Modelling the Relationship Between Blood Alcohol Concentration (BAC) and Breath Alcohol Concentration (BrAC)

Objective

This model evaluates the linear relationship between the concentration of ethanol in blood (BAC, in g/dL) and in exhaled breath (BrAC, in mg/L) using a partition ratio, often referred to as the blood-to-breath ratio (BBR).

The general model:

$$BAC = R \times BrAC$$

Where:

- R: Blood-to-breath ratio, typically 2100:1 (i.e., 1 mL of blood contains the same amount of alcohol as 2100 mL of breath).

The objective is to quantify and predict BAC based on BrAC measurements using a mathematical expression.

Assessment

We are assessing:

- The linear relationship and fit of this model using experimental data.
- The degree of correlation (R^2) between measured BAC and BrAC.
- How accurately BrAC can be used to estimate BAC.
- The potential variability or error in using breath analyzers to estimate blood alcohol levels.

Assumptions

- Ethanol diffuses from the blood into the alveoli and is exhaled in a predictable ratio.
- The partition ratio (BBR) is constant across individuals (usually taken as 2100).

- Temperature, breathing pattern, and timing post-consumption are not significantly distorting measurements.
- Measurement devices (blood and breath analyzers) are calibrated and accurate.
- Linear relationship exists in the physiological range of ethanol concentrations.

Principle

Ethanol in the bloodstream is volatile, meaning it readily evaporates and diffuses into the alveolar air (the deep lung air sacs) in the lungs. When exhaled deeply, the last portion of the breath comes from this alveolar region and this is the air that breathalyzers analyze.

This process relies on Henry's Law, which states:

At a constant temperature, the concentration of a gas dissolved in a liquid (like ethanol in blood) is proportional to the partial pressure of that gas in the air above the liquid (like ethanol vapor in breath).

Using this principle, breathalyzers measure the ethanol vapor in our breath and, applying the partition ratio, they estimate our Blood Alcohol Concentration (BAC).

Possible Errors

- **Physiological variability**: Individual partition ratios may vary from 1700 to 2400.
- **Measurement errors**: Device calibration.
- **Sampling errors**: Incomplete alveolar air sample or contamination.
- **Time-lag effect**: Delay between drinking and distribution of alcohol

Linear Expression

The Blood Alcohol Concentration (BAC) and Breath Alcohol Concentration (BrAC) follow a linear relationship given by:

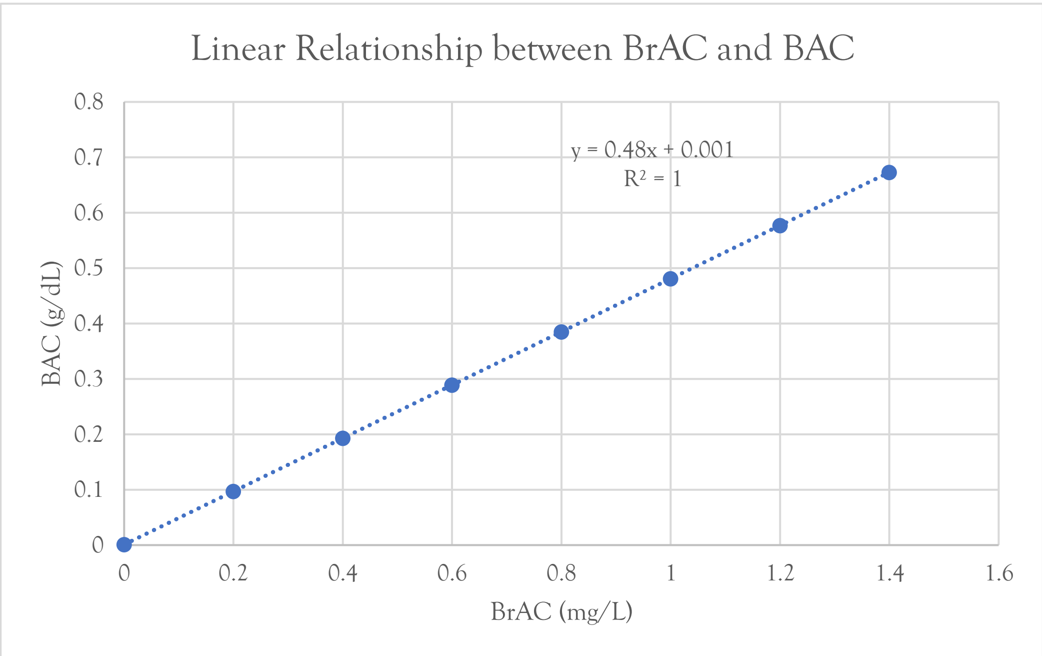
$$\boldsymbol{BAC\ (g/dL) = m \cdot BrAC\ (mg/L) + b}$$

Plot illustrating the relationship

Simulating data points for BrAC (mg/L) from 0 to 1.5 (typical range), and applying the model.

BrAC (mg/L)	BAC (g/dL)
0.0	0.001
0.2	0.097
0.4	0.193
0.6	0.289
0.8	0.385
1.0	0.481
1.2	0.577
1.4	0.673

Generating the plot and calculating R^2.



Fitted Equation and R^2 Value

Fitted Equation:

$$Y = mX + C$$

$$BAC = 0.48 \cdot BrAC + 0.001$$

Where:

BAC = Blood Alcohol Concentration, measured in g/dL

BrAC = Breath Alcohol Concentration, measured in mg/L

- 0.48 = Slope of the line
  → This tells us that for every 1 mg/L increase in BrAC, BAC increases by 0.48 g/dL
- 0.001= Y-intercept
  → Theoretically, this is the BAC when BrAC = 0. It's likely due to slight instrument bias or baseline correction.

R^2 Value:

- $R^2 = 1.000$
  → Means a perfect fit — all data points lie exactly on the line.

Prediction:

For example, BAC at BrAC of 0.78 mg/L can be estimated as:

$$BAC = 0.48 \cdot BrAC + 0.001$$

$$BAC = (0.48 \cdot 0.78) + 0.001$$

$$BAC = 0.3744 + 0.001$$

$$BAC = 0.3754\ g/dL$$

Final Assessment

This model demonstrates a strong linear correlation between BrAC and BAC under controlled conditions. The standard conversion factor (2100:1) works well in estimating blood alcohol concentration from breath samples, which supports the use of breathalyzers in legal and clinical settings.

Limitations of the Model and Process

- **Individual Variation**: The true blood-to-breath ratio may vary from 1300:1 to 3100:1 across individuals.
- **External Influences**: Recent alcohol consumption, mouth alcohol, or certain health conditions can skew BrAC readings.
- **Non-linearity at Extremes**: At very low or very high alcohol concentrations, the linear model might not hold well.
- **Time Dependence**: BAC and BrAC can vary based on metabolism, time since drinking, and absorption rates.
- **Temperature Sensitivity**: Breathalyzer readings are temperature-sensitive, a small increase in breath temperature can overestimate BAC.