $t<0$.

Stable because $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{\infty} e^{-3t}\,dt = \frac{e^{-3t}}{-3}\Big|_0^{\infty} = \frac{1}{3} < \infty$

b) $y(t) = x(t+1)$

Non Causal because $y(0)$ depends upon $x(1)$.

Laplace Transform

$h(t) = e^{-3t}u(t)$

$$H(s) = \int_0^{\infty} e^{-3t} e^{-st}\,dt$$

$$= \int_0^{\infty} e^{-(s+3)t}\,dt$$

$$= \frac{e^{-(s+3)t}}{-(s+3)}\Big|_0^{\infty}$$

$$= \frac{1}{s+3}$$

Region of convergence: $Re\{s\} > -3$

Pole: $s = -3$

Stable because all poles are in the left half plane.

Causal because h(t) is zero for all t < 0.

Characteristic mode: $c_1 = e^{-3t}$

Formula $c_i(t) = t^{k_i} e^{p_i t}$ $\quad 0 \le k_i < r_i$

p_i = poles

r_i = multiplicities

Significance: the zero input response is a linear sum of the characteristic modes.

Inverse Laplace Transform: Partial Fraction Expansion

$$H(s) = \frac{s+3}{(s+2)(s+4)^2} = \frac{A}{s+2} + \frac{B}{s+4} + \frac{C}{(s+4)^2}$$

$$\frac{s+3}{(s+4)^2} = A + \frac{B(s+2)}{s+4} + \frac{C(s+2)}{(s+4)^2}$$

$$s = -2$$

$$\frac{1}{4} = A + 0 + 0$$

$$\frac{s+3}{(s+2)} = A(s+4)^2 + B(s+4) + C$$

$$s = -4$$

$$\frac{-1}{-2} = 0 + 0 + C$$

$$\therefore \frac{s+3}{(s+2)(s+4)^2} = \frac{1/4}{s+2} + \frac{B}{s+4} + \frac{1/2}{(s+4)^2}$$

This must hold for all s. Therefore $s=0$

$$\frac{3}{2 \cdot 16} = \frac{1/4}{2} + \frac{B}{4} + \frac{1/2}{16}$$

$$3 = 4 + 8B + 1$$

$$-2 = 8B$$

$$B = -1/4$$

$$H(s) = \frac{1/4}{s+2} - \frac{1/4}{s+4} + \frac{1/2}{(s+4)^2}$$

Formula: Table 11.1

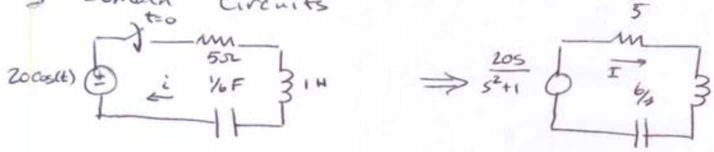
$$e^{pt}u(t) \longleftrightarrow \frac{1}{s-p}$$

$$te^{pt}u(t) \longleftrightarrow \frac{1}{(s-p)^2}$$

$$h(t) = \left[\frac{1}{4}e^{-2t} - \frac{1}{4}e^{-4t} + \frac{1}{2}te^{-4t}\right]u(t)$$

Stable because all poles are in the left half plane.
Causal because h(t) is zero for all t < 0.

S - Domain Circuits



Find transfer function, impulse response, output current

$$V = I\left(5 + s + \frac{6}{s}\right)$$

$$= I\left(\frac{s^2+5s+6}{s}\right)$$

$$\boxed{H = \frac{I}{V} = \frac{s}{s^2+5s+6}}$$

$$H = \frac{s}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \frac{s}{s+3}\bigg|_{s=-2} = \frac{-2}{1} = -2$$

$$B = \frac{s}{s+2}\bigg|_{s=-3} = \frac{-3}{-1} = 3$$

$$H = \frac{-2}{s+2} + \frac{3}{s+3}$$

$$h = \left[-2e^{-2t} + 3e^{-3t}\right]u(t)$$

$$I = HV$$

$$= \frac{s}{(s+2)(s+3)} \cdot \frac{20s}{s^2+1}$$

$$= \frac{20s^2}{(s^2+1)(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{Cs+D}{s^2+1}$$

