The above assumption is true when the distribution is symmetrical and the no. of class-intervals is not greater than $\frac{1}{20}$th of the range, otherwise the computation of moments will have certain error called **grouping error.**

This error is corrected by the following formulae given by **W.F. Sheppard**.

$$\mu_2 \text{ (corrected)} = \mu_2 - \frac{h^2}{12}$$

$$\mu_4 \text{ (corrected)} = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4$$

where h is the width of the class-interval while μ_1 and μ_3 require no correction.

These formulae are known as **Sheppard's corrections.**

Example 3. *Find the corrected values of the following moments using Sheppard's correction. The width of classes in the distribution is 10:*

$\mu_2 = 214,\quad \mu_3 = 468,\quad \mu_4 = 96712.$

Sol. We have $\mu_2 = 214,\quad \mu_3 = 468,\quad \mu_4 = 96712,\quad h = 10.$

Now, $$\mu_2 \text{ (corrected)} = \mu_2 - \frac{h^2}{12} = 214 - \frac{(10)^2}{12} = 214 - 8.333 = 205.667.$$

$$\mu_3 \text{ (corrected)} = \mu_3 = 468$$

$$\mu_4 \text{ (corrected)} = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4 = 96712 - \frac{(10)^2}{2}(214) + \frac{7}{240}(10)^4$$

$$= 96712 - 10700 - 291.667 = 86303.667.$$

3.8 MOMENTS ABOUT AN ARBITRARY NUMBER (Raw Moments)

If $x_1, x_2, x_3, ..., x_n$ are the values of a variable x with the corresponding frequencies $f_1, f_2, f_3, ..., f_n$ respectively then r^{th} moment μ_r' about the number $x = \text{A}$ is defined as

$$\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r \; ; r = 0, 1, 2, ... \quad \text{where,} \quad N = \sum_{i=1}^{n} f_i$$

For $r = 0$, $$\mu_0' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^0 = 1$$

For $r = 1$, $$\mu_1' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i - \frac{A}{N}\sum_{i=1}^{n} f_i = \bar{x} - A$$

For $r = 2$, $$\mu_2' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^2$$

For $r = 3$, $$\mu_3' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^3 \text{ and so on.}$$

In calculation work, if we find that there is some common factor h (> 1) in values of $x - A$, we can ease our calculation work by defining $u = \frac{x - A}{h}$. In that case, we have

$$\mu_r' = \frac{1}{N}\left(\sum_{i=1}^{n} f_i u_i^r\right) h^r \; ; r = 0, 1, 2, \ldots$$

Note. For an individual series,

1. $\mu_r' = \frac{1}{n}\sum_{i=1}^{n} (x_i - A)^r \; ; r = 0, 1, 2, \ldots$

2. $\mu_r' = \frac{1}{N}\left(\sum_{i=1}^{n} u_i^r\right) h^r \; ; r = 0, 1, 2, \ldots$ $\qquad \Big|$ for $u = \frac{x - A}{h}$

3.9 MOMENTS ABOUT THE ORIGIN

If $x_1, x_2, \ldots, x_n$ be the values of a variable x with corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively then r^{th} moment about the origin ν_r is defined as

$$\nu_r = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^r \; ; r = 0, 1, 2, \ldots \quad \text{where,} \quad N = \sum_{i=1}^{n} f_i$$

For $r = 0$, $\qquad \nu_0 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^0 = \frac{N}{N} = 1$

For $r = 1$, $\qquad \nu_1 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \bar{x}$

For $r = 2$, $\qquad \nu_2 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^2$ and so on.

3.10 RELATION BETWEEN μ_r AND μ_r'

We know that,

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i \left[(x_i - A) - (\bar{x} - A)\right]^r$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \left[(x_i - A) - \mu'_1\right]^r \qquad \Big| \because \mu_1' = \bar{x} - A$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i \left[(x_i - A)^r - {}^rC_1 (x_i - A)^{r-1} \mu_1' + {}^rC_2 (x_i - A)^{r-2} \mu_1'^2 - \ldots\ldots + (-1)^r \mu_1'^r\right]$$

$\Rightarrow$ $\qquad \Big|$ Using binomial theorem

$$\mu_r = \mu_r' - {}^rC_1 \mu'_{r-1}\mu_1' + {}^rC_2 \mu'_{r-2}\mu_1'^2 - \ldots\ldots + (-1)^r \mu_1'^r$$

