#sollutions by Goian Tudor George, IE1, gr913
#1
#a)

$eq1 := diff(x(t), t) = 1 - x(t)^2$

$$eq1 := \frac{d}{dt}\, x(t) = 1 - x(t)^2 \quad \textbf{(1)}$$

 # now we also know that the derivative of a constant is 0 and we solve the equation

$solve(-x^2 + 1, x)$

$$-1, 1 \quad \textbf{(2)}$$

#b)

$expr1 := rhs(dsolve(\{eq1, x(0) = \mathrm{eta}\}));$

$$expr1 := \tanh(t + \operatorname{arctanh}(\eta)) \quad \textbf{(3)}$$

$\mathrm{phi} := unapply(expr1, (t, \mathrm{eta}));$

$$\phi := (t, \eta) \mapsto \tanh(t + \operatorname{arctanh}(\eta)) \quad \textbf{(4)}$$

$\mathrm{phi}(t, \mathrm{eta});$

$$\tanh(t + \operatorname{arctanh}(\eta)) \quad \textbf{(5)}$$

 # if we try for -1, 1 we'll se that the it is not defined

$\mathrm{phi}(t, 1);$

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Error, (in arctanh) numeric exception: division by zero
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$\mathrm{phi}(t, -1);$

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Error, (in arctanh) numeric exception: division by zero
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#in order to find the sollutions we'll just apply dsolve

