

# Assignment 2

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## Question:

A(2,5), B(-1,2) and C(5,8) are the vertices of the triangle ABC, M is a point on AB such that AM:MB = 1:2. Find the co-ordinates of M. Hence find the equation of line passing through the points C and M.

**Solution:** Given, A, B, C form a triangle ABC.

$$A = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (1)$$

When the line segment AB, where the points are  $A = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, B = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ , is divided internally by C in the ratio  $m : n$ , from Section formula, we get the Coordinates of point C as,

$$C = \begin{pmatrix} \frac{mx_2 + nx_1}{m+n} \\ \frac{my_2 + ny_1}{m+n} \end{pmatrix}, \quad (2)$$

Given, M is a point on side AB such that AM : MB = 1 : 2. Using Section Formula, we get

$$M = \begin{pmatrix} \frac{1(-1) + 2(2)}{1+2} \\ \frac{1(2) + 2(5)}{1+2} \end{pmatrix} \quad (3)$$

$$\Rightarrow M = \begin{pmatrix} \frac{-1+4}{1+2} \\ \frac{2+10}{1+2} \end{pmatrix} \quad (4)$$

$$\Rightarrow M = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (5)$$

Let L be the line that passes through points  $C \begin{pmatrix} 5 \\ 8 \end{pmatrix}, M \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ . The direction vector of CM is,

$$m = C - M \quad (6)$$

$$\Rightarrow m = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (7)$$

$$\Rightarrow m = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (8)$$

Normal vector of the line is n, such that

$$m^\top n = 0 \quad (9)$$

$$\Rightarrow \begin{pmatrix} 4 & 4 \end{pmatrix} n = 0 \quad (10)$$

$$\Rightarrow n = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (11)$$

$$\Rightarrow n^\top = \begin{pmatrix} 1 & -1 \end{pmatrix} \quad (12)$$

The normal equation of the line L is given by,

$$n^\top (x - M) = 0 \quad (13)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \left( x - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right) = 0 \quad (14)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} x = -3 \quad (15)$$

Thus, the equation of line L passing through  $C \begin{pmatrix} 5 \\ 8 \end{pmatrix}$  and  $M \begin{pmatrix} 1 \\ 4 \end{pmatrix}$  is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} x + 3 = 0 \quad (16)$$

which can also be represented as

$$x - y + 3 = 0 \quad (17)$$

But, However, We get

1) The equation of the line joining  $A \begin{pmatrix} 2 \\ 5 \end{pmatrix}, B \begin{pmatrix} -1 \\ 2 \end{pmatrix}$  as  $\begin{pmatrix} 1 & -1 \end{pmatrix} x = -3$ .

2) The equation of the line joining  $B \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C \begin{pmatrix} 5 \\ 8 \end{pmatrix}$  as  $\begin{pmatrix} 1 & -1 \end{pmatrix} x = -3$  too.

Now, Let's find the area of triangle to verify if the points are collinear: Given,

$$A = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (18)$$

are the vertices of triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad (19)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, \quad (20)$$

The desired area is the magnitude of

$$\begin{vmatrix} 3 & -3 \\ 3 & -3 \end{vmatrix} \quad (21)$$

Thus the desired area is 0 units.

Hence, Given points  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  don't form a triangle.

Also Verified by plotting the graph of  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  and  $\mathbf{M}$  points :

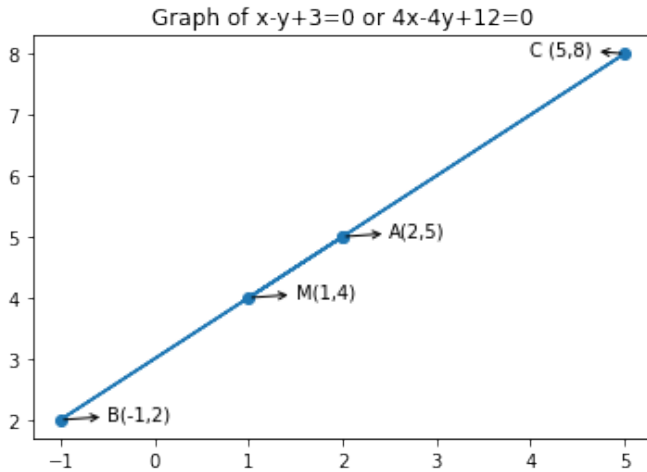


Fig. 1. Graph showing that the points  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{M}$  lie on the same line.