BMS INSTITUTE OF TECHNOLOGY & MANAGEMENT

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DEPARTMENT OF ELECTRONICS & TELECOMMUNICATION ENGINEERING

Academic Year 2021-22

REPORT

On

"POISSON DISTRIBUTION USING MATLAB"

Submitted By

USN	Name of the Student	Marks Awarded Max Marks: 05
1BY20ET048	S VARSHA	
Signature of faculty		

Course: Engineering Statistics and Linear Algebra

Course Code: 18EC44

Course Outcome: Perform in a group to simulate the different types of application related to engineering statistics and linear algebra

Under the guidance of

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Asst. Professor, Dept. of ETE

INTRODUCTION

- Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.
- A discrete random variable X is said to have a Poisson distribution, with parameter $\lambda > 0$, if it has a probability mass function given by

$$P_k = \frac{\lambda^k e^{-\lambda}}{k!}$$

  λ is the mean number of occurrences in the interval

  e is Euler's number ~2.71828

- PDF is given by

$$f_X(x) = \sum_{k=0}^{\infty} P_k \, \delta(x - k)$$

- CDF is given by

$$F_X(x) = \sum_{k=0}^{\infty} P_k \, u(x - k)$$

- The Poisson distribution is an appropriate model if the following assumptions are true:

1. k is the number of times an event occurs in an interval (k = 0, 1, 2, ...)
2. Events occur independently.
3. The average rate at which events occur is independent of any occurrences.
4. Two events cannot occur at exactly the same instant.

If these conditions are true, then k is a Poisson random variable, and the distribution of k is a Poisson distribution.

DERIVATION

Mean is given by

$$\mu_x = \sum_{k=0}^{\infty} k\, P_k$$

$$\mu_x = \sum_{k=0}^{\infty} k\, \frac{\lambda^k\, e^{-\lambda}}{k!}$$

n = 0 term is zero

$$\mu_x = e^{-\lambda} \sum_{k=1}^{\infty} k\, \frac{\lambda^k}{k!}$$

$$\mu_x = \lambda e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}$$

Consider m=k-1

$$\mu_x = \lambda e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^m}{m!}$$

By taylor series expansion

$$\mu_x = \lambda e^{-\lambda} e^{\lambda}$$

$$\mu_x = \lambda$$

To find **variance**

$$E[x^2] = \sum_{k=0}^{\infty} k(k-1)\frac{\lambda^k}{k!} e^{-\lambda}$$

n = 0 and n = 1 terms are zero

$$E[x^2] = \lambda^2 e^{-\lambda} \sum_{k=2}^{\infty} \frac{\lambda^{k-2}}{(k-2)!}$$

Consider m = k-2

$$E[x^2] = \lambda^2 e^{-\lambda} \sum_{m=2}^{\infty} \frac{\lambda^m}{m!} + \lambda$$

By taylor series expansion

$$E[x^2] = \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$$

$$E[x^2] = \lambda^2 + \lambda$$

$$\sigma^2 = E[x^2] - (\mu_x)^2$$

$$\sigma^2 = \lambda^2 + \lambda - \lambda^2$$

$$\sigma^2 = \lambda$$

$$\sigma = \sqrt{\lambda}$$

Hence the **standard deviation** of a Poisson distribution is equal to the square root of its mean.