$dsolve(\{eq1, x(0) = 1\});$

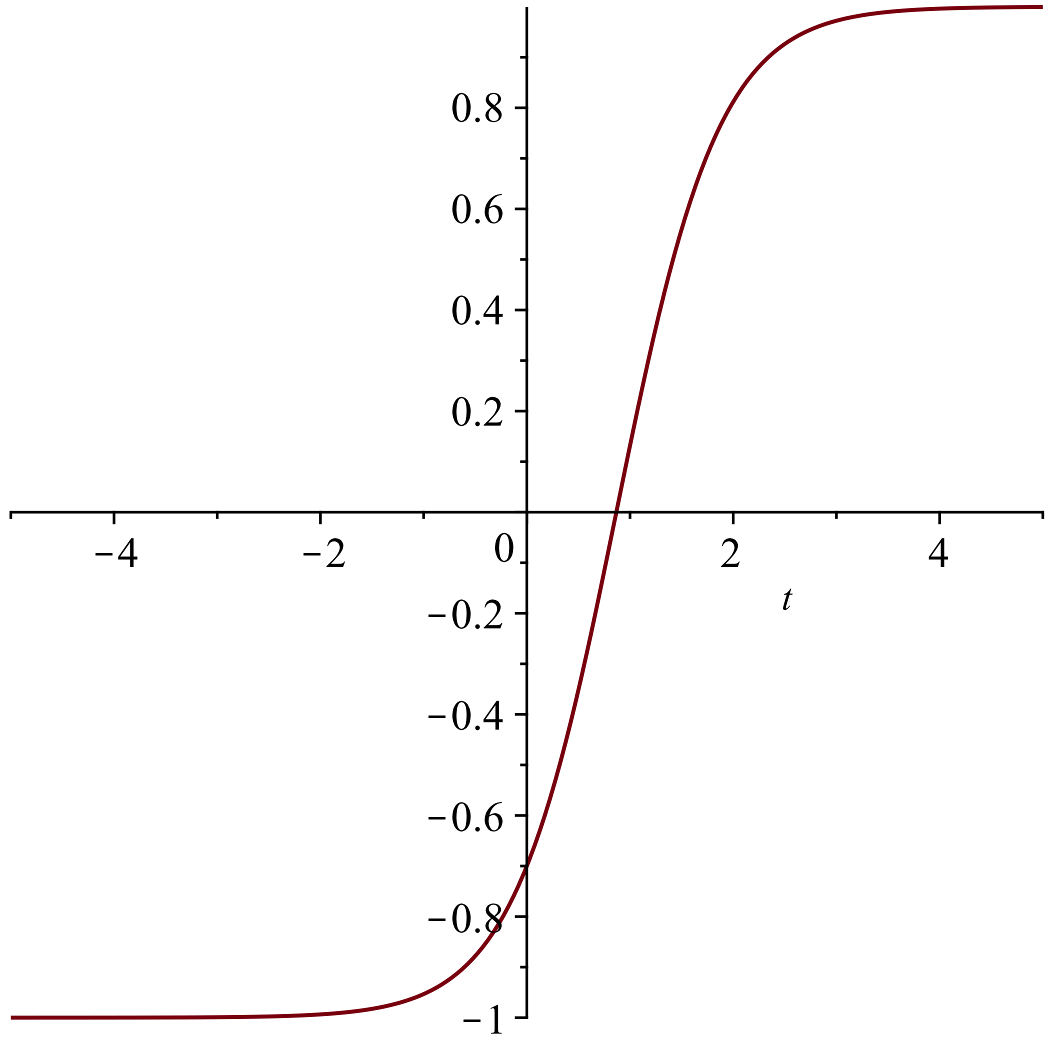
$$x(t) = 1 \quad \textbf{(6)}$$

$dsolve(\{eq1, x(0) = -1\});$

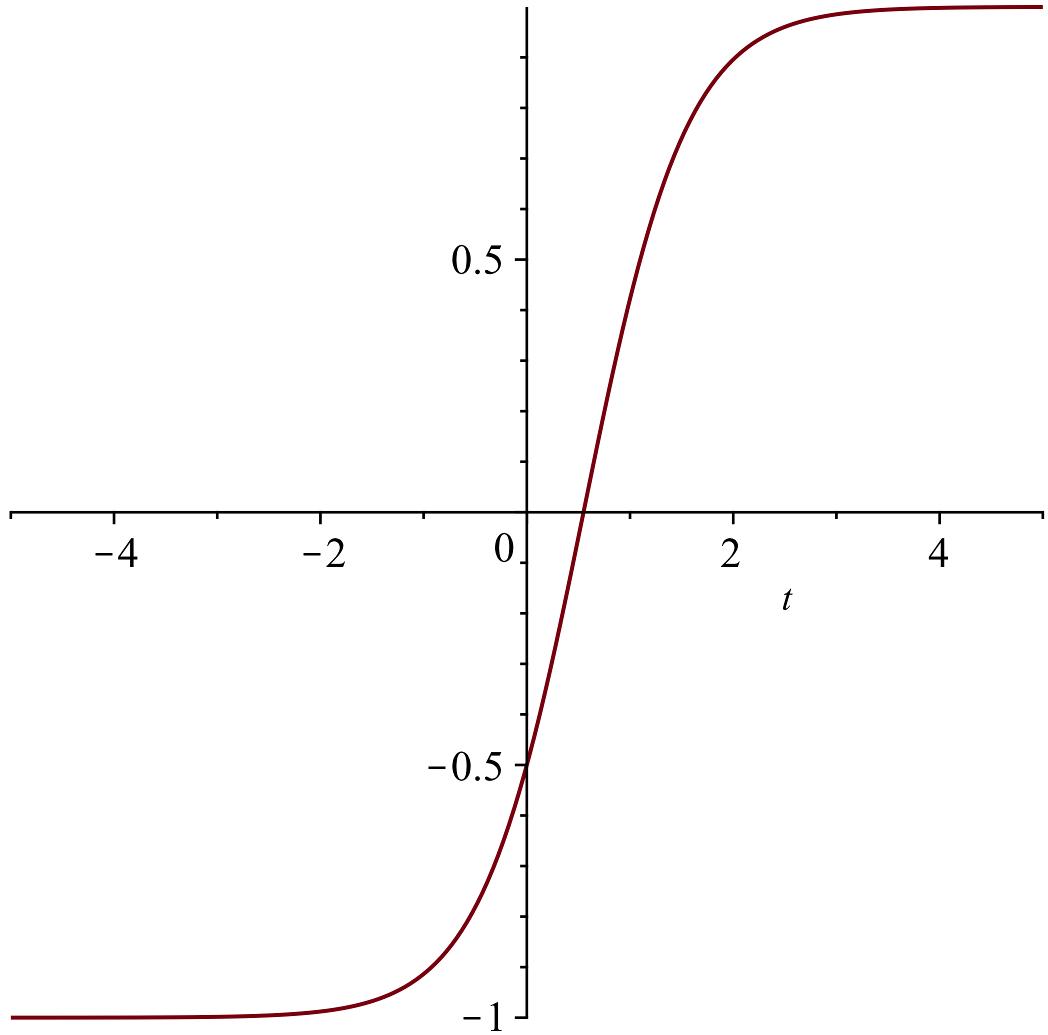
$$x(t) = -1 \quad \textbf{(7)}$$

#c)

