ANALOG ELECTRONICS - Project

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1 Active-RC Maximally Flat Low-Pass Filter

1.1 Hand calculation

(1) Given 3 dB Bandwidth, $\omega_n = 2\pi(1000)Hz = 2000\pi\ Hz$

also given Rejection > 20dB for $\omega_0 > 2\pi(2000)Hz$

$$20\left(\log\left|\frac{1}{\sqrt{1+\left(\frac{\omega_n}{\omega_n}\right)^{2n}}}\right| - \log\left|\frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega_n}\right)^{2n}}}\right|\right) > 20$$

$$\therefore \quad \log\left|\frac{\sqrt{1+2^{2n}}}{\sqrt{1+1^{2n}}}\right| > 1$$

$$\therefore \quad \left|\frac{\sqrt{1+4^n}}{\sqrt{2}}\right| > 10$$

$$\Rightarrow \quad 1+4^n > 200$$

$$\Rightarrow \quad 4^n > 199$$

$$\Rightarrow \quad \boxed{n \geq 4}$$

We need a 4^{th} order system to realise the maximally flat low pass filter

This can be obtained by cascading two second order systems

WK Poles of 4^{th} order system are given by

$$S = \omega_n e^{j\frac{2K+1}{2N}\pi + j\pi/2} \quad \text{where } K = 0, 1, \ldots N-1$$

here poles are $S_1 = \omega_n e^{j\frac{5\pi}{8}}$, $S_2 = \omega_n e^{j\frac{7\pi}{8}}$

$S_3 = \omega_n e^{j\frac{9\pi}{8}}$, $S_4 = \omega_n e^{j\frac{11\pi}{8}}$

$$S_{1,4} = \omega_n \left(\cos(5\pi/8) \pm j\sin(5\pi/8)\right) \quad \text{— (A)}$$

$$S_{2,3} = \omega_n \left(\cos(7\pi/8) \pm j\sin(7\pi/8)\right) \quad \text{— (B)}$$

$S_{1,4}$ are poles of 1st second order filter → Filter 1

$S_{2,3}$ are poles of 2nd second order filter → Filter 2

But we know from state-space biquad

$$\frac{V_{out}}{V_{in}} = \frac{R/R_i}{D(S)} \quad \text{where } D(S) = S^2 + 2\zeta\omega_n S + \omega_n^2$$

also $\boxed{\text{D.C gain} = 0\text{dB}}$ → $\boxed{R = R_i}$

Poles → Roots of $D(S) = 0$

$$\rightarrow \boxed{S = \omega_n\left(-\zeta \pm j\sqrt{1-\zeta^2}\right)} \quad \text{— (1)}$$

comparing (A), (1)

$-\zeta_1 = \cos(5\pi/8)$ → $\boxed{\zeta_1 = 0.382}$ ~ Filter (1)

comparing (B), (1)

$-\zeta_2 = \cos(7\pi/8)$ → $\boxed{\zeta_2 = 0.923}$ ~ Filter (2)

Component values of 1st state-space biquad filter

$\omega_n = 2000\pi$ Hz, $\zeta_1 = 0.382$

$\therefore$ we know $\omega_n = \frac{1}{RC}$ and $2\zeta_1 \omega_n = \frac{1}{R_Q C}$

Let us assume $\boxed{R = 5K\Omega} \Rightarrow C = \frac{1}{5\times10^3 \times 2\pi\times10^3}$

$\boxed{C = 0.0318\ \mu F}$

$\therefore R_Q = \frac{1}{0.0318\times10^{-6}\times2\times0.382\times2\pi\times10^3}$

$\boxed{R_Q = 6.55\ K\Omega}$ and $\boxed{R_i = 5K\Omega}$

III^ly component values of 2nd state-space biquad filter

$\omega_n = 2000\pi$ Hz, $\zeta_2 = 0.923$.

