

Postfix Expression

Postfix Expression

- Infix expression is the form AOB
 - A and B are numbers or also infix expression
 - O is operator $(+, -, *, /)$
- Postfix expression is the form ABO
 - A and B are numbers or also postfix expression
 - O is operator $(+, -, *, /)$

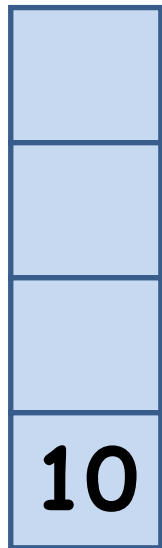
From Postfix to Answer

- The reason to convert infix to postfix expression is that we can compute the answer of postfix expression easier by using a stack.

From Postfix to Answer

Ex: 10 2 8 * + 3 -

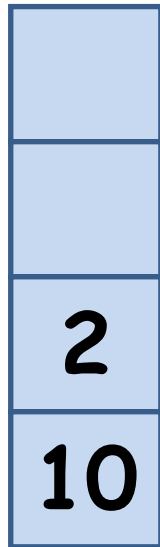
- First, push(10) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

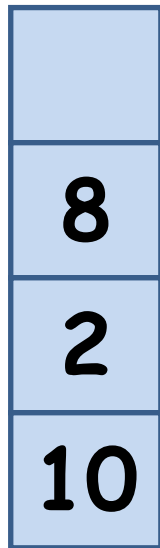
- Then, push(2) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

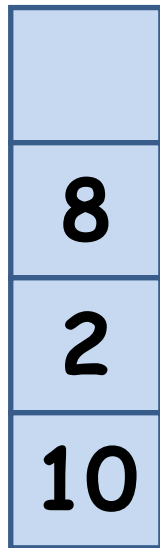
- Push(8) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

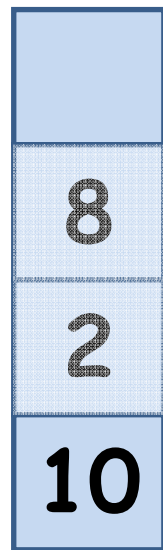
- Now we see an operator *, that means we can get a new number by calculation



From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get a new number by calculation
- Pop the first two numbers

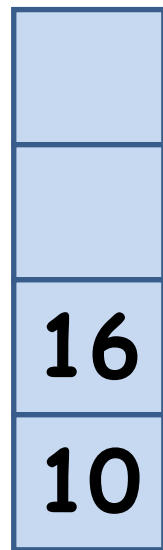


$2 * 8 = 16$

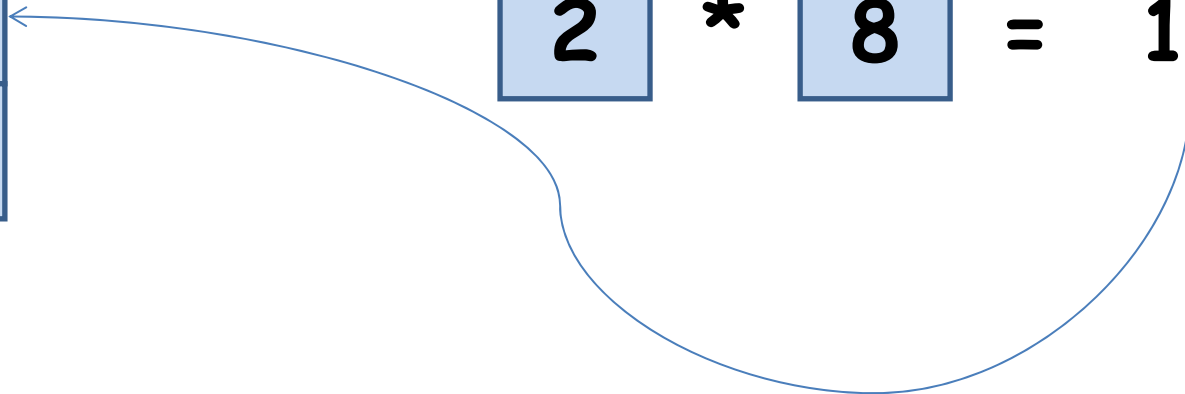
From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get a new number by calculation
- Push the new number back