$$A = \frac{20s^2}{(s^2+1)(s+3)}\bigg|_{s=-2} = \frac{80}{5 \cdot 1} = 16$$

$$B = \frac{20s^2}{(s^2+1)(s+2)}\bigg|_{s=-3} = \frac{180}{10 \cdot -1} = -18$$

True for all $s \Rightarrow$ true for $s=0$

$$0 = \frac{16}{2} - \frac{18}{3} + \frac{0+D}{1}$$

$$0 = 8 - 6 + D$$

$$D = -2$$

True for all $s \Rightarrow$ true for $s=-1$

$$\frac{20}{2 \cdot 1 \cdot 2} = \frac{A}{1} + \frac{B}{2} + \frac{-C+D}{2}$$

$$5 = 16 - 9 - \frac{C}{2} - 1$$

$$5 = 6 - C/2$$

$$-1 = -C/2$$

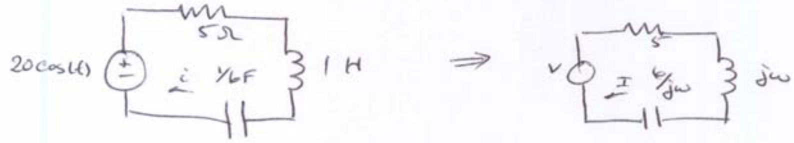
$$C = 2$$

$$I = \frac{16}{s+2} + \frac{-18}{s+3} + \frac{2s-2}{s^2+1}$$

Formula: $\cos(\omega_0 t)\,u(t) \leftrightarrow \frac{s}{s^2+\omega_0^2}$

$\sin(\omega_0 t)\,u(t) \leftrightarrow \frac{\omega_0}{s^2+\omega_0^2}$

$$i = \left[16e^{-2t} - 18e^{-3t} + 2\cos(t) - 2\sin(t)\right]u(t)$$



Find Frequency response and output current

$$V = I\left(j\omega + 5 + \frac{6}{j\omega}\right)$$

$$= I\left(\frac{-\omega^2 + j5\omega + 6}{j\omega}\right)$$

$$H = \frac{j\omega}{-\omega^2 + j5\omega + 6}$$

Note $H(j\omega) = H(s)\big|_{s=j\omega}$ ← transfer function; frequency response

Formula:
$x(t) = A\cos(\omega_0 t) \Rightarrow$
$y(t) = A \cdot |H(\omega_0)|\cos(\omega_0 t + \measuredangle H(\omega_0))$

$$H(1) = \frac{j}{-1 + j5 + 6} = \frac{j}{5 + j5}$$

$$|H(1)| = \frac{1}{5\sqrt{2}}$$

$$\measuredangle H(1) = \frac{\pi}{2} - \arctan\left(\frac{5}{5}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$i(t) = \frac{1}{5\sqrt{2}} \cdot 20 \cdot \cos(t + \pi/4)$$

$= 2\sqrt{2}\cos(t + \pi/4)$ ← perfectly correct answer, continued to show comparison with above.

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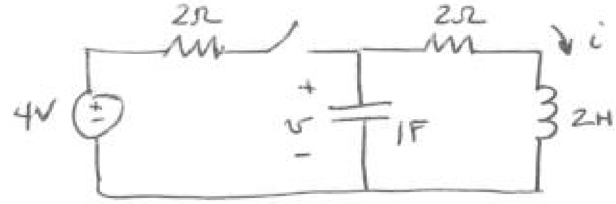
$$= 2\sqrt{2}\left[\cos(t)\cos(\pi/4) - \sin(t)\sin(\pi/4)\right]$$

$$= 2\sqrt{2}\left[\cos(t)\frac{\sqrt{2}}{2} - \sin(t)\frac{\sqrt{2}}{2}\right]$$

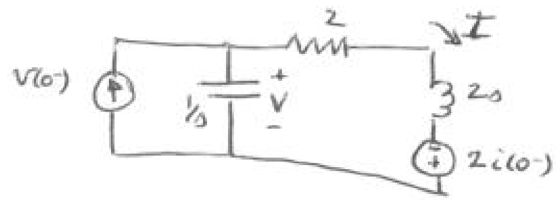
$$= 2\cos(t) - 2\sin(t)$$

In this picture, the switch is closed until time t=0. Then it is opened.

S- domain circuits



$$v(0^-) = 2V$$
$$i(0^-) = 1A$$



Loop equations:

$$-\frac{1}{s}\left(I - v(0^-)\right) - 2I - 2sI + 2i(0^-) = 0$$

$$I\left(-\frac{1}{s} - 2 - 2s\right) + \frac{v(0^-)}{s} + 2i(0^-) = 0$$

$$I\left(\frac{1+2s+2s^2}{s}\right) = \frac{2}{s} + 2$$

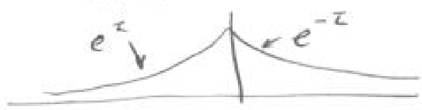
$$I\left(\frac{1+2s+2s^2}{s}\right) = \frac{2+2s}{s}$$

