Functions :- function from $A \to B$ is a relation $R_{A \to B}$ such that
(i) for every element of the domain is mapped to exactly one element.

→ domain

$A = \{1,2,3,5\}$ $B = \{4,6,9\}$

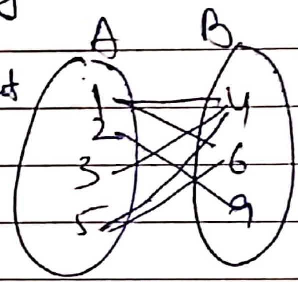
R is a relation from A to B

$R = \{(x,y)$ difference bt x and y is odd $x \in A, y \in B\}$.

$R = \{(1,4)\ (1,6)\ (2,9)\ (3,4)\ (5,4)\ (5,6)\}$

Domain is : first element in the odder pair $\{1,2,3,5\}$ it is set A.

Codomain is : 2nd element in the odder pair $(4,6,9)$ it is set B



Range : Range also 2nd element in the pair $\{4,6,9\} = B$. (But any other element is take in B. that is not connected to A it is not Range).

Ex : Find the domain and Range of the relation :

$R = \{(x,y) ;\ y = x + \frac{6}{x}$ where $x, y \in N \ \& \ x < 6\}$

Sol

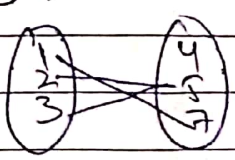
$R : N \to N$

$R = \{(1,7)\ (2,5)\ (3,5)\}$.

x=1 $y = 1 + \frac{6}{1} = 7$ x=2 $2+\frac{6}{2}$ x=3, $3+\frac{6}{3}$ x=4 $4+\frac{6}{4}$ 5, $\frac{6}{5}$ It is not no no So $4, 5$ is not consider

x=5

$5 + \frac{6}{5}$ It is not natural no



Domain $= \{1,2,3\}$

Range $\{5,7\}$

Codomain = 2nd set Completely so N

$R = \{(x,y)\ x,y \in W,\ x^2+y^2=25\}$, find the domain codomain and Range.

3^2+4^2
$9+16$

$R = \{(0,5),(5,0),$
$(3,4)\ (4,3)\}$

W stands for whole nos

Domain $= \{0,3,4,5\}$

Range: $\{5,0,4,3\}$

Codomain $=$ W (total 2nd set).

$1^2+y^2=25$
$y^2=24$
$y=\sqrt{24}$ ✗

$2^2+y^2=25$
$y^2=21$ ✗

$3^2+y^2=25$
$y^2=25-9$
$=16$
$y=4$.

$6^2+y^2=25$
That is not give whole nos.

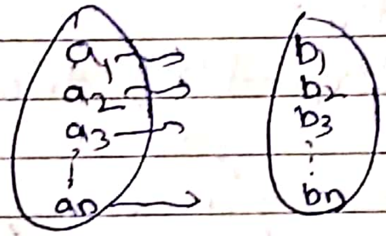
Functions

Def: Let X and Y be any two sets. A relation from X to Y is called a function if for every $x \in X$ there is a unique $y \in Y$ such that $(x, y) \in f$.

The number of relations possible from A to B where $|A| = m$, $|B| = n$?

2^{mn}.

How many fns possible from set A to set B
The no of fns from set A to set B where $|A| = m$, $|B| = n$?



n^m

→ Check the relation is function or not

1) $f(x) = x^2$ is a fn or not.
$R \to R$
$x \to x^2$ $\quad 1-1,\ 2-2^2,\ 3-3^2 - - - -$
all elements are mapping so it is function.

ii) $f(x) = \sqrt{x}$ is a fn or not.
$x \to \sqrt{x}$
$4 \to \sqrt{4}$ It is $+2$ and -2
So 4 is mapped to $+2$ and -2.

$\sqrt{-2} \Rightarrow$ It is not real no. It is Imaginary no
so some elements have multiple Images.
Some elements no Image. So it is not a fn.

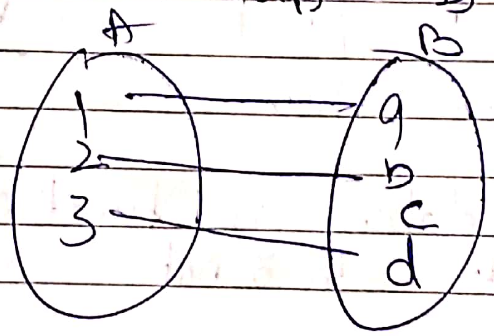
Functions are 3 type

one-one (Injective)
onto fn (surjective)
one-one correspondence (Bijective).

Def <u>one-one (Injective) :-</u>

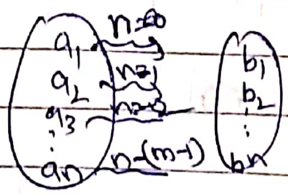
A mapping $f : X \to Y$ is called one-one (Injective) if distinct elements of X are mapped onto distinct elements of Y.

In other words,
f is one-one if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
or equivalently:
$f(x_1) = f(x_2)$ then $x_1 = x_2$.



every element in A mapped with different element in B

How many number of 1-1 fns from A to B where $|A| = m$, $|B| = n$.



$Ta_1 \to Ta_2 \to Ta_2$ ----
$|Ta_1| = n$. $|Ta_2| = (n-1)$

$\therefore n \times (n-1) \times (n-2) \cdots \times (n-(m-1))$

$= {}^nP_m$.

so total number of 1-1 fn from A to B is
${}^nP_m$

Camlin Page
Date / /

example :- find any given fn is one-one or not

(1) $f(x) = x^2$ $R \to R$

if two Images x_1 and x_2.
$f(x_1) = f(x_2)$ then $x_1 = x_2$

$f(x_1) = f(x_2)$
$x_1^2 = x_2^2 \Rightarrow x_1^2 - x_2^2 = 0 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$
$\therefore x_1 = +x_2$
$x_1 = -x_2$.
so for x_1 two value i.e x_2 and $-x_2$
$\therefore$ It is not 1-1 fn.

(2) $f(x) = x^2 + 2x$.

$f(x_1) = f(x_2)$
$x_1^2 + 2x_1 = x_2^2 + 2x_2$
$x_1^2 - x_2^2 + 2x_1 - 2x_2 = 0$
$x_1^2 - x_2^2 = -2(x_1 - x_2)$
$x_1 + x_2 = -2$
$\therefore$ It is not 1-1 fn.

$-2x_1 + 2x_2$
$-2(x_1 - x_1)$

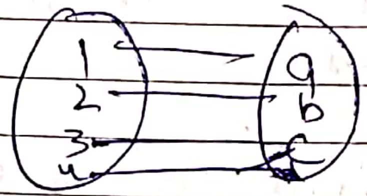
(3) $f(x) = 2x + 3$ $R \to R$.
$f(x_1) = f(x_2)$
$2x_1 + 3 = 2x_2 + 3$
$2x_1 = 2x_2$
$x_1 = x_2$.
$\therefore$ It is 1-1 fn.

Scanned by CamScanner

onto fn or Surjective :-

Def : A function is said to be onto or Surjection if every element of B has a pre Image in A.

onto fn is fn where every element in the codomain has pre Image.



ex find give fn is onto ?

→ $f(x) = 2x+3$

$R \to R$

$f(x) = y$

$2x+3 = y \Rightarrow x = \frac{y-3}{2}$ so every y in x.

y is any real no which is codomain so x becomes real no.

$\therefore f(x)$ is onto fn.

$f(x) = 2x+3$

$N \to R$

$y = 2x+3$

$x = \frac{y-3}{2}$

if $y = 1 \Rightarrow x = -1$, it is not part of natural no,

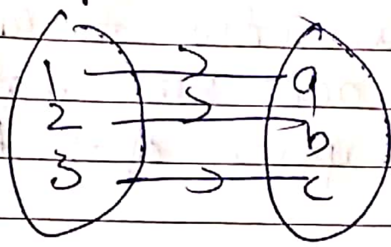
if $y = 6 \Rightarrow x = 1.5$ "

So it is not onto.