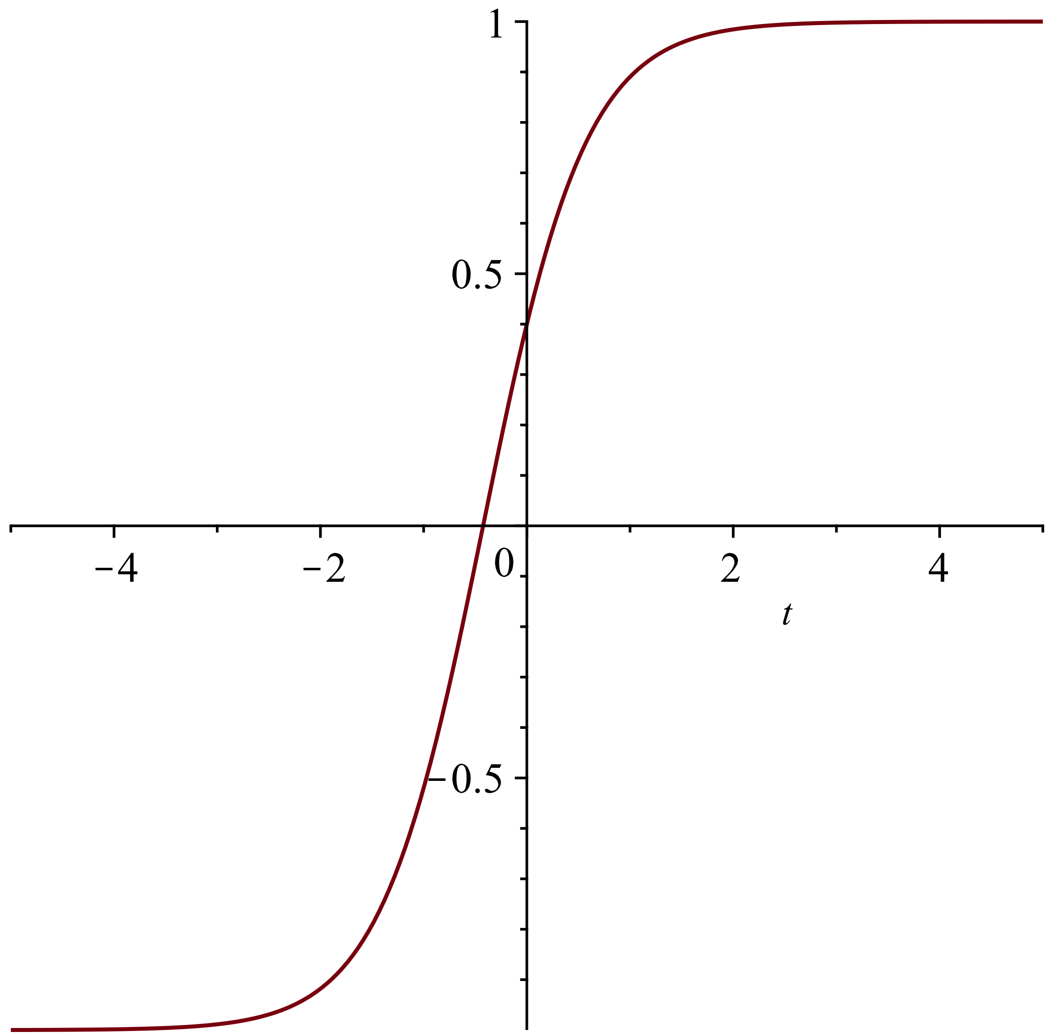
$plot(\mathrm{phi}(t, -0.7), t = -5..5);$



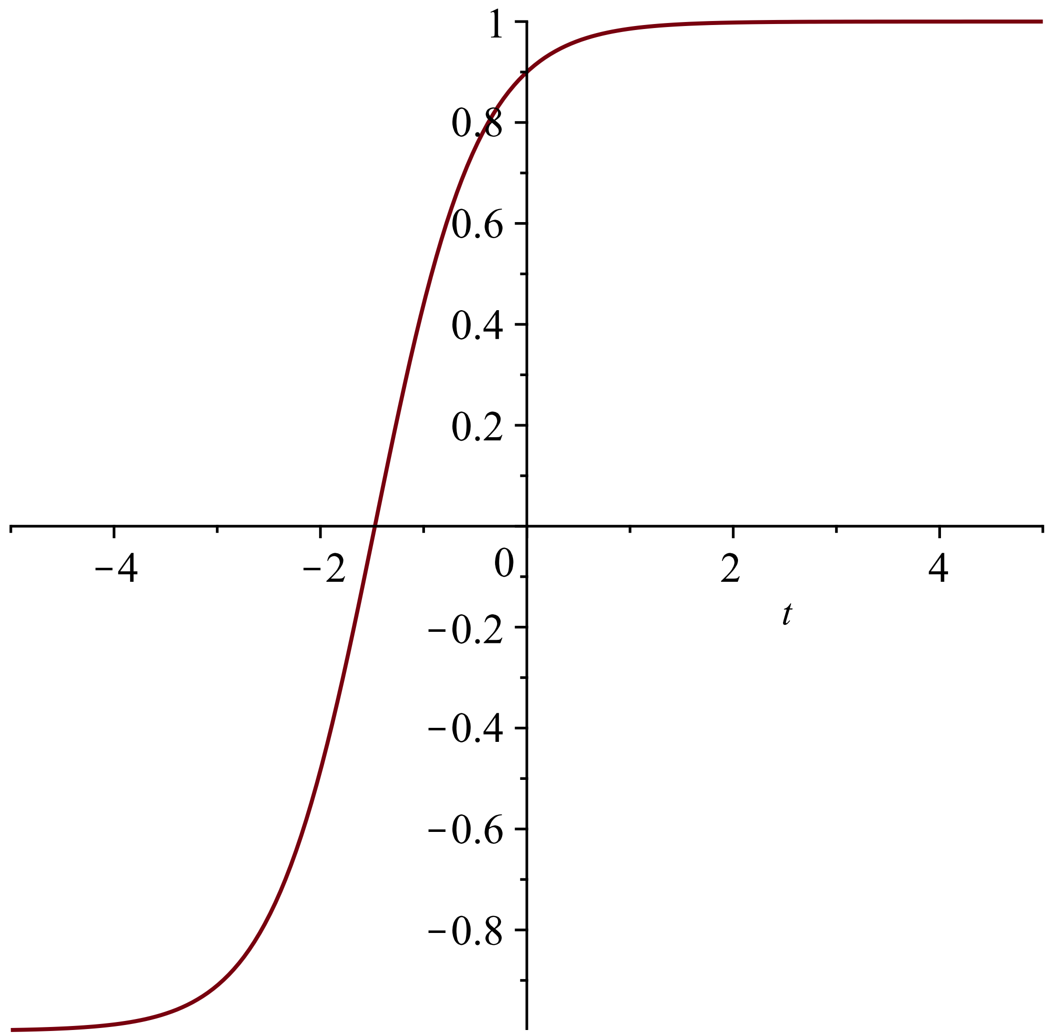
$plot(\mathrm{phi}(t, -0.5), t = -5..5);$



