Assignment 4

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Class 10 Probability (Ex - 15.1, Q-23)

Question: A game consists of tossing a ₹1 coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., 3 heads or 3 tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution:

1) Let the random variable $Y \in \{0, 1\}$ denote the outcome of trial of tossing a coin once, where $Y = 0, 1$ denote the outcomes of getting Tail, Head respectively.

$$\Pr(Y = 1) = p = 0.5 \tag{1}$$
$$\Pr(Y = 0) = 1 - p = 0.5 \tag{2}$$

2) On considering 3 Bernoulli trials for tossing a coin, let X be a Binomial random variable for the trials, with parameters n and p, such that $X = Y_1 + Y_2 + Y_3$, where
   a) n = No.of trials = 3
   b) p = probability with which it takes a favourable outcome(here say getting Head) = 0.5

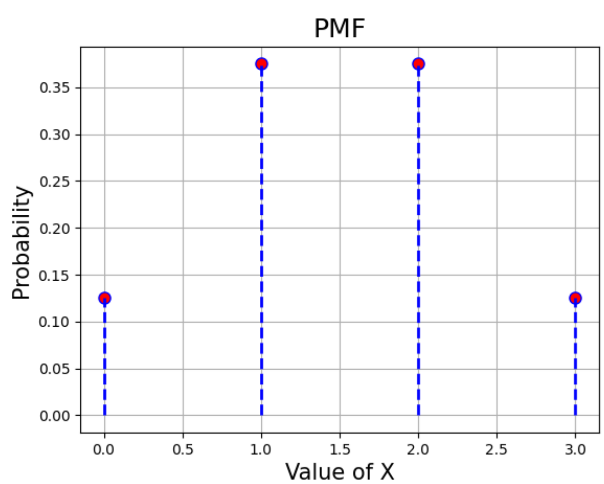

Fig. 1. Plot of the PMF

$$\boxed{\Pr(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}} \tag{3}$$

where $k = 0, 1, \ldots, n$ which is/are the number of Heads in the n trials.

Now, Let the random variable $Z \in \{0, 1\}$ denotes the outcome of the game such that :

Event	Description
$Z = 0$	Hanif losing the game
$Z = 1$	Hanif winning the game

TABLE I
DESCRIPTION OF EVENTS

Note: The above 2 events are mutually exclusive and exhaustive.

$$\implies \Pr(Z = 0) + \Pr(Z = 1) = 1 \tag{4}$$

Hence, Probability of Hanif winning the game i.e., all the 3 tosses resulting in either <u>3 Heads</u> or <u>3 Tails</u> is:

$$\Pr(Z = 1) = \Pr(X = 0) + \Pr(X = 3) \tag{5}$$

From the equation- 3,

$$\Pr(Z = 1) = \binom{3}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{3-0} + \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 \tag{6}$$

On calculating, we get,

$$\Pr(Z = 1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \tag{7}$$

From the equations- 4 and 7, The probability of Hanif losing the game is:

$$\Pr(Z = 0) = 1 - \Pr(Z = 1) = 1 - \frac{1}{4} \tag{8}$$
$$\implies \Pr(Z = 0) = \frac{3}{4} \tag{9}$$

Hence, the probability that Hanif will lose the game is <u>0.75</u>