ASSIGNMENT-3

For the flow in a 2D lid-driven square cavity, the governing equations in non-dimensional form is given by

Stream function equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\omega$$

Applying the finite difference discretization to replace all derivatives with the corresponding central difference expressions, we get the discretized equations as

$$\psi_{i,j}^{l+1} = \frac{1}{2(1+\beta^2)}\{\psi_{i+1,j}^{l} + \psi_{i-1,j}^{l+1} + \beta^2(\psi_{i,j+1}^{l} + \psi_{i,j-1}^{l+1}) + \Delta x^2 \omega_{i,j}\}$$

Vorticity equation:

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2}\right)$$

where $u = \frac{\partial \omega}{\partial y}$ and $v = -\frac{\partial \omega}{\partial x}$

Applying the finite difference discretization to replace all derivatives with the corresponding central difference expressions, we get the discretized equations as

$$\omega_{i,j} = \frac{1}{2(1+\beta^2)}[\{1 - (\Psi_{i,j+1} - \Psi_{i,j-1})\frac{\beta Re}{4}\}\omega_{i+1,j}^{l} + \{1 + (\Psi_{i,j+1} - \Psi_{i,j-1})\frac{\beta Re}{4}\}\omega_{i-1,j}^{l+1} + \{1 + (\Psi_{i+1,j} - \Psi_{i-1,j})\frac{Re}{4\beta}\}\beta^2 \omega_{i,j+1}^{l} + \{1 - (\Psi_{i+1,j} - \Psi_{i-1,j})\frac{Re}{4\beta}\}\omega_{i,j-1}^{l+1}]$$

B.C. for Ψ : Ψ is constant for all four boundaries. So, take $\psi = 0$ on all the walls.

B.C. for ω (Grid: $1 \le i \le i_{max}, 1 \le j \le j_{max}$):

Left Wall: $\omega_{1,j} = \frac{-2}{\Delta x^2}(\Psi_{2,j} - \Psi_{1,j})$ for $2 \le j \le j_{max-1}$

Right Wall: $\omega_{i_{max},j} = \frac{-2}{\Delta x^2}(\Psi_{m-1,j} - \Psi_{m,j})$ for $2 \le j \le j_{max-1}$

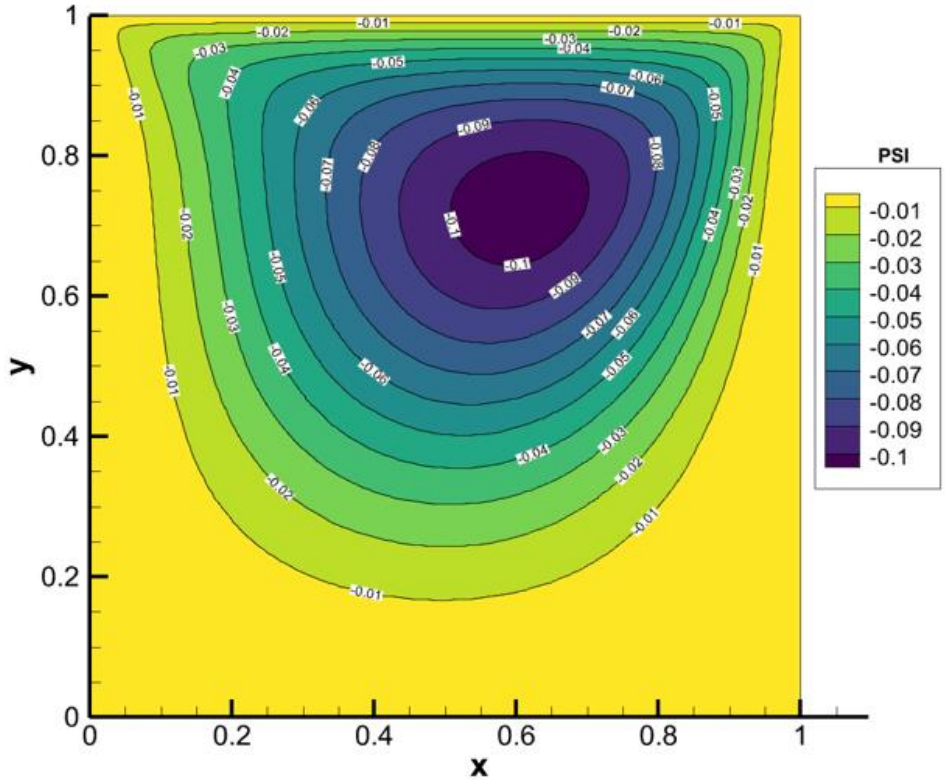
Bottom Wall: $\omega_{i,1} = \frac{-2}{\Delta y^2}(\Psi_{i,2} - \Psi_{i,1})$ for $2 \le j \le i_{max-1}$

