ASSIGNMENT-1

CS21BTECH11024 - Varshini Jonnala

ICSE 10 2018 - PROBLEM 7(C)

Question: $\mathbf{A}\begin{pmatrix}2, 5\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}-1, 2\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}5, 8\end{pmatrix}$ are the vertices of the triangle ABC, 'M' is a point on AB such that $AM : MB = 1 : 2$. Find the co-ordinates of 'M'. Hence find the equation of line passing through the points C and M.

Solution: According to the question, M is a point on the side AB such that

$$AM : MB = 1 : 2$$

When the line segment AB, where the points are $\mathbf{A} = \begin{pmatrix}x1\\y1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}x2\\y2\end{pmatrix}$, is divided internally by C in the ratio $m : n$, from Section formula, we get the Coordinates of point C as,

$$\mathbf{C} = \begin{pmatrix}\frac{mx2+nx1}{m+n}\\ \\ \frac{my2+ny1}{m+n}\end{pmatrix}, \tag{0.1}$$

From given data, Using (0.1) in finding M, we get

$$\mathbf{M} = \begin{pmatrix}\frac{-1+4}{1+2}\\ \\ \frac{2+10}{1+2}\end{pmatrix} \tag{0.2}$$

$$= \begin{pmatrix}1\\4\end{pmatrix} \tag{0.3}$$

The equation of the line joining two points $\begin{pmatrix}a\\b\end{pmatrix}$ and $\begin{pmatrix}c\\d\end{pmatrix}$ is

$$\begin{pmatrix}y-b\end{pmatrix} = \begin{pmatrix}\frac{d-b}{c-a}\end{pmatrix}\begin{pmatrix}x-a\end{pmatrix} \tag{0.4}$$

Here, the equation of the line joining points C$\begin{pmatrix}5\\8\end{pmatrix}$ and M$\begin{pmatrix}1\\4\end{pmatrix}$ will be

$$\begin{pmatrix}y-4\end{pmatrix} = \begin{pmatrix}\frac{8-4}{5-1}\end{pmatrix}\begin{pmatrix}x-1\end{pmatrix} \tag{0.5}$$

Simplified, we get the equation

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 3 = 0 \tag{0.6}$$

which can also be represented as

$$x - y + 3 = 0 \tag{0.7}$$

But, However,

On using (0.4), we get

1) The equation of the line joining $\mathbf{A}\begin{pmatrix}2\\5\end{pmatrix}, \mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 3 = 0.$
2) and the equation of the line joining $\mathbf{B}\begin{pmatrix}-1\\2\end{pmatrix}, \mathbf{C}\begin{pmatrix}5\\8\end{pmatrix}$ as $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 3 = 0$ too.

This implies that A, B, C points are 'collinear' and lie on the line $x - y + 3 = 0$ and Hence, given points A, B, C don't form a triangle.

Verified by plotting the graph of A,B,C and M points :