Department of Computer Science and Engineering
Quiz I

Course No.: CS 207   Course Name: Discrete Structures
Date: 1/9/2023   Time: 8-30 to 9-25 a.m.   Marks: 10

Note: The quiz is closed notes/books. Please DO NOT keep mobile phones or any other devices with you, and if you have one, switch it off. Anyone found using it will be disqualified. You can assume any results done in class/tutorials without proof but state it clearly whenever used. No extra sheet will be given, write clearly and precisely.

Q1 Let m, n be positive integers and let $k_1, k_2, \ldots, k_m$ be non-negative integers such that $k_1 + k_2 + \cdots + k_m = n$. Let $C(n; k_1, k_2, \ldots, k_m)$ be the multinomial coefficient $\frac{n!}{k_1! k_2! \ldots k_m!}$. Prove that $gcd(n, C(n; k_1, k_2, \ldots, k_m)) = 1$ if and only if $k_i = n$ for some $1 \leq i \leq m$. Note that the proof can be written in at most 5 lines, any long argument will not be corrected.   (3)

Q2 (a) Let $v_1 = (8, 3)$, $v_2 = (3, 7)$ and $v_3 = (6, 4)$ be two dimensional row vectors. Prove that every two dimensional row vector (m, n) with integer coordinates can be written as an integer linear combination $a_1 \cdot v_1 + a_2 \cdot v_2 + a_3 \cdot v_3$ for some integers a_1, a_2, a_3.   (4)

(b) Let $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$ be arbitrary two dimensional row vectors with integer coordinates. Give a necessary and sufficient condition, in terms of x_1, y_1, x_2, y_2, that the vectors must satisfy so that every two dimensional row vector with integer coordinates can be written as an integer linear combination of the two vectors. Prove that your condition is necessary and sufficient.   (3)