#lab2
#sollutions by Goian Tudor
#exercise 3

$eq1 := diff(x(t), t\$2) + x(t) = 0$

$$eq1 := \frac{d^2}{dt^2} x(t) + x(t) = 0 \quad \textbf{(1)}$$

$dsolve(eq1, x(t))$

$$x(t) = \_C1 \sin(t) + \_C2 \cos(t) \quad \textbf{(2)}$$

#exercise 7

$eq2 := 4 \cdot diff(x(t), t\$2) + 8 \cdot diff(x(t), t) + 5 \cdot x(t) = 0;$

$$eq2 := 4 \frac{d^2}{dt^2} x(t) + 8 \frac{d}{dt} x(t) + 5 x(t) = 0 \quad \textbf{(3)}$$

$ic := x(0) = 0, D(x)(0) = 0.5;$

$$ic := x(0) = 0, D(x)(0) = 0.5 \quad \textbf{(4)}$$

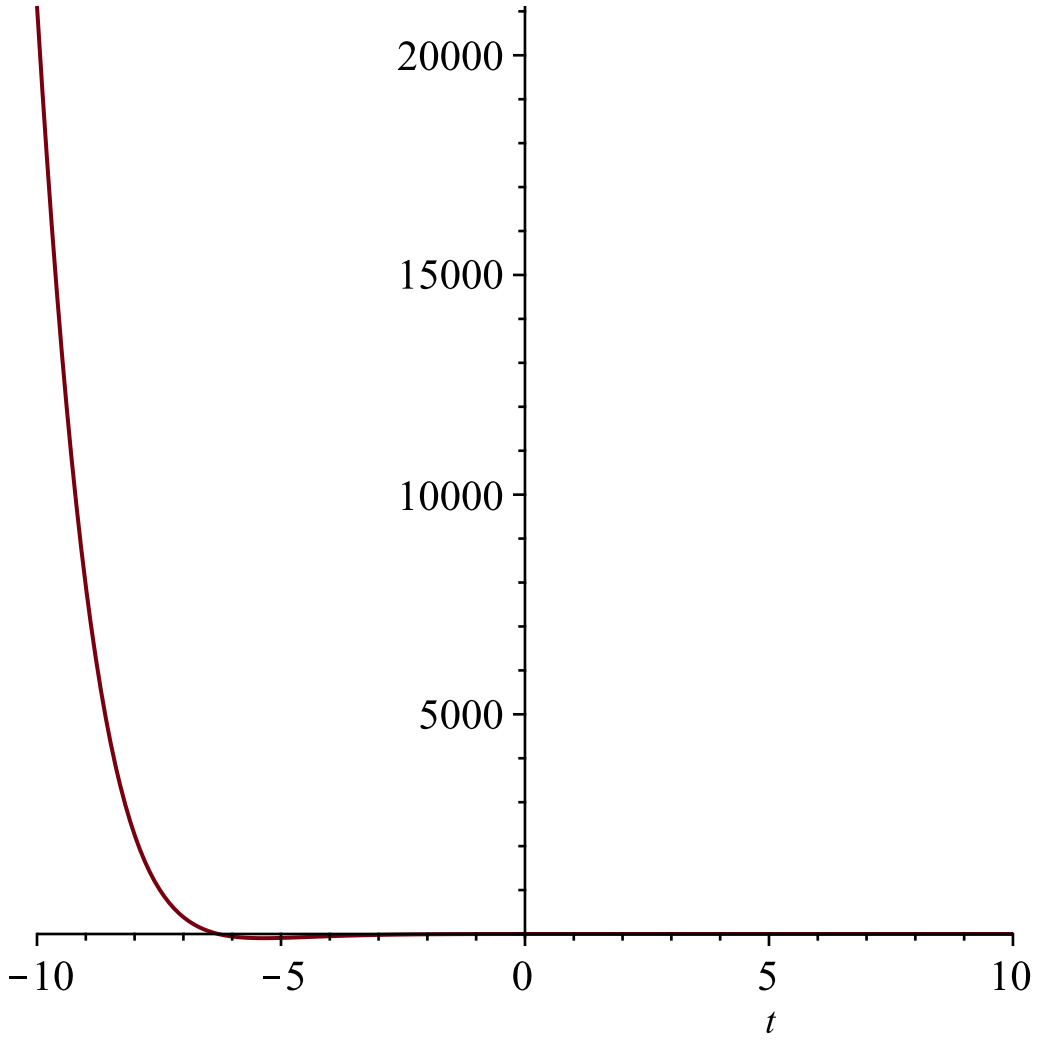
$sol1 := dsolve(\{eq2, ic\}, x(t));$

$$sol1 := x(t) = e^{-t} \sin\left(\frac{t}{2}\right) \quad \textbf{(5)}$$

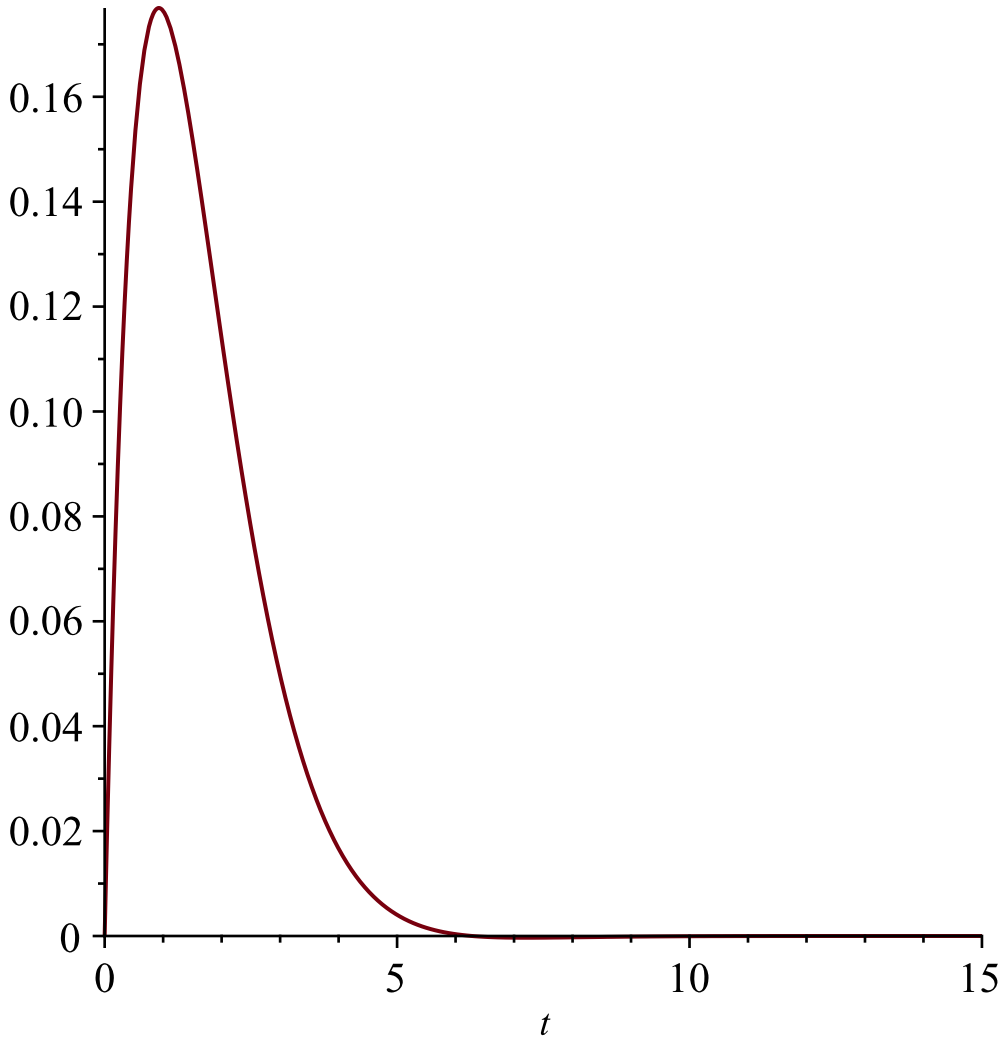
$exprsol1 := rhs(sol1)$

$$exprsol1 := e^{-t} \sin\left(\frac{t}{2}\right) \quad \textbf{(6)}$$

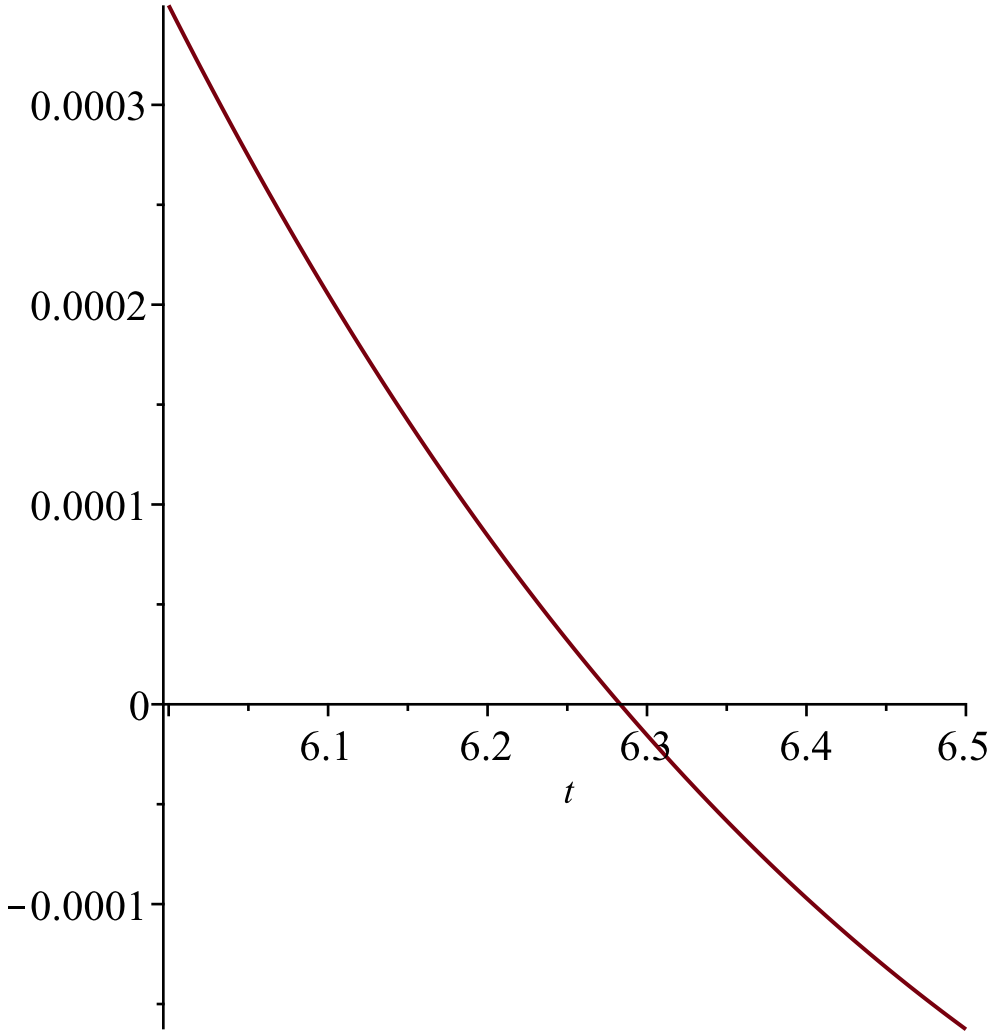
$plot(exprsol1, t = -10..10)$



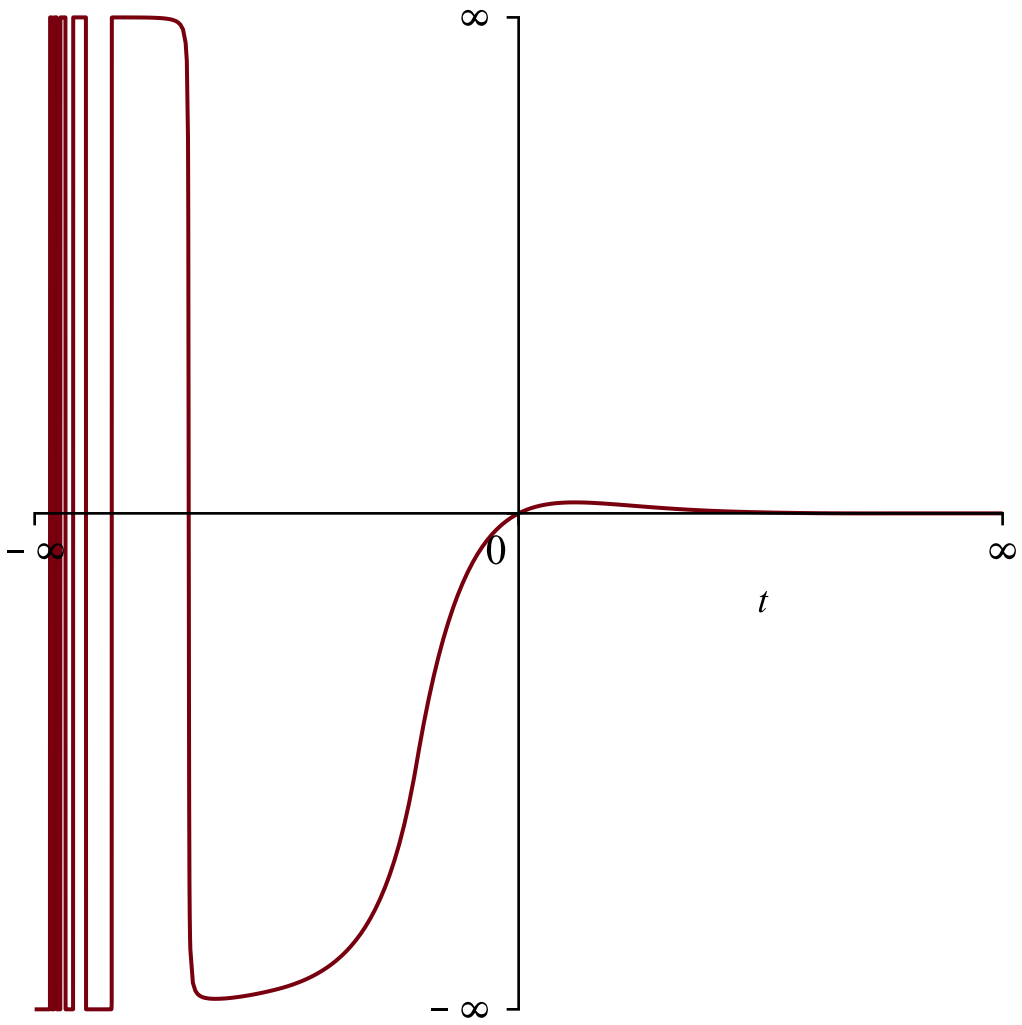
$plot(exprsol1, t = 0..15);$



$plot(exprsol1, t = 6..6.5);$



plot(*exprsol1*, *t* = − infinity .. infinity)



