TITLE

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Abstract. Abstract

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1. Introduction.

1.1. Contributions.

1.2. Previous studies / State-of-the-art.

1.3. Organization of the paper.

1.4. Notation.

2. .

3. .

$$\begin{aligned} &\min_{\boldsymbol{\beta},\boldsymbol{\tau}} \quad \frac{1}{2}\sum_{n=1}^{N} \left(F_n(\boldsymbol{\beta})\boldsymbol{\tau} - \tilde{w}_n\right)^2 + \frac{1}{2}\lambda_\alpha \left\|\boldsymbol{\tau}\right\|_2^2 + \lambda_\beta \mathcal{R}(\boldsymbol{\beta}) \\ &\text{s.t.} \quad \boldsymbol{\tau} \in \mathbb{R}_+^M\,,\ \boldsymbol{\beta} \in]0,1[^M\,. \end{aligned} \tag{3.1}$$

where $\mathcal{R}$ is a regularization that is proper, convex and lower-semicontinuous. For instance, $\mathcal{R}$ can be the total variation $\mathcal{R}(\mathbf{x}) = \left\|\mathbf{D}\mathbf{x}\right\|_1$ with $\mathbf{D}$ the discrete gradient matrix defined as the idendity matrix with coefficients -1 on its subdiagonal.

Equation (3.1) has the typical form of a constrained variable projection problem [1]. Indeed, it can be formulated as

$$\begin{aligned} &\min_{\boldsymbol{\beta},\boldsymbol{\tau}} \quad \frac{1}{2}\left\| \begin{pmatrix} \mathbf{F}(\boldsymbol{\beta}) \\ \sqrt{\lambda_\alpha}\,\mathrm{Id} \end{pmatrix} \boldsymbol{\tau} - \begin{pmatrix} \tilde{\mathbf{w}} \\ \mathbf{0} \end{pmatrix} \right\|_2^2 + \lambda_\beta \mathcal{R}(\boldsymbol{\beta}) \\ &\text{s.t.} \quad \boldsymbol{\tau} \in \mathbb{R}_+^M\,,\ \boldsymbol{\beta} \in]0,1[^M\,. \end{aligned} \tag{3.2}$$

Solving Problem 3.1 can be performed in a block-descent scheme. Minimization with respect to $\boldsymbol{\tau}$ consists in solving a non-negative least-squares problem. Minimization with respect to $\boldsymbol{\beta}$ can be performed with primal-dual methods [2]. Indeed, setting f as the indicator function of the set $[0+\epsilon, 1-\epsilon]$, g as the ℓ_1-norm multiplied by λ_β, and h as the least-squares term, Problem 3.1 can be written as an unconstrained problem of the form

$$\min_{\boldsymbol{\beta}\in\mathbb{R}^M} f(\boldsymbol{\beta}) + g(D\boldsymbol{\beta}) + h(\boldsymbol{\beta})\,, \tag{3.3}$$

where f, g, and h are proper, convex, and lower-semicontinuous functions, h is gradient 1-Lipschitz, and D is a linear operator from $\mathbb{R}^M$ to $\mathbb{R}^M$.

4. .

5. Conclusion.

Conclusion

Appendix A. Proof of blablabla.

REFERENCES

[1] G. Golub and V. Pereyra, *Separable nonlinear least squares: the variable projection method and its applications*, Inverse Problems, 19 (2003), pp. R1–R26, https://doi.org/10.1088/0266-5611/19/2/201.

[2] N. Komodakis and J.-C. Pesquet, *Playing with duality: An overview of recent primal-dual approaches for solving large-scale optimization problems*, IEEE Signal Process. Mag., 32 (2015), pp. 31–54, https://doi.org/10.1109/MSP.2014.2377273.