PROGRAM 1

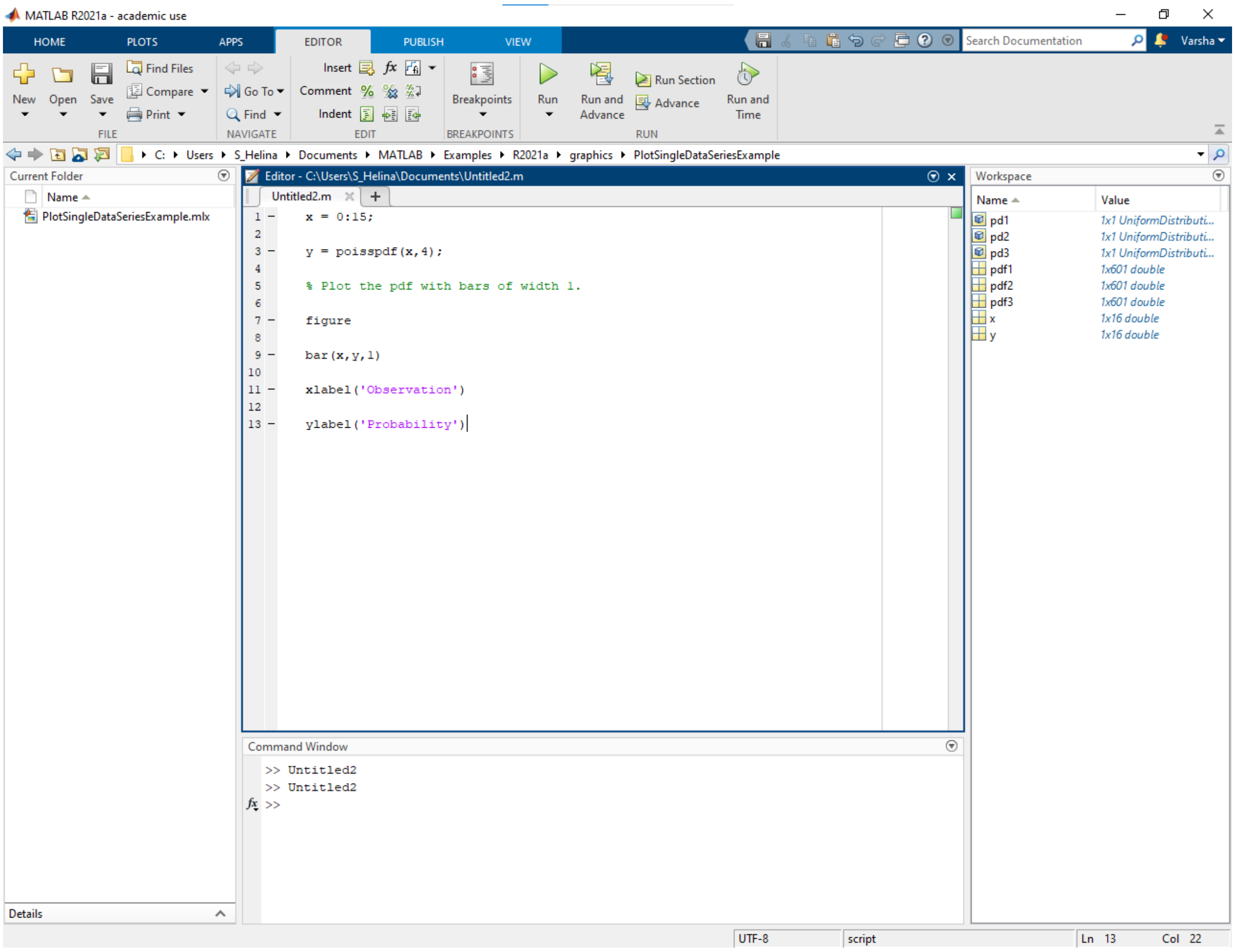
Probability Density Function of Poisson Distribution

Compute Poisson Distribution PDF (λ= 4)

```
x = 0:15;

y = poisspdf(x,4);

% Plot the pdf with bars of width 1.

figure

bar(x,y,1)

xlabel('Observation')

ylabel('Probability')
```

- In this program PDF of the poisson distribution for mean λ= 4 is plotted using the function `poisspdf(x,4)` in MATLAB.
- The PDF is plotted as a bar chart using the function `bar(x,y,1)` and is given labels using the `xlabel` and `ylabel` functions.

OUTPUT



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Breakpoints
BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
RUN
C: ▸ Users ▸ S_Helina ▸ Documents ▸ MATLAB ▸ Examples ▸ R2021a ▸ graphics ▸ PlotSingleDataSeriesExample
Current Folder
Name
PlotSingleDataSeriesExample.mlx
Details
Editor - C:\Users\S_Helina\Documents\Untitled2.m
Untitled2.m
x = 0:15;
y = poisspdf(x,4);
% Plot the pdf with bars of width 1.
figure
bar(x,y,1)
xlabel('Observation')
ylabel('Probability')
Command Window
>> Untitled2
>> Untitled2
>>
Workspace
Name
Value
pd1
1x1 UniformDistributi...
pd2
1x1 UniformDistributi...
pd3
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pdf1
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pdf2
1x601 double
pdf3
1x601 double
x
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y
1x16 double
UTF-8
script
Ln 13
Col 22

