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2022561

AI ASSIGNMENT 2

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THEORY

Q1) Let us assume some ~~predicate~~ propositions.

let G_0, Y_0, R be true if the light at general state is Green/Yellow/Red.
let G', Y', R' be true if the light at the next state is G/Y/R.
let G'', Y'', R'' be true for the next to next state and G''', Y''', R''' for the state after that and so on.

a) $(G \wedge \neg Y \wedge \neg R) \vee (\neg G \wedge Y \wedge \neg R) \vee (\neg G \wedge \neg Y \wedge R)$

b) $(G \to G' \vee Y') \wedge (Y \to Y' \vee R') \wedge (R \to R' \wedge G')$

c) If we consider the (b) part let us write it as $(G \to G' \vee C_2')$, which means that if G is the current colour, C_2 is the only colour which can be switched to.

$\therefore C_1 \to C_2' \vee (C_1' \wedge C_2'') \vee (C_1' \wedge C_1'' \wedge C_2''') \vee (C_1' \wedge C_1'' \wedge C_1''' \wedge C_2'''')$

So it becomes

$(G_0 \to Y' \vee (G' \wedge Y'') \vee (G' \wedge G'' \wedge Y''') \vee (G' \wedge G'' \wedge G''' \wedge Y''''))$
$\wedge (Y \to R' \vee (Y' \wedge R'') \vee (Y' \wedge Y'' \wedge R''') \vee (Y' \wedge Y'' \wedge Y''' \wedge R''''))$
$\wedge (R \to G' \vee (R' \wedge G'') \vee (R' \wedge R'' \wedge G''') \vee (R' \wedge R'' \wedge R''' \wedge G''''))$

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Q2) Let $E(x,y)$ denote an edge from x to y.
Let $C(x, c_1)$ denote colour of c_1 alloted to x.

a) $\forall x, y, c_1 \exists c_2 . (x \neq y) \wedge (c_1 \neq c_2) \wedge E(x,y) \wedge C(x,c_1) \rightarrow C(y,c_2)$

b) Let Y be "yellow" colour.

$$\forall x, y, z . (x \neq y) \wedge (y \neq z) \wedge (z \neq x) \wedge C(x, Y) \wedge C(y, Y) \rightarrow \neg C(z, Y)$$

c) Let R, G be "red", "green" colours.

$$\forall x, y, z, a, b \; E(x,y) \wedge E(y,z) \wedge E(z,a) \wedge E(a,b) \wedge C(x,R) \longrightarrow C(y,R) \vee (\neg C(y,R) \wedge C(z,R)) \vee (\neg C(y,R) \wedge \neg C(z,R) \wedge C(a,R)) \vee (\neg C(y,R) \wedge \neg C(z,R) \wedge \neg C(a,R) \wedge C(b,R))$$

d) $\forall c_1 \exists x \; C(x, c_1)$

e) $\forall x \; \exists c_1 \in C \; C(x, c_1)$

By this, we know that every node can have any one of the $|C|$ colours only.

$$\forall x \; \forall c_1, c_2 \; C(x,c_1) \wedge C(x,c_2) \rightarrow (c_1 = c_2)$$

By this, we imply that every node can have only one colour and all nodes can be divided into $|C|$ disjoint sets

$\forall c_1 \; \exists x \; C(x, c_1)$

This implies that these sets are non empty.

Spiral

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A clique is a ~~graph~~ graph where every node is connected to every other node.

$$\forall x, y, c_1 \quad C(x, c_1) \wedge C(y, c_1) \rightarrow E(x, y) \wedge E(y, x)$$

This implies that these sets are now cliques.

$$\left(\forall x \; \exists c_1 \in C \quad C(x, c_1)\right) \wedge$$

$$\left(\forall x, c_1, c_2 \quad C(x, c_1) \wedge C(x, c_2) \rightarrow (c_1 = c_2)\right) \wedge$$

$$\left(\forall c_1 \; \exists x \quad C(x, c_1)\right) \wedge$$

$$\left(\forall x, y, c_1 \quad C(x, c_1) \wedge C(y, c_1) \rightarrow E(x, y) \wedge E(y, x)\right)$$

Q3) ~~Let Read(x)~~ mean ~~x can Read~~
~~Let Literate(x)~~ m

For Propositional Logic,
$D_1, D_2, D_3 \ldots$ are propositions for if Creatures 1, 2, 3 are Dolphins
$L_1, L_2, L_3 \ldots$ " " " " " are Literate
I_1, I_2, I_3 " " " " " " " are intelligent
R_1, R_2, R_3 " " " " " " " can read.

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For Predicate Logic, ~~Read~~ R(x) means x can Read,
~~Dolphin~~ D(x) means x is a Dolphin,
L(x) means x is Literate,
I(x) means x is Intelligent.

a) PL:
$(R_1 \rightarrow L_1) \wedge (R_2 \rightarrow L_2) \wedge (R_3 \rightarrow L_3) \wedge \ldots$

FOL:
$\forall x$ ~~Read(x) ⇒~~ $R(x) \rightarrow L(x)$

b) PL:
~~$(D_1 \wedge \neg L_1) \wedge (D_2 \wedge \neg L_2) \wedge (D_3 \wedge \neg L_3) \wedge \ldots$~~
$(D_1 \rightarrow \neg L_1) \wedge (D_2 \rightarrow \neg L_2) \wedge (D_3 \rightarrow \neg L_3) \wedge \ldots$
FOL:
$\forall x \quad D(x) \rightarrow \neg L(x)$

c) PL:
$(D_1 \wedge I_1) \vee (D_2 \wedge I_2) \vee (D_3 \wedge I_3) \vee \ldots$

FOL:
$\exists x$ ~~$D(x) \rightarrow I(x)$~~ $D(x) \wedge I(x)$

d) PL:
$(I_1 \wedge \neg R_1) \vee (I_2 \wedge \neg R_2) \vee (I_3 \wedge \neg R_3) \vee \ldots$

FOL:
$\exists x$ ~~$I(x) \rightarrow \neg R(x)$~~ $I(x) \wedge \neg R(x)$

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PL:

e) $\left((D_1 \wedge I_1 \wedge R_1) \vee (D_2 \wedge I_2 \wedge R_2) \vee (D_3 \wedge I_3 \wedge R_3) \vee \ldots\right) \wedge$

$\left((I_1 \wedge D_1 \wedge R_1 \rightarrow \neg L_1) \wedge (I_2 \wedge D_2 \wedge R_2 \rightarrow \neg L_2) \wedge (I_3 \wedge D_3 \wedge R_3 \rightarrow \neg L_3) \wedge \ldots\right)$

FOL:

$\left(\exists x \ D(x) \wedge I(x) \wedge R(x)\right) \wedge$

$\left(\forall x \ D(x) \wedge I(x) \wedge R(x) \rightarrow \neg L(x)\right)$

Visualizing the stop-route graph using Plotly: