DISCRETE STRUCTURES

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CS207

Quiz 2 Solutions

1- An $n \times n$ board with n^2 unit squares is supposed to be coloured by 2 colours red or blue.

The group of symmetries are:

Rotating about centre by $90°, 180°, 270°, 360°$

Reflecting about each of the 2 diagonals

Reflecting about a line passing through midpoints of pair of opposite sides: There are 2 two such lines

$\therefore |G| = 8$

There are 2 cases: n is even; n is odd.

Now do case work.

2. Given a poset of size $km+1$ prove that there exists a chain of length $k+1$ or antichain of length $m+1$.

Proof:

For every poset, the minimum number of antichains that the poset can be partitioned into equal to the length of the longest maximal chain in the poset.

Let the length of the longest chain $\leq k$.

There exist a minimum of k antichains into which the poset can be partitioned.

In every such partition one of the antichains must contain an antichain of length $\geq m+1$ $\square$

We have proved,

$\sim$(A chain of length $k+1$ exists)

$\Rightarrow$ (A chain of length $m+1$ exists)

Which is equivalent to proving what was asked

Q1. Define a relation $\Leftarrow\Rightarrow$, with $\wedge, \vee$ operations that are commutative, associative, and are idempotent and absorptive.

Finite boolean lattice: Set of all subsets of a finite set ordered by a relation.

Distributive properties are not satisfied by $\vee, \wedge$ in general.

There are lattices that are not Boolean lattices but satisfy these properties. Additional properties are required to define a Boolean lattice.

Identity: $\exists$ elements 0 and 1 $a \vee 0 = a$, $a \wedge 1 = a$ $\forall a$

0 – Empty set, 1 – whole set

Complement: $\forall a \; \exists \; a^c \ni a \vee a^c = 1$, $a \wedge a^c = 0$

A partial order with $\wedge, \vee$ with the above properties (on a finite set) is a Boolean lattice with $\wedge$ corresponding to intersection and $\vee$ corresponding to union.