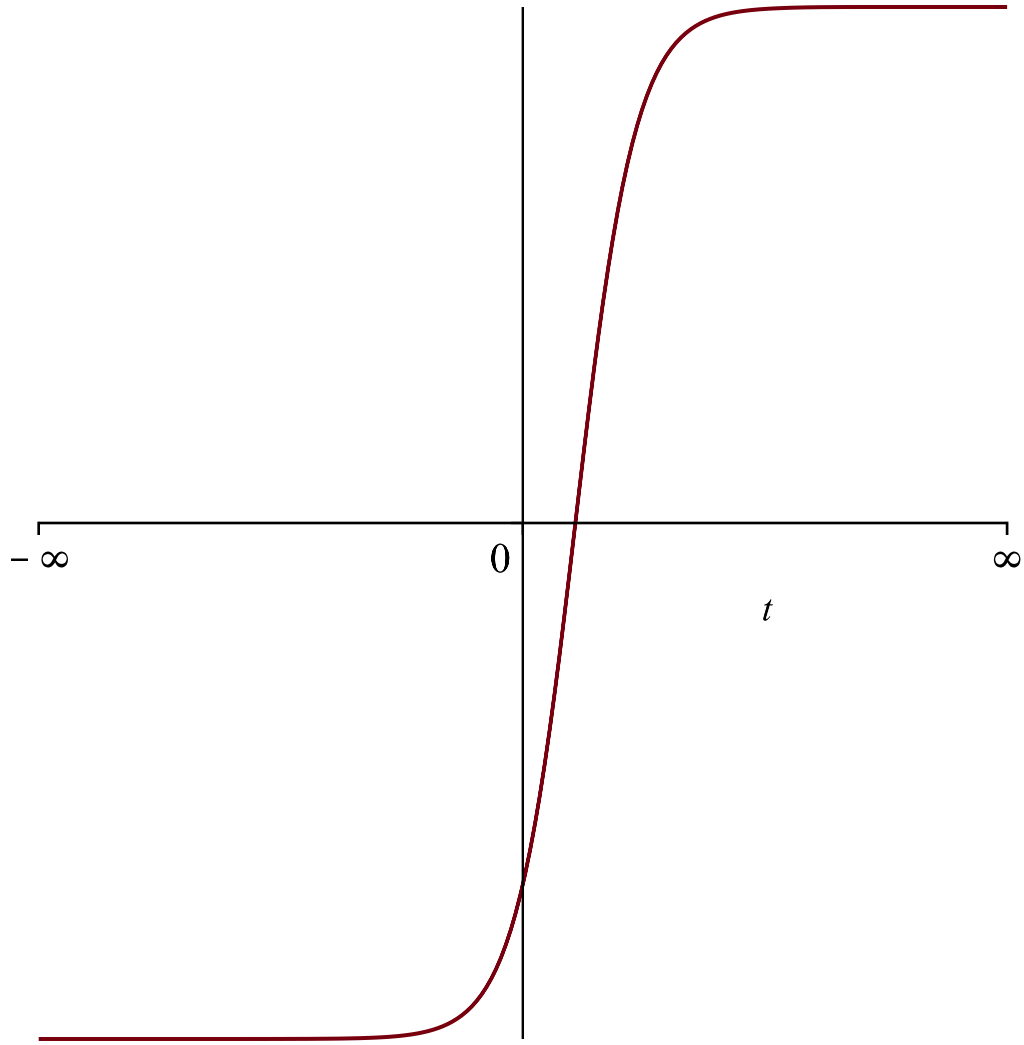
plot(phi(*t*, 0.4), *t* =−5 ..5);