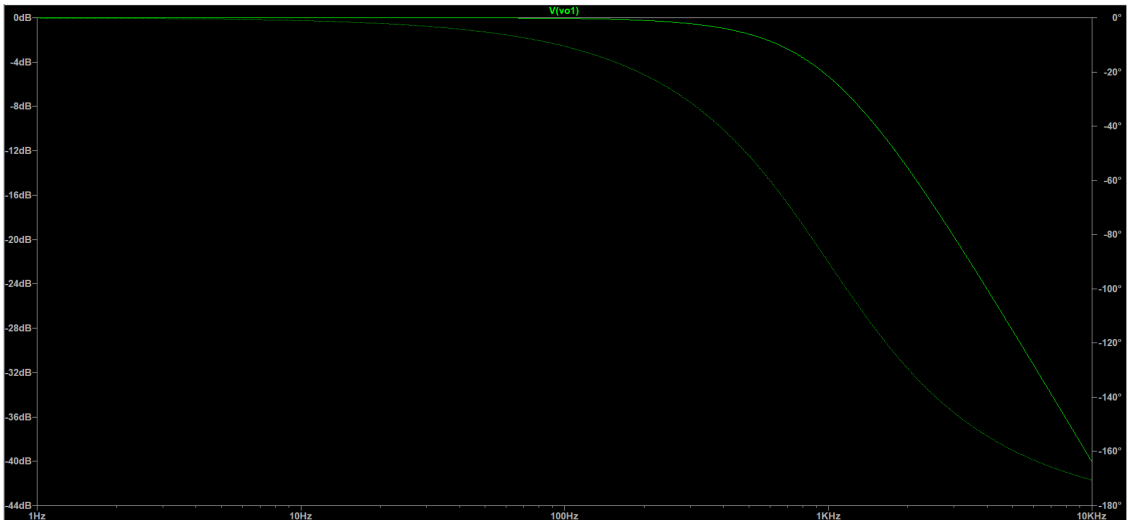
consider $\boxed{R' = 5K\Omega} \Rightarrow \boxed{C' = \frac{1}{5\times10^3\times2\pi\times10^3} = 0.0318\mu F}$

$R_Q = \frac{1}{2\times0.923\times2\pi\times10^3\times0.0318\times10^{-6}} \Rightarrow \boxed{R_Q' = 2.711K\Omega}$

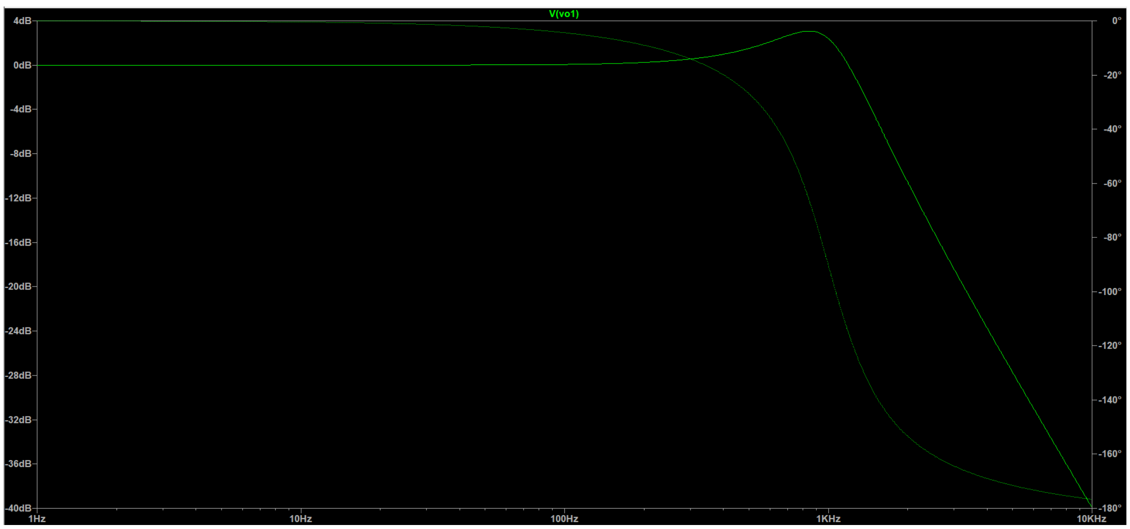
$\boxed{R' = R_i' = 5K\Omega}$

1.2 Ordering the filters in the cascade

- As shown in the hand calculations one of the filters has damping factor = 0.923, while the other has damping factor = 0.382.
- So the filter with damping factor of 0.923 ($> \frac{1}{\sqrt{2}}$) shows no peaking.
- Whereas the filter with damping factor of 0.382 ($< \frac{1}{\sqrt{2}}$) shows peaking.
- Hence the filter with damping factor of 0.923 is ordered first in position of cascade, to ensure no peaking at both the output nodes. (V_{O_1} and V_{O_2})
- Also the output of the cascaded system would be maximally flat response without any peaking.