limit(*exprsol1*, *t* = infinity);

$$0 \tag{7}$$

#*exercise 11*

eq11 := *diff*(*x*(*t*), *t*$2) + *x*(*t*) = 0;

$$eq11 := \frac{d^2}{dt^2} x(t) + x(t) = 0 \tag{8}$$

bc111 := *x*(0) = 0;

$$bc111 := x(0) = 0 \tag{9}$$

bc112 := *x*(1) = 0;

$$bc112 := x(1) = 0 \tag{10}$$

dsolve({*eq1*, *bc1*, *bc2*}, *x*(*t*))

$$x(t) = 0 \tag{11}$$

#*exercise 15*

eq15 := *diff*(*x*(*t*), *t*) + *x*(*t*) = $-t^2 + 3 \cdot t - 7$;

$$eq15 := \frac{d}{dt} x(t) + x(t) = -t^2 + 3\,t - 7 \tag{12}$$

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dsolve(eq15);
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$$x(t) = -t^2 + 5\,t - 12 + \mathrm{e}^{-t}\_C1 \tag{13}$$

```
#exercise 19
eq19 := diff(x(t), t) + x(t) = 2/sqrt(Pi)*e^(-t^2 - t);
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$$eq19 := \frac{\mathrm{d}}{\mathrm{d}t}\,x(t) + x(t) = \frac{2\,e^{-t^2 - t}}{\sqrt{\pi}} \tag{14}$$

```
dsolve(eq19);
```

$$x(t) = \left(\int \frac{2\,e^{-t\,(t+1)}\,\mathrm{e}^{t}}{\sqrt{\pi}}\,\mathrm{d}t + \_C1 \right) \mathrm{e}^{-t} \tag{15}$$