Top wall: $\omega_{i,j_{max}} = \frac{-2}{\Delta y^2}(\Psi_{i,n-1} - \Psi_{i,n} + u\,\Delta\, y)$ for $2 \le j \le i_{max-1}$

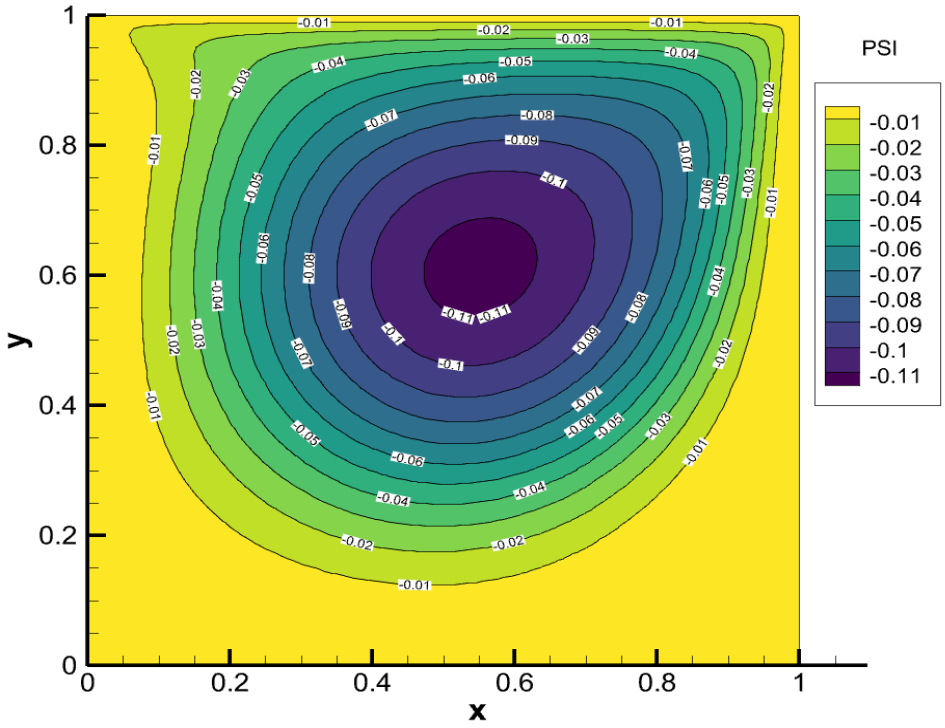
RESULTS :

Streamlines Contour:

Re = 100

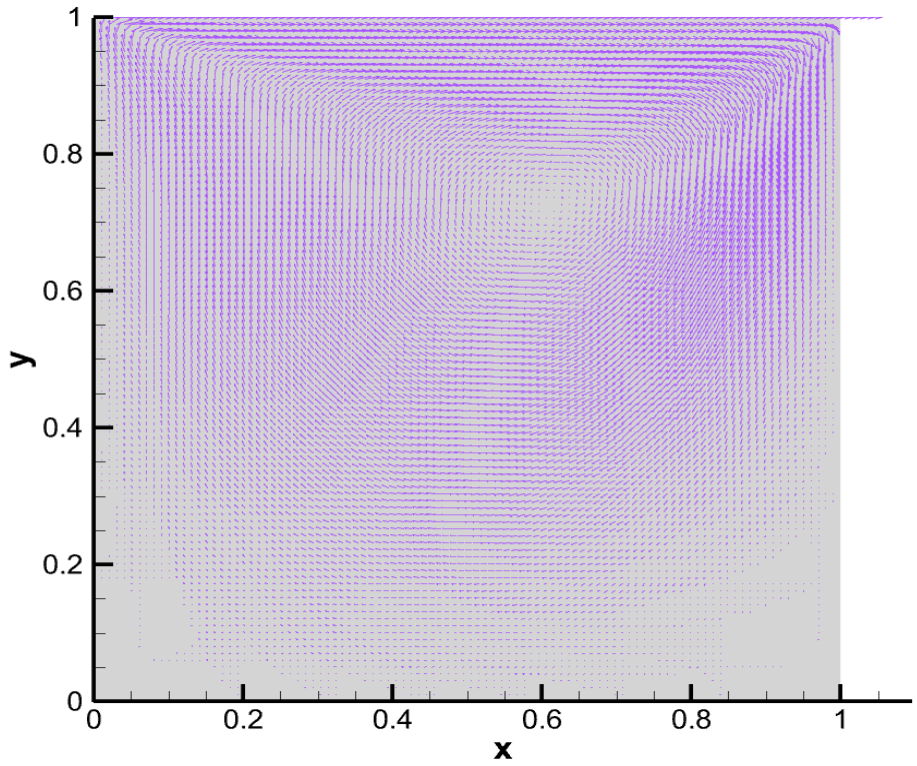


Re = 400



velocity vectors:

Re = 100



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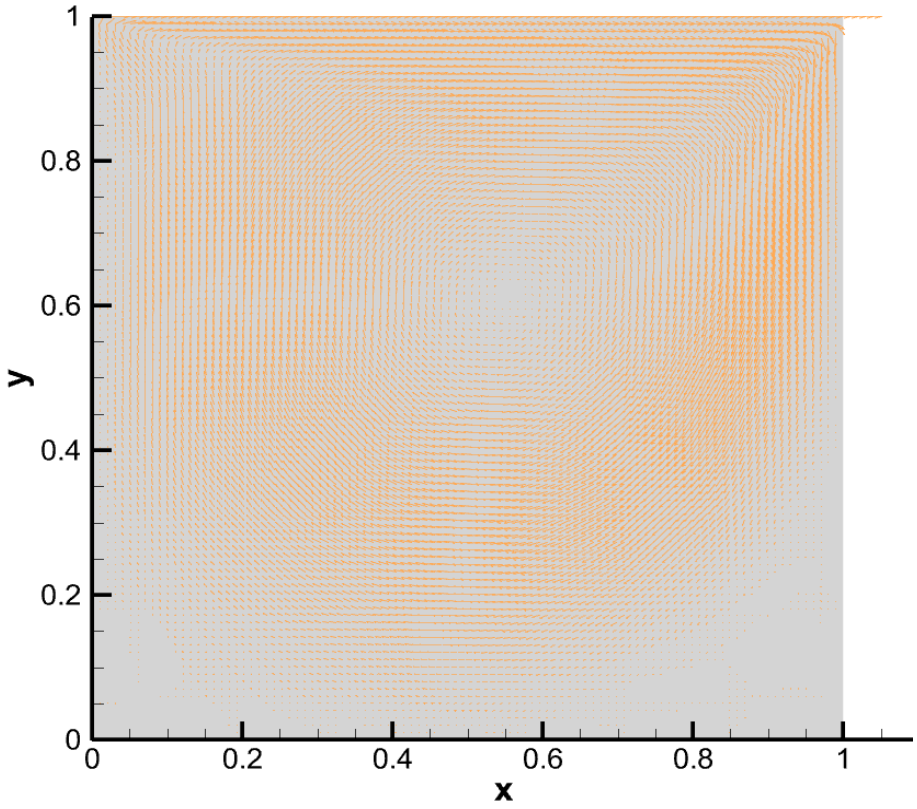
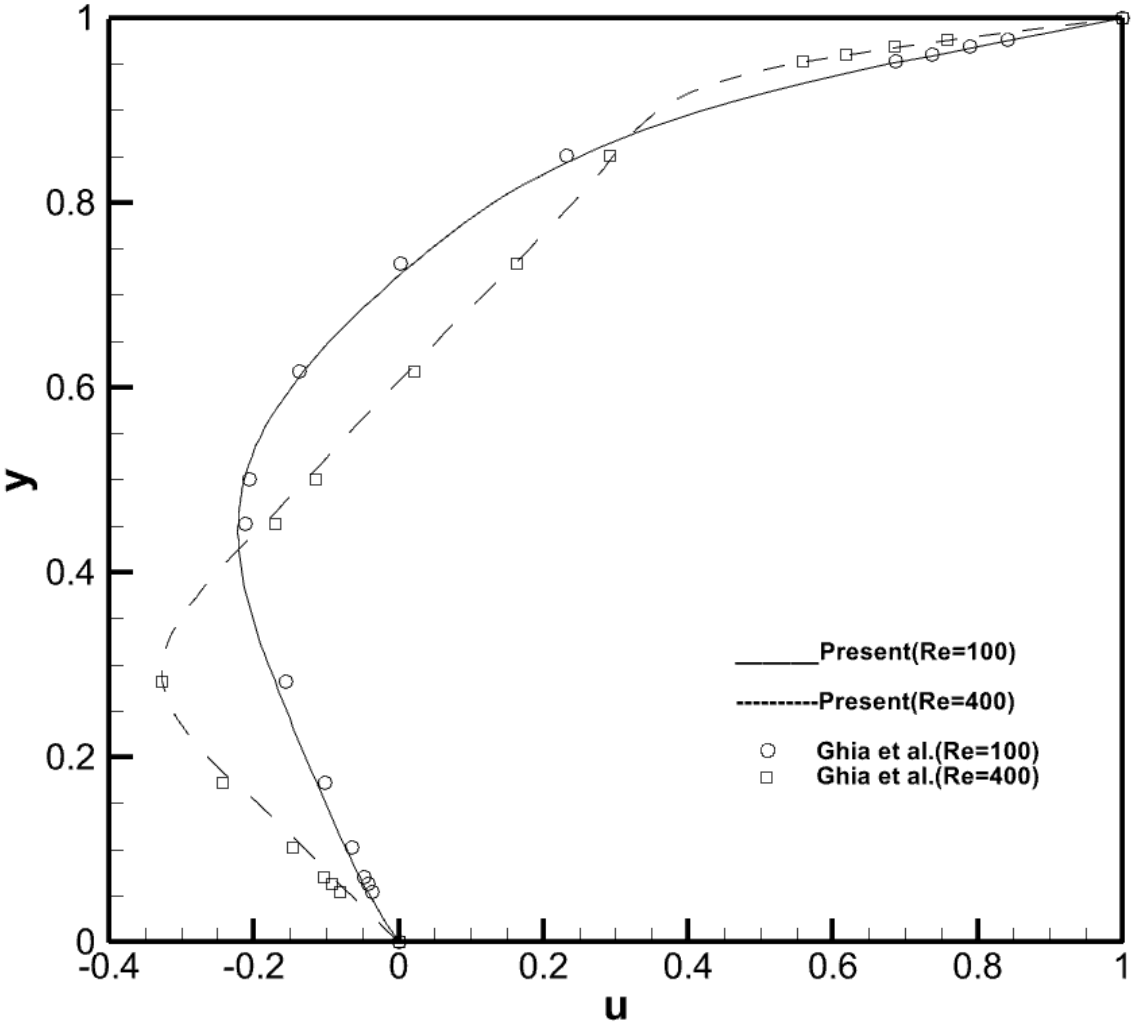


u-velocity profile along the vertical centreline of the square cavity



v-velocity profile along the horizontal centreline of the square cavity