Assignment 1

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Question:
A(2,5), B(-1,2) and C(5,8) are the vertices of the triangle ABC, $\mathbf{M}$ is a point on AB such that AM:MB = 1:2. Find the co-ordinates of $\mathbf{M}$. Hence find the equation of line passing through the points $\mathbf{C}$ and $\mathbf{M}$.

Solution: Given, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle ABC.

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{1}$$

Let R divides $\overrightarrow{PQ}$ internally in the ratio of k : 1 i.e, $\frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{k}{1}$ such that

$$\overrightarrow{a} = \overrightarrow{OP} \tag{2}$$
$$\overrightarrow{b} = \overrightarrow{OQ} \tag{3}$$
$$\overrightarrow{r} = \overrightarrow{OR} \tag{4}$$

Then, we get

$$\overrightarrow{r} = \frac{k\overrightarrow{b} + 1\overrightarrow{a}}{k+1} \tag{5}$$

Given, $\mathbf{M}$ is a point on side AB such that $AM : MB = 1 : 2$. Using (5), we get

$$\overrightarrow{m} = \frac{\overrightarrow{b} + 2\overrightarrow{a}}{3} \tag{6}$$

On substituting, we get

$$\mathbf{M} = \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix}}{3} \tag{7}$$
$$\implies \mathbf{M} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}. \tag{8}$$

Let L be the line that passes through points $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$,

The direction vector of CM is,

$$\mathbf{m} = \mathbf{C} - \mathbf{M} \tag{9}$$
$$\implies \mathbf{m} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{10}$$
$$\implies \mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{11}$$

Normal vector of the line is $\mathbf{n}$, such that

$$\mathbf{m}^\top \mathbf{n} = 0 \tag{12}$$
$$\implies \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{n} = 0 \tag{13}$$
$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{14}$$
$$\implies \mathbf{n}^\top = \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{15}$$

The normal equation of the line L is given by,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{M}) = 0 \tag{16}$$
$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right) = 0 \tag{17}$$
$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3 \tag{18}$$

Thus, the equation of line $\mathbf{L}$ passing through $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and $\mathbf{M}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{19}$$

which can also be represented as

$$x - y + 3 = 0 \tag{20}$$

But, However, We get

1) The equation of the line joining $\mathbf{A}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{B}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ as $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3$.
2) The equation of the line joining $\mathbf{B}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ as $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3$ too.

Now, Let's find the area of triangle to verify if the points are collinear: Given,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{21}$$

are the vertices of triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \tag{22}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, \tag{23}$$

The desired area is the magnitude of

$$\begin{vmatrix} 3 & -3 \\ 3 & -3 \end{vmatrix} \tag{24}$$

Thus the desired area is 0 units.

Hence, Given points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ don't form a triangle.

Also Verified by plotting the graph of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{M}$ points :

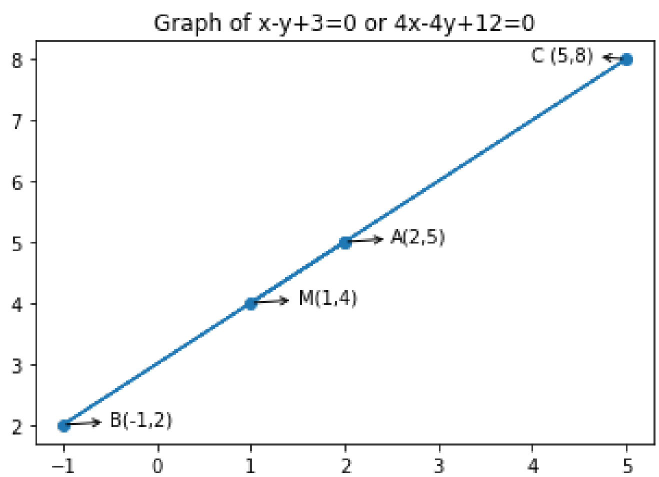

Fig. 1. Graph showing that the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$, $\mathbf{M}$ lie on the same line.