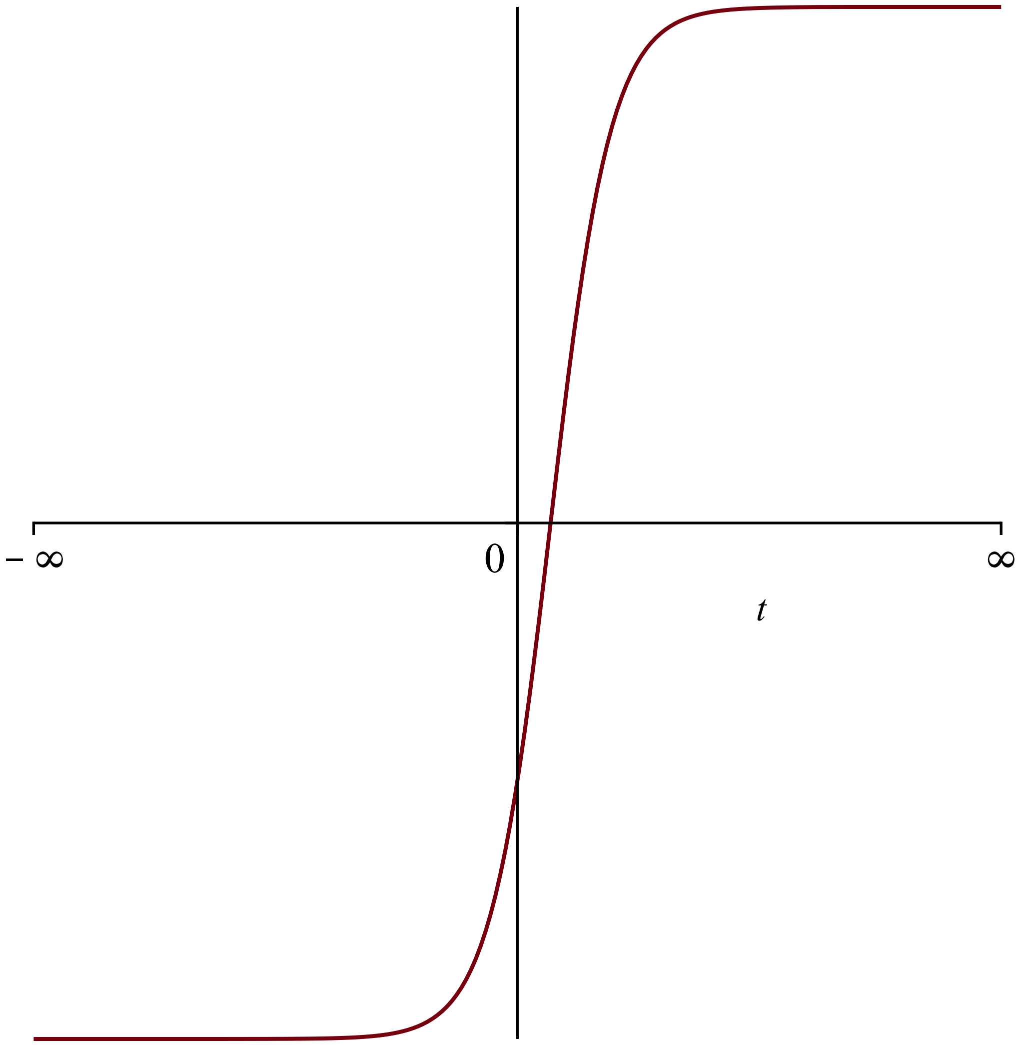
plot(phi(*t*, 0.9), *t* =−5 ..5);



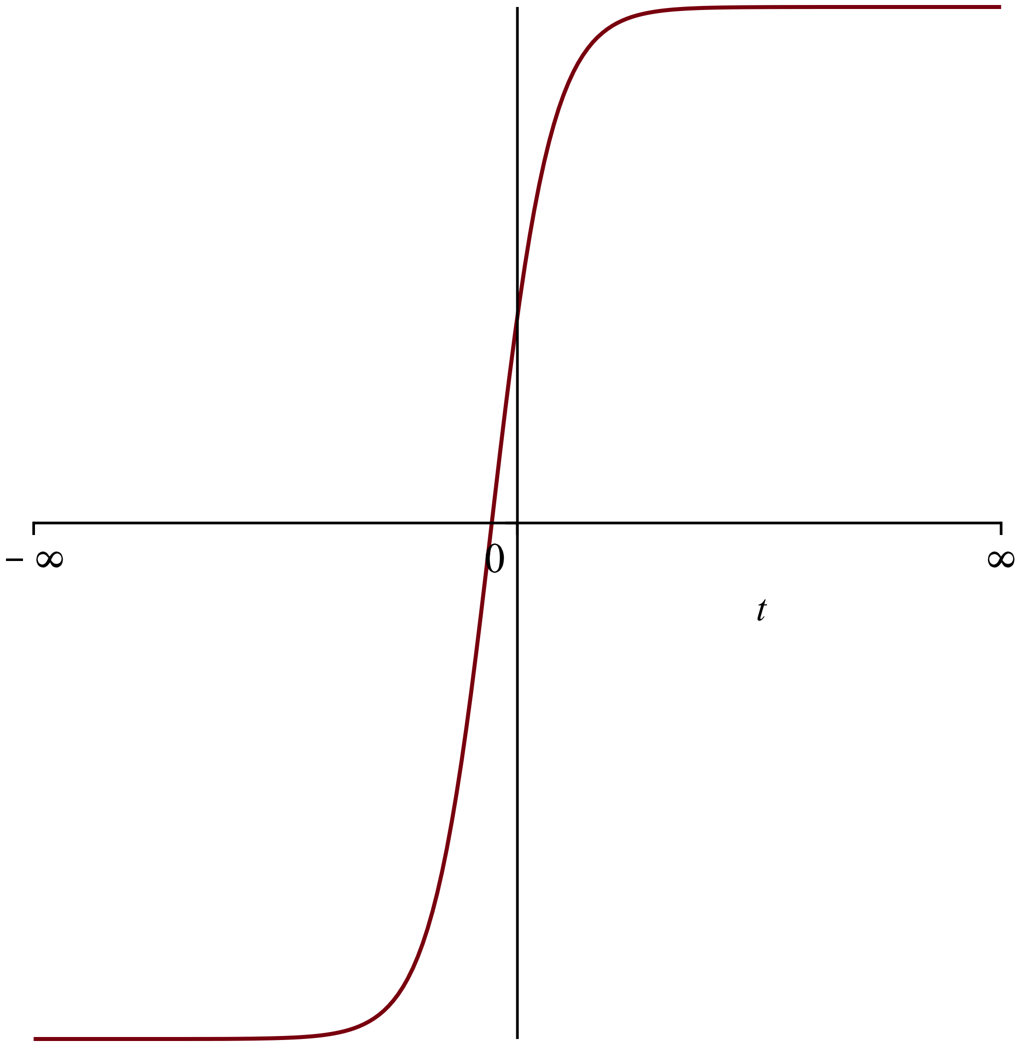
$plot(\mathrm{phi}(t, -0.7), t = -\mathrm{infinity}..\mathrm{infinity});$



