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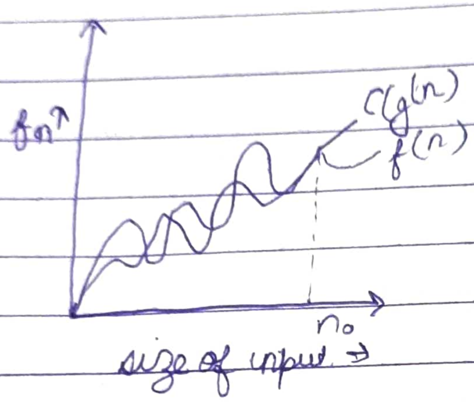
Semester - 4

A1. <u>Asymptotic Notations</u>
↳ Tending to Infinity

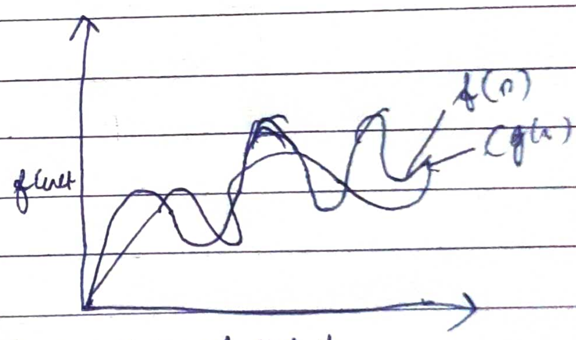
They help you find the complexity an algorithm when input is very large

① Big O(o)



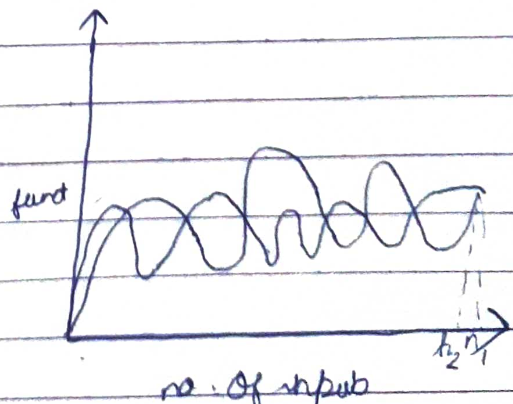
$f(n) = O(g(n))$
iff $f(n) \leq Cg(n)$
$\forall n \geq n_0$
for some constant $C > 0$
⇒ g(n) is tight upper bound of f(n)

② Big Omega (Ω)
$f(n) = \Omega(g(n))$
g(n) is tight lower bound of f(n)



$f(n) = \Omega(g(n))$
iff $f(n) \geq Cg(n)$
$\forall n \geq n_0$ for some cont $c > 0$ n no of inputs

③ Theta (θ)
$f(n) = \Theta(g(n))$
g(n) is both "tight" upper & lower bound of fn f(n)



$f(n) = \Theta(g(n))$
iff
$C_1 g(n) \leq f(n) \leq C_2 g(n)$
$\forall n \geq \max(n_1, n_2)$
for some constant $c_1 > 1$ & $c_2 > 0$

4. small $o(o)$

$f(n) = o(g(n))$

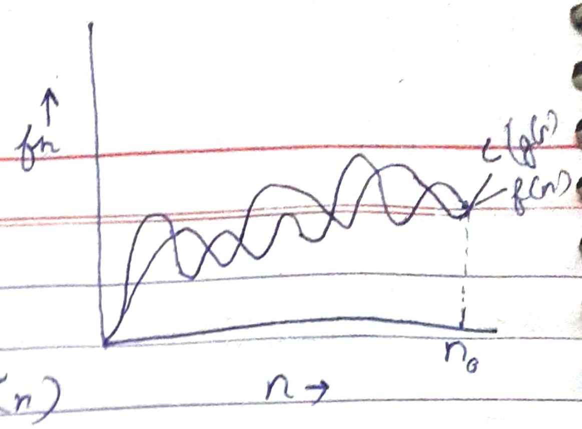
$g(n)$ is upper bound of fn $f(n)$

$f(n) = o(g(n))$

when $f(n) \leq c \cdot g(n)$

$\forall\ n > n$

$\&\ \forall\ c > 0$



5. small omega (ω)

$f(n) = \omega(g(n))$

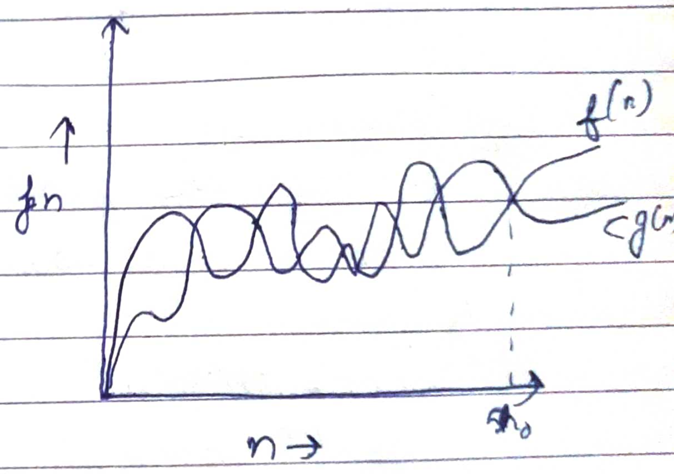
$g(n)$ is lower bound of fn $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall\ n > n_0$