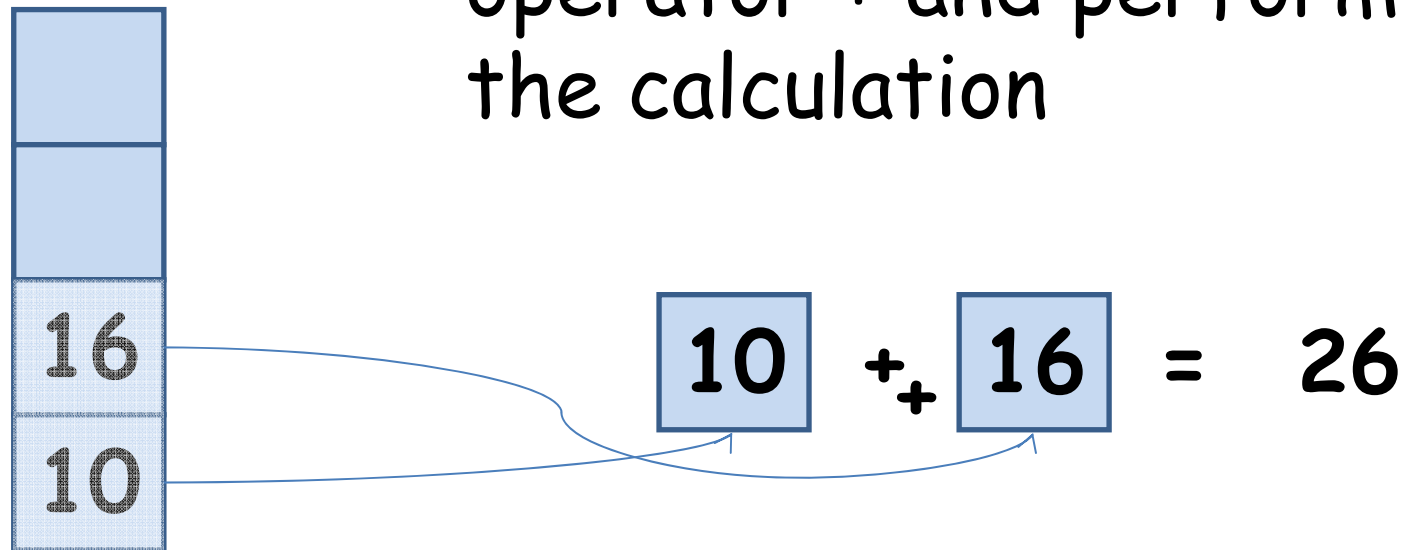
$$\boxed{2} * \boxed{8} = 16$$



From Postfix to Answer

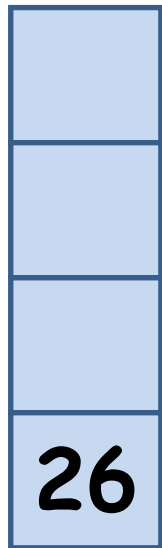
Ex: 10 2 8 * + 3 -

- Then we see the next operator + and perform the calculation



From Postfix to Answer

Ex: 10 2 8 * + 3 -



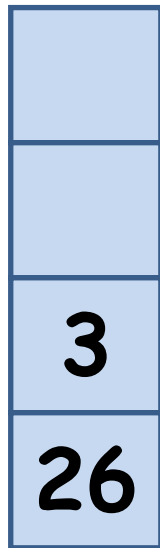
- Then we see the next operator + and perform the calculation
- Push the new number back

$$\boxed{10} + \boxed{16} = 26$$

From Postfix to Answer

Ex: 10 2 8 * + 3 -

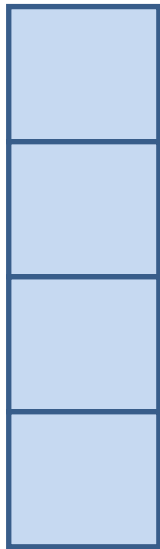
- We see the next number 3
- Push (3) into the stack



Compute the Answer

Ex: 10 2 8 * + 3 -

- The last operation

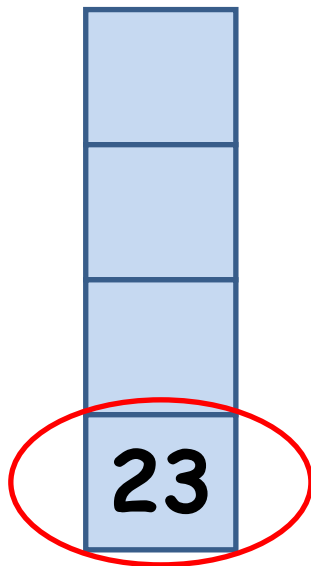


$$\boxed{26} - \boxed{3} = 23$$

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- The last operation



$$\boxed{26} - \boxed{3} = \boxed{23}$$

answer!

