

Assignment 1

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Question:

A(2,5), B(-1,2) and C(5,8) are the vertices of the triangle ABC, M is a point on AB such that AM:MB = 1:2. Find the co-ordinates of M. Hence find the equation of line passing through the points C and M.

Solution: Given, A, B, C form a triangle ABC.

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (1)$$

Let M divides AB internally in the ratio of k : 1
Then, we get

$$\mathbf{M} = \frac{k\mathbf{B} + 1\mathbf{A}}{k + 1} \quad (2)$$

Given, M is a point on side AB such that AM : MB = 1:2. Using (2), we get

$$\mathbf{M} = \frac{\mathbf{B} + 2\mathbf{A}}{3} \quad (3)$$

On substituting, we get

$$\mathbf{M} = \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix}}{3} \quad (4)$$

$$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}. \quad (5)$$

Let L be the line that passes through points C $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$, The direction vector of CM is,

$$\mathbf{m} = \mathbf{C} - \mathbf{M} \quad (6)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (7)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (8)$$

Normal vector of the line is \mathbf{n} , such that

$$\mathbf{m}^\top \mathbf{n} = 0 \quad (9)$$

$$\Rightarrow \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{n} = 0 \quad (10)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (11)$$

$$\Rightarrow \mathbf{n}^\top = \begin{pmatrix} 1 & -1 \end{pmatrix} \quad (12)$$

The normal equation of the line L is given by,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{M}) = 0 \quad (13)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right) = 0 \quad (14)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3 \quad (15)$$

Thus, the equation of line L passing through C $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and M $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \quad (16)$$

Now, Let's find the area of triangle: Given,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (17)$$

are the vertices of triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad (18)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, \quad (19)$$

The desired area is the magnitude of

$$\begin{vmatrix} 3 & -3 \\ 3 & -3 \end{vmatrix} \quad (20)$$

(6) Thus the desired area is 0 units which implies that the given 3 points do not form a triangle, but are collinear.

And, The points A, B, C and M lie on the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \quad (21)$$

Also Verified by plotting the graph that shows that A, B, C and M points are collinear:

