Experiment no - 06

Solutions to the problems on different types (I, II, III and clairat's eq^n) of P.d.e.

Type I eq^{n}s containing p and q only $f(p,q)=0$ — ①

Sln:- let the solution of eq^n ① be $z = ax+by+c$

$$\Rightarrow P = \frac{dz}{dx} = a, \quad q = \frac{dz}{dy} = b.$$

put $p=a$ and $q=b$ in eqn ①

$$f(a,b)=0 \Rightarrow b = \phi(a)$$

$\therefore$ Soln of ① is $z = ax + \phi(a)y + c$.

① Solve $p^2 + q^2 = 1$

let $z = ax+by+c$. be the solution

$\therefore P=a \quad q=b$

① $\Rightarrow a^2+b^2=1 \Rightarrow b^2 = 1-a^2 \Rightarrow b = \pm\sqrt{1-a^2}$

$\therefore$ Soln:- $z = ax \pm \sqrt{1-a^2}\cdot y + c$

② $pq = 1$

$P=a \quad q=b$

$Pb=1 \Rightarrow ab=1 \Rightarrow b = 1/a$

$$z = ax + \frac{1}{a}y + c$$

③ $pq + p + q = 0$

$P=a \quad q=b$

$ab + a + b = 0 \Rightarrow a(b+1)+a=0 \Rightarrow b = \frac{-a}{a+1}$

$$z = ax - \frac{a}{a+1}y + c$$

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①
```
from sympy.abc import x,y,a,b,c.
from sympy import *
f=function('f')
u=f(x,y)
P=u.diff(x).
q=u.diff(y)
eq=p**2+q**2-1
Eq=eq.subs(p,a).subs(q,a)
Print("Equation becomes," Eq)
b_val=solve(Eq,b)
Print("b-", b-value)
z=(a*x+b*y+c)
ans= z.subs(b, b-val[0])
print(" The required solution is z=", ans).
```

②
```
eq=p*q-1
```

③
```
Eq=p*q+p+q
```

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Type II : Eq^n of the form $f(p,q,z)=0$ — ①
This eq^n is independent of variable x & y

Sln:- Let z be a function of u where $u=x+ay$

$$p=\frac{dz}{dx}=\frac{dz}{du}\frac{du}{dx}=\frac{dz}{du}$$

$$q=\frac{dz}{dy}=\frac{dz}{du}\frac{du}{dy}=a\frac{dz}{du}$$

$$\Rightarrow f\left(\frac{dz}{du},\ a\frac{dz}{du},\ z\right)=0$$

This is an ordinary diff eq of first order which can be solved.

① Solve $p(1-q^2)=q(1-z)$.

Let z be function of u where $u=x+ay$

$$① \Rightarrow \frac{dz}{du}\left(1-a^2\left(\frac{dz}{du}\right)^2\right)=a\frac{dz}{du}\left(1-z\right).$$

$$1-a^2\left(\frac{dz}{du}\right)^2=a(1-z)$$

$$a^2\left(\frac{dz}{du}\right)^2=1-a+az$$

$$a\frac{dz}{du}=\sqrt{1-a+az}$$

$$\int\frac{a\,dz}{\sqrt{az-a+1}}=\int du \qquad\qquad \int\frac{f'(x)}{\sqrt{f(x)}}dx=2\sqrt{f(x)}$$

$$2\sqrt{az-a+1}=u+c$$

$$4(az-a+1)=(x+ay+c)^2$$

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2\. Solve $p(1+q) = zq$ $\qquad p = dz/du \qquad q = a\, dz/du$

$$\frac{dz}{du}\left(1 + a\frac{dz}{du}\right) = z\,a\frac{dz}{du}$$

$$a\frac{dz}{du} = az - 1$$

integrate $\displaystyle\int a\frac{dz}{az-1} = \int du$

$$\log(az-1) = u + c$$

$$\log(az-1) = x + ay + c.$$

3\. $p = qz$

$p = dz/du \qquad q = a\,dz/du$

$$\frac{dz}{du} = a\frac{dz}{du}\cdot z$$

$$1 = az$$

$$z = \frac{1}{a}$$

```
from sympy import *
from sympy.abc import x, y, u, p, q, a, b
z = Function ("z")(u)
eqn = p*(1- q**2)*(1-z)
eqn1 = eqn.subs(p.diff(z,u)).subs(q, a*diff(z,u))
dzdu = solve(eqn1, diff(z, u))
print("All posible values of dz/du are")
pprint(dzdu[0])
pprint(dzdu[1])
```

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```
print (dzdu [2])
sol = dsolve (dzdu[2] - diff (z,u))
print (sol)
ans = sol. subs (u, x+a*y)
print ("The solution is z=", ans.rhs).
```

(2)
```
Eqn = p*(1+q) - (z*q
dzdu = solve (eqn 1, diff (z,u))
print ("All possible value of dz/du are", dzdu [0], 'and',
                                           dzdu [1])
```

(3)
```
z = function ('z')(u)
eqn = p/q - z
z = solve (eqn 1, z)
print ("the solution is z=", Z.[0])
```

Type III eqn of the form $f_1(x,p) = f_2(y,q)$

The eq^n where variable z is about absent the term can be separated from those containing q & y can be written in form $f_1(x,p) = f_2(y,q)$

Soln. put $f_1(x,p) = f_2(y,q) = a$

express $p = f(x,a)$, $q = g(y,a)$

sub those values of p & q in the eq^n

$$dz = p\,dx + q\,dy$$

integrate & get the complete solution.

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$$dz = (a + \sin x)\,dx + (\sin y - a)\,dy$$

integrating

$$z = ax - \cos x - ay - \cos y + c.$$

1.
```
from sympy import *
from sympy.abc import x,y,p,q
lhs = p+x
rhs = q+y
r1 = Eq (lhs, a)
r2 = Eg (rhs, a)
h1 = solve (r1, p)
h2 = solve (r2, q)
print ("p =", h1, "and", "q = ", h2)
z = integrate (h1 [0], x)+integrate (h2[0], y)
print ("The solution is z =", z).
```

2.
```
lhs = p-cos(x)
rhs = cos(y)/q
```

3.
```
lhs = p-sin(x)
rhs = -q + sin(y)
```

Type IV Eq's of the form $z = px + qy + f(p,q)$. This eq^n is called clairant's equation.

solⁿ Replace p by a & q by b
$z = ax + by + f(a,b)$

① Solve $z = px + qy + (p^2+q^2)$
put $p=a$ $q=b$
$z = ax + by + (a^2+b^2)$

② $z = px + qy + \frac{pq}{p-q}$
put $p=a$ $q=b$
$z = ax + by + \frac{ab}{a-b}$

③ $z = px + qy + \log(pq)$
put $p=a$ $q=b$
$z = ax + by + \log(ab)$.

```
from sympy.abc import x,y,a,b,p,q
from sympy import *
z= p*x + q*y + (p**2 - q**2)
sol= z.subs([(p,a), (q,b)])
print ("General solution is z=", sol)
```

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(2) $z = p^*x + q^*y + (p^*q/p\text{-}q)$

(3) $z = p^*x + q^*y + \log(p^*q)$

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