CONTROL SYSTEMS

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EE19BTECH11020

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Question

•**Find the transfer function $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit shown in figures given below**

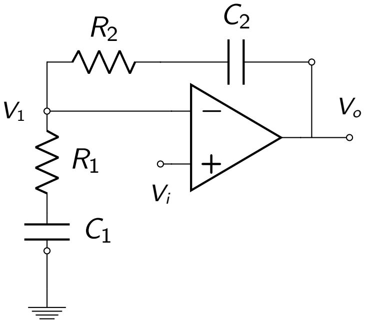


Fig.(a)

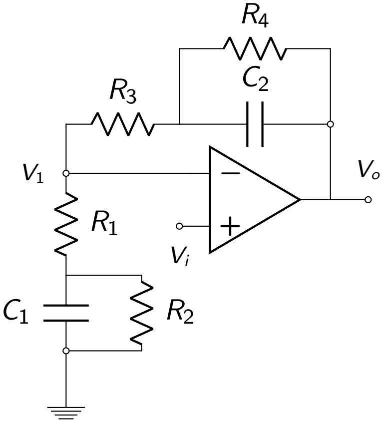
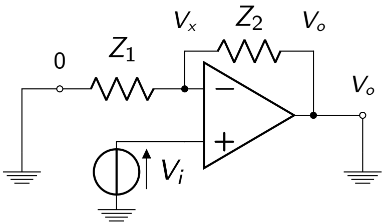


Fig.(b)

Non Inverting Op-Amp

It is the operational amplifier in which the output is in phase with input signal.The input signal is applied to "+" terminal of OpAmp:



Using the voltage divider rule in above circuit :

$$V_x = \frac{Z_1}{Z_1 + Z_2} * V_o \tag{2.1}$$

As the ideal op-amp's input impedance is infinite, its positive and negative terminal are virtually short, implies

$$V_+ = V_- \tag{2.2}$$

TRANSFER FUNCTION of Non Inverting Op-Amp

From the above circuit we know:

$$V_+ = V_i \text{ and } V_- = V_x \tag{2.3}$$

So from Eq.(2.2) and Eq.(2.3), this implies

$$V_i = V_x$$

From Eq.(2.1):

$$V_i = \frac{Z_1}{Z_1 + Z_2} * V_o$$

$$\boxed{\frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}} \tag{2.4}$$

We apply this formula of transfer function of Non inverting Op-amp [Eq.(2.4)] in Fig.(a) and Fig.(b) to obtain solution.

Resistor and Capacitor values for Fig.(a)

Here in Fig.(a), resistor and capacitor values are given as following:

- $R_1 = 4*10^5\ \Omega$
- $C_1 = 4*10^{-6}$ F
- $R_2 = 1.1*10^5\ \Omega$
- $C_2 = 4*10^{-6}$ F

Hence comparing with general terms :

1. Impedance Z_1 is given by series combination of resistance R_1 and capacitance C_1
2. Impedance Z_2 is given by series combination of resistance R_2 and capacitance C_2

Calculating Z_1 and Z_2 for Fig.(a)

We know Impedance of Capacitor(C) in Laplace form $= \frac{1}{sC}$.
Hence, :

$$Z_1(s) = R_1 + \frac{1}{sC_1}$$

$$Z_1(s) = 4 * 10^5 + \frac{1}{4s * 10^{-6}} \tag{3.1}$$

$$Z_2(s) = R_2 + \frac{1}{sC_2}$$

$$Z_2(s) = 1.1 * 10^5 + \frac{1}{4s * 10^{-6}} \tag{3.2}$$

Solution(a)

From Eq.(2.4) :

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_1 + Z_2}{Z_1}$$

From Eq.(3.1) and Eq.(3.2):

$$G(s) = \frac{(4 * 10^5) + (1.1 * 10^5) + \frac{1}{4s*10^{-6}} + \frac{1}{4s*10^{-6}}}{4 * 10^5 + \frac{1}{4s*10^{-6}}}$$

Therefore on further simplification:

$$G(s) = \frac{51s + 50}{40s + 25}$$

$$\boxed{G(s) = 1.275 \left(\frac{s + 0.98}{s + 0.625} \right)}$$

Resistor and Capacitor values for Fig.(b)

Here in Fig.(a), resistor and capacitor values are given as following:

- $R_1 = 4*10^5\ \Omega$
- $C_1 = 4*10^{-6}$ F
- $R_2 = 6*10^5\ \Omega$
- $R_3 = 6*10^5\ \Omega$
- $C_2 = 4*10^{-6}$ F
- $R_4 = 1.1*10^5\ \Omega$

