# Assignment 2

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**Question:**
A(2,5), B(-1,2) and C(5,8) are the vertices of the triangle $\mathbf{ABC}$, $\mathbf{M}$ is a point on $\mathbf{AB}$ such that AM:MB = 1:2. Find the co-ordinates of $\mathbf{M}$. Hence find the equation of line passing through the points $\mathbf{C}$ and $\mathbf{M}$.

**Solution:** Given, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle $ABC$.

$$\mathbf{A} = \begin{pmatrix}2\\5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-1\\2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}5\\8\end{pmatrix} \tag{1}$$

Given, $\mathbf{M}$ is a point on side $\mathbf{AB}$ such that $AM : MB = 1 : 2$. Using Section Formula, we get

$$\mathbf{M} = \begin{pmatrix}\frac{-1+4}{1+2}\\ \\ \frac{2+10}{1+2}\end{pmatrix} \tag{2}$$

$$= \begin{pmatrix}1\\4\end{pmatrix} \tag{3}$$

Let $\mathbf{L}$ be the line that passes through points $\mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}, \mathbf{M}\begin{pmatrix}1\\4\end{pmatrix}$. The direction vector of $CM$ is,

$$\mathbf{m} = \mathbf{C} - \mathbf{M} \tag{4}$$

$$\implies \mathbf{m} = \begin{pmatrix}5\\8\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix} \tag{5}$$

$$\implies \mathbf{m} = \begin{pmatrix}4\\4\end{pmatrix} \tag{6}$$

Normal vector of the line is $\mathbf{n}$, such that

$$\mathbf{m}^\top \mathbf{n} = 0 \tag{7}$$

$$\implies \begin{pmatrix}4 & 4\end{pmatrix}\mathbf{n} = 0 \tag{8}$$

$$\implies \mathbf{n} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{9}$$

$$\implies \mathbf{n}^\top = \begin{pmatrix}1 & -1\end{pmatrix} \tag{10}$$

The normal equation of the line $\mathbf{L}$ is given by,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{M}) = 0 \tag{11}$$

$$\implies \begin{pmatrix}1 & -1\end{pmatrix}\left(\mathbf{x} - \begin{pmatrix}1\\4\end{pmatrix}\right) = 0 \tag{12}$$

$$\implies \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3 \tag{13}$$

Thus, the equation of line $\mathbf{L}$ passing through $\mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ and $\mathbf{M}\begin{pmatrix}1\\4\end{pmatrix}$ is

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 3 = 0 \tag{14}$$

which can also be represented as

$$x - y + 3 = 0 \tag{15}$$

But, However, We get

1) The equation of the line joining $\mathbf{A}\begin{pmatrix}2\\5\end{pmatrix}, \mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ and
2) The equation of the line joining $\mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}, \mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$ too.

This shows that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ points are collinear and lie on the same line $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -3$.
Hence, Given points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ don't form a triangle.

Verified by plotting the graph of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{M}$ points :

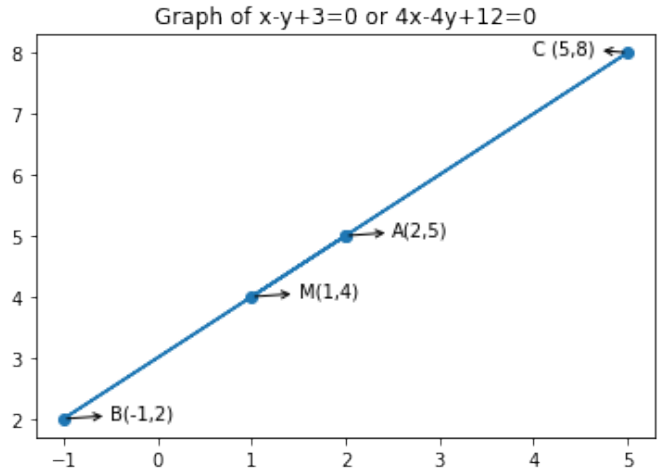


Fig. 1. Graph showing that the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$, $\mathbf{M}$ lie on the same line.