Assignment 12

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June 15, 2022

Question

Using Schwarz's inequality, show that $\left|\int_a^b f(x)dx\right|^2 \leq (b-a)\int_a^b \left|f(x)\right|^2 dx$

Solution I

The Schwarz's inequality for integration is:

$$\left(\int f(x)g(x)dx\right)^2 \leq \int f^2(x)dx \int g^2(x)dx \tag{1}$$

Here, Using equation - (1), Taking $g(x) = 1$,

$$\left|\int_a^b f(x).1dx\right|^2 \leq \int_a^b \left|f(x)\right|^2 dx. \int_a^b 1^2 dx \tag{2}$$

$$\leq (b-a)\int_a^b \left|f(x)\right|^2 dx \tag{3}$$

Hence, proved.