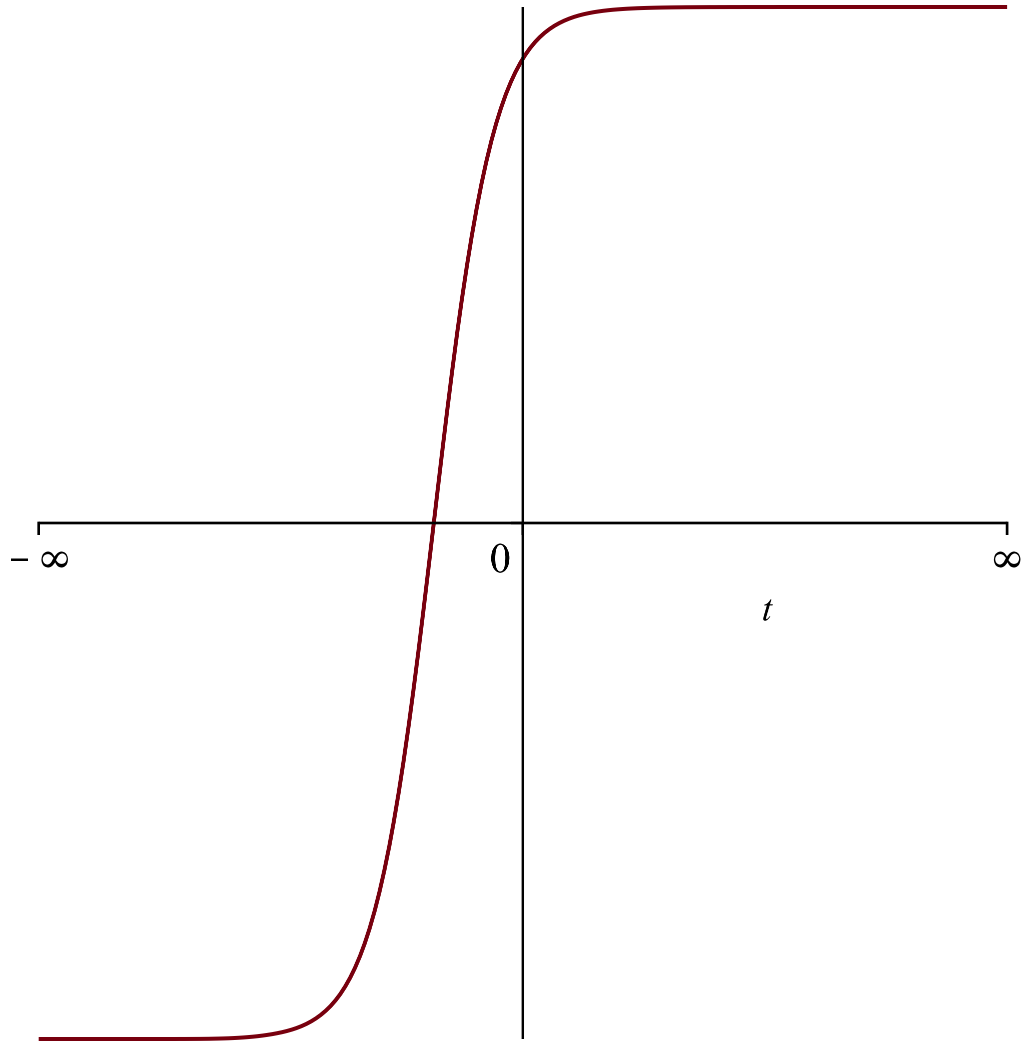
$plot(\mathrm{phi}(t, -0.5), t = -\mathrm{infinity}..\mathrm{infinity});$



$plot(\mathrm{phi}(t, 0.4), t = -\mathrm{infinity}..\mathrm{infinity});$



plot(phi(*t*, 0.9), *t* =− infinity ..infinity);



assume(-1 < eta **and** eta < 1);
limit(phi(*t*, eta), *t* = infinity);

$$1 \tag{8}$$

limit(phi(*t*, eta), *t* =- infinity);

$$-1 \tag{9}$$

#d)
hasassumptions(eta);

