

Assignment 1

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Question:

A(2,5), B(-1,2) and C(5,8) are the vertices of the triangle ABC, M is a point on AB such that AM:MB = 1:2. Find the co-ordinates of M. Hence find the equation of line passing through the points C and M.

Solution: Given, A, B, C form a triangle ABC.

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (1)$$

Let

$$\vec{a} = \overrightarrow{OA} \quad (2)$$

$$\vec{b} = \overrightarrow{OB} \quad (3)$$

$$\vec{m} = \overrightarrow{OM} \quad (4)$$

and M divides \overrightarrow{AB} internally in the ratio of k : 1
Then, we get

$$\vec{m} = \frac{k\vec{b} + 1\vec{a}}{k + 1} \quad (5)$$

Given, M is a point on side AB such that AM : MB = 1 : 2. Using (5), we get

$$\vec{m} = \frac{\vec{b} + 2\vec{a}}{3} \quad (6)$$

On substituting, we get

$$\mathbf{M} = \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 5 \end{pmatrix}}{3} \quad (7)$$

$$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}. \quad (8)$$

Let L be the line that passes through points C $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$, The direction vector of CM is,

$$\mathbf{m} = \mathbf{C} - \mathbf{M} \quad (9)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (10)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (11)$$

Normal vector of the line is n, such that

$$\mathbf{m}^T \mathbf{n} = 0 \quad (12)$$

$$\Rightarrow \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{n} = 0 \quad (13)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (14)$$

$$\Rightarrow \mathbf{n}^T = \begin{pmatrix} 1 & -1 \end{pmatrix} \quad (15)$$

The normal equation of the line L is given by,

$$\mathbf{n}^T (\mathbf{x} - \mathbf{M}) = 0 \quad (16)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right) = 0 \quad (17)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3 \quad (18)$$

Thus, the equation of line L passing through C $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and M $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 3 = 0 \quad (19)$$

which can also be represented as

$$x - y + 3 = 0 \quad (20)$$

But, However, We get

1) The equation of the line joining A $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$, B $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ as $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3$.

2) The equation of the line joining B $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, C $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ as $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -3$ too.

Now, Let's find the area of triangle to verify if the points are collinear: Given,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (21)$$

are the vertices of triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad (22)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}, \quad (23)$$

The desired area is the magnitude of

$$\begin{vmatrix} 3 & -3 \\ 3 & -3 \end{vmatrix} \quad (24)$$

Thus the desired area is 0 units.

Hence, Given points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ don't form a triangle.

Also Verified by plotting the graph of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and \mathbf{M} points :

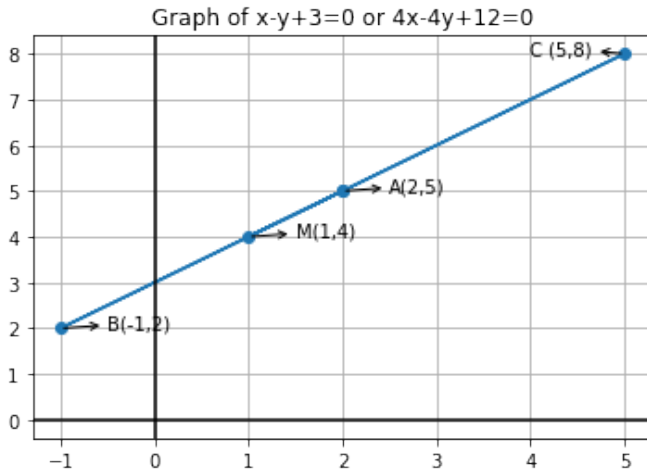


Fig. 1. Graph showing that the points $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{M}$ lie on the same line.