Expt. No. 2 Date 20/04/22

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Linear Dependence, Independence Basis and Dimensions

Problems:

I Linear Dependence:

1) $S=\{(1,0),(2,0)\}$ is linearly dependent or independent in $\mathbb{R}^2$.

Soln $c_1(1,0)+c_2(2,0)=0$

$c_1=1 \qquad c_2=-1/2$

$1(1,0) \; -\frac{1}{2}(2,0)=(0,0)$

$\therefore$ S is linearly dependent

2) $S=\{(1,0),(0,2)\}$ is linearly dependent or independent in $\mathbb{R}^2$.

Soln $c_1(1,0)+c_2(0,2)=(0,0)$

$c_1=0 \qquad c_2=0$

$\therefore$ S is linearly independent

3) $S=\{(1,0,0,-1)(0,1,0,-1)\ (0,0,1,-1)\ (0,0,0,1)\}$ is linearly dependent or independent in $\mathbb{R}^4$.

Soln $c_1(1,0,0,-1)+c_2(0,1,0,-1)+c_3(0,0,1,-1)+c_4(0,0,0,1)=(0,0,0,0)$

$c_1=0 \qquad c_2=0 \qquad c_3=0$

$-c_1+c_2-c_3+c_4=0$

$c_4=0$

$\therefore$ S is linearly independent

4) $S=\{1+x+x^2,\ x+x^2,\ x^2\}$

$c_1(1+x+x^2)+c_2(x+x^2)+c_3(x^2)=0$

$c_1(c_1+c_2)x+(c_1+c_2+c_3)x^2=0$

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True

$c_1 = 0$

$c_1 + c_2 = 0$

$c_2 = 0$

$c_1 + c_2 + c_3 = 0$

$c_3 = 0$

$\therefore$ S is linearly independent

II Basis:

1) Is $S = \{(1,0,0,-1), (0,1,0,-1), (0,0,1,-1), (0,0,0,1)\}$ a basis of $\mathbb{R}^4$.

Sol^n S is linearly Independent

No. of elts in the subset is 4.

No of Dimensions in the vector space is 4.

$\therefore$ It is a basis

2) Is $S = \{1+x+x^2, x+x^2, x^2\}$ a basis of $\mathbb{R}^4$.

Sol^n S is linearly Independent

No. of elements in the subset is 3.

No. of Dimensions in the vector space is 4.

$\therefore$ It is not a basis

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PROGRAM:

```
import numpy as np
import sympy as sp
def is_independent(S):
    variables = sp.symbols(f'a:{len(S)}')
    M = None
    for i, var in enumerate(variables):
        if M is None:
            M = var * S[i]
        else:
            M = M + var * S[i]
    Soln = sp.solve(M, variables, manual=True)
    if np.abs(Soln[0]).sum()==0:
        print("Yes! The given set is linearly independent")
        return True
    else:
        print("NO! The given set is linearly dependent")
        print("And the scalars are", Soln)
        return False
```

1. S = np.array([[1,0], [2,0]])
   is_independent(S)
2. S = np.array([[1,0], [0,2]])
   is_independent(S)
3. S = np.array([[1,0,0,-1], [0,1,0,-1], [0,0,1,-1], [0,0,0,-1]])
4. x = sp.Symbols('x')
   S = np.array([[1+x+x**2, x+x**2, x**2]])
   is_independent(S)

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II
```
def is_basis(B, dim):
    if is_independent(B):
        if len(B)==int(dim):
            print("And the dimension of V is", dim,
                  "=number of elements of B.").
            print("So, the given set is a basis")
        else:
            print("But, the dimension is not matched.
                  So not a basis")
    else:
        print("so, the given set is not a basis")
```

1. B = sp.array([[1,0,0,-1],[0,1,0,-1],[0,0,1,-1],[0,0,0,0]])
   dim = 4
   is_basis(B, dim)

2. x = sp.Symbols('x')
   B = sp.array([1+x+x**2, x+x**2, x**2])
   dim = 4
   is_basis(B, dim)

22/5/22

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