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A Novel Chimp Optimization Algorithm with Refraction Learning and Its Engineering Applications

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Abstract: The Chimp Optimization Algorithm (ChOA) is a heuristic algorithm proposed in recent years. It models the cooperative hunting behaviour of chimpanzee populations in nature and can be used to solve numerical as well as practical engineering optimization problems. ChOA has the problems of slow convergence speed and easily falling into local optimum. In order to solve these problems, this paper proposes a novel chimp optimization algorithm with refraction learning (RL-ChOA). In RL-ChOA, the Tent chaotic map is used to initialize the population, which improves the population's diversity and accelerates the algorithm's convergence speed. Further, a refraction learning strategy based on the physical principle of light refraction is introduced in ChOA, which is essentially an Opposition-Based Learning, helping the population to jump out of the local optimum. Using 23 widely used benchmark test functions and two engineering design optimization problems proved that RL-ChOA has good optimization performance, fast convergence speed, and satisfactory engineering application optimization performance.

Keywords: chimp optimization algorithm; refraction learning; tent chaos; global optimization; engineering design optimization

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1. Introduction

Over the past twenty years, the heuristic optimization algorithms have been widely appreciated for their simplicity, flexibility, and robustness. Common heuristic optimization algorithms include genetic algorithm (GA) [1], simulated annealing [2], crow search algorithm [3], ant colony optimization [4], differential evolution (DE) [5], particle swarm optimization (PSO) [6], bat algorithm (BA) [7], cuckoo search algorithm (CSA) [8], whale optimization algorithm (WOA) [9], firefly algorithm (FA) [10], grey wolf optimizer (GWO) [11], teaching-learning-based optimization [12], artificial bee colony (ABC) [13], and chimp optimization algorithm (ChOA) [14]. As technology continues to evolve and update, the heuristic optimization algorithms are currently widely applied in various areas of real life, such as the welded beam problem [15], feature selection [16–18], the welding shop scheduling problem [19], economic dispatch problem [20], training neural networks [21], path planning [15,22], churn prediction [23], image segmentation [24], 3D reconstruction of porous media [25], bankruptcy prediction [26], tuning of fuzzy control systems [27–29], interconnected multi-machine power system stabilizer [30], power systems [31,32], large scale unit commitment problem [33], combined economic and emission dispatch problem [34], multi-robot exploration [35], training multi-layer perceptron [36], parameter estimation of photovoltaic cells [37], and resource allocation in wireless networks [38].

Although there are many metaheuristic algorithms, each has its shortcomings. GWO does not have a good local and global search balance. In [39,40], the authors studied the possibility of enhancing the exploration process in GWO by changing the original control parameters. GWO lacks population diversity, and in [41], the authors replaced the typical real-valued encoding method with a complex-valued one, which increases the diversity of the population. ABC has the disadvantages of slow convergence and lack of population diversity. In [42], the authors' proposed approach refers to the procedure of differential mutation in DE and produces uniform distributed food sources in the employed bee phase to avoid the local optimal solution. In [43], in order to speed up the convergence of ABC, the authors proposed a new chaos-based operator and a new neighbour selection strategy improving the standard ABC. WOA has the disadvantage of premature convergence and easily falls into the local optimum. In [37], the authors modified WOA using chaotic maps to prevent the population from falling into local optima. In [44] WOA was hybridized with DE, which has a good exploration ability for function optimization problems to provide a promising candidate solution. CSA faces the problem of getting stuck in local minima. In [45], the authors addressed this problem by introducing a new concept of time varying flight length in CSA. BA has many challenges, such as a lack of population diversity, insufficient local search ability, and poor performance on high-dimensional optimization problems. In [46], Boltzmann selection and a monitor mechanism were employed to keep the suitable balance between exploration and exploitation ability. FA has a few drawbacks, such as computational complexity and convergence speed. In order to overcome such obstacles, in [47], the chaotic form of two algorithms, namely the sine–cosine algorithm and the firefly algorithms, are integrated to improve the convergence speed and efficiency, thus minimizing several complexity issues. DE is an excellent algorithm for dealing with nonlinear and complex problems, but the convergence rate is slow. In [48], the authors proposed that oppositional-based DE employs OBL to help population initialization and generational jumping.

There are many ways to help improve the performance of the metaheuristic algorithm, for example, Opposition-Based Learning (OBL) [49], chaotic [50], Lévy flight [51], etc.

Tizhood introduced OBL [49] in 2005. The main idea of OBL is to find a better candidate solution and use this candidate solution to find a solution closer to the global optimum. Hui et al. [52] added generalized OBL to the PSO to speed up the convergence speed. Ahmed et al. [53] proposed an improved version of the grasshopper optimization algorithm based on OBL. First, they use the OBL population for better distribution in the initialization phase, and then they use it in the iteration of the algorithm to help the population jump out of the local optimum, etc.

Paul introduced Lévy flight [51]. In Lévy flight, the small jumps are interspersed with longer jumps or "flights", which causes the variance of the distribution to diverge. As a consequence, Lévy flight do not have a characteristic length scale. Lévy flight is widely used in meta-heuristic algorithms. The small jumps help the algorithm with local exploration, and the longer jumps help with the global search. Ling [51] et al. proposed an improvement to the whale optimization algorithm based on a Lévy flight, which helps increase the diversity of the population against premature convergence and enhances the capability of jumping out of local optimal optima. Liu et al. [54] proposed a novel ant colony optimization algorithm with the Lévy flight mechanism that guarantees the search speed extends the searching space to improve the performance of the ant colony optimization, etc.

The chaotic sequence [50] is a commonly used method for initializing the population in the meta-heuristic algorithm, which can broaden the search space of the population and speed up the convergence speed of the algorithm. Kuang et al. [55] added the Tent map to the artificial bee colony algorithm to make the population more diverse and obtain a better initial population. Suresh et al. [56] proposed novel improvements to the Cuckoo Search, and one of these improvements is the use of the logistic chaotic [50] function to initialize the population. Afrabandpey et al. [57] used chaotic sequences in the Bat Algorithm for population initialization instead of random initialization, etc.

This paper mainly focuses on the Chimp Optimization Algorithm (ChOA), which was proposed in 2020 by Khishe et al. [14] as a heuristic optimization algorithm based on the social behaviour of the chimp population. The ChOA has the advantages of fewer parameters, easier implementation, and higher stability than other types of heuristic optimization algorithms. Although different heuristic optimization algorithms adopt different approaches to the search, the common goal is mostly to explore the balance between population diversity and search capacity; convergence accuracy and speed are guaranteed while avoiding premature maturity. Since the ChOA was proposed, researchers have used various strategies to improve its performance and further apply it to practical problems. Khishe et al. [58] proposed a weighted chimp optimization algorithm (WChOA), which uses a position-weighted equation in the individual update position to improve the convergence speed and help jump out of the local optimum. Kaur et al. [59] proposed a novel algorithm that fuses ChOA and sine–cosine functions to solve the problem of poor balance during development and applied it to the engineering problems of vessel pressure, clutch brakes, and digital filters design, etc. Jia et al. [60] initialized the population through highly destructive polynomial mutation and then used the beetle antenna search algorithm on weak individuals to obtain visual ability and improve the ability of the algorithm to jump out of the local optimum. Houssein et al. [61] used the opposition-based learning strategy and the Lévy flight strategy in ChOA to improve the diversity and optimization ability of the population in the search space and applied the proposed algorithm to image segmentations. Wang et al. [62] proposed a novel binary ChOA. Hu et al. [63] used ChOA to optimize the initial weights and thresholds of extreme learning machines and then applied the proposed model to COVID-19 detection to improve the prediction accuracy. Wu et al. [64] combined the improved ChOA [60] and support vector machines (SVM) and proposed a novel SVM model that outperforms other methods in classification accuracy.

In summary, there are numerous heuristic optimization algorithms and improvement mechanisms, each with its advantages; however, the No-free-Lunch (NFL) theorem [65] logically proves that no single population optimization algorithm can be used to solve all kinds of optimization problems and that it is suitable for solving some optimization problems, but reveals shortcomings for solving others. Therefore, to improve the performance and applicability of ChOA and explore a more suitable method for solving practical optimization problems, an improved ChOA (called RL-ChOA) is proposed in this paper.

The main contributions of this paper are summarized as follows:

1. A new ChOA framework is proposed that does not affect the configuration of the traditional ChOA.
2. Use Tent chaotic map to initialize the population to improve the diversity of the population.
3. A strategy of Refraction learning of light is proposed to prevent the population from falling into local optima.
4. In comparing RL-ChOA with five other state-of-the-art heuristic algorithms, the experimental results show that the RL-ChOA is more efficient and accurate than other algorithms in most cases.
5. The experimental comparison of two engineering design optimization problems shows that RL-ChOA can be applied more effectively to practical engineering problems.

The rest of this paper is organized as follows: Section 2 introduces the preliminary knowledge, including the original ChOA and the principles of light refraction. Section 3 illustrates the proposed RL-ChOA in detail. Sections 4 and 5 detail the experimental simulations and experimental results, respectively. Section 6 discusses two engineering case application analyses of RL-ChOA. Finally, Section 7 concludes this paper.

2. Preliminary Knowledge

2.1. Chimp Optimization Algorithm

ChOA is a heuristic optimization algorithm based on the chimp population's hunting behaviour. Within the chimp population, the groups of chimps are classified as "Attacker", "Barrier", "Chaser", and "Driver", according to the diversity of intelligence and abilities that individuals display during hunting. Each chimp species has the ability to think independently and use its search strategy to explore and predict the location of prey. While they have their tasks, they are also socially motivated to acquire sex and benefits in the final stages of the hunts. During this process, the chaotic individual hunting behaviour occurs.

The standard ChOA is as follows: Suppose there are N chimps and the position of the i-th chimp is X_i. The optimal solution is "Attacker", the second optimal solution is "Barrier", the third optimal solution is "Chaser," and the fourth optimal solution is "Driver". The behaviour of chimps approaching and surrounding the prey and their position update equations are as follows:

$$D = |C \cdot X_{prey}(t) - m \cdot X_{chimp}(t)| \tag{1}$$

$$X_{chimp}(t+1) = X_{chimp}(t) - A \cdot D \tag{2}$$

$$A = f \cdot (2 \cdot r_1 - 1), C = 2 \cdot r_2 \tag{3}$$

$$m = chaotic\_value \tag{4}$$

where r_1 and r_2 are random vectors with values in the range of $[0, 1]$. f is the non-linear decay factor, and its value decreases linearly from 2.5 to 0 with the increase in the number of iterations. t represents the current number of iterations. A is a random vector, and its value is a random number between $[-f, f]$. m is a chaotic factor representing the influence of sexual behaviour incentives on the individual position of chimps. C is a random variable that influences the position of prey within $[0, 2]$ on the individual position of chimps (when $C < 1$, the degree of influence weakens; when $C > 1$, the degree of influence strengthens).

The positions of other chimps in the population are determined by the positions of Attacker, Barrier, Chaser, and Driver, and the position update equations are as follows:

$$\begin{cases} D_{Attacker} = |C_1 * X_{Attacker} - m_1 * X| \\ D_{Barrier} = |C_2 * X_{Barrier} - m_2 * X| \\ D_{Chaser} = |C_3 * X_{Chaser} - m_3 * X| \\ D_{Driver} = |C_4 * X_{Driver} - m_4 * X| \end{cases} \tag{5}$$

$$\begin{cases} X_1 = X_{Attacker} - A_1 * D_{Attacker} \\ X_2 = X_{Barrier} - A_2 * D_{Barrier} \\ X_3 = X_{Chaser} - A_3 * D_{Chaser} \\ X_4 = X_{Driver} - A_4 * D_{Driver} \end{cases} \tag{6}$$

$$X(t+1) = (X_1 + X_2 + X_3 + X_4)/4 \tag{7}$$

where C_1, C_2, C_3 and C_4 are similar to C; A_1, A_2, A_3 and A_4 are similar to A; And m_1, m_2, m_3 and m_4 are similar to m.

The pseudo-code of the ChOA is listed in Algorithm 1.

2.2. Principle of Light Refraction

The principle of light refraction says that when light is obliquely incident from one medium into another medium, the propagation direction changes so that the speed of light changes at the junction of different media, and the deflection of the route occurs [66].

As shown in Figure 1, the light from the point P in medium 1 passes through the refraction point O, is refracted, and finally reaches the point Q in medium 2. Suppose the refractive indexes of light in medium 1 and medium 2 are n_1 and n_2, respectively; the angle of incidence and refraction are α and β, respectively; and the velocities of light in medium 1 and medium 2 are v_1 and v_2. The ratio of the sine of the angle of incidence to the angle of refraction is equal to the ratio of the reciprocals of the refraction path or the ratio of the velocities of light in two media. That is, as shown in Formula (8):

$$\delta = \frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1} = \frac{v_1}{v_2} \tag{8}$$

where δ is called the refractive index of medium 2 relative to medium 1.

Algorithm 1: Pseudo-code of the Chimp Optimization Algorithm (ChOA).

Input: Population size N, Maximum number of iteration $MaxIt$.
Output: The best location $Xbest$, the best fitness value $F(Xbest)$.

t = 0;
Randomly initialize the chimp population $X_i (i = 1, 2, 3 \ldots, N)$;
Initialize the parameters A, C, and f;
foreach X_i **do**
    Calculate fitness $F(X_i)$
end
$X_{Attacker}$ = the best solution;
$X_{Barrier}$ = the second best solution;
X_{Chaser} = the third best solution;
X_{Driver} = the fourth best solution;
while $t < MaxIt$ **do**
    foreach *each chimp* **do**
        update the position of the current chimp using Equation (7);
    end
    Update the parameter A, C, and f;
    Update $D_{Attacker}$, $D_{Barrier}$, D_{Chaser}, and D_{Driver} using Equation (5);
    Update $X_{Attacker}$, $X_{Barrier}$, X_{Chaser}, and X_{Driver} using Equation (6);
    $t = t + 1$;
end

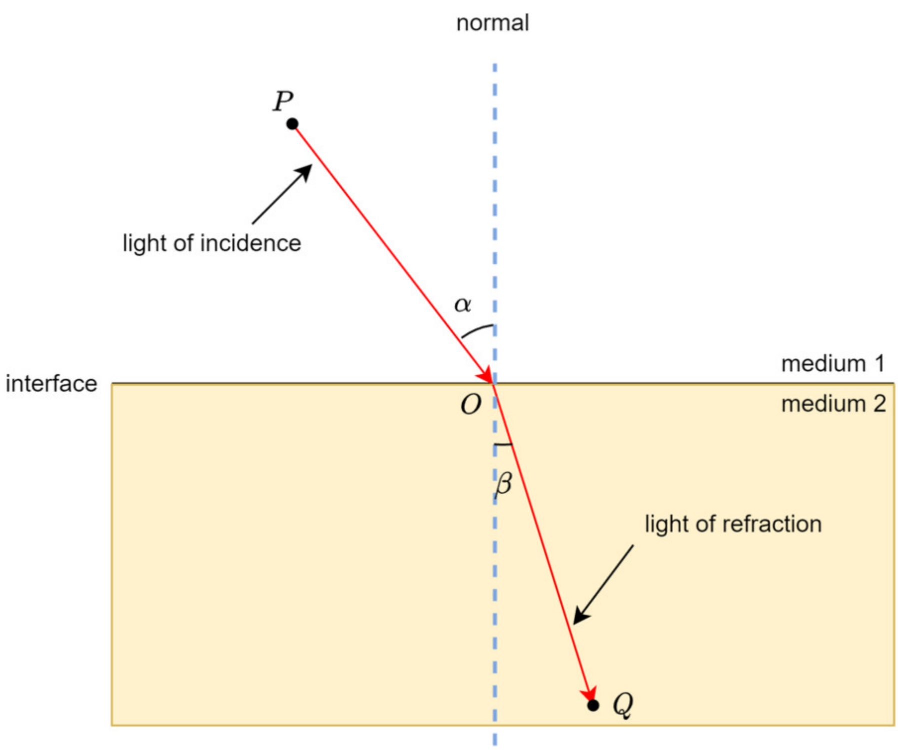


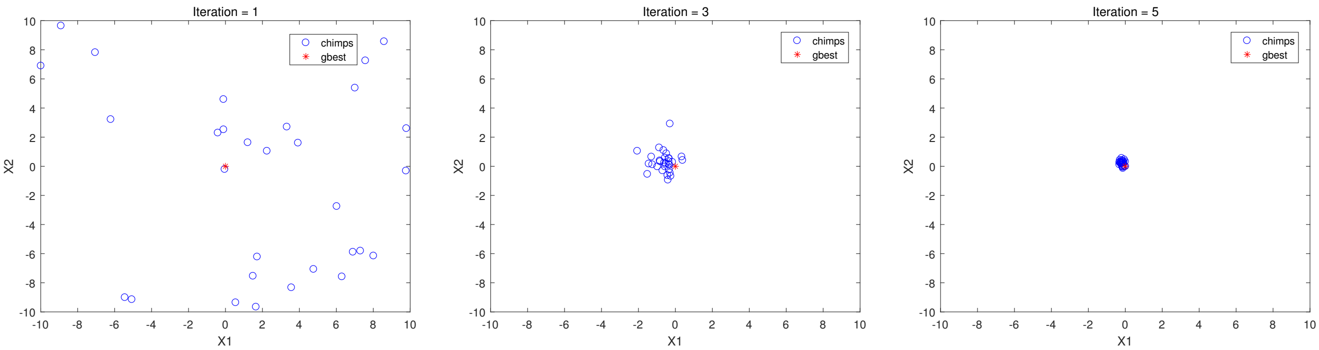
Figure 1. Principle of light refraction.

3. Proposed RL-ChOA

3.1. Motivation

According to Equation (7), the chimp population completed the hunting process under the guidance of "Attacker", "Barrier", "Chaser", and "Driver". Mathematically, the fitter chimps ("Attacker", "Barrier", "Chaser", and "Driver") help the other chimps in the population update their positions. Therefore, the fitter chimps are mainly responsible for prey search and providing prey direction for the population. Thus, these leading chimps must have good fitness values. This way of updating the population position is conducive to attracting the population to the fitter chimps more quickly, but the diversity of the population is hindered. In the end, ChOA is trapped in a local optimum.

Figure 2 illustrates the 30 population distribution of the Sphere function with two dimensions in [−10, 10] observed at various stages of ChOA.



(**a**) Population distribution iteration = 1 (**b**) Population distribution iteration = 3 (**c**) Population distribution iteration = 5

Figure 2. Population distribution observed at various stages in RL-ChOA.

As shown in Figure 2, after initialization, the chimp population of ChOA begins to explore the entire search space. Then, led by "Attacker", "Barrier", "Chaser", and "Driver", ChOA can accumulate many chimpanzees into the optimal search space. However, due to the poor exploratory ability of other chimps in the population, if the current one is in a local optimum, then ChOA is easy to fall into a local optimum. In other words, the global exploration ability of ChOA mainly depends on the well-fitted chimps. In addition, it is noted that there is still room for further improvement in the global exploration ability of ChOA. Therefore, the population leader needs to improvise to avoid being premature due to locally optimal solutions.

3.2. Tent Chaos Sequence

The Tent chaos sequence has the characteristics of randomness, ergodicity, and regularity [67]. Using these characteristics to optimize the search can effectively maintain the diversity of the population, suppress the algorithm from falling into the local optimum, and improve the global search ability. The original chimp optimization algorithm adopts The rand function and randomly initializes the population, and the resulting population distribution is not broad enough. This paper studies Logistic chaotic map [50] and Tent chaotic map; the results demonstrate that the Tent chaotic map has better population distribution uniformity and faster search speed than the Logistic chaotic map. Figures 3 and 4 represent the histograms of the Tent chaotic sequence and the Logistics chaotic sequence, respectively. From these two figures, it can be found that the value probability of the Logistic chaotic map between $[0, 0.05]$ and $[0.95, 1]$ is higher than that of other segments, while the value probability of the Tent chaotic map in each segment of the feasible region is relatively uniform. Therefore, Tent chaotic map is better than the Logistic chaotic map in population initialization. In this paper, the ergodicity of the Tent chaotic map is used to generate a uniformly distributed chaotic sequence, which reduces the influence of the initial population distribution on the algorithm optimization.

The expression of the Tent chaotic map is as follows:

$$x_{i+1} = \begin{cases} 2x_i, 0 \leq x \leq 0.5 \\ 2(1 - x_i), 0.5 < x \leq 1 \end{cases} \tag{9}$$

After the Bernoulli shift transformation, the expression is as follows:

$$x_{i+1} = (2x_i)mod1 \tag{10}$$

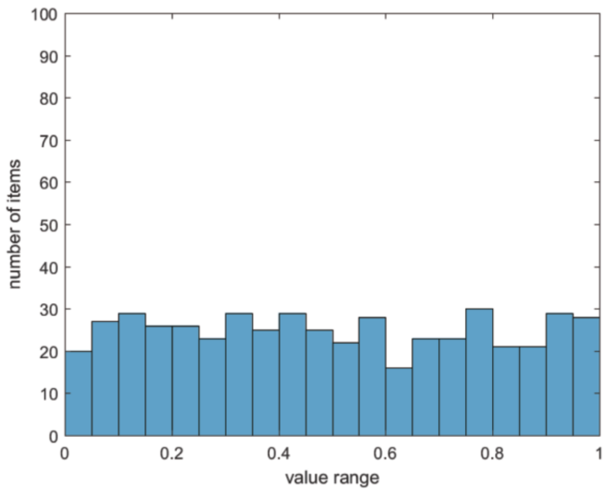


Figure 3. Tent chaotic sequence distribution histogram.