$$I = \frac{1+s}{s^2+s+\frac{1}{2}}$$

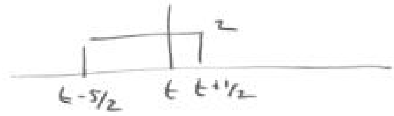
Convolution

$$x(t) = e^{-|t|}, \quad h(t) = 2\,\mathrm{rect}\left(\frac{t-1}{3}\right)$$

$x(\tau)$



$h(t-\tau)$



Case 1: $t + 1/2 < 0 \Rightarrow t < -1/2$

$$y(t) = \int_{t-5/2}^{t+1/2} 2e^{\tau}\,d\tau$$

$$= 2\left(e^{t+1/2} - e^{t-5/2}\right)$$

Case 2: $t + 1/2 > 0 \Rightarrow t > -1/2$

$t - 5/2 < 0 \Rightarrow t < 5/2$

$$y(t) = \int_{t-5/2}^{0} 2e^{\tau}\,d\tau + \int_{0}^{t+1/2} 2e^{-\tau}\,d\tau$$

$$= 2\left(2 - e^{t-5/2} - e^{-t-1/2}\right)$$

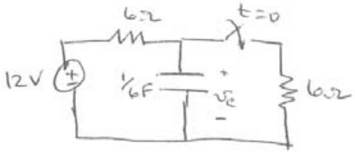
Case 3: $t - 5/2 > 0 \Rightarrow t > 5/2$

$$y(t) = \int_{t-5/2}^{t+1/2} 2e^{-\tau}\,d\tau$$

$$= 2\left(e^{-t+5/2} - e^{-t-1/2}\right)$$

$$y(t) = \begin{cases} 2\left(e^{t+1/2} - e^{t-5/2}\right) & t < -1/2 \\ 2\left(2 - e^{t-5/2} - e^{-t-1/2}\right) & -1/2 < t < 5/2 \\ 2\left(e^{-t+5/2} - e^{-t-1/2}\right) & 5/2 < t \end{cases}$$

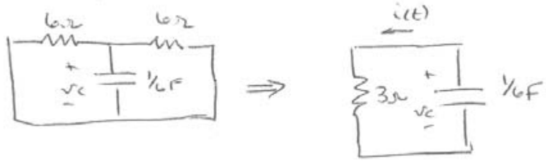
(6) Find $v_c(t)$ for $t \geq 0$. Assume DC steady state at $t=0$.



At $t=0^-$, $v_c = 12V$

$\therefore$ at $t=0^+$, $v_c = 12V$

Zero Input



$$i(t) = -\frac{1}{6}\frac{dv_c(t)}{dt}$$

$$i(t) = \frac{v_c(t)}{3}$$

$$\frac{v_c(t)}{3} = -\frac{1}{6}\frac{dv_c(t)}{dt}$$

$$\frac{dv_c(t)}{dt} + 2v_c(t) = 0$$

Assume $v_c(t) = Ae^{mt}$

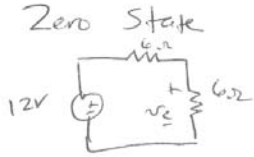
$$Ame^{mt} + 2Ae^{mt} = 0$$

$$\underbrace{Ae^{mt}}_{\neq 0}(m+2) = 0 \quad \therefore m+2=0$$

$$m = -2$$

$$v_c(t) = Ae^{-2t}$$

Zero State



$$v_c(t) = \frac{12V}{2} = 6V$$

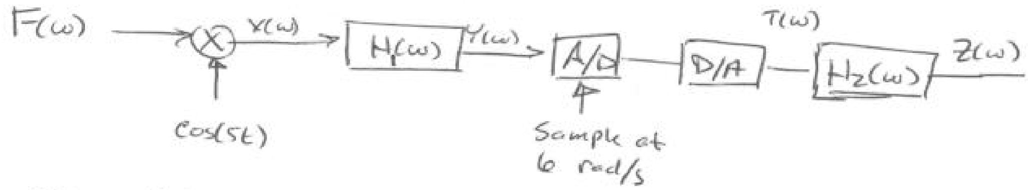
$$v_c(t) = 6 + Ae^{-2t}$$

$$v_c(0) = 6 + A = 12$$

$$A = 6$$

$$v_c(t) = (6 + 6e^{-2t}) \quad t \geq 0$$

Modulation and Sampling



$F(\omega) = \delta(\omega - 10) + \delta(\omega + 10)$

$H_1(\omega) = \text{rect}\left(\frac{\omega}{16}\right)$

$H_2(\omega) =$ ideal anti-imaging filter

$$X(\omega) = \tfrac{1}{2}\delta(\omega - 15) + \tfrac{1}{2}\delta(\omega - 5) + \tfrac{1}{2}\delta(\omega + 5) + \tfrac{1}{2}\delta(\omega + 15)$$

