Assignment 11

Varshini Jonnala - CS21BTECH11024

June 15, 2022

Question

Show that, if the process $X(w)$ is white noise with zero mean and autocovariance $Q(u)\delta(u-v)$, then its inverse Fourier transform $x(t)$ is WSS with power spectrum $Q(w)/2\pi$.

Solution I

$$E\left\{x(t_1)x^*(t_2)\right\} = \frac{1}{4\pi^2}E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left\{X(u)X^*(v)\right\}e^{j(ut_1-vt_2)}dudv\right\} \tag{1}$$

$$= \frac{1}{4\pi^2}E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(u)\delta(u-v)e^{j(ut_1-vt_2)}dudv\right\} \tag{2}$$

We know that, A random process $\{X(t), t \in R\}$ whose mean is independent of t, is weak-sense stationary or wide-sense stationary (WSS) for all $t \in R$, if

$$R_X(t_1, t_2) = R_X(t_1 - t_2) \tag{3}$$

Solution II

Now, This depends only on $\tau = t_1 - t_2$:

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Q(u)e^{jur}du \tag{4}$$

Hence, Power Spectrum $S_{XX}(\omega)$ of $X(\omega)$ would be:

$$S_{XX}(\omega) = \frac{Q(\omega)}{2\pi} \tag{5}$$

Hence, proved.