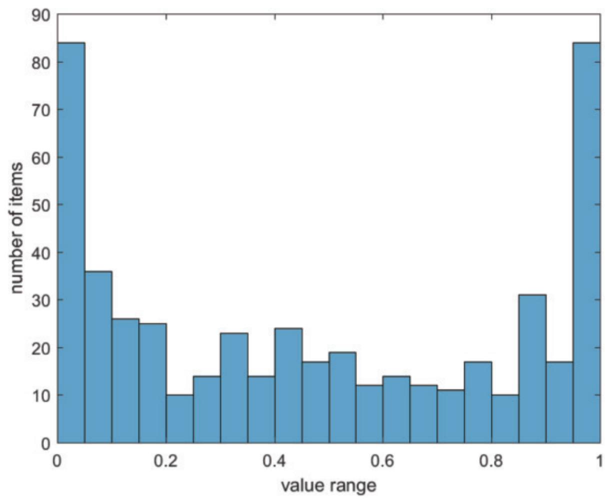


Figure 4. Logistic chaotic sequence distribution histogram.

By analyzing the Tent chaotic iterative sequence, it can be found that there are short periods and unstable period points in the sequence. To avoid the Tent chaotic sequence falling into small period points and unstable period points during iteration, a random variable $rand(0,1) \times \frac{1}{N}$ is introduced into the Tent chaotic map expression. Then, the improved Tent chaotic map expression is as follows:

$$x_{i+1} = \begin{cases} 2x_i + rand(0,1) \times \frac{1}{N}, 0 \leq x \leq 0.5 \\ 2(1 - x_i) + rand(0,1) \times \frac{1}{N}, 0.5 < x \leq 1 \end{cases} \tag{11}$$

The transformed expression is as follows:

$$x_{i+1} = (2x_i)mod1 + rand(0,1) \times \frac{1}{N} \tag{12}$$

where N is the number of particles in the sequence. $rand(0,1)$ is a random number in the range $[0,1]$. The introduction of the random variable $rand(0,1) \times \frac{1}{N}$ not only maintains the randomness, ergodicity, and regularity of the Tent chaotic map but can also effectively avoid iterative falling into small periodic points and unstable periodic points.

3.3. Refraction of Light Learning Strategy

Mathematically speaking, by Equations (5)–(7), in standard ChOA, the Attacker, Barrier, Chaser, and Driver are primarily responsible for searching for prey and guiding other chimps in the population to complete location updates during the hunt. However, when the Attacker, Barrier, Chaser, and Driver search for the best prey in the current local area, they will guide the entire population to gather near these four types of chimps, which will cause the algorithm to fall into local optimum and reduce population diversity. This makes the algorithm appear to be "Prematurity".

Therefore, to solve this problem, this paper proposes a new learning strategy based on the refraction principle of light, which is used to help the population jump out of the local optimum and maintain the diversity of the population. The basic principle of refraction learning is shown in Figure 5.

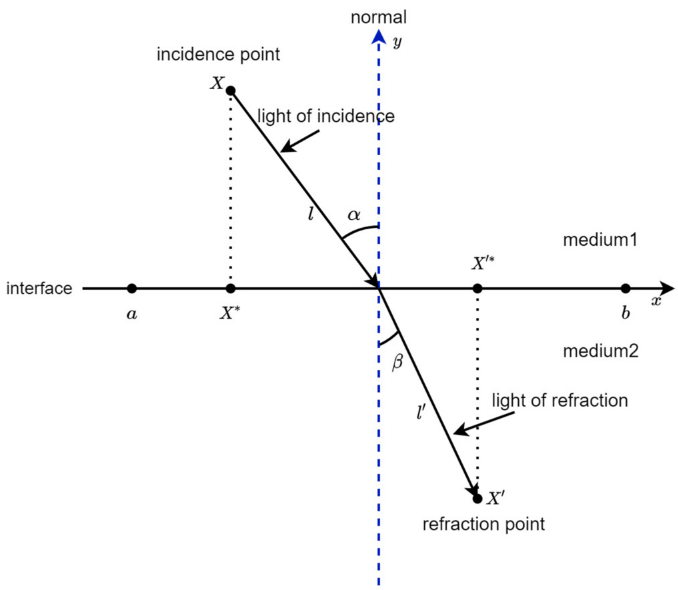


Figure 5. One-dimensional spatial refraction learning process for the current global optima X^*.

In Figure 5, the search space for the solution on the x-axis is $[a,b]$; the incident ray and the refracted ray are l and l', respectively; the y-axis represents the normal; α and β are the angles of incidence and refraction, respectively; X and X' are the points of incidence and the point of reflection, respectively; the projection points of X and X' on the x-axis are X^* and X'^*, respectively. From the geometric relationship of the line segments in Figure 5, the following formulas can be obtained:

$$\sin\alpha = ((a+b)/2 - X^*)/l \tag{13}$$

$$\sin\beta = (X'^* - (a+b)/2)/l' \tag{14}$$

From the definition of the refractive index, we know that $\delta = \sin\alpha / \sin\beta$. Combining it with the above Equations (13) and (14), the following formula can be obtained:

$$\delta = \frac{\sin\alpha}{\sin\beta} = \frac{((a+b)/2 - X^*)/l}{(X'^* - (a+b)/2)/l'} \tag{15}$$

Let $k = l/l'$, then, the above Equation (15) can be changed into the following formula:

$$\delta = ((a+b)/2 - X^*)/(X'^* - (a+b)/2) \tag{16}$$

Convert Equation (16), and the calculation formula for the solution based on refraction learning is as follows:

$$X'^* = (a+b)/2 + (a+b)/(2k\delta) - X^*/k\delta \tag{17}$$

When the dimension of the optimization problem is n-dimensional, and $\eta = k\delta$, the Equation (17) can be modified as follows:

$$X_j'^* = (a_j + b_j)/2 + (a_j + b_j)/(2\eta) - X_j^*/\eta \tag{18}$$

where, x_j^* and $x_j'^*$ are the original value of the j-th dimension in the n-dimensional space and the value after refraction learning, respectively; a_j and b_j are the maximum and minimum values of the j-th dimension in the n-dimensional space, respectively. When $\eta = 1$, Equation (18) becomes as follows:

$$X_j'^* = a_j + b_j + X_j^* \tag{19}$$

Equation (19) is a typical OBL, so it can be observed that OBL is a special kind of refraction learning.

3.4. The Framework of RL-ChOA

The framework of the proposed RL-ChOA is shown in Algorithm 2.

Algorithm 2: Framework of the Proposed Algorithm (RL-ChOA).

Input: Population size N, Maximum number of iteration $MaxIt$.
Output: The best location $Xbest$, the best fitness value $F(Xbest)$.

t = 0;
Initialize the chimp population $X_i (i = 1, 2, 3 \ldots, N)$ by Equation (12);
Initialize the parameters A, C, and f;
foreach X_i **do**
    Calculate fitness $F(X_i)$
end
$X_{Attacker}$ = the best solution;
$X_{Barrier}$ = the second best solution;
X_{Chaser} = the third best solution;
X_{Driver} = the fourth best solution;
while $t < MaxIt$ **do**
    foreach *each chimp* **do**
        Update the position of the current chimp using Equation (7);
    end
    Update the parameter A, C, and f;
    Update $D_{Attacker}$, $D_{Barrier}$, D_{Chaser}, and D_{Driver} using Equation (5);
    Update $X_{Attacker}$, $X_{Barrier}$, X_{Chaser}, and X_{Driver} using Equation (6);
    Carry out the refraction leaning strategy for the current optimal solution by using Equation (18);
    $t = t + 1$;
end

3.5. Time Complexity Analysis

The time complexity [68] indirectly reflects the convergence speed of the algorithm. In the original ChOA, it is assumed that the time required for initialization of parameters (population size is N, search space dimension is n, coefficients are A, f, C, and other parameters) is α_1; and the time required for each individual to update its position is α_2; the time to solve the objective function is $f(n)$. Then, the time complexity of the original ChOA is as follows:

$$T_1(n) = O(\alpha_1 + N(n\alpha_2 + f(n))) = O(N(n\alpha_2 + f(n))) \tag{20}$$

In the RL-ChOA proposed in this paper, the Tent chaos sequence is used in the population initialization, and the time spent is consistent with the original population initialization. In the algorithm loop stage, the time required to use the refraction learning strategy of light is α_3; and the time needed to execute the greedy mechanism is α_4. Then, the time complexity of RL-ChOA is as follows:

$$T_2(n) = O(\alpha_1 + \alpha_4 + N(n\alpha_2 + \alpha_3 + f(n))) = O(N(n\alpha_2 + f(n))) \tag{21}$$

The time complexity of RL-ChOA proposed in this paper is consistent with the original ChOA:

$$T_1(n) = T_2(n) = O(N(n\alpha_2 + f(n))) \tag{22}$$

In summary, the improved strategy proposed in this paper for ChOA defects does not increase the time complexity.

3.6. Remark

In Section 3.1, the reasons why ChOA is prone to fall into local optimum and the motivation for improvement are analyzed. In Sections 3.2 and 3.3, the strategies for improving ChOA are analyzed in detail. Based on the principle of light refraction, we propose refraction learning, and apply it to ChOA to help jump out of the local optimum and achieve a balance between a global and local search. In Sections 3.4 and 3.5, the process framework and time complexity of RL-ChOA are analyzed.

4. Experimental Simulations

In this section, to test the optimization performance of RL-ChOA, we conducted a set of experiments using benchmark test functions. Section 4.1 describes the benchmark test suite used in the experiments, Section 4.2 describes the experimental simulation environment, and Section 4.3 describes the parameter settings.

4.1. Benchmark Test Suite

Twenty-three classical benchmark test functions, widely verified by many researchers, are used in experiments to verify the algorithm's performance. These benchmark test functions $F1$–$F23$ can be found in [9]. Among them, $F1$–$F7$ are seven continuous unimodal benchmark test functions, and their dimensions are set to 30, which are used to test the optimization accuracy of the algorithm; $F8$–$F13$ are six continuous multi-modal benchmark test functions, and their dimensions are also set to 30, to test the algorithm's ability to avoid prematurity; $F14$–$F23$ are nine fixed-dimension multi-modal benchmark test functions, which are used to test the global search ability and convergence speed of the algorithm. Details of the benchmark test functions, for example, the name of the function, the expression of the function, the search range of the function, and the optimal value of the function, can be found in [9].

4.2. Experimental Simulation Environment

The computer configuration used in the experimental simulation is Intel "Core i7-4720HQ", the main frequency is "2.60 GHz", "8 GB memory", "Windows10 64bit" operating system and the computing environment is "Matlab2016 (a)".

4.3. Parameter Setting

The common parameters of all algorithms are set to the same size. The population size is set to 30, the maximum fitness function call is set to 15,000, and each algorithm runs independently 30 times. The parameter settings in different algorithms are shown in Table 1.

Table 1. Algorithm parameter setting.

Algorithm	Parameter
SMIGWO	$A_{max} = 2, A_{min} = 0$
IGWO	$W = 0.75$
SPTLBO	$C = 0.2, PF = 0.3$
ChOA	$m = chaos(3, 1, 1)$
WChOA	$m = chaos(3, 1, 1)$
RL-ChOA	$m = chaos(3, 1, 1), \delta = 100, k = 100$

5. Experimental Results

In this section, to verify the algorithm's performance proposed in this paper, it is compared with five state-of-the-art algorithms, namely the Grey Wolf Algorithm Based on L/'evy Flight and Random Walk Strategy (MGWO) [69], improved Grey Wolf Optimization Algorithm based on iterative mapping and simplex method (SMIGWO) [70], Teaching-learning-based Optimization Algorithm with Social Psychology Theory (SPTLBO) [71], original ChOA, and WChOA. Tables 2–4 present the statistical results of the best value, average value, worst value, and standard deviation obtained by the six different algorithms on the three types of test problems. In order to accurately get statistically reasonable conclusions, Tables 5–7 use Friedman's test [72] to rank three different types of benchmark test functions. In addition, Wilcoxon's rank-sum test [73] is a nonparametric statistical test that can detect more complex data distributions. Table 8 shows the Wilcoxon's rank-sum test for independent samples at the p = 0.05 level of significant difference [68]. The symbols "+", "=", and "−" indicate that RL-ChOA is better than, similar to, and worse than the corresponding comparison algorithms, respectively. In Section 5.2, we analyze and illustrate the convergence of RL-ChOA on 23 widely used test functions. In Section 5.3, the different values of parameters k and δ in the light refraction learning strategy are analyzed.

Table 2. Results of unimodal benchmark test functions.

Func	Criteria	SMIGWO	MGWO	SPTLBO	ChOA	WChOA	RL-ChOA
F1	Best	9.29×10^{-30}	3.45×10^{-47}	4.30×10^{-104}	4.82×10^{-10}	6.70×10^{-288}	$0.00 \times 10^{+00}$
	Mean	9.41×10^{-28}	8.27×10^{-45}	7.17×10^{-101}	2.78×10^{-06}	7.14×10^{-281}	$0.00 \times 10^{+00}$
	Worst	6.18×10^{-27}	8.33×10^{-44}	7.74×10^{-100}	1.94×10^{-05}	1.16×10^{-279}	$0.00 \times 10^{+00}$
	Std	1.43×10^{-27}	1.91×10^{-44}	1.88×10^{-100}	4.35×10^{-06}	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
F2	Best	2.02×10^{-17}	4.11×10^{-28}	7.53×10^{-55}	6.41×10^{-09}	2.19×10^{-146}	$0.00 \times 10^{+00}$
	Mean	6.61×10^{-17}	7.16×10^{-27}	2.61×10^{-53}	2.83×10^{-05}	1.22×10^{-144}	$0.00 \times 10^{+00}$
	Worst	1.47×10^{-16}	3.75×10^{-26}	2.37×10^{-52}	1.48×10^{-04}	9.24×10^{-144}	$0.00 \times 10^{+00}$
	Std	3.84×10^{-17}	7.71×10^{-27}	4.81×10^{-53}	3.67×10^{-05}	1.77×10^{-144}	$0.00 \times 10^{+00}$
F3	Best	2.63×10^{-08}	4.51×10^{-16}	1.04×10^{-72}	$2.40 \times 10^{+00}$	4.12×10^{-273}	$0.00 \times 10^{+00}$
	Mean	6.53×10^{-06}	1.23×10^{-08}	1.24×10^{-67}	$8.59 \times 10^{+01}$	1.94×10^{-208}	$0.00 \times 10^{+00}$
	Worst	8.17×10^{-05}	3.66×10^{-07}	2.63×10^{-66}	$5.66 \times 10^{+02}$	5.83×10^{-207}	$0.00 \times 10^{+00}$
	Std	1.57×10^{-05}	6.67×10^{-08}	4.86×10^{-67}	$1.21 \times 10^{+02}$	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
F4	Best	4.31×10^{-08}	1.62×10^{-14}	2.64×10^{-50}	9.46×10^{-03}	3.29×10^{-141}	$0.00 \times 10^{+00}$
	Mean	5.99×10^{-07}	6.12×10^{-13}	2.30×10^{-48}	4.15×10^{-01}	1.32×10^{-137}	$0.00 \times 10^{+00}$
	Worst	1.92×10^{-06}	7.62×10^{-12}	1.00×10^{-47}	$2.88 \times 10^{+00}$	1.02×10^{-136}	$0.00 \times 10^{+00}$
	Std	4.63×10^{-07}	1.38×10^{-12}	2.62×10^{-48}	5.30×10^{-01}	2.36×10^{-137}	$0.00 \times 10^{+00}$

Table 2. *Cont.*

Func	Criteria	SMIGWO	MGWO	SPTLBO	ChOA	WChOA	RL-ChOA
F5	Best	$2.56 \times 10^{+01}$	$2.62 \times 10^{+01}$	$2.73 \times 10^{+01}$	$2.76 \times 10^{+01}$	$2.90 \times 10^{+01}$	$2.84 \times 10^{+01}$
	Mean	$2.71 \times 10^{+01}$	$2.73 \times 10^{+01}$	$2.82 \times 10^{+01}$	$2.88 \times 10^{+01}$	$2.90 \times 10^{+01}$	$2.88 \times 10^{+01}$
	Worst	$2.85 \times 10^{+01}$	$2.88 \times 10^{+01}$	$2.87 \times 10^{+01}$	$2.90 \times 10^{+01}$	$2.90 \times 10^{+01}$	$2.90 \times 10^{+01}$
	Std	7.67×10^{-01}	7.32×10^{-01}	3.38×10^{-01}	2.86×10^{-01}	4.91×10^{-05}	1.35×10^{-01}
F6	Best	2.50×10^{-01}	9.55×10^{-01}	8.99×10^{-01}	$2.91 \times 10^{+00}$	$2.00 \times 10^{+00}$	$1.96 \times 10^{+00}$
	Mean	9.75×10^{-01}	$1.54 \times 10^{+00}$	$1.48 \times 10^{+00}$	$3.70 \times 10^{+00}$	$2.33 \times 10^{+00}$	$2.62 \times 10^{+00}$
	Worst	$1.75 \times 10^{+00}$	$2.27 \times 10^{+00}$	$2.21 \times 10^{+00}$	$4.87 \times 10^{+00}$	$2.89 \times 10^{+00}$	$3.33 \times 10^{+00}$
	Std	3.78×10^{-01}	3.95×10^{-01}	2.92×10^{-01}	4.18×10^{-01}	2.04×10^{-01}	3.30×10^{-01}
F7	Best	8.49×10^{-04}	1.65×10^{-04}	2.78×10^{-05}	1.65×10^{-04}	1.37×10^{-05}	1.81×10^{-04}
	Mean	1.81×10^{-03}	8.40×10^{-04}	3.63×10^{-04}	2.80×10^{-03}	1.28×10^{-04}	9.34×10^{-04}
	Worst	3.71×10^{-03}	2.04×10^{-03}	1.24×10^{-03}	2.04×10^{-02}	4.21×10^{-04}	6.91×10^{-03}
	Std	6.75×10^{-04}	5.03×10^{-04}	2.54×10^{-04}	4.04×10^{-03}	1.18×10^{-04}	1.22×10^{-03}

Table 3. Results of multi-modal benchmark functions.

Func	Criteria	SMIGWO	MGWO	SPTLBO	ChOA	WChOA	RL-ChOA
F8	Best	$-5.65 \times 10^{+03}$	$-8.14 \times 10^{+03}$	$-6.26 \times 10^{+03}$	$-5.94 \times 10^{+03}$	$-3.33 \times 10^{+03}$	$-5.93 \times 10^{+03}$
	Mean	$-5.22 \times 10^{+03}$	$-5.74 \times 10^{+03}$	$-4.48 \times 10^{+03}$	$-5.73 \times 10^{+03}$	$-2.58 \times 10^{+03}$	$-5.74 \times 10^{+03}$
	Worst	$-4.58 \times 10^{+03}$	$-3.40 \times 10^{+03}$	$-3.44 \times 10^{+03}$	$-5.64 \times 10^{+03}$	$-2.02 \times 10^{+03}$	$-5.61 \times 10^{+03}$
	Std	$1.60 \times 10^{+02}$	$1.22 \times 10^{+03}$	$6.00 \times 10^{+02}$	$7.11 \times 10^{+01}$	$3.59 \times 10^{+02}$	$7.12 \times 10^{+01}$
F9	Best	5.68×10^{-14}	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$	5.36×10^{-06}	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
	Mean	$3.23 \times 10^{+00}$	1.89×10^{-14}	$0.00 \times 10^{+00}$	$1.19 \times 10^{+01}$	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
	Worst	$1.31 \times 10^{+01}$	5.12×10^{-13}	$0.00 \times 10^{+00}$	$4.72 \times 10^{+01}$	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
	Std	$4.02 \times 10^{+00}$	9.36×10^{-14}	$0.00 \times 10^{+00}$	$1.30 \times 10^{+01}$	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
F10	Best	5.95×10^{-06}	7.99×10^{-15}	8.88×10^{-16}	$2.00 \times 10^{+01}$	8.88×10^{-16}	8.88×10^{-16}
	Mean	$1.74 \times 10^{+01}$	1.44×10^{-14}	3.85×10^{-15}	$2.00 \times 10^{+01}$	4.32×10^{-15}	8.88×10^{-16}
	Worst	$2.01 \times 10^{+01}$	2.22×10^{-14}	4.44×10^{-15}	$2.00 \times 10^{+01}$	4.44×10^{-15}	8.88×10^{-16}
	Std	$6.60 \times 10^{+00}$	3.54×10^{-15}	1.35×10^{-15}	1.31×10^{-03}	6.49×10^{-16}	$0.00 \times 10^{+00}$
F11	Best	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$	6.44×10^{-10}	$0.00 \times 10^{+00}$	$0.00 \times 10^{+00}$
	Mean	4.25×10^{-03}	2.89×10^{-04}	$0.00 \times 10^{+00}$	1.94×10^{-02}	2.69×10^{-04}	$0.00 \times 10^{+00}$
	Worst	2.50×10^{-02}	8.67×10^{-03}	$0.00 \times 10^{+00}$	1.08×10^{-01}	8.06×10^{-03}	$0.00 \times 10^{+00}$
	Std	8.43×10^{-03}	1.58×10^{-03}	$0.00 \times 10^{+00}$	2.92×10^{-02}	1.47×10^{-03}	$0.00 \times 10^{+00}$
F12	Best	1.74×10^{-02}	4.82×10^{-02}	3.19×10^{-02}	2.56×10^{-01}	8.84×10^{-02}	1.44×10^{-01}
	Mean	5.27×10^{-02}	1.19×10^{-01}	5.56×10^{-02}	5.73×10^{-01}	1.70×10^{-01}	2.30×10^{-01}
	Worst	1.55×10^{-01}	6.40×10^{-01}	1.18×10^{-01}	9.69×10^{-01}	2.28×10^{-01}	6.68×10^{-01}
	Std	3.10×10^{-02}	1.06×10^{-01}	2.01×10^{-02}	2.32×10^{-01}	2.97×10^{-02}	9.06×10^{-02}
F13	Best	2.08×10^{-01}	6.49×10^{-01}	6.18×10^{-01}	$2.32 \times 10^{+00}$	$3.00 \times 10^{+00}$	$2.99 \times 10^{+00}$
	Mean	7.03×10^{-01}	$1.06 \times 10^{+00}$	$1.13 \times 10^{+00}$	$2.75 \times 10^{+00}$	$3.00 \times 10^{+00}$	$2.99 \times 10^{+00}$
	Worst	$1.37 \times 10^{+00}$	$1.52 \times 10^{+00}$	$1.72 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$2.99 \times 10^{+00}$
	Std	2.46×10^{-01}	2.19×10^{-01}	2.68×10^{-01}	1.34×10^{-01}	1.18×10^{-04}	1.94×10^{-03}

Table 4. Results of fixed-dimension multi-modal benchmark functions.

