# Assignment 8

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# Question

- For geometric distribution, $\Pr(x) = 2^{-x}$ ; $x = 1, 2, 3, \ldots$. Prove that Chebychev' s inequality gives $\Pr(|X - 2| \leq 2) > \frac{1}{2}$ while the actual probability is $\frac{15}{16}$.

# Solution I

$$E(X) = \sum_{x=1}^{\infty} \frac{x}{2^x} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots \tag{1}$$

$$= \frac{1}{2}\left(1 + 2A + 3A^2 + 4A^3 + \dots\right) \text{ where A } = \frac{1}{2} \tag{2}$$

$$= \frac{1}{2}\left(1 - A\right)^{-2} = 2 \tag{3}$$

$$E(X^2) = \sum_{x=1}^{\infty} \frac{x^2}{2^x} = \frac{1}{2^2} + \frac{4}{2^3} + \frac{9}{2^4} + \dots \tag{4}$$

$$= \frac{1}{2}\left(1 + 4A + 9A^2 + \dots\right) \text{ where A } = \frac{1}{2} \tag{5}$$

$$= \frac{1}{4}\left(1 + A\right)\left(1 - A\right)^{-3} = 6 \tag{6}$$

## Solution II

Calculating the value of $\sigma$

$$\sigma^2 = Var(X) = E(X^2) - E(X)^2 = 6 - 4 = 2 \tag{7}$$

$$\sigma = \sqrt{2} \tag{8}$$

Using Chebychev's inequality, we get

$$\Pr\left(|X - E(X)| \leq k\sigma\right) > 1 - \frac{1}{k^2} \tag{9}$$

$$\text{With } k = \sqrt{2}, \text{ we get } \Pr\left(|X - 2| \leq \sqrt{2}\sqrt{2}\right) > 1 - \frac{1}{2} = \frac{1}{2} \tag{10}$$

$$\implies \Pr\left(|X - 2| \leq 2\right) > \frac{1}{2} \tag{11}$$

# Solution III

And, the actual probability is given by

$$\Pr\left(|X-2| \le 2\right) = \Pr\left(0 \le X \le 4\right) \tag{12}$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 \tag{13}$$

$$= \frac{15}{16} \tag{14}$$

Hence, proved.