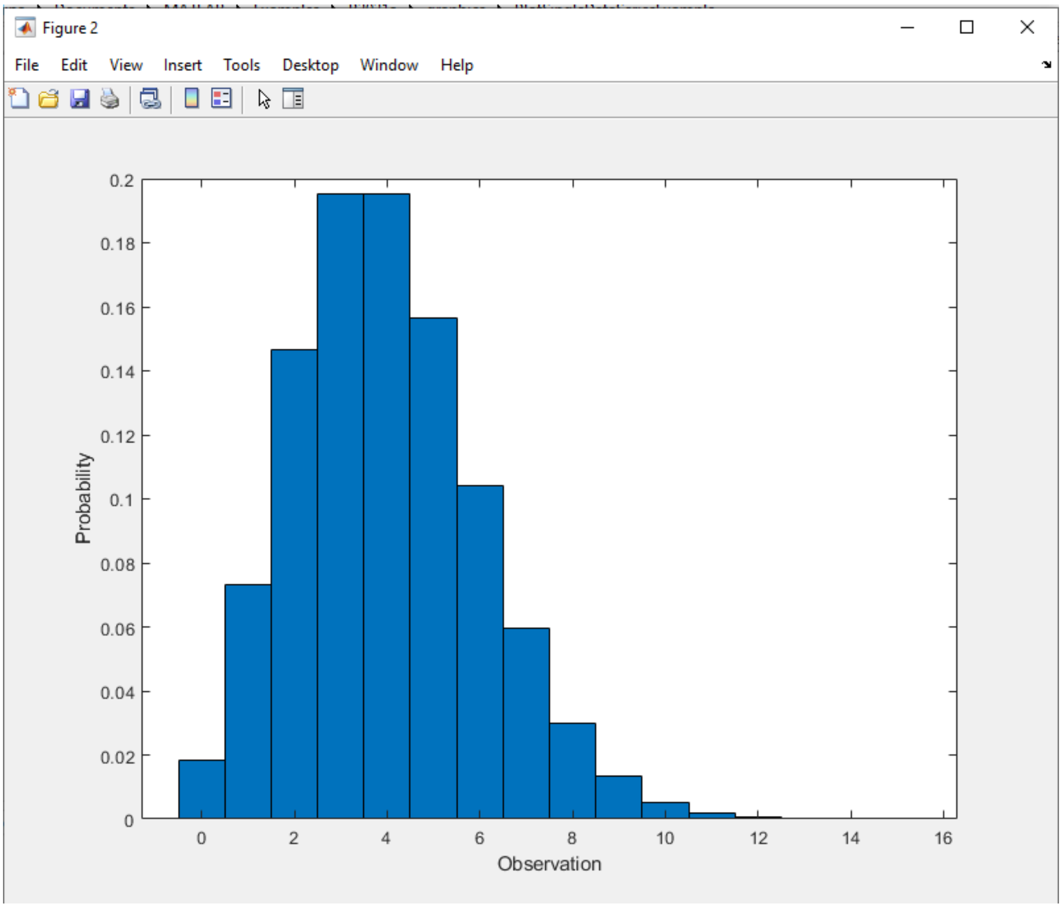


Figure 2
File
Edit
View
Insert
Tools
Desktop
Window
Help
Probability
Observation
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
12
14
16