Func	Criteria	SMIGWO	MGWO	SPTLBO	ChOA	WChOA	RL-ChOA
F14	Best	9.98×10^{-01}	9.98×10^{-01}	9.98×10^{-01}	9.98×10^{-01}	9.98×10^{-01}	9.98×10^{-01}
	Mean	$4.95 \times 10^{+00}$	$4.46 \times 10^{+00}$	9.98×10^{-01}	$1.03 \times 10^{+00}$	$1.51 \times 10^{+00}$	$1.03 \times 10^{+00}$
	Worst	$1.27 \times 10^{+01}$	$1.27 \times 10^{+01}$	$1.01 \times 10^{+00}$	$2.00 \times 10^{+00}$	$2.98 \times 10^{+00}$	$1.99 \times 10^{+00}$
	Std	$4.24 \times 10^{+00}$	$4.26 \times 10^{+00}$	2.34×10^{-03}	1.83×10^{-01}	5.30×10^{-01}	1.81×10^{-01}

Table 4. *Cont.*

Func	Criteria	SMIGWO	MGWO	SPTLBO	ChOA	WChOA	RL-ChOA
F15	Best	3.08×10^{-04}	3.08×10^{-04}	3.30×10^{-04}	1.22×10^{-03}	1.03×10^{-03}	1.24×10^{-03}
	Mean	6.78×10^{-04}	3.09×10^{-03}	4.89×10^{-04}	1.31×10^{-03}	4.49×10^{-02}	1.32×10^{-03}
	Worst	1.22×10^{-03}	2.04×10^{-02}	7.24×10^{-04}	1.39×10^{-03}	1.19×10^{-01}	1.52×10^{-03}
	Std	4.25×10^{-04}	6.89×10^{-03}	1.17×10^{-04}	5.84×10^{-05}	4.03×10^{-02}	5.65×10^{-05}
F16	Best	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$
	Mean	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.01 \times 10^{+00}$	$-1.03 \times 10^{+00}$
	Worst	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.03 \times 10^{+00}$	$-1.00 \times 10^{+00}$	$-1.03 \times 10^{+00}$
	Std	2.40×10^{-08}	4.54×10^{-07}	9.14×10^{-16}	1.30×10^{-05}	9.31×10^{-03}	1.62×10^{-05}
F17	Best	3.98×10^{-01}	3.98×10^{-01}	3.98×10^{-01}	3.98×10^{-01}	4.10×10^{-01}	3.98×10^{-01}
	Mean	3.98×10^{-01}	3.98×10^{-01}	3.98×10^{-01}	3.98×10^{-01}	9.99×10^{-01}	3.98×10^{-01}
	Worst	3.98×10^{-01}	3.98×10^{-01}	3.98×10^{-01}	4.00×10^{-01}	$3.54 \times 10^{+00}$	4.00×10^{-01}
	Std	1.32×10^{-06}	6.47×10^{-06}	$0.00 \times 10^{+00}$	4.94×10^{-04}	8.06×10^{-01}	4.74×10^{-04}
F18	Best	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$
	Mean	$5.70 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$
	Worst	$8.40 \times 10^{+01}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$	$3.00 \times 10^{+00}$
	Std	$1.48 \times 10^{+01}$	3.10×10^{-05}	2.52×10^{-15}	1.20×10^{-04}	7.84×10^{-05}	2.41×10^{-04}
F19	Best	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.85 \times 10^{+00}$	$-3.86 \times 10^{+00}$
	Mean	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.44 \times 10^{+00}$	$-3.86 \times 10^{+00}$
	Worst	$-3.85 \times 10^{+00}$	$-3.85 \times 10^{+00}$	$-3.86 \times 10^{+00}$	$-3.85 \times 10^{+00}$	$-2.18 \times 10^{+00}$	$-3.85 \times 10^{+00}$
	Std	1.88×10^{-03}	2.80×10^{-03}	3.04×10^{-15}	2.57×10^{-03}	3.93×10^{-01}	1.88×10^{-03}
F20	Best	$-3.32 \times 10^{+00}$	$-3.32 \times 10^{+00}$	$-3.32 \times 10^{+00}$	$-3.31 \times 10^{+00}$	$-2.79 \times 10^{+00}$	$-3.31 \times 10^{+00}$
	Mean	$-3.21 \times 10^{+00}$	$-3.24 \times 10^{+00}$	$-3.30 \times 10^{+00}$	$-2.81 \times 10^{+00}$	$-1.77 \times 10^{+00}$	$-3.28 \times 10^{+00}$
	Worst	$-3.04 \times 10^{+00}$	$-3.07 \times 10^{+00}$	$-3.20 \times 10^{+00}$	$-1.92 \times 10^{+00}$	-6.61×10^{-01}	$-3.23 \times 10^{+00}$
	Std	8.25×10^{-02}	8.04×10^{-02}	4.15×10^{-02}	3.43×10^{-01}	5.81×10^{-01}	2.28×10^{-02}
F21	Best	$-1.02 \times 10^{+01}$	$-1.01 \times 10^{+01}$	$-5.06 \times 10^{+00}$	$-5.04 \times 10^{+00}$	$-2.95 \times 10^{+00}$	$-5.05 \times 10^{+00}$
	Mean	$-8.33 \times 10^{+00}$	$-8.88 \times 10^{+00}$	$-5.06 \times 10^{+00}$	$-4.02 \times 10^{+00}$	$-1.02 \times 10^{+00}$	$-4.81 \times 10^{+00}$
	Worst	$-2.52 \times 10^{+00}$	$-2.63 \times 10^{+00}$	$-5.06 \times 10^{+00}$	-8.81×10^{-01}	-5.50×10^{-01}	-8.82×10^{-01}
	Std	$2.67 \times 10^{+00}$	$2.61 \times 10^{+00}$	1.33×10^{-09}	$1.76 \times 10^{+00}$	5.81×10^{-01}	7.44×10^{-01}
F22	Best	$-1.04 \times 10^{+01}$	$-1.04 \times 10^{+01}$	$-8.83 \times 10^{+00}$	$-5.08 \times 10^{+00}$	$-3.27 \times 10^{+00}$	$-5.07 \times 10^{+00}$
	Mean	$-9.87 \times 10^{+00}$	$-1.02 \times 10^{+01}$	$-5.26 \times 10^{+00}$	$-3.59 \times 10^{+00}$	$-1.16 \times 10^{+00}$	$-5.01 \times 10^{+00}$
	Worst	$-5.09 \times 10^{+00}$	$-5.11 \times 10^{+00}$	$-5.09 \times 10^{+00}$	-9.07×10^{-01}	-5.16×10^{-01}	$-4.89 \times 10^{+00}$
	Std	$1.61 \times 10^{+00}$	9.62×10^{-01}	6.99×10^{-01}	$1.94 \times 10^{+00}$	6.80×10^{-01}	4.65×10^{-02}
F23	Best	$-1.05 \times 10^{+01}$	$-1.05 \times 10^{+01}$	$-1.05 \times 10^{+01}$	$-6.96 \times 10^{+00}$	$-4.06 \times 10^{+00}$	$-5.10 \times 10^{+00}$
	Mean	$-1.02 \times 10^{+01}$	$-1.05 \times 10^{+01}$	$-6.25 \times 10^{+00}$	$-4.60 \times 10^{+00}$	$-1.27 \times 10^{+00}$	$-5.04 \times 10^{+00}$
	Worst	$-5.17 \times 10^{+00}$	$-1.05 \times 10^{+01}$	$-5.13 \times 10^{+00}$	-9.46×10^{-01}	-7.02×10^{-01}	$-4.92 \times 10^{+00}$
	Std	$1.36 \times 10^{+00}$	1.62×10^{-02}	$2.03 \times 10^{+00}$	$1.38 \times 10^{+00}$	6.46×10^{-01}	4.62×10^{-02}

Table 5. Average rankings of the algorithms (Friedman) on fixed-dimension multi-modal benchmark functions.

Algorithm	Ranking
SMIGWO	3.01
MGWO	2.94
SPTLBO	1.65
ChOA	4.35
WChOA	5.25
RL-ChOA	3.80

Table 6. Average rankings of the algorithms (Friedman) on unimodal benchmark test functions.

Algorithm	Ranking
SMIGWO	4.11
MGWO	3.68
SPTLBO	2.71
ChOA	5.64
WChOA	2.45
RL-ChOA	2.41

5.1. Experiments on 23 Widely Used Test Functions

As shown in Table 2, in the unimodal benchmark test functions ($F1$–$F7$): the overall performance of the proposed algorithm RL-ChOA is better than other algorithms on the unimodal benchmark problem. RL-ChOA can perform best compared to other algorithms except for three benchmark test functions ($F5$–$F7$) and find the optimal value in four benchmark test functions ($F1$–$F4$). The SMIGWO achieves the best performance on two benchmark test functions ($F5$ and $F6$). The WChOA obtains the best solution on the one benchmark test function ($F7$). In addition, KEEL software [74] was used for the Friedman rank, and the results are shown in Table 6 RL-ChOA achieved the best place.

As can be observed from Table 3, in the multi-modal benchmark test functions ($F8$–$F13$): RL-ChOA can obtain the best solution on the four benchmark test functions ($F8$–$F11$). SPTLBO can obtain the best solution on the one benchmark test function ($F12$). WChOA can get the best solution on the three benchmark test functions ($F9$, $F11$, and $F12$). SPTLBO can obtain the best solution on the one benchmark test function ($F13$). RL-ChOA outperforms other algorithms overall, and in Table 7, RL-ChOA ranks first in the Friedman rank.