$\&\ \forall\ c > 0$



Ques 2

for (i=1 to n)   // i = 1, 2, 4, 8, ..., n
{ i = i * 2 }   // O(1)

$\Rightarrow \sum_{i=1} 1+2+4+8+ \ldots n$

GP kth value $\Rightarrow T_k = a r^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow n = 2^k$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\Rightarrow \log n + 1 = k$

$\Rightarrow O(k) = O(1 + \log n)$

$= O(\log n)$

Q3. $T(n) = 3T(n-1)$ — ①

$n = n-1$

$T(n-1) = 3T(n-2)$ — ②

from 1 & 2

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ — ③

putting $n = n-2$ in ①

$T(n) = 3(T(n-3))$

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^k(T(n-k))$

putting $n-k=0$

$n = k$

$\Rightarrow T(n) = 3^n[T(n-n)]$

$\Rightarrow T(n) = 3^n T(0)$

$T(n) = 3^n \times 1$ $\qquad T(0) = 1$

$T(n) = O(3^n)$

4. $T(n) = \{ 2T(n-1) - 1$ — ①

Let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1$ — ②

from ① & ②

$\Rightarrow T(n) = 2[2T(n-2) - 1] - 1$

$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ — ③

Let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ — ④

from ③ & ④

$\Rightarrow T(n) = 4[2T(n-3) - 1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

$\Rightarrow T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2}$

$\Rightarrow GP = 2^{k-1} + 2^{k-2} + 2^{k-3} + \ldots\ldots 1$

$a = 2^{k-1}$

$r = 1/2$

$\Rightarrow \quad = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{k-1}(1-(1/2)^n)}{1}$

$= 2^k(1-(1/2)^k)$

$= 2^k - 1$

Let $n - k = 0$

$\Rightarrow n = k$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n - 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

$T(n) = O(1)$

Ans 5.

Sum of $s = 1 + 3 + 6 + 10 + \dots + n$

also $s = 1 + 3 + 6 + 10 + \dots + T_m + T_n$.

from ① - ②

$0 = 1 + 2 + 3 + 4 + \dots n - T_n$

$\Rightarrow T_k = 1 + 2 + 3 + 4 + \dots k$

$\Rightarrow T_k = \frac{1}{2} k(k+1)$

$\Rightarrow$ for k iterations

$1 + 2 + 3 + \dots + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\Rightarrow \frac{k^2 + k}{2} <= n$

$\Rightarrow O(k^2) <= n$

$\Rightarrow k = O(\sqrt{n})$

$\Rightarrow T(n) = O(\sqrt{n})$

A6 as $i^2 <= n$

$i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots\ldots, \sqrt{n}$

$\sum_{i=1} 1+2+3+4+\ldots\ldots+\sqrt{n}$

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = O(n)$

~~A8.~~

A7. for $k = k^{*2}$

$k = 1, 2, 4, 8, \ldots\ldots, n$

GP $\Rightarrow$ $a=1$, $r=2$

$R_o = \frac{a(r^n - 1)}{r-1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k$

$\Rightarrow \log n = k$

$\Rightarrow$

i	j	k
1	log n	log n * log n
2	log n	log n * log n
⋮	⋮	⋮
n	log n	log n * log n

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

(8)

$\Rightarrow T(n) = T(n/3) + n^2$

$\Rightarrow a = 1,\ b = 3,\quad f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow n^0 = 1 < (f(n) = n^2)$

$\Rightarrow T(n) = \theta(n^2)$

9. for $i = 1 \Rightarrow j = 1, 2, 3, 4, \ldots n = n$

for $i = 2 \Rightarrow j = 1, 3, 5 \ldots n = n/2$

for $i = 3 \Rightarrow j = 1, 4, 7, \ldots n = n/3$

for $i = n \Rightarrow j = 1$

$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \ldots + 1$

$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n}\right]$

$\sum_{j=n} n[\log n]$

$\Rightarrow T(n) = [n \log n]$

$T(n) = O(n \log n)$

10- as given n^k & c^n
relation b/w n^k & c^n is

$n^k = O(c^n)$

as $n^k \le ac^n$

$\forall\, n \ge n_0$ & some constant $a > 0$

for $n_0 = 1$

$c = 2$

$1^k \le a^1$

$\Rightarrow n_0 = 1$ & $c = 2$