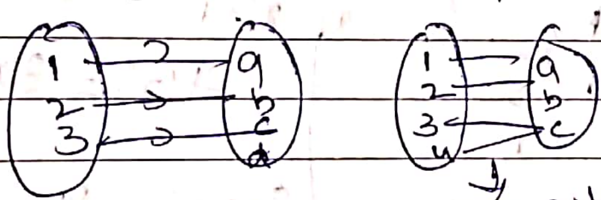
1-1 correspondence (Bijective):-

A fn f is said to be one to one or a bijection, if f is one-one and onto.

f is one to one, then different elements of A have different Images and every element of B has a pre Image in A, so the no of elements of A is equal to the no of elements of B.



necessary condition for 1-1 fn is

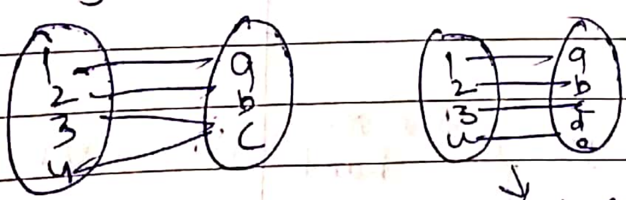


It is not one-one.

so

$$|A| \le |B| \quad -① $$

neccessary condition for onto fn



It is not onto.

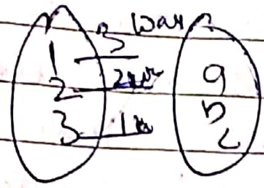
so
nec condition is $|A| \ge |B| \quad -②$

from ① & ②

so Bijection satisfies $|A| \le |B|$ and
$|A| \ge |B|$

$\therefore |A| = |B|$.

The no of bijections from set A to set B where $|A| = n$, $|B| = n$?



$3 \times 2 \times 1$ -

n elements $\underline{n!}$ ways.

Inverse fn:- Let $f: A \to B$ be a 1-1 and onto mapping. Then the mapping $f^{-1}: B \to A$ which associates to each element $b \in B$, the element $a \in A$, such that $f(a) = b$ is called Inverse mapping of $f: A \to B$.

Ex:-

Find the Inverse function of ① $f(x) = x^3+1$ ② $f(x) = 3x+2 / 4x-1$, ③ $f(x) = x^3+1$

$f(x) = \frac{x-3}{2}$

Step ① replace $y = f(x)$

$y = \frac{x+3}{2}$

Step ② Swap x with y (interchange)

$x = \frac{y-3}{2}$

Step ③ solve for y: { y in one hand

$2x = y-3$

$\therefore y = 2x+3$

here x & y are swap

Step ④ $\boxed{\therefore y = f^{-1}(x)}$

$\boxed{f^{-1}(x) = 2x+3}$

$f(x) = 3x+2 / 4x-1$

Step 1: $y = \frac{3x+2}{4x-1}$

②: $x = \frac{3y+2}{4y-1}$

③ $4xy - x = 3y+2$

$y(4x-3) = x+2$

$y = \frac{x+2}{4x-3}$

Step 4: $y = f^{-1}(x)$

$f^{-1}(x) = \frac{x+2}{4x-3}$.

Scanned by CamScanner

Composite functions :- let $f: X \to Y$ and $g: Y \to Z$ be two fns. The composite relation gof such that

$$gof = \{(x,z) \mid (x \in X) \wedge (z \in Z) \wedge (\exists y (y \in Y \wedge y = f(x) \wedge z = g(y))\}$$

is called the composition of function or relative product of function f and g.

gof is called the left composition of g with f.

Ex: Let $f(x) = x+2$, $g(x) = x-2$, and $h(x) = 3x$ for $x \in R$ where R is the set of real nos. find gof, fog, fof, gog, foh, hog, hof and $fohog$.

$f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$

Sol:

→ $gof = g(f(x)) = g(x+2)$
$= x-2+2 = x$

→ $fog = f(g(x)) = f(x-2)$
$= x-2+2 = x$

→ $fof = f(f(x)) = f(x+2)$
$= x+2+2 = x+4$

→ $gog(x) = g(g(x)) = g(x-2)$
$= x-2-2 = x-4$.

→ $foh = f(h(x))$
$= f(3x)$
$= 3(x+2) = 3x+2$

$hog = h(g(x))$
$= h(x-2) = 3x-2$

$hof = h(f(x))$
$= h(x+2) = 3x+2$

$fohog = f(h(g(x)))$
$= f(h(x-2))$
$= f(3x-2)$
$= 3(x+2)-2$
$= 3x+4$