As can be observed from Table 4, in the fixed-dimension multi-modal benchmark functions ($F14$–$F23$): SPTLBO can get the best solution on the six benchmark test functions ($F14$–$F19$). MGWO can get the best solution on the four benchmark test functions ($F16$–$F19$ and $F23$). SMIGWO can obtain the best solution on the five benchmark test function ($F16$, $F17$, $F19$, $F21$, and $F22$); RL-ChOA cannot perform well on these test functions, but RL-ChOA outperforms both original ChOA and WChOA in overall performance. As shown in Table 5, RL-ChOA ranks third in the Friedman rank.

Wilcoxon's rank-sum test was used to verify the significant difference between RL-ChOA and the other five algorithms. The statistical results are shown in Table 8. The results prove that $\alpha = 0.05$, a significant difference can be observed in all cases, and RL-ChOA outperforms 12, 11, 7, 15, and 19 benchmark functions of SMIGWO, MGWO, SPTLBO, ChOA, and WChOA, respectively.

Table 7. Average rankings of the algorithms (Friedman) on multi-modal benchmark functions.

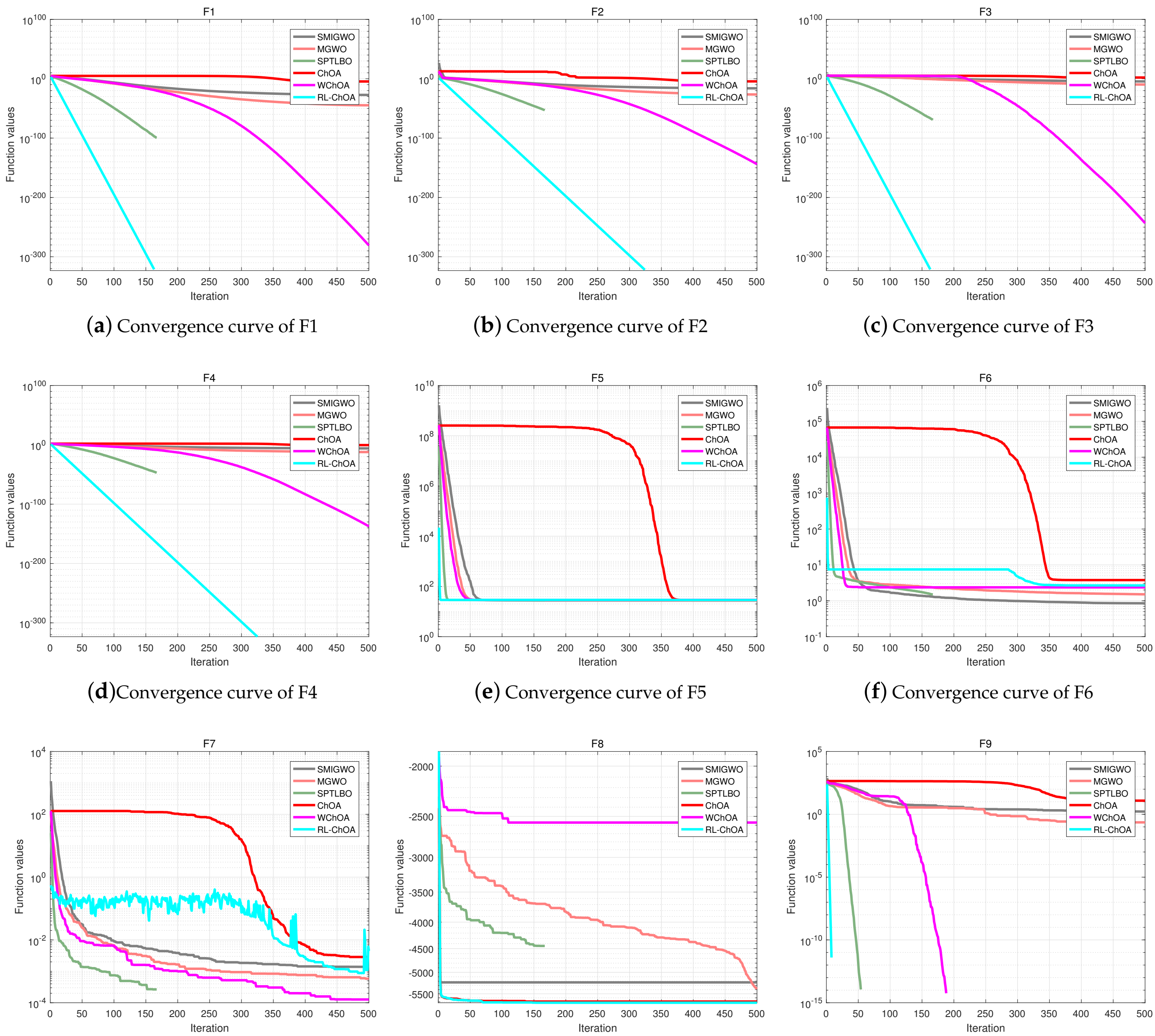
Algorithm	Ranking
SMIGWO	3.75
MGWO	3.52
SPTLBO	2.75
ChOA	4.96
WChOA	3.46
RL-ChOA	2.56

Table 8. Test statistical results of Wilcoxon's rank-sum test.

Benchmark	RL-ChOA vs. SMIGWO			RL-ChOA vs. MGWO			RL-ChOA vs. SPTLBO			RL-ChOA vs. ChOA			RL-ChOA vs. WChOA		
	H	***p*-Value**	**Winner**	**H**	***p*-Value**	**Winner**	**H**	***p*-Value**	**Winner**	**H**	***p*-Value**	**Winner**	**H**	***p*-Value**	**Winner**
F1	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+
F2	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+
F3	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+
F4	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+	1	1.21×10^{-12}	+
F5	1	5.49×10^{-11}	-	1	1.17×10^{-09}	-	1	1.78×10^{-10}	-	1	1.78×10^{-04}	-	1	2.99×10^{-11}	+
F6	1	3.02×10^{-11}	-	1	1.33×10^{-10}	-	1	4.50×10^{-11}	-	1	1.09×10^{-10}	+	1	2.01×10^{-04}	-
F7	1	1.87×10^{-07}	+	0	5.79×10^{-01}	-	1	1.04×10^{-04}	-	1	3.51×10^{-02}	+	1	2.37×10^{-10}	-
F8	1	3.69×10^{-11}	+	0	2.58×10^{-01}	+	1	2.23×10^{-09}	+	0	3.55×10^{-01}	+	1	3.02×10^{-11}	+
F9	1	1.20×10^{-12}	+	0	1.61×10^{-01}	+	0	NaN	=	1	1.21×10^{-12}	+	0	NaN	=
F10	1	1.21×10^{-12}	+	1	3.17×10^{-13}	+	1	8.99×10^{-11}	+	1	1.21×10^{-12}	+	1	1.17×10^{-13}	+
F11	1	5.58×10^{-03}	+	0	3.34×10^{-01}	+	0	NaN	=	1	1.21×10^{-12}	+	0	3.34×10^{-01}	+
F12	1	3.69×10^{-11}	-	1	1.69×10^{-09}	-	1	3.02×10^{-11}	-	1	2.87×10^{-10}	+	1	6.28×10^{-06}	+
F13	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	1	5.57×10^{-10}	-	1	2.79×10^{-11}	+
F14	1	1.02×10^{-05}	+	1	7.29×10^{-03}	+	1	4.31×10^{-08}	+	0	8.19×10^{-01}	=	1	3.20×10^{-09}	+
F15	1	3.02×10^{-11}	-	1	1.11×10^{-06}	+	1	3.02×10^{-11}	-	0	7.17×10^{-01}	-	1	8.89×10^{-10}	+
F16	1	3.02×10^{-11}	=	1	4.08×10^{-11}	=	1	1.99×10^{-11}	=	0	6.31×10^{-01}	=	1	3.02×10^{-11}	+
F17	1	5.49×10^{-11}	=	1	2.87×10^{-10}	=	1	1.21×10^{-12}	=	0	2.58×10^{-01}	=	1	3.02×10^{-11}	+
F18	1	1.17×10^{-05}	+	1	4.44×10^{-07}	=	1	2.78×10^{-11}	=	0	5.40×10^{-01}	=	1	3.85×10^{-03}	=
F19	1	2.37×10^{-10}	=	1	2.03×10^{-09}	=	1	1.22×10^{-11}	=	0	9.94×10^{-01}	=	1	3.02×10^{-11}	+
F20	1	7.96×10^{-03}	+	0	6.63×10^{-01}	+	1	9.79×10^{-05}	-	1	1.17×10^{-09}	+	1	3.02×10^{-11}	+
F21	1	7.12×10^{-09}	-	1	8.35×10^{-08}	-	1	3.02×10^{-11}	-	0	7.96×10^{-01}	+	1	7.39×10^{-11}	+
F22	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	1	1.78×10^{-04}	+	1	3.02×10^{-11}	+
F23	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	1	3.02×10^{-11}	-	0	3.11×10^{-01}	+	1	3.02×10^{-11}	+
+/−/=		12/8/3			11/8/4			7/10/6			15/3/5			19/2/2	

5.2. Convergence Analysis

To analyze the convergence of the proposed RL-ChOA, Figure 6 shows the convergence curves of SMIGWO, MGWO, SPTLBO, ChOA, WChOA, and RL-ChOA with the increasing number of iterations. In Figure 6, the x-axis represents the number of algorithm iterations, and the y-axis represents the optimal value of the function. It can be observed that the convergence speed of RL-ChOA is significantly better than that of other algorithms because the learning strategy, based on light refraction, can enable the optimal individual to find a better position in the solution space and lead other individuals to approach this position quickly. RL-ChOA has apparent advantages in unimodal benchmark functions and multi-modal benchmark functions. However, despite poor performance on the fixed-dimensional multi-modal benchmark function RL-ChOA, it outperforms the original ChOA and the improved ChOA-based WChOA. RL-ChOA is sufficiently competitive with other state-of-the-art heuristic algorithms in terms of overall performance.