$$\mathit{true} \tag{10}$$

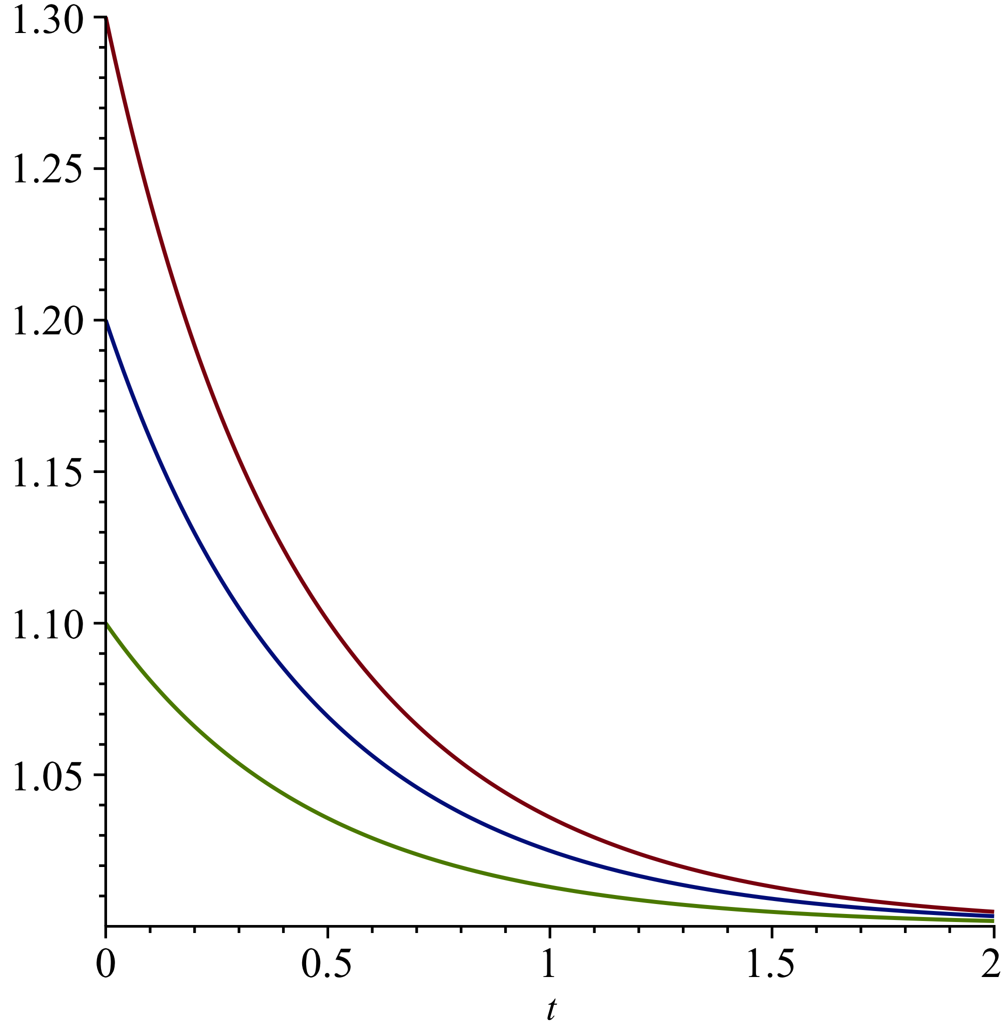
eta := 'eta';

$$\eta := \eta \tag{11}$$

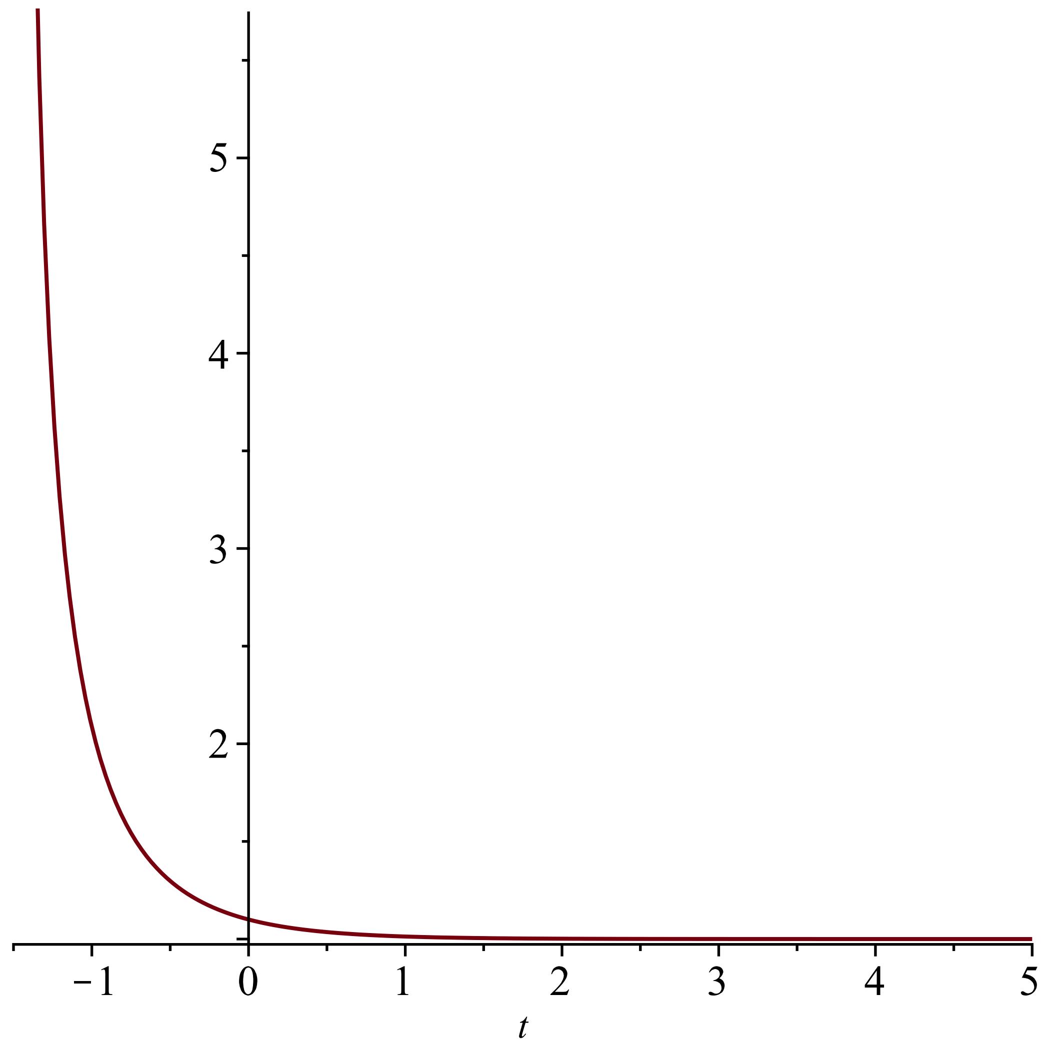
hasassumptions(eta);