PROGRAM 2

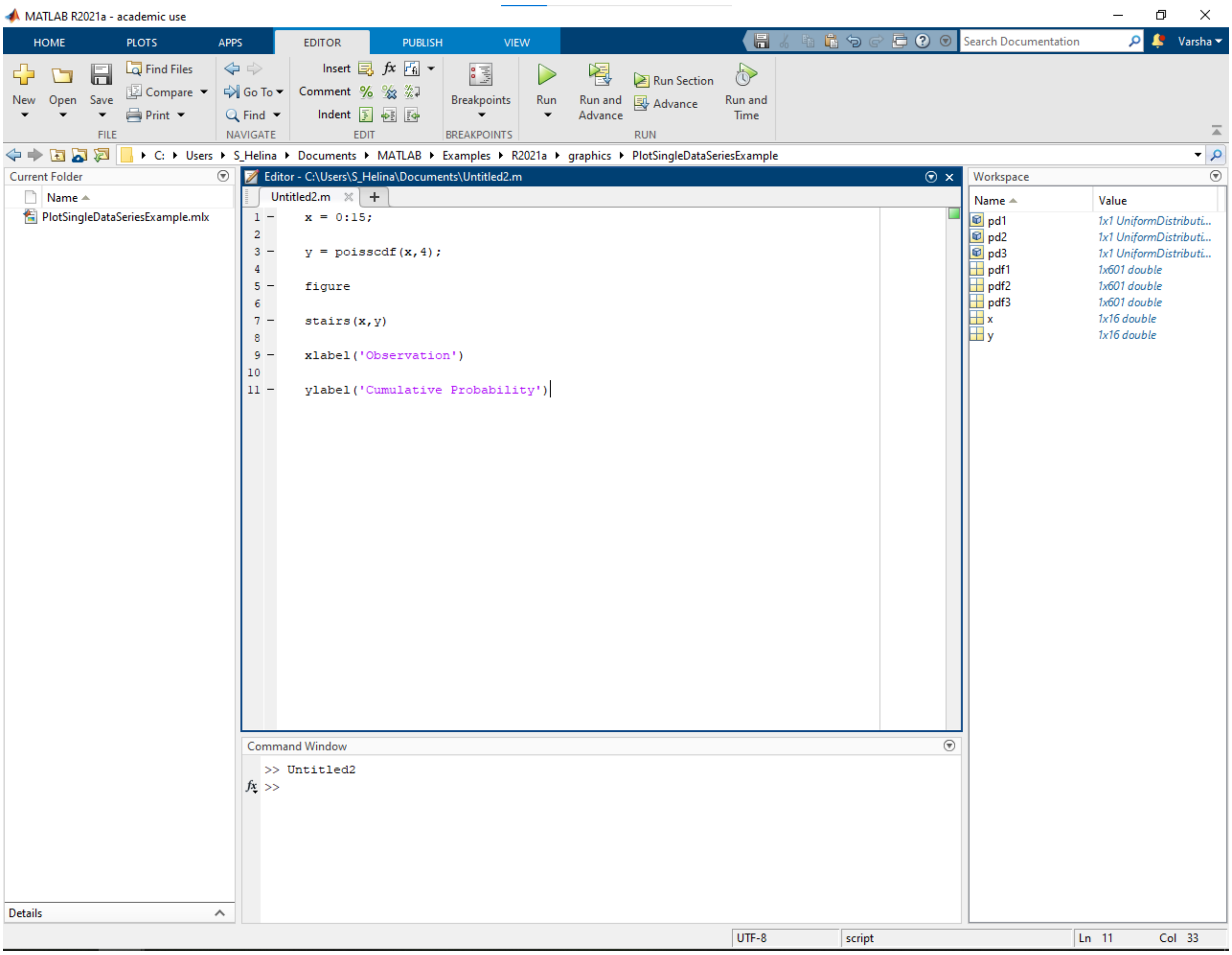
Cumulative Density Function of Poisson Distribution

Compute Poisson Distribution CDF (λ= 4)

```
x = 0:15;

y = poisscdf(x,4);

figure

stairs(x,y)

xlabel('Observation')

ylabel('Cumulative Probability')
```

- In this program CDF of the poisson distribution for mean λ= 4 is plotted using the function `poisscdf(x,4)` in MATLAB.
- The CDF is plotted as a stair chart using the function `stairs(x,y)` and is given labels using the `xlabel` and `ylabel` functions.

