Problem Solving:

I. Verify whether the given transformation are linear:

1) $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T(x_0, x_1) = (x_0, 2x_0 + x_1) \ \forall (x_0, x_1) \in \mathbb{R}^2$$

Soln://

Let $\lambda \in \mathbb{R}$

$(x_0, x_1), (y_0, y_1) \in \mathbb{R}^2$

chandra's

Consider

$$T(\lambda(x_0, x_1) + (y_0, y_1))$$
$$T((\lambda x_0, \lambda x_1) + (y_0, y_1))$$
$$T(\lambda x_0 + y_0, \lambda x_1 + y_1)$$
$$= (\lambda x_0 + y_0, 2(\lambda x_0 + y_0) + (\lambda x_1 + y_1))$$
$$= \lambda x_0 + y_0 ; \lambda(2x_0 + x_1) + 2y_0 + y_1) \quad — (1)$$

Consider

$$\lambda . T(x_0, x_1) + T(y_0, y_1)$$
$$\lambda(x_0, 2x_0 + x_1) + (y_0, 2y_0 + y_1)$$
$$(\lambda x_0 + y_0, \lambda(2x_0 + x_1) + (2y_0 + y_1)) \quad — (2)$$

from (1) & (2) we have

$$T(\lambda(x_0, x_1) + (y_0, y_1) = \lambda T(x_0, x_1) + T(y_0, y_1)$$

$\therefore$ By theorem 1 T is linear

(2) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x_0, x_1, x_2) = (x_0 - x_1, 2x_2, 1)$

sln// Here $T(0,0,0) = (0-0, 2 \cdot 0, 1)$
$$= (0, 0, 1)$$
$$T(0,0,0) \neq (0,0,1)$$
$\therefore$ By theorem 2, T is not linear

3) Find the range space, rank, nullspace & nullity for the linear transformation

$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$T(e_1) = (1, -1, 0)$ $\quad T(e_2) = (2, 0, 1)$ $\quad T(e_3) = (1, 1, 1)$

where $e_1 = (1,0,0)$ $\quad e_2 = (0,1,0)$ $\quad e_3 = (0,0,1)$

and hence verify Rank - nullity theorem also write a

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Python program to verify the answers obtained

Soln. Here,

$$T(x,y,z) = T(x(1,0,0) + y(0,1,0) + z(0,0,1))$$
$$= x.T(1,0,0) + y.T(0,1,0) + zT(0,0,1)$$
$$= x(1,-1,0) + y(2,0,1) + z(1,1,1)$$
$$= (x+2y+z, -x+z, y+z)$$

To find $N(T)$:

W.K.T $N(T) = \{(x,y,z) / T(x,y,z) = (0,0,0)\}$
$= \{(x,y,z) / (x+2y+z, -x+z, y+z) = (0,0,0)\}$

$$x+2y+z=0$$
$$-x+z=0$$
$$y+z=0$$

Consider the co-eff- matrix of the above system:

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_3 \to R_3 - 1/2 R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x+2y+z=0$ —— ①

$2y+2z=0$

$\boxed{y=-z}$

; $x+2(-z)+z=0$

$\boxed{x=z}$

& z is arbitrary

$\therefore$ The solution is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ -z \\ z \end{pmatrix} = z \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \forall z \in \mathbb{R}$

$\therefore N(T) = \{ z(1,-1,1) / z \in \mathbb{R} \}$

So, Nullity $(T) = 1$

To find rank (T):

Fact: If B is a basis for a v.s V, then

Span $(T(B)) = R(T)$

In this problem

Span $(T(e_1), T(e_2), T(e_3)) = R(T)$

i.e, Span$((1,-1,0), (2,0,1), (1,1,1)) = R(T)$

let $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$

$\therefore \dim(R(T)) + \dim(N(T)) = \dim V = \dim(\mathbb{R}^3) = 2+1 = 3$

Hence verified