ASSIGNMENT-1

CS21BTECH11024 - Varshini Jonnala

7(c) Question: $\mathbf{A}\begin{pmatrix}2,5\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}-1,2\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}5,8\end{pmatrix}$ are the vertices of the triangle $\mathbf{ABC}$, $'\mathbf{M}'$ is a point on $\mathbf{AB}$ such that $AM : MB = 1 : 2$. Find the co-ordinates of $'\mathbf{M}'$. Hence find the equation of line passing through the points $\mathbf{C}$ and $\mathbf{M}$.

Solution: We know that, When the line segment $\mathbf{AB}$, where the points are $\mathbf{A} = \begin{pmatrix}x1\\y1\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}x2\\y2\end{pmatrix}$, is divided internally by $\mathbf{C}$ in the ratio $m : n$, from Section formula, we get the Coordinates of point $\mathbf{C}$ as,

$$\mathbf{C} = \begin{pmatrix}\frac{mx2+nx1}{m+n}\\ \\ \frac{my2+ny1}{m+n}\end{pmatrix}, \tag{1}$$

Given, $\mathbf{M}$ is a point on side $\mathbf{AB}$ such that $AM : MB = 1 : 2$. Using (1) in finding $\mathbf{M}$, we get

$$\mathbf{M} = \begin{pmatrix}\frac{-1+4}{1+2}\\ \\ \frac{2+10}{1+2}\end{pmatrix} \tag{2}$$

$$= \begin{pmatrix}1\\4\end{pmatrix} \tag{3}$$

Let $\mathbf{L}$ be the line that passes through points $\mathbf{X}, \mathbf{Y}$,Then, $\mathbf{L}$ can be expressed as

$$\mathbf{L} = \mathbf{X} + \lambda.\hat{XY}, \tag{4}$$

$$\hat{XY} = \frac{\mathbf{Y} - \mathbf{X}}{\left|\mathbf{Y} - \mathbf{X}\right|} \tag{5}$$

Using (4) and (5) to find the equation of the line passing through the points $\mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$,$\mathbf{M}\begin{pmatrix}1\\4\end{pmatrix}$

$$\hat{CM} = \frac{\begin{pmatrix}5\\8\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}}{\left|\begin{pmatrix}5\\8\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}\right|} \tag{6}$$

$$= \frac{\begin{pmatrix}4\\4\end{pmatrix}}{\left|\begin{pmatrix}4\\4\end{pmatrix}\right|} \tag{7}$$

$$= \frac{\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{2}} \tag{8}$$

The slope of line $\mathbf{L}$ is 1.
Thus, Line $\mathbf{L}$ is $\begin{pmatrix}1\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\1\end{pmatrix}$.
For y-intercept, from the above equation of line $\mathbf{L}$, it can be written as

$$\begin{pmatrix}0\\y\end{pmatrix} = \begin{pmatrix}1+\lambda\\4+\lambda\end{pmatrix} \tag{9}$$

On equating, we get λ as $-\mathbf{1}$, and hence, y-intercept would be 3.
Thus, the equation of line $\mathbf{L}$ passing through $\mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ and $\mathbf{M}\begin{pmatrix}1\\4\end{pmatrix}$ is

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 3 = 0 \tag{10}$$

which can also be represented as

$$x - y + 3 = 0 \tag{11}$$

But, However, We get

a) The equation of the line joining $\mathbf{A}\begin{pmatrix}2\\5\end{pmatrix}, \mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ and

b) The equation of the line joining $\mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}, \mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ too.

This implies that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ points are 'collinear' and lie on the line $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ (or) $x-y+3=0$ and Hence, Given points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ don't form a triangle.

Verified by plotting the graph of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{M}$ points :