$$\mathit{false} \tag{12}$$

plot({ phi(*t*, 1.1), phi(*t*, 1.2), phi(*t*, 1.3) }, *t* = 0 ..2);



plot(phi(*t*, 1.1), *t* =− 1.5 ..5);

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assume(1 < eta);
limit(phi(t, eta), t = infinity);
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$$1 \tag{13}$$

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eta ='eta';
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$$\eta\sim = \eta \tag{14}$$

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plot({phi(t,-1.1), phi(t,-1.5), phi(t,-3)}, t =-2 ..0);
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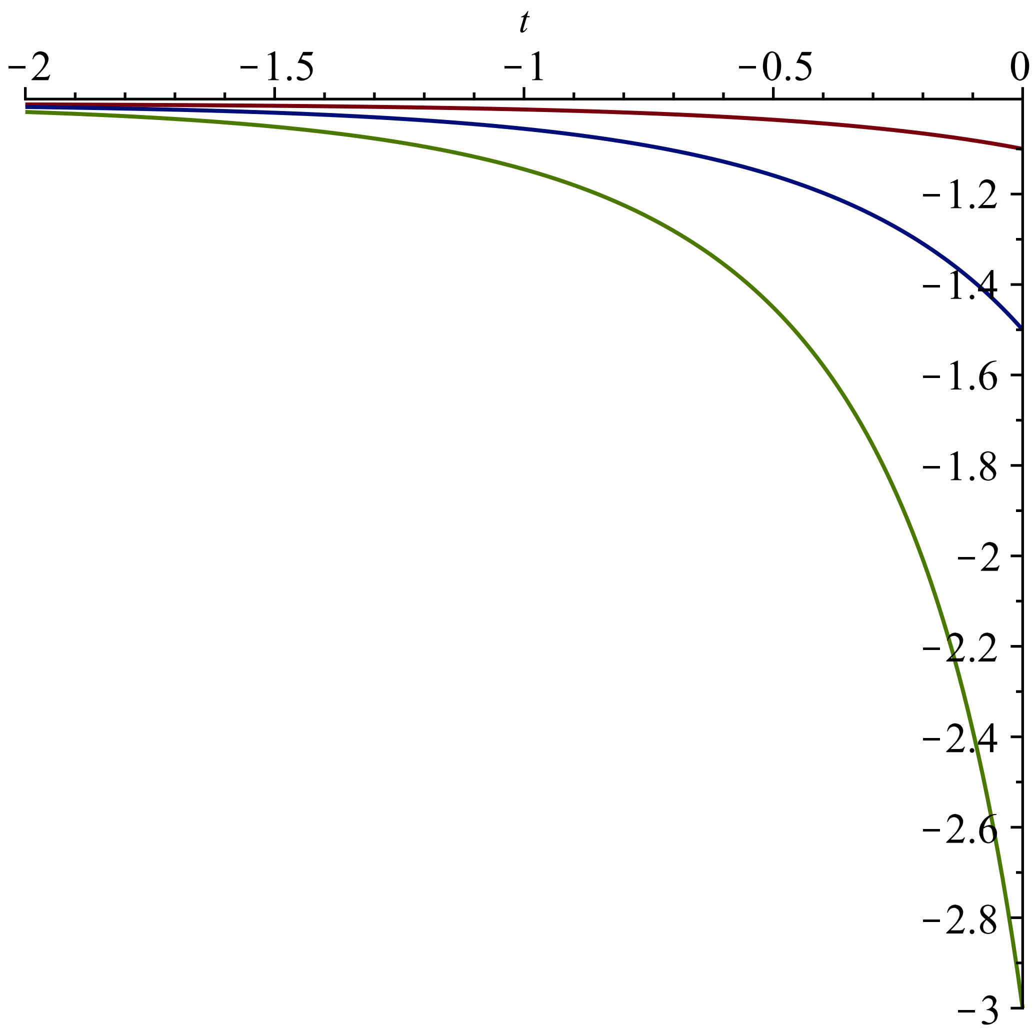

assume(eta < 1);
limit(phi(*t*, eta), *t* =− infinity);

$$-1 \tag{15}$$

eta ='eta;'

$$\eta\sim = \eta \tag{16}$$