Hence comparing with general terms :
1. Impedance Z_1 is given by series combination of resistance R_1 and another impedance which is parallel combination of R_2 and C_1
2. Impedance Z_2 is given by series combination of resistance R_3 and another impedance which is parallel combination of R_4 and C_2

Calculating Z_1 and Z_2 for Fig.(b)

We know Impedance of Capacitor(C) in Laplace form $= \frac{1}{sC}$.
Hence, :

$$Z_1(s) = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{\frac{1}{sC_1}}} = R_1 + \frac{R_2}{sR_2C_1 + 1}$$

$$Z_1(s) = 4 * 10^5 + \frac{6 * 10^5}{6s * 10^5 * 4 * 10^{-6} + 1} \tag{3.3}$$

$$Z_2(s) = R_3 + \frac{1}{\frac{1}{R_4} + \frac{1}{\frac{1}{sC_2}}} = R_3 + \frac{R_4}{sR_4C_2 + 1}$$

$$Z_2(s) = 6 * 10^5 + \frac{1.1 * 10^5}{1.1s * 10^5 * 4 * 10^{-6} + 1} \tag{3.4}$$

Solution(b)

From Eq.(2.4) :

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_1 + Z_2}{Z_1}$$

From Eq.(3.3) and Eq.(3.4):

$$G(s) = \frac{(4 * 10^5) + (6 * 10^5) + \frac{6*10^5}{6s*10^5*4*10^{-6}+1} + \frac{1.1*10^5}{1.1s*10^5*4*10^{-6}+1}}{4 * 10^5 + \frac{6*10^5}{6s*10^5*4*10^{-6}+1}}$$

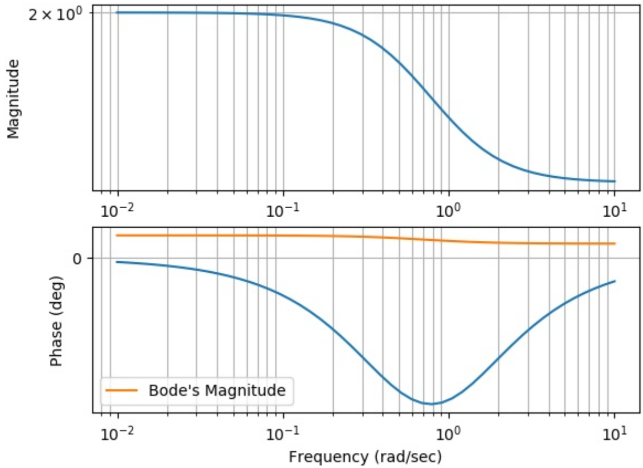
Therefore on further simplification:

$$G(s) = \frac{10.56s^2 + 33.68s + 17.1}{4.224s^2 + 14s + 10}$$

$$\boxed{G(s) = \frac{2640s^2 + 8420s + 4275}{1056s^2 + 3500s + 2500}}$$

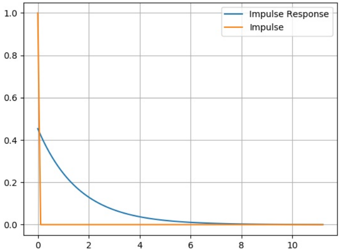
Bode Plot

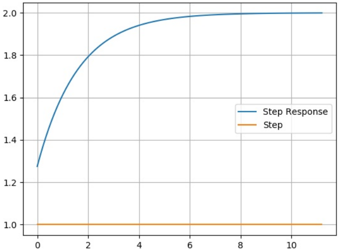
This is the Bode plot for the transfer function of Fig.(a)



Response Plots

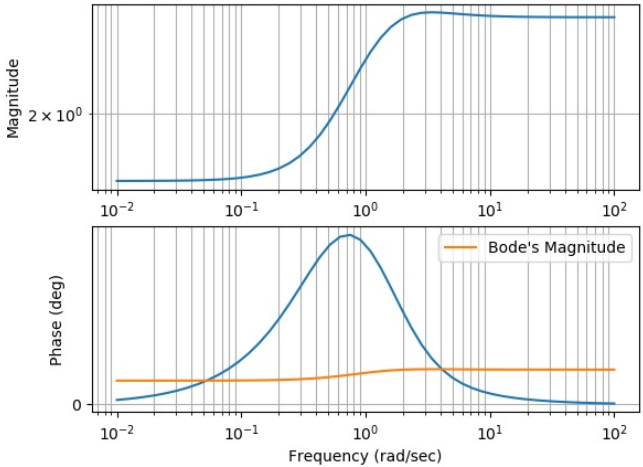
These are the respective plots of impulse and step response for Fig.(a)





Bode Plot

This is the Bode plot for the transfer function of Fig.(b)



Response Plots

These are the respective plots of impulse and step response for Fig.(b)