05

# Total Differential Equations

A differential equation of the form $Pdx + Qdy + Rdz$, where $P, Q, R$ are functions of $x, y, z$ is called Total differential eqn

Theorem:- A necessary and sufficient condition for $Pdx + Qdy + Rdz = 0$ to be integrable is

$$P\left(\frac{dQ}{dz} - \frac{dR}{dy}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{dP}{dy} - \frac{dQ}{dx}\right) = 0$$

1. Test the integrability and solve $(y+z)dx + (z+x)dy + (x+y)dz = 0$

$P = y+z \qquad \partial P/\partial y = 1 \qquad \partial P/\partial z = 1$

$Q = z+x \qquad \partial Q/\partial x = 1 \qquad \partial Q/\partial z = 1$

$R = x+y \qquad \partial R/\partial y = 1 \qquad \partial R/\partial x = 1$

$$\therefore P\left(\frac{dQ}{dz} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

$$y + z(1-1) + z + x(1-1) + x + y(1-1) = 0$$

$\therefore$ given eqn is integrable

$$ydx + zdx + zdy + xdy + xdz + ydz = 0$$

$$(ydx + xdy) + (zdx + xdz) + (zdy + ydz) = 0$$

$$d(yx) + d(zx) + d(zy) = d(c)$$

$$xy + xz + zy = c$$

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2. $yzdx + 2xz\,dy - 3xy\,dz = 0$

$P = yz \quad Q = 2xz \quad R = -3xy$

$yz(2x - (-3x)) + 2xz(-3y - y) + (-3xy)(z - 3z)$

$yz(5x) + 2xz(-4y) + 3xy(2)$

$5xyz - 8xyz + 3xyz = 0$

$\therefore$ it is integrable

$yzdx + 2xzdy - 3xydz = 0$ — ①

let z be constant $\rightarrow dz = 0$

① $\Rightarrow yzdx + 2xzdy = 0$

$y\,dx + 2x\,dy = 0$

$\frac{dx}{x} + 2\frac{dy}{y} = 0$

integrate $\int \frac{dx}{x} + 2\int \frac{dy}{y} = 0$

$\log x + 2\log y = \log c$

$xy^2 = c$

let $xy^2 = f(z)$ — *

diff totally

$y^2dx + 2xy\,dy = f'(dz)$

x/y equn with z/y.

$ydx + 2xzdy = \frac{z}{y} f'(dz)$ — ②

Comparing ① & ②

$\frac{z}{y} f'(dz) = 3xy\,dz$

$z/y\, f' = 3xy$

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$$z f' = 3xy^2 = 3f \Rightarrow z\frac{df}{dz} = 3f$$

This is an ordinary diff eqn in $f$ and $z$

$$\frac{df}{f} = \frac{3dz}{z}$$

$$\int \frac{df}{f} = 3\int \frac{dz}{z}$$

$$\log f = 3\log z + \log k$$

$$f = z^3 k$$

$$\therefore xy^2 f = kz^3$$

3) Test the integrability & solve $(x+z)^2 dy + y^2(dx+dz) = 0$

$$(x+z)^2 dy + y^2 dx + y^2 dz = 0 \quad -①$$

$$p = y^2 \qquad q = (x+z)^2. \qquad R = y^2$$

$$y^2(2x+2z-2y) + (x+z)^2(0-0) + y^2(2y-2x-2z)$$

$$2xy^2 + 2zy^2 - 2y^3 + 2y^3 - 2xy^2 - 2zy^2 = 0$$

$\therefore$ it is integrable.

Let $y$ be constant $\Rightarrow dy = 0$

$$① \Rightarrow y^2 dx + y^2 dz = 0$$

$$dx + dz = 0$$

integrate $x + z = c$

let $x + z = f(y) \longrightarrow$

diff totally

$$dx + dz = f'(dy)$$

$$y^2 dx + y^2 dz = y^2 f'(dy) \quad -②$$

Comparing ① & ② $\Rightarrow$ $-(x+z)\,dy = y^2 f'(dy)$

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$$-(x+y)^2 = y^2 f'$$

$$-f' = y^2 f'$$

$$\frac{df}{f^2} + \frac{dy}{y^2} = 0$$

$$\int \frac{df}{f^2} + \int \frac{dy}{y^2} = 0$$

$$\frac{1}{f} + \frac{1}{y} = k$$

$$\frac{1}{x+y} + \frac{1}{y} = k$$

$$x+y+y = k(x+y)y$$

⇒ **Programs :-**

1. 
```
from sympy import *
x, y, z = symbols('x y z')
P = y+z
Q = z+x
R = x+y
f = simplify (P*(diff (R,z) - diff(R,y)) + Q*(diff (R,
    diff (P,z)) + R*(diff (P,y) - diff (Q,x)))
if f==0:
    print ('The given equation is integrable')
else :
    print ('The given equation is not integrable')
```