4528 - Schedule Pairs of Jobs

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In a factory, there are n pairs of jobs, (p_i, q_i), $i = 1, 2,..., n$, to be scheduled. Each job, p_i or q_i, needs 1 unit of time to process. All the jobs p_i, $i = 1, 2,..., n$, must be scheduled before of all the jobs q_i, $i = 1, 2,..., n$. The order among the jobs p_i, $i = 1, 2,..., n$, as well as the order among the jobs q_i, $i = 1, 2,..., n$, is not important. However, it is required that the time between p_i and q_i, measured from the starting time of p_i to the starting time of q_i, should be at most d_i, for $i = 1, 2,..., n$.

Given a sequence of n positive integers $d_1, d_2,..., d_n$, we want to know whether these n pairs of jobs can be scheduled in the time interval $[0, 2n]$ or not. We say that the problem is solvable if the n pairs of jobs can be scheduled in a time interval of length $2n$ units, in such a way that the time between p_i and q_i is at most d_i, for $i = 1, 2,..., n$.

For example, for $n = 3$, the sequence 1, 3, 5 is solvable, since we can schedule these 3 pairs of jobs as follows:

The sequence 3, 3, 4, 6 is also solvable, since we can schedule the jobs in the following way:

In this problem, you are going to design a computer program to schedule pairs of jobs with the above constraints.

Technical Specification

Assume that $n < 16$, and each $d_i < 2^{31}$. For simplicity, assume that $d_1 \leq d_2 \leq ... \leq d_n$, $\sum_{i=1}^{k} d_i \geq k^2$ for $1 \leq k < n$, and $\sum_{i=1}^{n} d_i = n^2$. Note that, in this case, if the problem is solvable then the time between each pair of jobs (p_i, q_i) is exactly d_i.

If the solution is not unique, try to schedule the jobs so that the job q_i with smaller index is finished as early as possible. For example, let the input requirements be `3 3 4 6`. Then print out the solution `p4 p1 p2 p3 q1 q2 q4 q3`.

Input

Input file contains a set of test cases. Each test case contains a positive integer n, followed by n integers d_i, $1 \leq i \leq n$. The last test case is followed by a line containing only one integer 0.

Output

Print the job in ascending order of their starting time. Print one line for each test case and for readability print a space before each ``p'' and ``q''. If the pairs of jobs cannot be scheduled, then print the message ``no solution'' in that line.

Sample Input

```
3
1 3 5
4
3 3 4 6
6
4 4 4 8 8 8
0
```

Sample Output

```
 p3 p2 p1 q1 q2 q3
 p4 p1 p2 p3 q1 q2 q4 q3
no solution
```

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