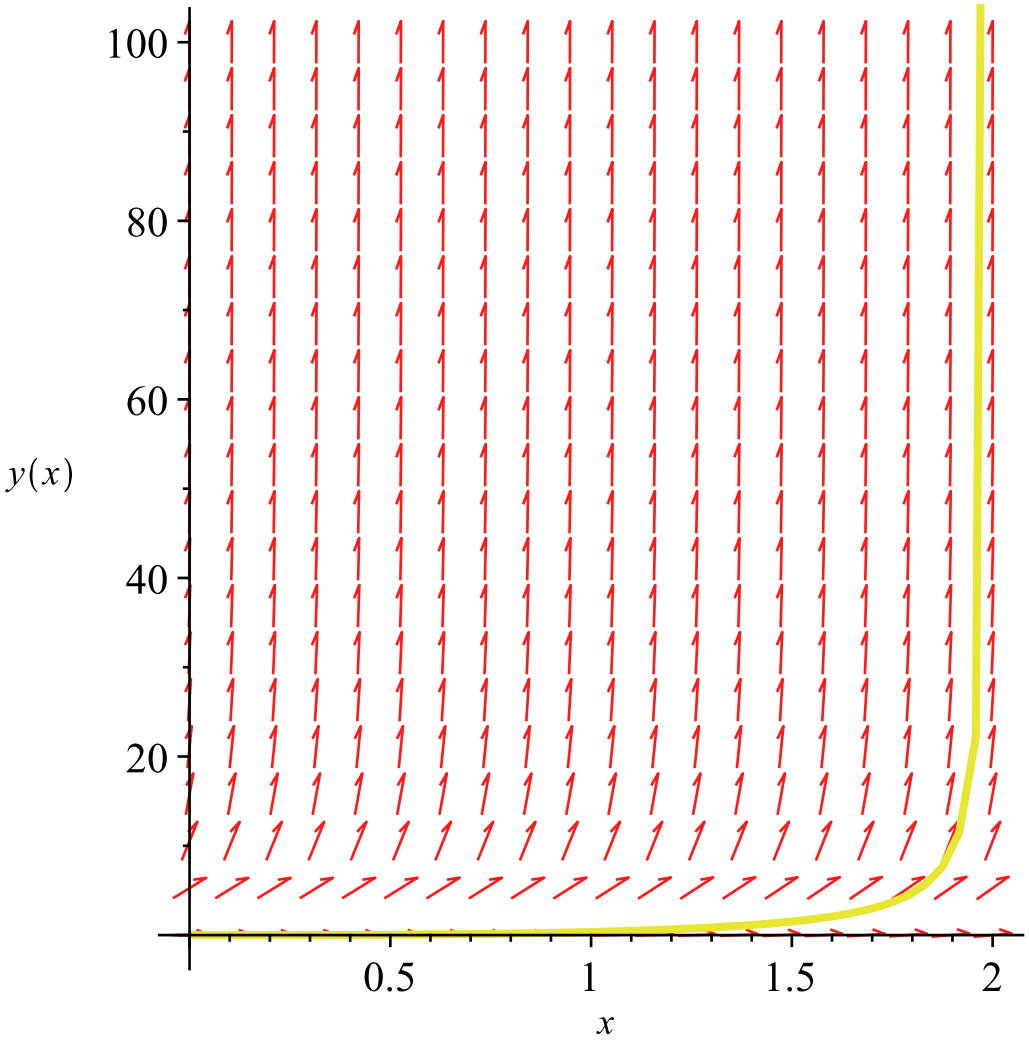
#problem 5

```
restart : with(DEtools) : f := (x, y) → x^2 + y^2; dsolve( {diff(y(x), x) = y(x)^2 + x^2, y(0) = 0} );
```