From Postfix to Answer

- Algorithm: maintain a stack and scan the postfix expression from left to right
 - If the element is a number, push it into the stack
 - If the element is a operator O , pop twice and get A and B respectively. Calculate BOA and push it back to the stack
 - When the expression is ended, the number in the stack is the final answer

Transform Infix to Postfix

- Now, we have to design an algorithm to transform infix expression to postfix

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators
 - The parentheses must be added according to the priority of operations.
 - The priority of operator $*$ and $/$ is higher than those of operation $+$ and $-$
 - If there are more than one equal-priority operators, we assume that the left one's priority is higher than the right one's
 - This is called left-to-right parsing.

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators (cont.)
 - For example, to add parentheses for the expression $10 + 2 * 8 - 3$,
 - we first add parenthesis to $2 * 8$ since its priority is highest in the expression.
 - Then we add parenthesis to $10 + (2 * 8)$ since the priorities of $+$ and $-$ are equal, and $+$ is on the left of $-$.
 - Finally, we add parenthesis to all the expression and get $((10 + (2 * 8)) - 3)$.

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators (cont.)
 - The computation order of expression $((10 + (2 * 8)) - 3)$ is:
 - $2 * 8 = 16$ $\rightarrow ((10 + 16) - 3)$
 - $10 + 16 = 26$ $\rightarrow (26 - 3)$
 - $26 - 3 = 23$ $\rightarrow 23$

Transform Infix to Postfix

- Simplify the problem, how if there are only $+/-$ operators?

Transform Infix to Postfix

- Simplify the problem, how if there are only +/− operators?
- The leftmost operator will be done first
 - Ex: $10 - 2 + 3 \rightarrow 8 + 3 \rightarrow 11$

Transform Infix to Postfix

- Simplify the problem, how if there are only $+/-$ operators?
- Algorithm: maintain a stack and scan the postfix expression from left to right
 - When we get a number, output it
 - When we get an operator O , pop the top element in the stack if the stack is not empty and then push(O) into the stack

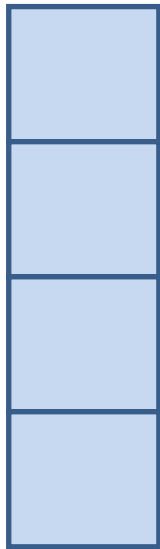
Transform Infix to Postfix

- Simplify the problem, how if there are only $+/-$ operators?
- Algorithm: maintain a stack and scan the postfix expression from left to right
 - When we get a number, output it
 - When we get an operator O , pop the top element in the stack if the stack is not empty and then push(O) into the stack
 - When the expression is ended, pop all the operators remain in the stack

Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

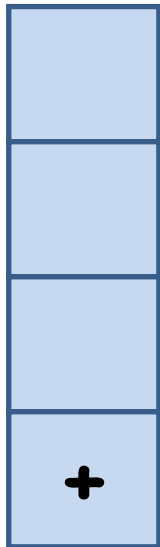
- We see the first number 10, output it



Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

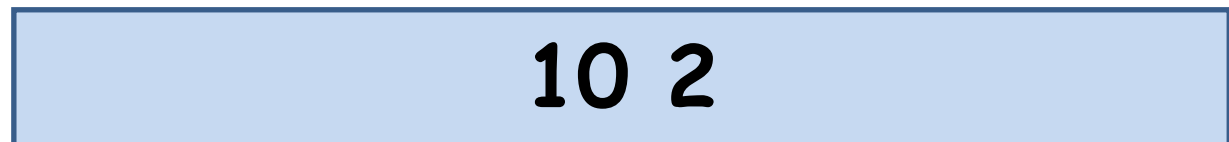
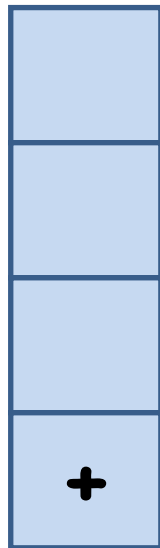
- We see the first operator $+$, push($+$) into the stack because at this moment the stack is empty



Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

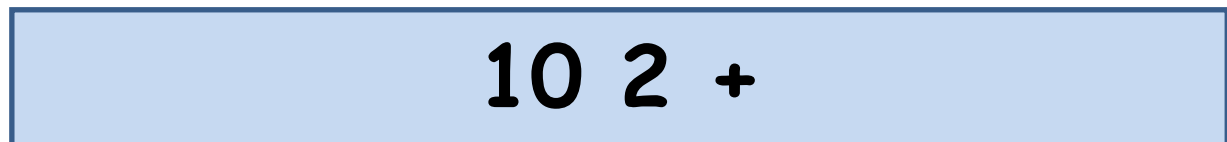
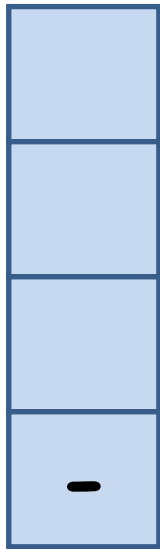
- We see the number 2,
output it



Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

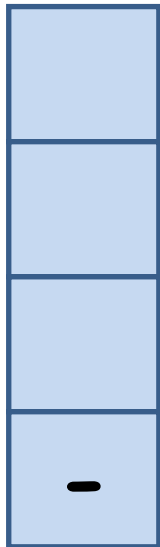
- We see the operator $-$, pop the operator $+$ and push $-$ into the stack



Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

- We see the number 8,
output it

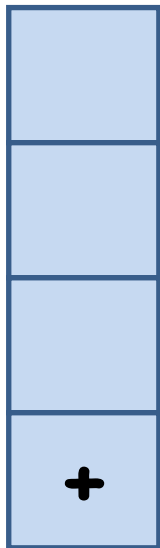


10 2 + 8

Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

- We see the operator $+$, pop the operator $-$ and push($+$) into the stack

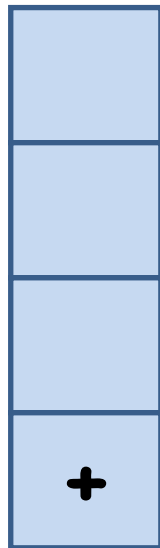


10 2 + 8 -

Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

- We see the number 3,
output it

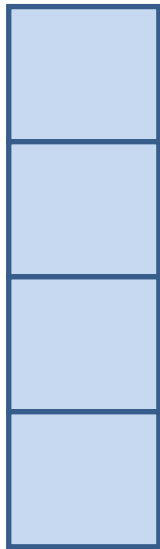


10 2 + 8 - 3

Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$

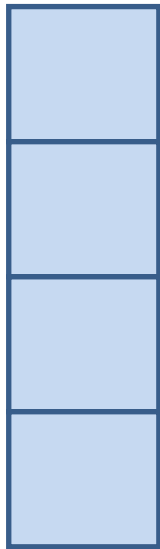
- We come to the end of the expression, then we pop all the operators in the stack



10 2 + 8 - 3 +

Transform Infix to Postfix

Ex: $10 + 2 - 8 + 3$



- When we get an operator, we have to push it into the stack and pop it when we see the next operator.
- The reason is, we have to "wait" for the second operand of the operator

Transform Infix to Postfix

- How to solve the problem when there are operators $+$, $-$, $*$, $/$?

Transform Infix to Postfix

- Observation 2: scan the infix expression from left to right, if we see higher-priority operator after lower-priority one, we know that the second operand of the lower-priority operator is an expression
 - Ex: $a + b * c = a + (b * c) \rightarrow a \text{ } b \text{ } c \text{ } * \text{ } +$
 - That is, the expression $b \text{ } c \text{ } *$ is the second operand of the operator “+”

Transform Infix to Postfix

- So, we modify the algorithm to adapt the situation

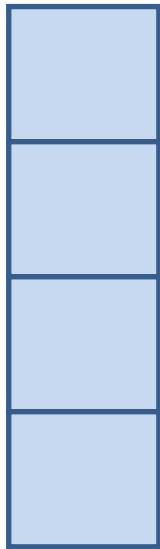
Transform Infix to Postfix

- Algorithm: maintain a stack and scan the postfix expression from left to right
 - When we get a number, output it
 - When we get an operator O , pop the top element in the stack until there is no operator having higher priority than O and then push(O) into the stack
 - When the expression is ended, pop all the operators remain in the stack

Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

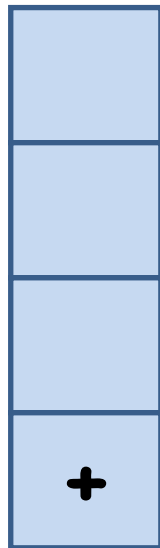
- We see the first number 10, output it



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

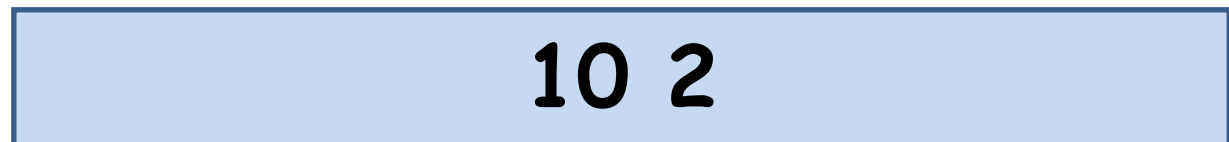
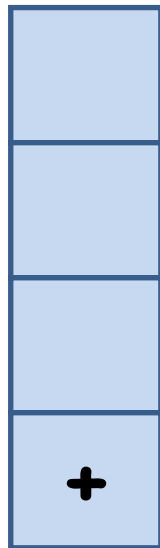
- We see the first operator $+$, push it into the stack



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

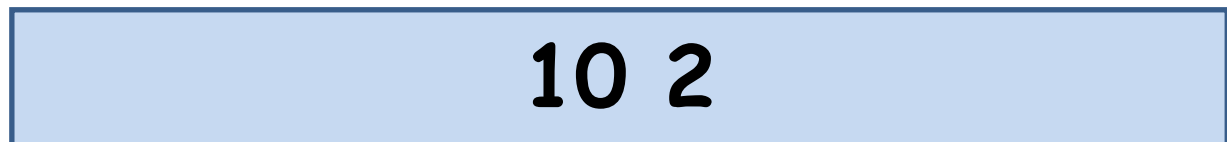
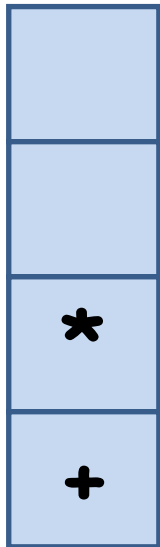
- We see the number 2,
output it



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

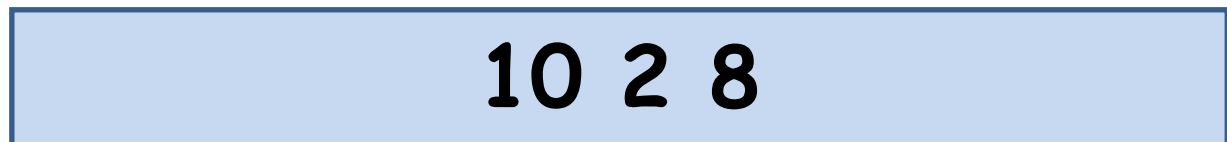
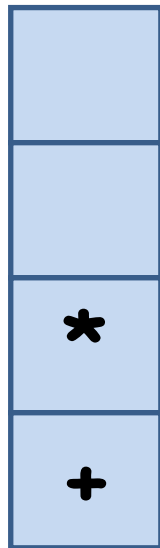
- We see the operator $*$, since the top operator in the stack, $+$, has lower priority than $*$, push($*$)



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

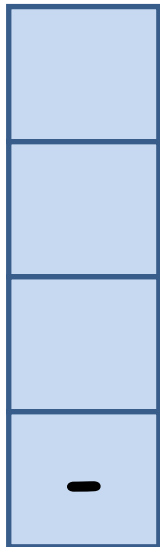
- We see the number 8,
output it



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

- We see the operator -, because its priority is lower than *, we pop. Also, because + is on the left of it, we pop +, too. Then we push(-)

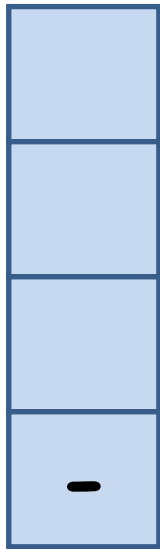


10 2 8 * +

Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

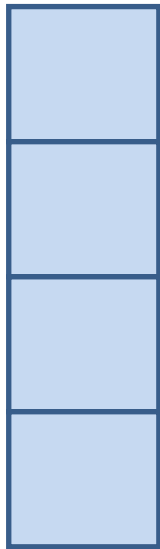
- We see the number 3,
output it



Transform Infix to Postfix

Ex: $10 + 2 * 8 - 3$

- Because the expression is ended, we pop all the operators in the stack



10 2 8 * + 3 -