OUTPUT


MATLAB R2021a - academic use
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Varsha
New
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Find Files
Compare
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FILE
Go To
Find
NAVIGATE
Insert
Comment
Indent
EDIT
Breakpoints
BREAKPOINTS
Run
Run and Advance
Run Section
Advance
Run and Time
RUN
C: ▸ Users ▸ S_Helina ▸ Documents ▸ MATLAB ▸ Examples ▸ R2021a ▸ graphics ▸ PlotSingleDataSeriesExample
Current Folder
Name
PlotSingleDataSeriesExample.mlx
Editor - C:\Users\S_Helina\Documents\Untitled2.m
Untitled2.m
x = 0:15;
y = poisscdf(x,4);
figure
stairs(x,y)
xlabel('Observation')
ylabel('Cumulative Probability')
Workspace
Name
Value
pd1
1x1 UniformDistributi...
pd2
1x1 UniformDistributi...
pd3
1x1 UniformDistributi...
pdf1
1x601 double
pdf2
1x601 double
pdf3
1x601 double
x
1x16 double
y
1x16 double
Command Window
>> Untitled2
>>
Details
UTF-8
script
Ln 11
Col 33

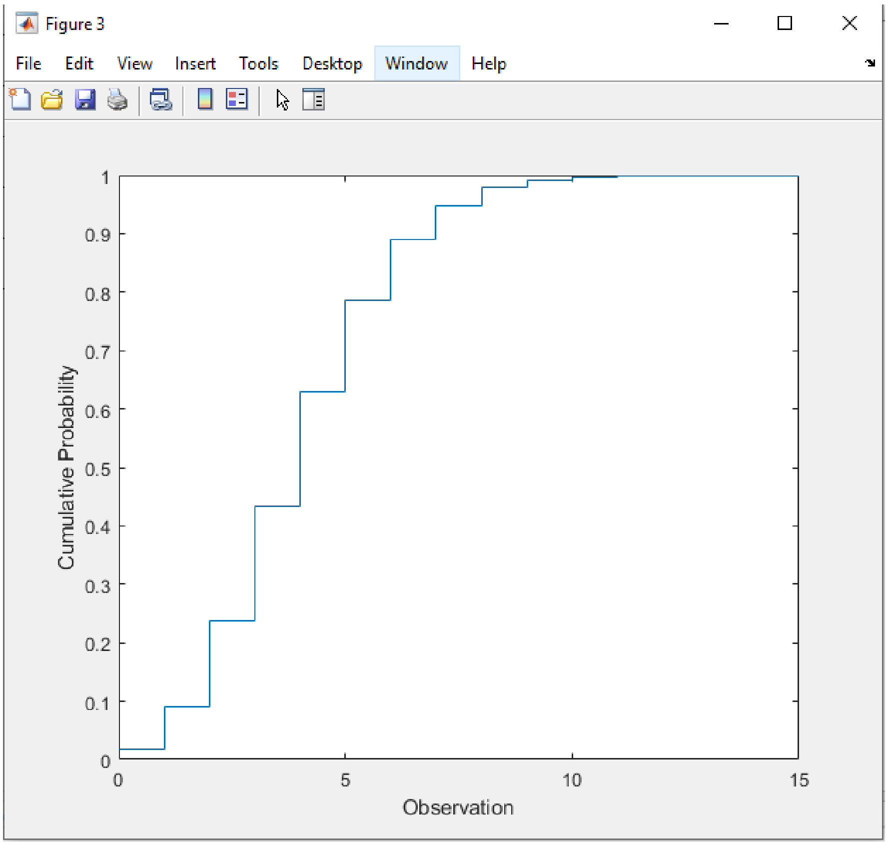
Figure 3
File
Edit
View
Insert
Tools
Desktop
Window
Help
Cumulative Probability
Observation
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15

APPLICATIONS

Applications of the Poisson distribution can be found in many fields including

- Telecommunication example: telephone calls arriving in a system.
- Astronomy example: photons arriving at a telescope.
- Biology example: the number of mutations on a strand of DNA per unit length.
- Management example: customers arriving at a counter or call centre.
- Finance and insurance example: number of losses or claims occurring in a given period of time.
- Earthquake seismology example: an asymptotic Poisson model of seismic risk for large earthquakes.
- Internet traffic.
- The number of deaths per year in a given age group.
- The number of jumps in a stock price in a given time interval.
- The number of bacteria in a certain amount of liquid.
- The proportion of cells that will be infected at a given multiplicity of infection.
- Radioactivity example: number of decays in a given time interval in a radioactive sample.