Assignment 4

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Abstract—This document contains the solution to Question-23 of Exercise 15.1(Probability) in the Class 10 NCERT Textbook.

Question: A game consists of tossing a ₹1 coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., 3 heads or 3 tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution:

1) Let the random variable $Y \in \{0,1\}$ denote the outcome of trial of tossing a coin once, where $Y = 0, 1$ denote the outcomes of getting Tail, Head respectively.

$$\Pr(Y = 1) = p = 0.5 \quad (1)$$
$$\Pr(Y = 0) = 1 - p = 0.5 \quad (2)$$

2) On considering 3 Bernoulli trials for tossing a coin, let X be a Binomial random variable for the trials, with parameters n and p, such that $X = Y_1 + Y_2 + Y_3$, where
   a) n = No.of trials = 3
   b) p = probability with which it takes a favourable outcome(here say getting Head) = 0.5

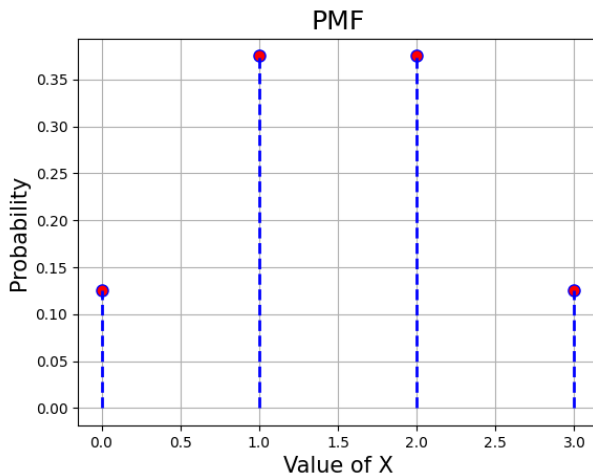Fig. 1. Plot of the PMF

$$\boxed{\Pr(X = k) = {}^{n}C_k \, (p)^k \, (1 - p)^{n-k}} \quad (3)$$

where $k = 0, 1, \ldots, n$ which is/are the number of Heads in the n trials.

Now, Let the random variable $Z \in \{0,1\}$ denotes the outcome of the game such that :

Event	Description
$Z = 0$	Hanif losing the game
$Z = 1$	Hanif winning the game

TABLE I
DESCRIPTION OF EVENTS

Note: The above 2 events are mutually exclusive and exhaustive.

$$\implies \Pr(Z = 0) + \Pr(Z = 1) = 1 \quad (4)$$

Hence, Probability of Hanif winning the game i.e., all the 3 tosses resulting in either <u>3 Heads</u> or <u>3 Tails</u> is:

$$\Pr(Z = 1) = \Pr(X = 0) + \Pr(X = 3) \quad (5)$$

From the equation- 3,

$$\Pr(Z = 1) = {}^{3}C_0 \, (0.5)^0 \, (0.5)^{3-0} + \\ {}^{3}C_3 \, (0.5)^3 \, (0.5)^0 \quad (6)$$

On calculating, we get,

$$\Pr(Z = 1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \quad (7)$$

From the equations- 4 and 7, The probability of Hanif losing the game is:

$$\Pr(Z = 0) = 1 - \Pr(Z = 1) = 1 - \frac{1}{4} \quad (8)$$

$$\implies \Pr(Z = 0) = \frac{3}{4} \quad (9)$$

Hence, the probability that Hanif will lose the game is <u>0.75</u>