ASSIGNMENT-1

CS21BTECH11024 - Varshini Jonnala

ICSE 10 2018 - Problem 7(c)

Question: $\mathbf{A}\left(2,5\right)$, $\mathbf{B}\left(-1,2\right)$, $\mathbf{C}\left(5,8\right)$ are the vertices of the triangle $\mathbf{ABC}$, '$\mathbf{M}'$ is a point on $\mathbf{AB}$ such that $AM : MB = 1 : 2$. Find the co-ordinates of '$\mathbf{M}'$. Hence find the equation of line passing through the points $\mathbf{C}$ and $\mathbf{M}$.

Solution: According to the question, $\mathbf{M}$ is a point on the side $\mathbf{AB}$ such that

$$AM : MB = 1 : 2$$

When the line segment $\mathbf{AB}$, where the points are $\mathbf{A} = \begin{pmatrix} x1 \\ y1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x2 \\ y2 \end{pmatrix}$, is divided internally by $\mathbf{C}$ in the ratio $m : n$, from Section formula, we get the Coordinates of point $\mathbf{C}$ as,

$$\mathbf{C} = \begin{pmatrix} \frac{mx2+nx1}{m+n} \\ \\ \frac{my2+ny1}{m+n} \end{pmatrix}, \tag{1}$$

From given data, Using (1) in finding $\mathbf{M}$, we get

$$\mathbf{M} = \begin{pmatrix} \frac{-1+4}{1+2} \\ \\ \frac{2+10}{1+2} \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{3}$$

Let L be the line that passes through $\mathbf{X}, \mathbf{Y}$, Then, L can be expressed as

$$\mathbf{L} = \mathbf{X} + \lambda \hat{XY} \tag{4}$$

$$\hat{XY} = \frac{\mathbf{Y} - \mathbf{X}}{\left|\mathbf{Y} - \mathbf{X}\right|} \tag{5}$$

Using (4) and (5) to find the equation of the line passing through the points $\mathbf{C}\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and $\mathbf{M}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$$\hat{CM} = \frac{\begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}\right|} \tag{6}$$

$$= \frac{\begin{pmatrix} 4 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 4 \\ 4 \end{pmatrix}\right|} \tag{7}$$

$$= \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\sqrt{2}} \tag{8}$$

Thus, Line $\mathbf{L}$ is $\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.This implies that the slope of line $\mathbf{L}$ is 1.

For y-intercept, from the above equation of line $\mathbf{L}$, it can be written as

$$\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 1+\lambda \\ 4+\lambda \end{pmatrix} \tag{9}$$

On equating, we get λ as $-\mathbf{1}$, and hence, y-intercept would be 3

Thus, the equation of line $\mathbf{L}$ passing through $\mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ and $\mathbf{M}\begin{pmatrix}1\\4\end{pmatrix}$ is

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x}+3=0 \tag{10}$$

which can also be represented as

$$x-y+3=0 \tag{11}$$

But, However,

Similarly, we get

1) The equation of the line joining $\mathbf{A}\begin{pmatrix}2\\5\end{pmatrix}, \mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ and
2) The equation of the line joining $\mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}, \mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ too.

This implies that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ points are 'collinear' and lie on the line $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ (or) $x-y+3=0$ and Hence, given points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ don't form a triangle.
Verified by plotting the graph of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{M}$ points :