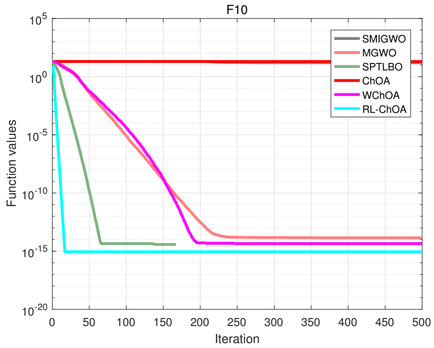


(**a**) Convergence curve of F1 (**b**) Convergence curve of F2 (**c**) Convergence curve of F3

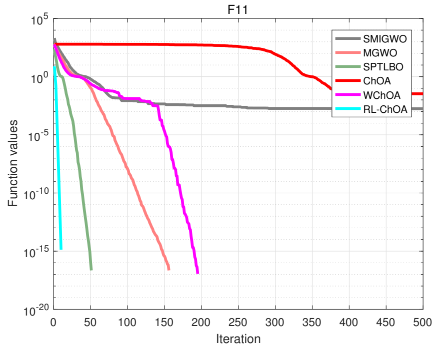
(**d**)Convergence curve of F4 (**e**) Convergence curve of F5 (**f**) Convergence curve of F6

(**g**) Convergence curve of F7 (**h**) Convergence curve of F8 (**i**) Convergence curve of F9

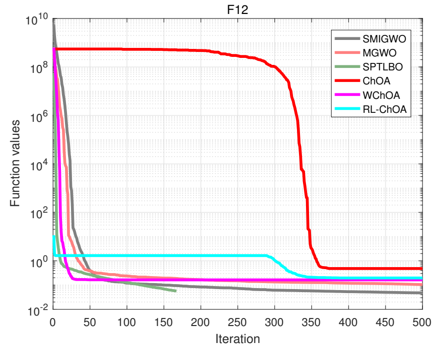
Figure 6. *Cont.*



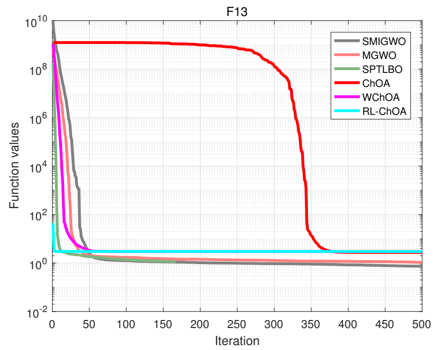
(**j**) Convergence curve of F10



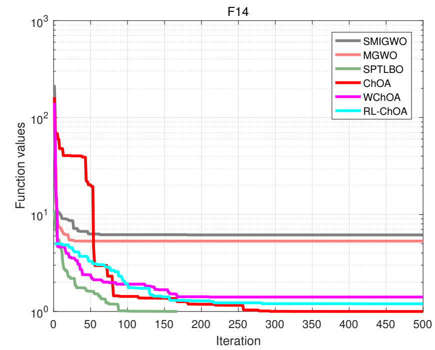
(**k**) Convergence curve of F11



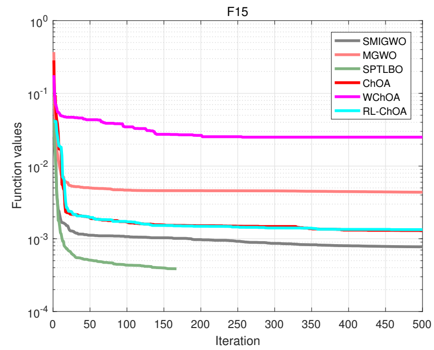
(**l**) Convergence curve of F12



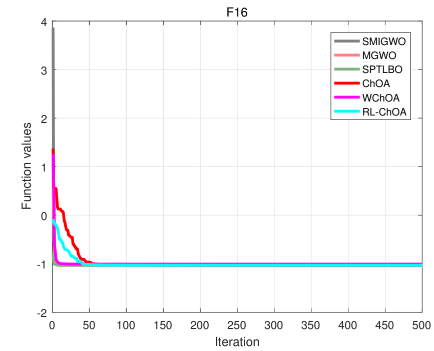
(**m**) Convergence curve of F13



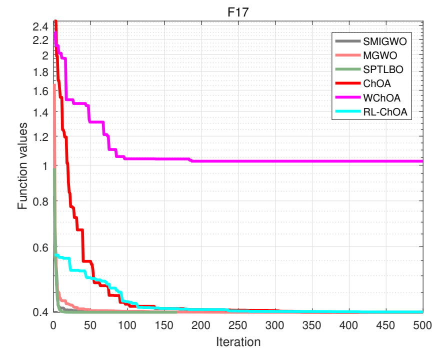
(**n**) Convergence curve of F14



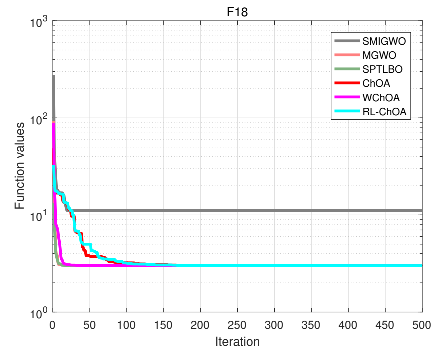
(**o**) Convergence curve of F15



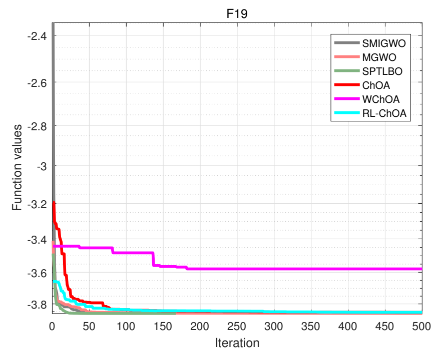
(**p**) Convergence curve of F16



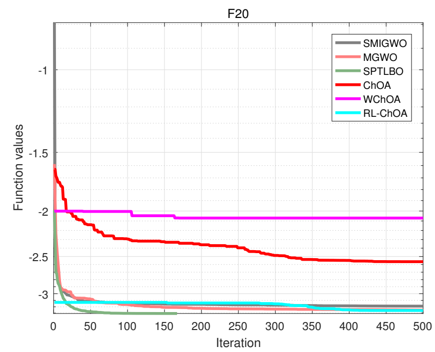
(**q**) Convergence curve of F17



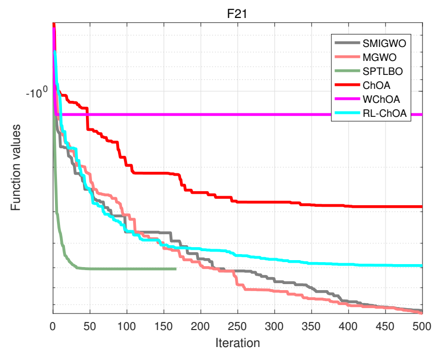
(**r**) Convergence curve of F18



(**s**) Convergence curve of F19

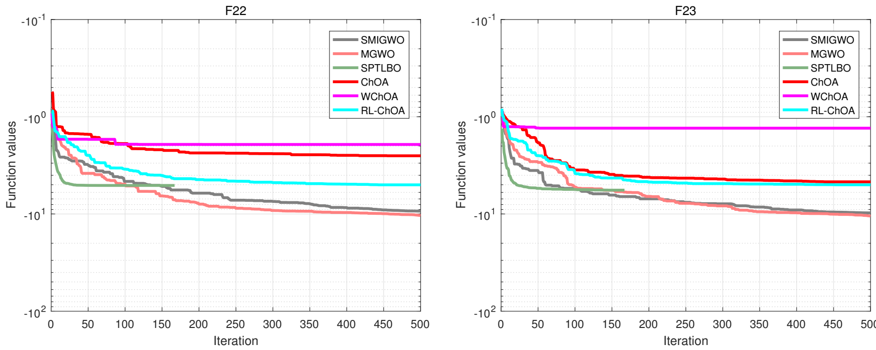


(**t**) Convergence curve of F20



(**u**) Convergence curve of F21

Figure 6. *Cont.*



(**v**) Convergence curve of F22 (**w**) Convergence curve of F23

Figure 6. Convergence figures on test functions $F1$–$F23$.

5.3. Parameter Sensitivity Analysis

The refraction learning of light as described in Section 3.3, and the parameters δ and k in Equation (18), are the keys to improving the performance of RL-ChOA. In this subsection, to investigate the sensitivity of parameters δ and k, a series of experiments are conducted, and it is concluded that RL-ChOA performs best when the values of parameters δ and k are in $[1, 1000]$. We tested RL-ChOA with different δ: 1, 10, 100 and 1000, and different k: 1, 10, 100 and 1000. The benchmark functions are the same as those selected in Section 4.1. The parameter settings are the same as in Section 4.3. Table 5 summarizes the mean and standard deviation of the test function values for RL-ChOA using different δ and k combinations. As shown in Table 9, when parameters δ and k are set to 100 and 100, respectively, RL-ChOA outperforms other parameter settings in most test functions.

5.4. Remarks

According to the above results: (1) RL-ChOA has excellent performance on the unimodal benchmark test functions and the multi-modal benchmark test function because the best individual uses the light refraction-based learning strategy to improve the algorithm's global exploration ability. In addition, Tent chaos sequence was introduced to increase the diversity of the population and improve search accuracy and convergence speed. However, the performance needs to improve on fixed-dimension multi-modal benchmark functions. (2) RL-ChOA outperforms the original ChOA and the improved WChOA based on ChOA, whether on the multi-modal benchmark test functions, the unimodal benchmark test functions, or the fixed-dimension multi-modal benchmark functions.

Table 9. Experimental results of RL-ChOA using different combinations of k and δ.