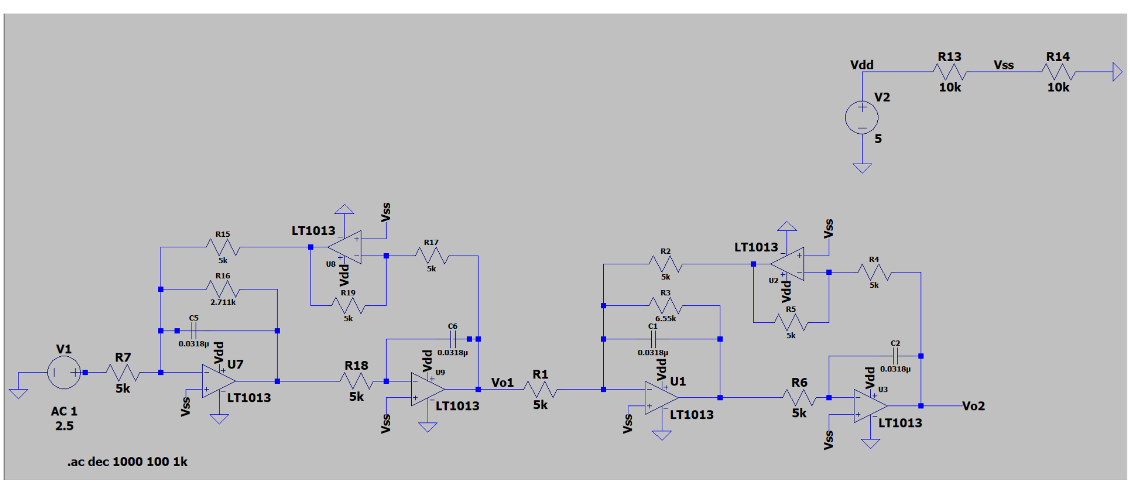


Output of filter with damping factor 0.923

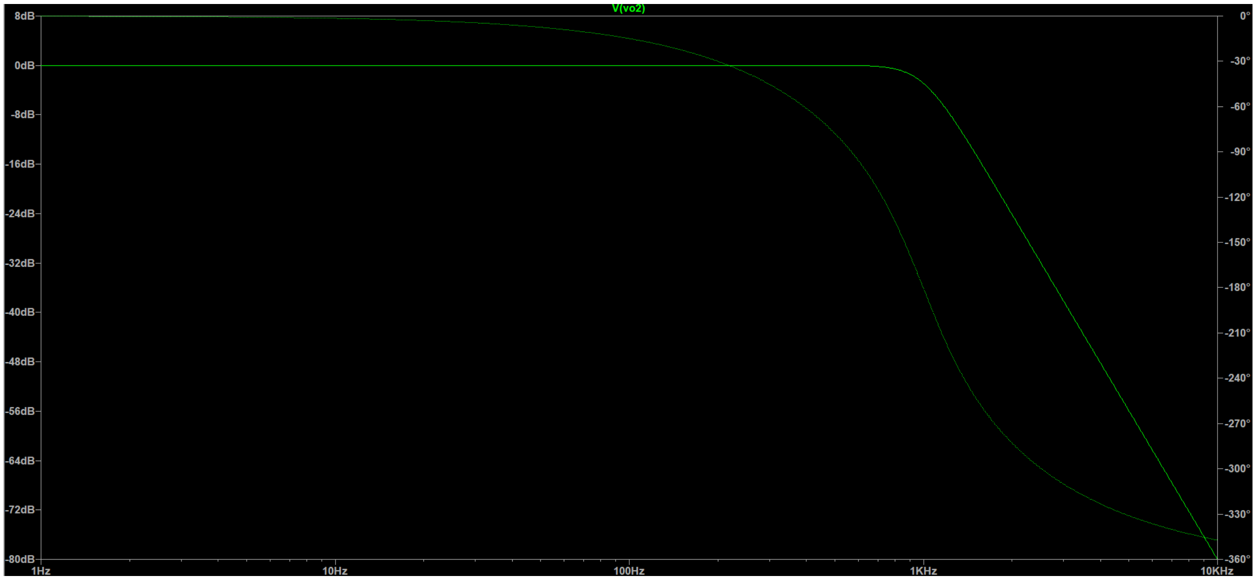


Output of filter with damping factor 0.382

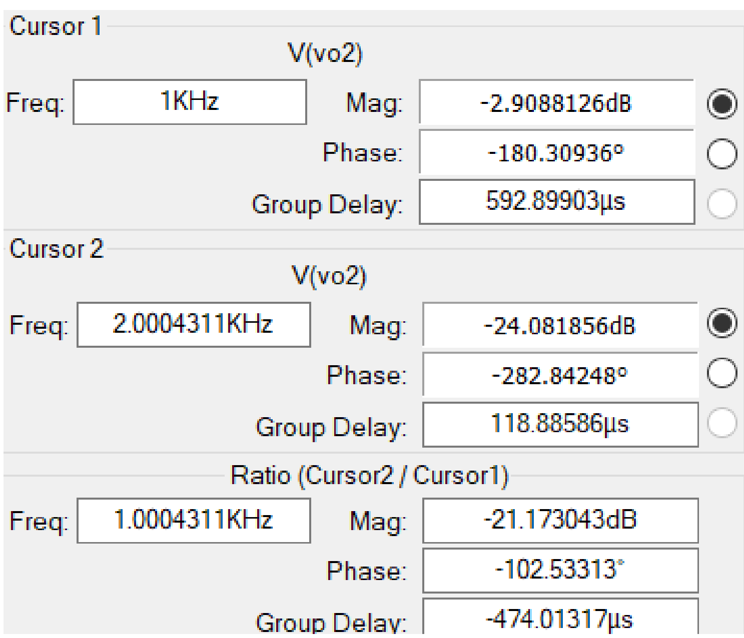
1.3 Circuit Diagram



1.4 Frequency Response



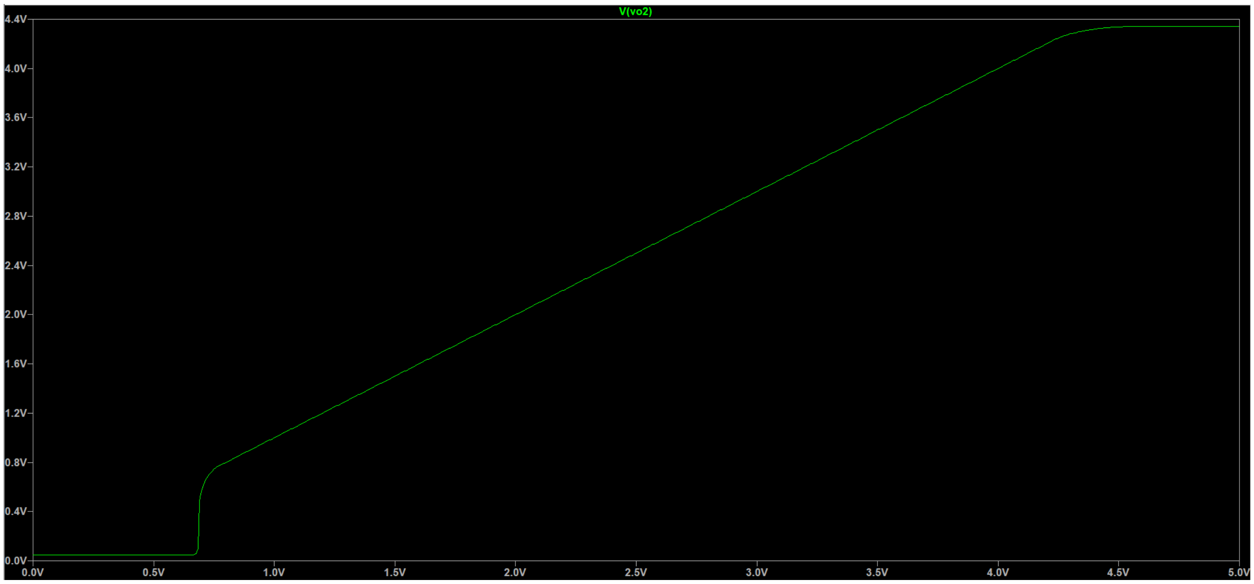
Magnitude and phase frequency response till 10 kHz



Indicating bandwidth and rejection

- We can observe the 3db bandwidth at approximately 1kHz.
- We can also observe the magnitude of around -24db at 2kHz, which implies rejection is greater than 20dB.

