Assignment 1

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Question: $\mathbf{A}$(2,5), $\mathbf{B}$(-1,2) and $\mathbf{C}$(5,8) are the vertices of the triangle ABC, $\mathbf{M}$ is a point on AB such that AM:MB = 1:2. Find the co-ordinates of $\mathbf{M}$. Hence find the equation of line passing through the points $\mathbf{C}$ and $\mathbf{M}$.

Solution: Given, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle ABC.

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{1}$$

Let $\mathbf{M}$ divides AB internally in the ratio of k : 1 Then, we get

$$\mathbf{M} = \frac{k\mathbf{B} + 1\mathbf{A}}{k+1} \tag{2}$$

Given, $\mathbf{M}$ is a point on side AB such that $AM : MB$ = 1:2. Using (2), we get

$$\mathbf{M} = \frac{\mathbf{B} + 2\mathbf{A}}{3} \tag{3}$$

On substituting, we get

$$\mathbf{M} = \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix}}{3} \tag{4}$$

$$\implies \mathbf{M} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}. \tag{5}$$

Let L be the line that passes through points $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$, The direction vector of CM is,

$$\mathbf{m} = \mathbf{C} - \mathbf{M} \tag{6}$$

$$\implies \mathbf{m} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{8}$$

Normal vector of the line is $\mathbf{n}$, such that

$$\mathbf{m}^\top \mathbf{n} = 0 \tag{9}$$

$$\implies \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{n} = 0 \tag{10}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{11}$$

$$\implies \mathbf{n}^\top = \begin{pmatrix} 1 & -1 \end{pmatrix} \tag{12}$$

The normal equation of the line L is given by,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{M}) = 0 \tag{13}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}\right) = 0 \tag{14}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3 \tag{15}$$

Thus, the equation of line $\mathbf{L}$ passing through $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and $\mathbf{M}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{16}$$

Now, Let's find the area of triangle: Given,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{17}$$

are the vertices of triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \tag{18}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, \tag{19}$$

The desired area is the magnitude of

$$\begin{vmatrix} 3 & -3 \\ 3 & -3 \end{vmatrix} \tag{20}$$

Thus the desired area is 0 units which implies that the given 3 points do not form a triangle, but are collinear.

And, The points $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{M}$ lie on the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{21}$$

Also Verified by plotting the graph that shows that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{M}$ points are collinear:

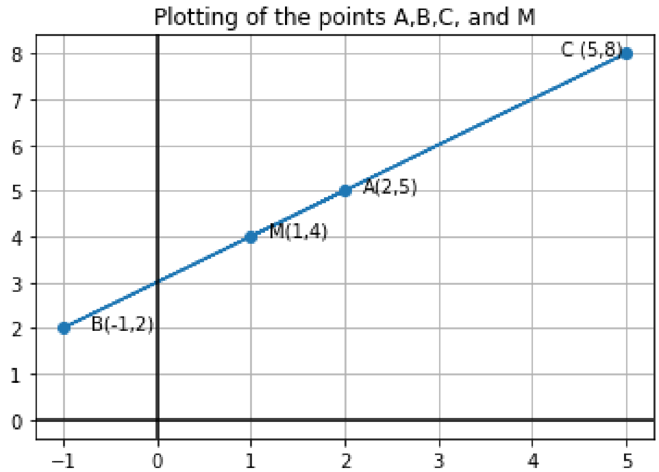

Fig. 1. Plot of the points A,B,C,M.