Function	δ = 1, k = 1		δ = 10, k = 10		δ = 100, k = 100		δ = 1000, k = 1000	
	Mean	**St.dev**	**Mean**	**St.dev**	**Mean**	**St.dev**	**Mean**	**St.dev**
F1	2.9363×10^{-44}	9.7403×10^{-44}	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F2	3.6343×10^{-25}	4.0394×10^{-25}	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F3	1.0063×10^{-24}	5.4903×10^{-24}	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F4	3.1650×10^{-19}	7.3138×10^{-19}	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F5	$2.8968 \times 10^{+01}$	6.6944×10^{-02}	$2.8975 \times 10^{+01}$	5.5369×10^{-02}	$2.8732 \times 10^{+01}$	3.0079×10^{-01}	$2.8974 \times 10^{+01}$	5.6955×10^{-02}
F6	$4.9292 \times 10^{+00}$	3.0194×10^{-01}	$4.2253 \times 10^{+00}$	6.1582×10^{-01}	$4.1277 \times 10^{+00}$	5.4895×10^{-01}	$4.2277 \times 10^{+00}$	7.4070×10^{-01}
F7	2.6141×10^{-04}	2.0053×10^{-04}	4.1955×10^{-04}	1.2170×10^{-03}	2.5403×10^{-04}	2.5588×10^{-04}	3.5305×10^{-04}	3.8636×10^{-04}
F8	$-6.1749 \times 10^{+03}$	$3.5394 \times 10^{+02}$	$-6.0819 \times 10^{+03}$	$3.4058 \times 10^{+02}$	$-6.2563 \times 10^{+03}$	$4.2758 \times 10^{+02}$	$-6.1964 \times 10^{+03}$	$4.3861 \times 10^{+02}$
F9	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F10	1.2375×10^{-14}	4.7283×10^{-15}	8.8818×10^{-16}	$0.0000 \times 10^{+00}$	8.8818×10^{-16}	$0.0000 \times 10^{+00}$	8.8818×10^{-16}	$0.0000 \times 10^{+00}$
F11	9.1245×10^{-03}	2.8466×10^{-02}	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$	$0.0000 \times 10^{+00}$
F12	7.1215×10^{-01}	2.2662×10^{-01}	6.1241×10^{-01}	1.5222×10^{-01}	5.9170×10^{-01}	1.5623×10^{-01}	6.1394×10^{-01}	1.8421×10^{-01}
F13	$2.7715 \times 10^{+00}$	9.6490×10^{-02}	$2.9987 \times 10^{+00}$	4.0780×10^{-04}	$2.9988 \times 10^{+00}$	3.1025×10^{-04}	$2.9989 \times 10^{+00}$	4.7321×10^{-04}
F14	$1.2980 \times 10^{+00}$	6.0103×10^{-01}	$1.0710 \times 10^{+00}$	2.5954×10^{-01}	$1.0123 \times 10^{+00}$	5.8488×10^{-02}	$1.2268 \times 10^{+00}$	5.5093×10^{-01}
F15	1.3967×10^{-03}	8.3618×10^{-05}	1.3964×10^{-03}	9.0421×10^{-05}	1.3763×10^{-03}	7.3362×10^{-05}	1.4112×10^{-03}	7.8338×10^{-05}
F16	$-1.0316 \times 10^{+00}$	4.0525×10^{-05}	$-1.0316 \times 10^{+00}$	2.1053×10^{-05}	$-1.0316 \times 10^{+00}$	3.7042×10^{-05}	$-1.0316 \times 10^{+00}$	2.9538×10^{-05}
F17	3.9821×10^{-01}	3.8502×10^{-04}	3.9820×10^{-01}	4.5274×10^{-04}	3.9822×10^{-01}	2.9042×10^{-04}	3.9816×10^{-01}	2.7264×10^{-04}
F18	$3.0002 \times 10^{+00}$	2.7370×10^{-04}	$3.0003 \times 10^{+00}$	3.9395×10^{-04}	$3.0002 \times 10^{+00}$	3.5075×10^{-04}	$3.0003 \times 10^{+00}$	4.1256×10^{-04}
F19	$-3.8555 \times 10^{+00}$	2.2133×10^{-03}	$-3.8551 \times 10^{+00}$	2.0785×10^{-03}	$-3.8551 \times 10^{+00}$	1.9819×10^{-03}	$-3.8553 \times 10^{+00}$	2.0139×10^{-03}
F20	$-3.2830 \times 10^{+00}$	1.9182×10^{-02}	$-3.2941 \times 10^{+00}$	1.4701×10^{-02}	$-3.2940 \times 10^{+00}$	1.5499×10^{-02}	$-3.2893 \times 10^{+00}$	1.7582×10^{-02}
F21	$-2.9032 \times 10^{+00}$	$2.0892 \times 10^{+00}$	$-3.6353 \times 10^{+00}$	$1.9805 \times 10^{+00}$	$-4.2423 \times 10^{+00}$	$1.5862 \times 10^{+00}$	$-3.3565 \times 10^{+00}$	$2.0556 \times 10^{+00}$
F22	$-2.1130 \times 10^{+00}$	$1.9578 \times 10^{+00}$	$-4.3551 \times 10^{+00}$	$1.5668 \times 10^{+00}$	$-4.4875 \times 10^{+00}$	$1.4273 \times 10^{+00}$	$-4.0791 \times 10^{+00}$	$1.7789 \times 10^{+00}$
F23	$-5.0784 \times 10^{+00}$	2.5185×10^{-02}	$-4.9409 \times 10^{+00}$	7.5430×10^{-01}	$-5.0880 \times 10^{+00}$	1.9410×10^{-02}	$-4.9393 \times 10^{+00}$	7.5488×10^{-01}

6. RL-ChOA Engineering Example Application Analysis

This section aims further to explore the excellent performance of RL-ChOA in practical engineering. Welded beam engineering [59] is discussed in Section 6.1, tension/compression spring optimal design [59] is addressed in summary Section 6.2, and the results are compared with PSO, GA, ChOA, and WChOA.

6.1. Welded Beam Design Problem

The structure of the welded beam [59] is shown in Figure 7. The goal of structural design optimization of welded beams is to minimize the total cost under certain constraints. The design variables are: $x_1(h)$, $x_2(l)$, $x_3(t)$ and $x_4(b)$, and the constraints are the shear stress τ, the bending stress σ on the beam, the buckling critical load Pc and the tail of the beam, and beam deflection δ. The objective function and constraints are as follows:

Consider:

$$\begin{aligned}&\mathbf{x} = [x_1\ x_2\ x_3\ x_4] = [h\ l\ t\ b];\\ &\text{Minimize} f(\mathbf{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2).\end{aligned} \tag{23}$$

Subject to:

$$\begin{aligned}
g_1(\mathbf{x}) &= \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} - \tau_{max} \le 0;\\
g_2(\mathbf{x}) &= \frac{6PL}{x_3^2x_4} - \sigma_{max} \le 0;\\
g_3(\mathbf{x}) &= x_1 - x_4 \le 0;\\
g_4(\mathbf{x}) &= 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \le 0;\\
g_5(\mathbf{x}) &= 0.125 - x_1 \le 0;\\
g_6(\mathbf{x}) &= \frac{4PL^3}{Ex_3^3x_4} - \delta_{max} \le 0;\\
g_7(\mathbf{x}) &= P - \frac{4.013Ex_3x_4^3}{6L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \le 0;
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
&\tau' = \frac{P}{2x_1x_2}, \tau'' = MRJ, M = P(L + \frac{x_2}{2}),\\
&J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\},\\
&R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, P = 6000lb,\\
&L = 14in, E = 30 \times 10^6psi, G = 12 \times 10^6psi,\\
&\tau_{max} = 13,600psi, \sigma_{max} = 30,000psi,\\
&\delta_{max} = 0.25in.
\end{aligned}$$

with

$$0.1 \le x_1, x_4 \le 2.0 \ and \ 0.1 \le x_2, x_3 \le 10.0$$

Table 10 shows the average value of each optimization result of the five algorithms for solving the welded beam design problem. Each algorithm is independently run 50 times, the maximum number of iterations is 1000, and the population size is 30.

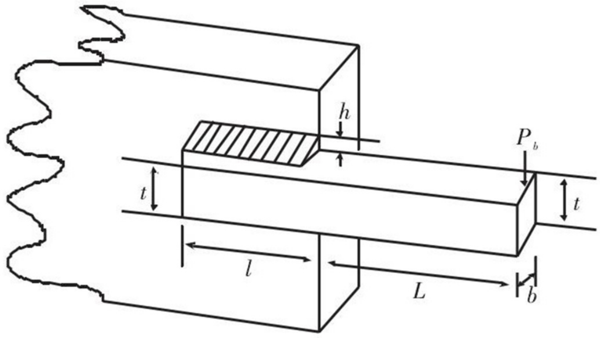


Figure 7. Design principle of the welded beam.

Table 10. Comparison of welded beam design.

Algoritm	h	l	t	b	Mean
GA	0.2455	6.1986	8.1264	0.2247	2.4414
PSO	0.2024	3.5441	9.048	0.2057	1.7315
ChOA	0.2214	3.5358	8.9115	0.2127	1.7737
WChOA	0.2021	3.4707	9.0366	0.2057	1.7249
RL-ChOA	0.2036	3.4715	9.0286	0.2058	1.7227

It can be observed from Table 10 that RL-ChOA shows superior performance on the optimization problem of welded beams. Although the single variable is not optimal, it is optimal in terms of the total cost.

6.2. Extension/Compression Spring Optimization Design Problem

The optimization goal of the extension/compression spring design problem [59] is to reduce the weight of the spring, the schematic diagram of which is shown in Figure 8. Constraints include subject to minimum deviation (g_1), shear stress (g_2), shock frequency (g_3), outer diameter limit (g_4), and decision variables include wire diameter d, average coil diameter D, and effective coil number P, $f(x)$ to minimize spring weight. The objective functions and constraints of these three optimization variables are as follows:

$$\begin{aligned}
&\text{Consider}: \\
&\mathbf{x} = [x_1\ x_2\ x_3] = [d\ D\ P]; \\
&\text{Minimize} f(\mathbf{x}) = x_1^2 x_2 (2 + x_3);
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\text{subject to}: \\
&g_1(\mathbf{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0; \\
&g_2(\mathbf{x}) = \frac{4x_2^2 - x_1 x_2}{12{,}566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} \le 0; \\
&g_3(\mathbf{x}) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0; \\
&g_4(\mathbf{x}) = \frac{x_1 + x_2}{1.5} - 1 \le 0; \\
&\text{with} \\
&0.05 \le x_1 \le 2.0, 0.25 \le x_2 \le 1.3, \mathit{and}\ 2.0 \le x_3 \le 15.0.
\end{aligned} \tag{26}$$

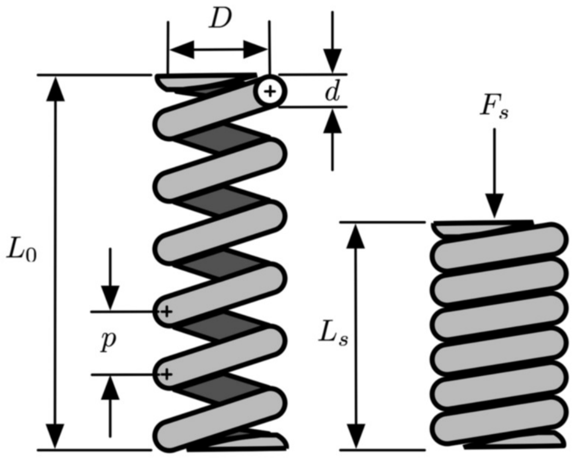


Figure 8. Extension/compression spring structure.

Table 11 shows the average value of each optimization result of the five algorithms for solving the extension/compression spring design optimization problem. Each algorithm is independently run 50 times, the maximum number of iterations is 1000, and the population size is 30.

Table 11. Comparison of tension/compression spring design.

Algoritm	*d*	*D*	*p*	Mean
GA	0.0524	0.3521	11.5980	0.032
PSO	0.0500	0.3104	14.998	0.0131
ChOA	0.0500	0.3159	14.3999	0.0132
WChOA	0.0508	0.3142	14.4314	0.0134
RL-ChOA	0.0613	0.6259	4.1889	0.0129

It can be observed from Table 11 that RL-ChOA shows superior performance on the optimization problem of welded beams.

7. Conclusions

This paper proposes an improved ChOA based on the learning strategy of light reflection (RL-ChOA) based on the original ChOA. An improved Tent chaos sequence was introduced to increase the diversity of the population and improve search accuracy and convergence speed. In addition, a new refraction learning strategy is proposed and introduced into the original ChOA based on the principle of physical light refraction to help the population jump out of the local optimum. First, it uses 23 widely used benchmark functions and Wilcoxon's rank-sum test verify that RL-ChOA has better optimization performance and stronger robustness. Our source code is available on https://github.com/zhangquan123-zq/RL-ChOA (accessed on 20 April 2022).

In the future, some work needs to be conducted to further improve the algorithm's performance. An adaptive parameter strategy is designed to improve the optimization performance of RL-ChOA. The optimization effectiveness of RL-ChOA is tested on some difficult benchmark functions [75–77]. Some more multi-dimensional engineering problems are used to verify the engineering validity of RL-ChOA.

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