CONTROL SYSTEMS

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Question

Theoretical background

Solution

- Solution(a)
- Solution(b)

Question

•Find the transfer function $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit shown in figures given below

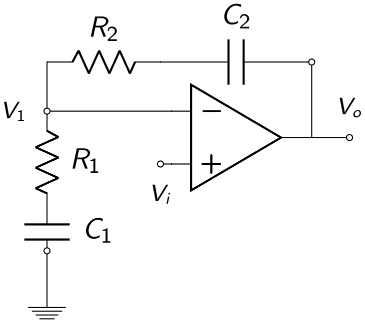


Fig.(a)

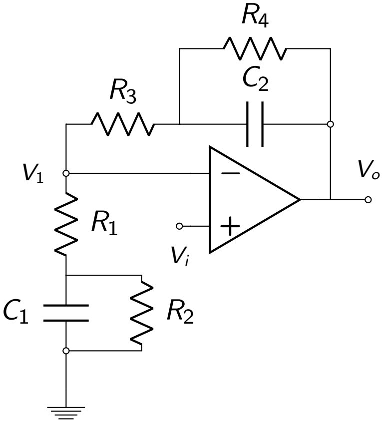
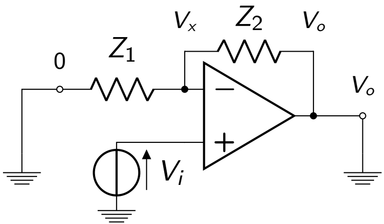


Fig.(b)

Non Inverting Op-Amp

It is the operational amplifier in which the output is in phase with input signal.The input signal is applied to "+" terminal of OpAmp:



Using the voltage divider rule in above circuit :

$$V_x = \frac{Z_1}{Z_1 + Z_2} * V_o \tag{2.1}$$

As the ideal op-amp's input impedance is infinite, its positive and negative terminal are virtually short, implies

$$V_+ = V_- \tag{2.2}$$

TRANSFER FUNCTION of Non Inverting Op-Amp

From the above circuit we know:

$$V_+ = V_i \ and \ V_- = V_x \tag{2.3}$$

So from Eq.(2.2) and Eq.(2.3), this implies

$$V_i = V_x$$

From Eq.(2.1):

$$V_i = \frac{Z_1}{Z_1 + Z_2} * V_o$$

$$\boxed{\frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}} \tag{2.4}$$

We apply this formula of transfer function of Non inverting Op-amp [Eq.(2.4)] in Fig.(a) and Fig.(b) to obtain solution.

Resistor and Capacitor values for Fig.(a)

Here in Fig.(a), resistor and capacitor values are given as following:

- $R_1 = 4*10^5\ \Omega$
- $C_1 = 4*10^{-6}$ F
- $R_2 = 1.1*10^5\ \Omega$
- $C_2 = 4*10^{-6}$ F

Hence comparing with general terms :

1. Impedance Z_1 is given by series combination of resistance R_1 and capacitance C_1
2. Impedance Z_2 is given by series combination of resistance R_2 and capacitance C_2

Calculating Z_1 and Z_2 for Fig.(a)

We know Impedance of Capacitor(C) in Laplace form $= \frac{1}{sC}$.
Hence, :

$$Z_1(s) = R_1 + \frac{1}{sC_1}$$

$$Z_1(s) = 4 * 10^5 + \frac{1}{4s * 10^{-6}} \tag{3.1}$$

$$Z_2(s) = R_2 + \frac{1}{sC_2}$$

$$Z_2(s) = 1.1 * 10^5 + \frac{1}{4s * 10^{-6}} \tag{3.2}$$

Solution(a)

From Eq.(2.4) :

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_1 + Z_2}{Z_1}$$

From Eq.(3.1) and Eq.(3.2):

$$G(s) = \frac{(4 * 10^5) + (1.1 * 10^5) + \frac{1}{4s*10^{-6}} + \frac{1}{4s*10^{-6}}}{4 * 10^5 + \frac{1}{4s*10^{-6}}}$$

Therefore on further simplification:

$$G(s) = \frac{51s + 50}{40s + 25}$$

$$\boxed{G(s) = 1.275 \left(\frac{s + 0.98}{s + 0.625} \right)}$$

Resistor and Capacitor values for Fig.(b)

Here in Fig.(a), resistor and capacitor values are given as following:

- $R_1 = 4*10^5\ \Omega$
- $C_1 = 4*10^{-6}$ F
- $R_2 = 6*10^5\ \Omega$
- $R_3 = 6*10^5\ \Omega$
- $C_2 = 4*10^{-6}$ F
- $R_4 = 1.1*10^5\ \Omega$

Hence comparing with general terms :

1. Impedance Z_1 is given by series combination of resistance R_1 and another impedance which is parallel combination of R_2 and C_1
2. Impedance Z_2 is given by series combination of resistance R_3 and another impedance which is parallel combination of R_4 and C_2

Calculating Z_1 and Z_2 for Fig.(b)

We know Impedance of Capacitor(C) in Laplace form $= \frac{1}{sC}$.
Hence, :

$$Z_1(s) = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{\frac{1}{sC_1}}} = R_1 + \frac{R_2}{sR_2C_1 + 1}$$

$$Z_1(s) = 4 * 10^5 + \frac{6 * 10^5}{6s * 10^5 * 4 * 10^{-6} + 1} \tag{3.3}$$

$$Z_2(s) = R_3 + \frac{1}{\frac{1}{R_4} + \frac{1}{\frac{1}{sC_2}}} = R_3 + \frac{R_4}{sR_4C_2 + 1}$$

$$Z_2(s) = 6 * 10^5 + \frac{1.1 * 10^5}{1.1s * 10^5 * 4 * 10^{-6} + 1} \tag{3.4}$$

Solution(b)

From Eq.(2.4) :

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_1 + Z_2}{Z_1}$$

From Eq.(3.3) and Eq.(3.4):

$$G(s) = \frac{(4 * 10^5) + (6 * 10^5) + \frac{6*10^5}{6s*10^5*4*10^{-6}+1} + \frac{1.1*10^5}{1.1s*10^5*4*10^{-6}+1}}{4 * 10^5 + \frac{6*10^5}{6s*10^5*4*10^{-6}+1}}$$

Therefore on further simplification:

$$G(s) = \frac{10.56s^2 + 33.68s + 17.1}{4.224s^2 + 14s + 10}$$

$$\boxed{G(s) = \frac{2640s^2 + 8420s + 4275}{1056s^2 + 3500s + 2500}}$$