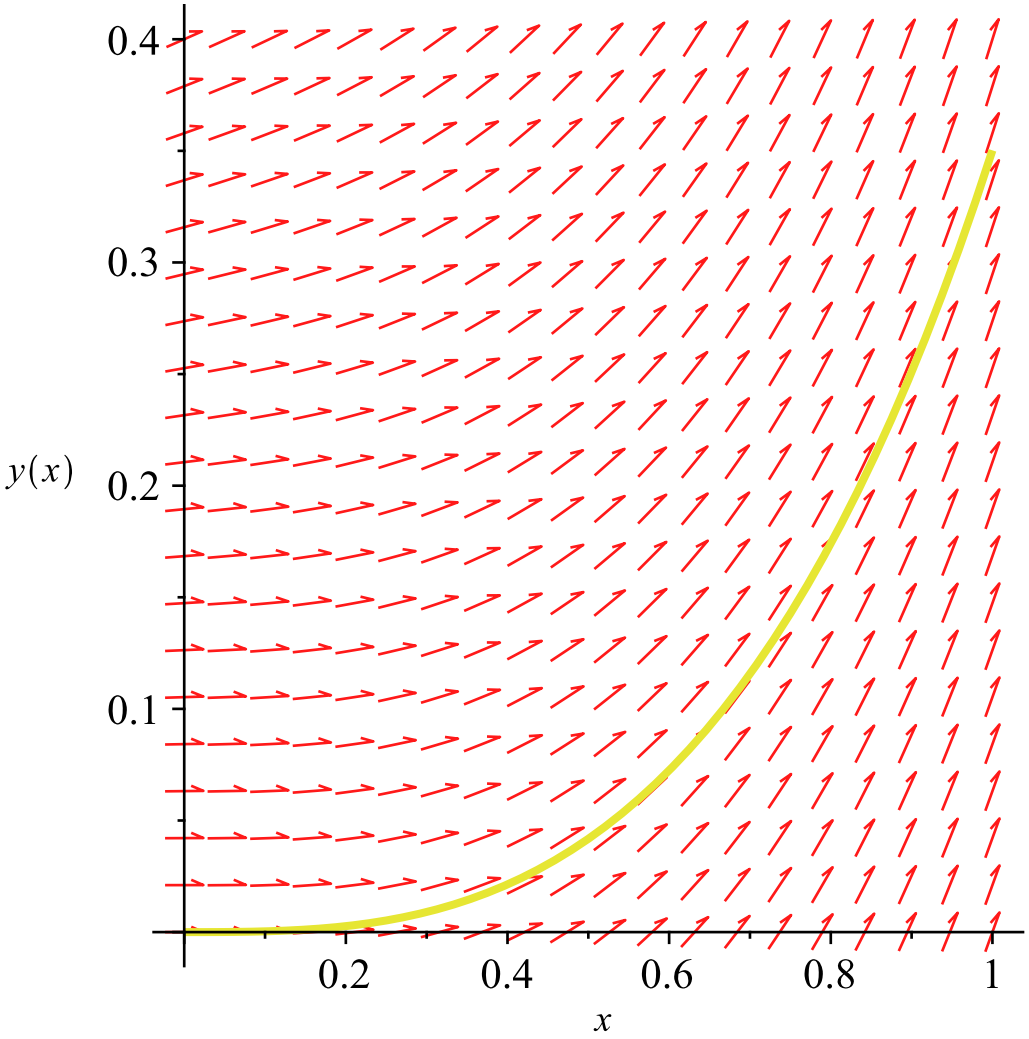
$$f := (x, y) \mapsto y^2 + x^2$$

$$y(x) = \begin{cases} 0 & x = 0 \\ \dfrac{\left(\text{BesselJ}\left(-\frac{3}{4}, \frac{x^2}{2}\right) - \text{BesselY}\left(-\frac{3}{4}, \frac{x^2}{2}\right)\right) x}{-\text{BesselJ}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \text{BesselY}\left(\frac{1}{4}, \frac{x^2}{2}\right)} & \textit{otherwise} \end{cases} \tag{1}$$

```
DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0..2, [[y(0) = 0]], y = 0..100);
```




```
DEplot(diff(y(x), x) = f(x, y(x)), y(x), x = 0..1, [[y(0) = 0]], y = 0..0.4);
```



$h := 0.1; x := 0; y := 0;$

$$h := 0.1$$
$$x := 0$$
$$y := 0$$ **(2)**

$h := 0.1; x := 0; y := 0;$
for i **from** 1 **to** 20 **do** $y := y + h * f(x, y) : x := x + h : print(x, y)$; **od**:

$$h := 0.1$$
$$x := 0$$
$$y := 0$$
$$0.1, 0.$$
$$0.2, 0.001$$
$$0.3, 0.0050001$$
$$0.4, 0.01400260010$$
$$0.5, 0.03002220738$$
$$0.6, 0.05511234067$$

0.7, 0.09141607768

0.8, 0.1412517676

0.9, 0.2072469738

1.0, 0.2925421046

1.1, 0.4011001929

1.2, 0.5381883294

1.3, 0.7111529972

1.4, 0.9307268557

1.5, 1.213352104

1.6, 1.585574437

1.7, 2.092979066

1.8, 2.820035203

1.9, 3.939295058

2.0, 5.852099613 **(3)**

$h := 0.1; x := 0; y := 0;$

$$h := 0.1$$

$$x := 0$$

$$y := 0$$ **(4)**

for i **from** 1 **to** 20 **do** $y := y + h/2 * f(x, y) + h/2 * f(x + h, y + h * f(x, y)) : x := x + h : print(x, y)$; **od**:

0.1, 0.0005000000000

0.2, 0.003000125004

0.3, 0.009503025760

0.4, 0.02202467595

0.5, 0.04262140864

0.6, 0.07344210066

0.7, 0.1168165840

0.8, 0.1753963673

0.9, 0.2523742135

1.0, 0.3518301326

1.1, 0.4792938348

1.2, 0.6427029949

1.3, 0.8541363558

1.4, 1.133184603

1.5, 1.514119178

1.6, 2.062972003

1.7, 2.924894430

1.8, 4.487143656

1.9, 8.165117641