Putting $r = 2, 3, 4$, we get

$$\mu_2 = \mu_2' - 2\mu_1'^2 + \mu_1'^2 = \mu_2' - \mu_1'^2 \qquad | \because \mu_0' = 1$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 3\mu_1'^3 - \mu_1'^3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Hence, we have the following relations:

$$\boxed{\mu_1 = 0}$$

$$\boxed{\mu_2 = \mu_2' - \mu_1'^2}$$

$$\boxed{\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3}$$

and

$$\boxed{\mu_4 = \mu_4' - 4\mu'_3\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4}$$

3.11 RELATION BETWEEN ν_r AND μ_r

We know that,

$$\nu_r = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^{\,r} \; ; r = 0, 1, 2, \ldots$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A + A)^r$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i [(x_i - A)^r + {}^rC_1 (x_i - A)^{r-1} . A + \ldots + A^r]$$

$$= \mu_r' + {}^rC_1 \mu_{r-1} A + \ldots + A^r$$

If we take, $A = \bar{x}$ (for μ_r) then ...(1)

$$\nu_r = \mu_r + {}^rC_1\mu_{r-1}\,\bar{x} + {}^rc_2\mu_{r-2}\,\bar{x}^2 + \ldots + \bar{x}^r$$

Putting, $r = 1, 2, 3, 4$ in (1), we get $\qquad | \because \mu_1 = 0, \mu_0 = 1$

$$\nu_1 = \mu_1 + \mu_0\bar{x} = \bar{x}$$

$$\nu_2 = \mu_2 + {}^2c_1\mu_1\bar{x} + {}^2c_2\,\mu_0\bar{x}^2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + {}^3c_1\mu_2\bar{x} + {}^3c_2\mu_1\bar{x}^2 + {}^3c_3\mu_0\bar{x}^3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + {}^4c_1\mu_3\bar{x} + {}^4c_2\mu_2\bar{x}^2 + {}^4c_3\mu_1\bar{x}^3 + {}^4c_4\mu_0\bar{x}^4$$

$$= \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

Hence, we have the following relations:

$$\boxed{\nu_1 = \bar{x}} \qquad \boxed{\nu_2 = \mu_2 + \bar{x}^2}$$

$$\boxed{\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3} \quad \text{and} \quad \boxed{\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4.}$$

3.12 KARL PEARSON'S β AND γ COEFFICIENTS

Karl Pearson defined the following four coefficients based upon the first four moments of a frequency distribution about its mean:

$$\left.\begin{aligned}\beta_1 &= \frac{\mu_3^2}{\mu_2^3}\\ \beta_2 &= \frac{\mu_4}{\mu_2^2}\end{aligned}\right\} \qquad (\beta\text{-coefficients})$$

$$\left.\begin{aligned}\gamma_1 &= +\sqrt{\beta_1}\\ \gamma_2 &= \beta_2 - 3\end{aligned}\right\} \qquad (\gamma\text{-coefficients})$$

The practical use of these coefficients is to measure the skewness and kurtosis of a frequency distribution. These coefficients are pure numbers independent of units of measurement.

Example 4. *The first three moments of a distribution, about the value '2' of the variable are 1, 16 and – 40. Show that the mean is 3, variance is 15 and $\mu_3 = -86$.*

Sol. We have $A = 2,\ \mu_1' = 1,\ \mu_2' = 16,$ and $\mu_3' = -40$

We know that $\mu_1' = \bar{x} - A \Rightarrow \bar{x} = \mu_1' + A = 1 + 2 = 3$

$$\text{Variance} = \mu_2 = \mu_2' - \mu_1'^2 = 16 - (1)^2 = 15$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -40 - 3(16)(1) + 2(1)^3 = -40 - 48 + 2 = -86.$$

Example 5. *The first four moments of a distribution, about the value '35' are – 1.8, 240, – 1020 and 144000. Find the values of $\mu_1, \mu_2, \mu_3, \mu_4$.*

Sol.

$$\mu_1 = 0.$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 240 - (-1.8)^2 = 236.76$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -1020 - 3(240)(-1.8) + 2(-1.8)^3 = 264.36$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 144000 - 4(-1020)(-1.8) + 6(240)(-1.8)^2 - 3(-1.8)^4 = 141290.11.$$

Example 6. *Calculate the variance and third central moment from the following data:*

x_i	0	1	2	3	4	5	6	7	8
f_i	1	9	26	59	72	52	29	7	1

Sol.

Calculation of Moments

x	f	$u = \frac{x - A}{h}$, $A = 4, h = 1$	fu	fu^2	fu^3
0	1	– 4	– 4	16	– 64
1	9	– 3	– 27	81	– 243
2	26	– 2	– 52	104	– 208
3	59	– 1	– 59	59	– 59
4	72	0	0	0	0