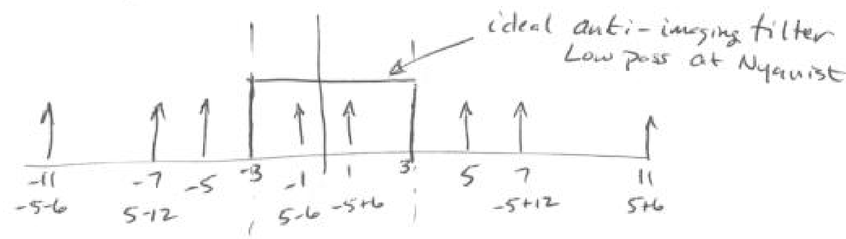
$$Y(\omega) = \underbrace{\tfrac{1}{2}\delta(\omega - 5) + \tfrac{1}{2}\delta(\omega + 5)}_{}$$

Signal at 5 rad/s $> \frac{6}{2} = 3$ ← Nyquist

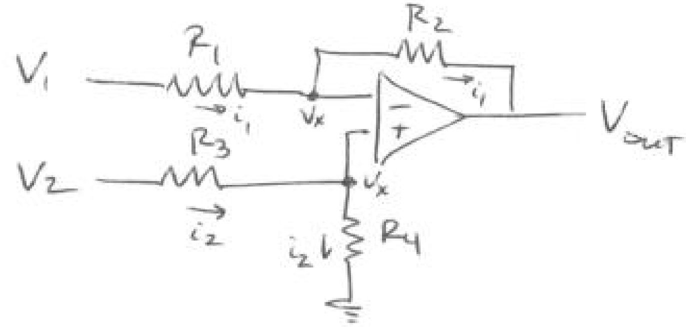
$\therefore$ aliasing

$$Z(\omega) = \tfrac{1}{2}\delta(\omega - 1) + \tfrac{1}{2}\delta(\omega + 1)$$

$T(\omega)$:



Op-amps and Superposition



Set $V_2 = 0$

$$\frac{V_1 - \cancel{V_x}^{0}}{R_1} = i_1 = \frac{\cancel{V_x}^{0} - V_{out}}{R_2}$$

$$V_{out} = -\frac{R_2}{R_1} V_1$$

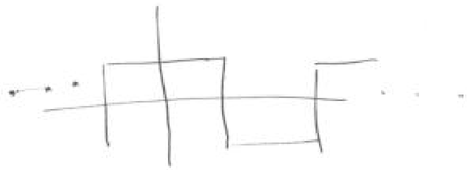
Set $V_1 = 0$

$$V_x = V_2 \left(\frac{R_4}{R_3 + R_4}\right) \qquad V_x = V_{out}\left(\frac{\cancel{R_2}^{R_1}}{R_1 + R_2}\right)$$

$$V_{out} = V_2 \left(\frac{R_1 + R_2}{R_2}\right)\left(\frac{R_4}{R_3 + R_4}\right)$$

$$V_{out} = -\frac{R_2}{R_1} V_1 + \left(1 + \frac{R_1}{R_2}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_2$$

Fourier Series



$$F_n = \frac{1}{T}\int_{-T/4}^{T/4} A e^{-j\frac{2\pi n t}{T}}\,dt + \frac{1}{T}\int_{T/4}^{3T/4} -A e^{-j\frac{2\pi n t}{T}}\,dt$$

$$= \frac{A}{T}\,\frac{e^{-j\frac{2\pi n t}{T}}}{-j\frac{2\pi n}{T}}\Bigg|_{-T/4}^{T/4} - \frac{A}{T}\,\frac{e^{-j\frac{2\pi n t}{T}}}{-j\frac{2\pi n}{T}}\Bigg|_{T/4}^{3T/4} \qquad \underline{\underline{n \neq 0}}$$

$$= \frac{A}{-j2\pi n}\left(e^{-j\frac{\pi n}{2}} - e^{j\frac{\pi n}{2}} - e^{-j\frac{3\pi n}{2}} + e^{-j\frac{\pi n}{2}} \right)$$

complex exponentials are periodic $\therefore e^{+j\frac{\pi n}{2}}$

$$= \frac{2A}{\pi n}\left(\frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{j2} \right)$$

$$= \frac{2A}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$F_0 = \frac{1}{T}\int_{-T/4}^{T/4} A\,dt + \frac{1}{T}\int_{T/4}^{3T/4} -A\,dt$$

$$= \frac{A}{T}\left[\frac{T}{4} + \frac{T}{4} - \left(\frac{3T}{4} - \frac{T}{4}\right)\right]$$

$$= 0$$

$$F_n = \begin{cases} \frac{2A}{\pi n}\sin\left(\frac{\pi n}{2}\right) & n \neq 0 \\ 0 & n = 0 \end{cases}$$