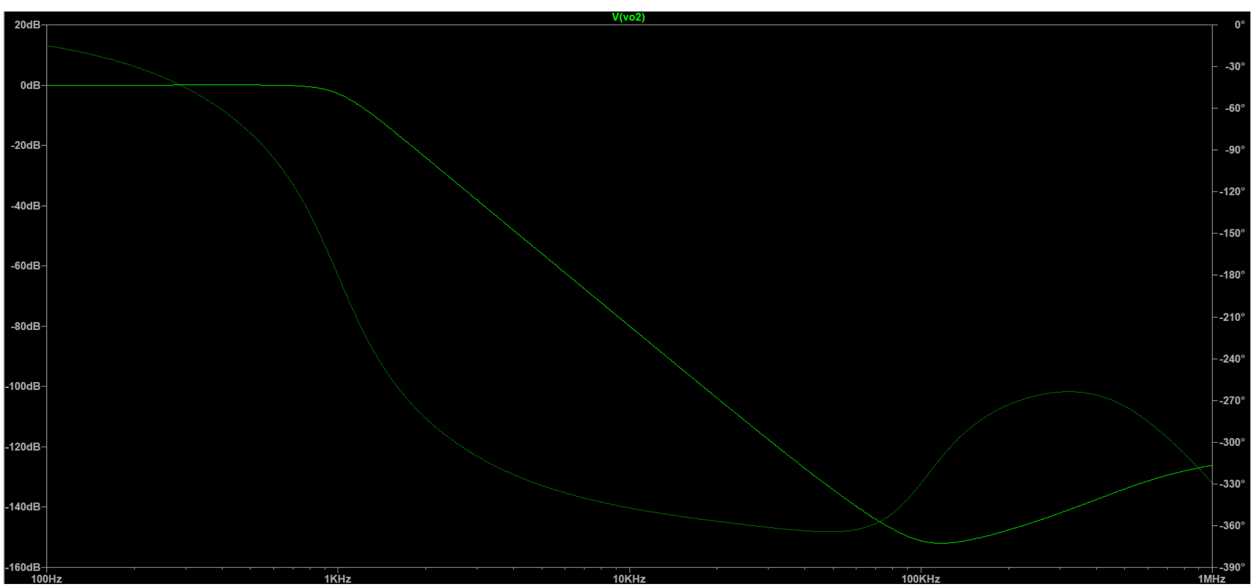
1.5 Input-output DC characteristics



DC characteristics with input varying from 0 V to 5 V.

- We can see that output is constant at 0.145V for a while ($V_{in} < 0.8$V) and then increases suddenly, due to non ideality of Op-amp.
- Later it suddenly increases and becomes equal to input (until 4.4V) , since it is a low pass filter and also the DC gain is zero db.
- Finally it settles at constant value (≈ 4.4V), this is because the voltage difference between terminals of Op-amp LT1013 settles at some non-zero constant value.

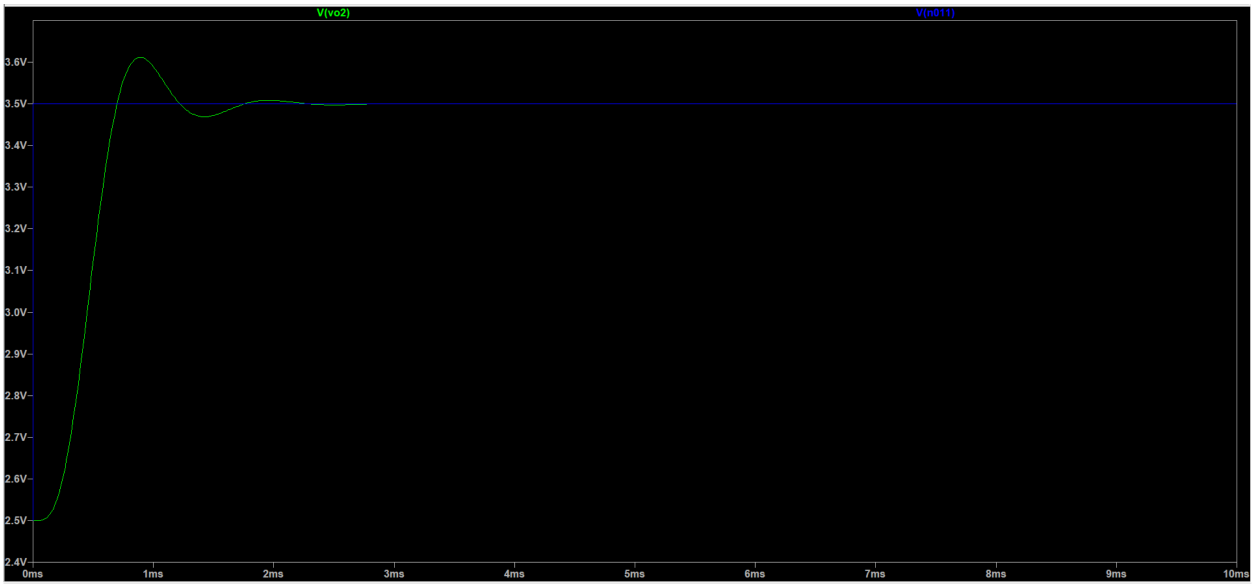
1.6 Magnitude response till 10 MHz



The high frequency behaviour

- We can observe that the above plot is abnormal.
- This is because of non ideal nature of Op-amp LT1013, as it has finite UGB unlike the ideal Op-amp with infinite UGB.

1.7 Step response



Step response for input going from 2.5 V to 3.5 V in 1 ns

- We can observe output settles to 3.5V at steady state once all the transients die out.
- We can also observe it is a under-damped response because all the poles are complex conjugates.