

# ASSIGNMENT-1

CS21BTECH11024 - Varshini Jonnala

**7(c) Question:** A  $(2, 5)$ , B  $(-1, 2)$ , C  $(5, 8)$  are the vertices of the triangle ABC, 'M' is a point on AB such that  $AM : MB = 1 : 2$ . Find the co-ordinates of 'M'. Hence find the equation of line passing through the points C and M.

**Solution:** We know that, When the line segment AB, where the points are  $A = \begin{pmatrix} x1 \\ y1 \end{pmatrix}$ ,  $B = \begin{pmatrix} x2 \\ y2 \end{pmatrix}$ , is divided internally by C in the ratio  $m : n$ , from Section formula, we get the Coordinates of point C as,

$$C = \begin{pmatrix} \frac{mx2+nx1}{m+n} \\ \frac{my2+ny1}{m+n} \end{pmatrix}, \quad (1)$$

Given, M is a point on side AB such that  $AM : MB = 1 : 2$ . Using (1) in finding M, we get

$$M = \begin{pmatrix} \frac{-1+4}{1+2} \\ \frac{2+10}{1+2} \end{pmatrix} \quad (2)$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (3)$$

Let L be the line that passes through points X, Y, Then, L can be expressed as

$$L = X + \lambda.XY, \quad (4)$$

$$XY = \frac{Y - X}{|Y - X|} \quad (5)$$

Using (4) and (5) to find the equation of the

line passing through the points C  $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ , M  $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$\hat{CM} = \frac{\begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\left| \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|} \quad (6)$$

$$= \frac{\begin{pmatrix} 4 \\ 4 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ 4 \end{pmatrix} \right|} \quad (7)$$

$$= \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\sqrt{2}} \quad (8)$$

The slope of line L is 1.

Thus, Line L is  $\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ .

For y-intercept, from the above equation of line L, it can be written as

$$\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 1 + \lambda \\ 4 + \lambda \end{pmatrix} \quad (9)$$

On equating, we get  $\lambda$  as  $-1$ , and hence, y-intercept would be 3.

Thus, the equation of line L passing through C  $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$  and M  $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$  is

$$(1 \ -1)x + 3 = 0 \quad (10)$$

which can also be represented as

$$x - y + 3 = 0 \quad (11)$$

But, However, We get

- a) The equation of the line joining  $A \begin{pmatrix} 2 \\ 5 \end{pmatrix}, B \begin{pmatrix} -1 \\ 2 \end{pmatrix}$  as  $(1 \ -1) \mathbf{x} = -3$  and
- b) The equation of the line joining  $B \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C \begin{pmatrix} 5 \\ 8 \end{pmatrix}$  as  $(1 \ -1) \mathbf{x} = -3$  too.

This implies that A, B, C points are collinear and lie on the line  $(1 \ -1) \mathbf{x} = -3$  (or)  $x - y + 3 = 0$  and Hence, Given points A, B, C don't form a triangle.

Verified by plotting the graph of A, B, C and M points :