```
int(e^(t^2), t);
```

$$\frac{\sqrt{\pi}\ \mathrm{erf}\left(\sqrt{-\ln(e)}\ t\right)}{2\sqrt{-\ln(e)}} \tag{16}$$

```
int(2/sqrt(Pi)*e^(-t^2), t);
```

$$\frac{\mathrm{erf}\left(\sqrt{\ln(e)}\ t\right)}{\sqrt{\ln(e)}} \tag{17}$$

```
#erf is the error function

#exercise 23
 fc23 := piecewise(t <= 2, t, t > 2, 3 - t);
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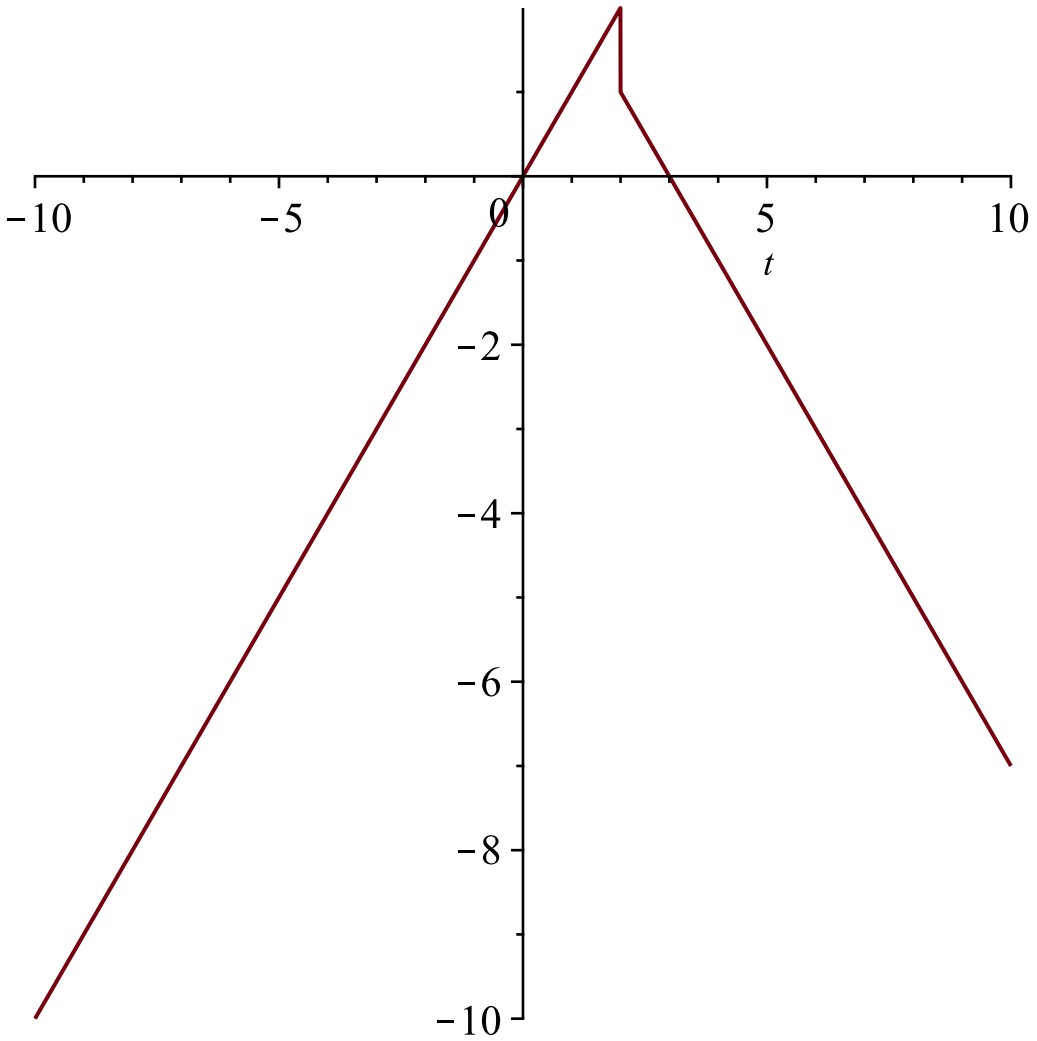
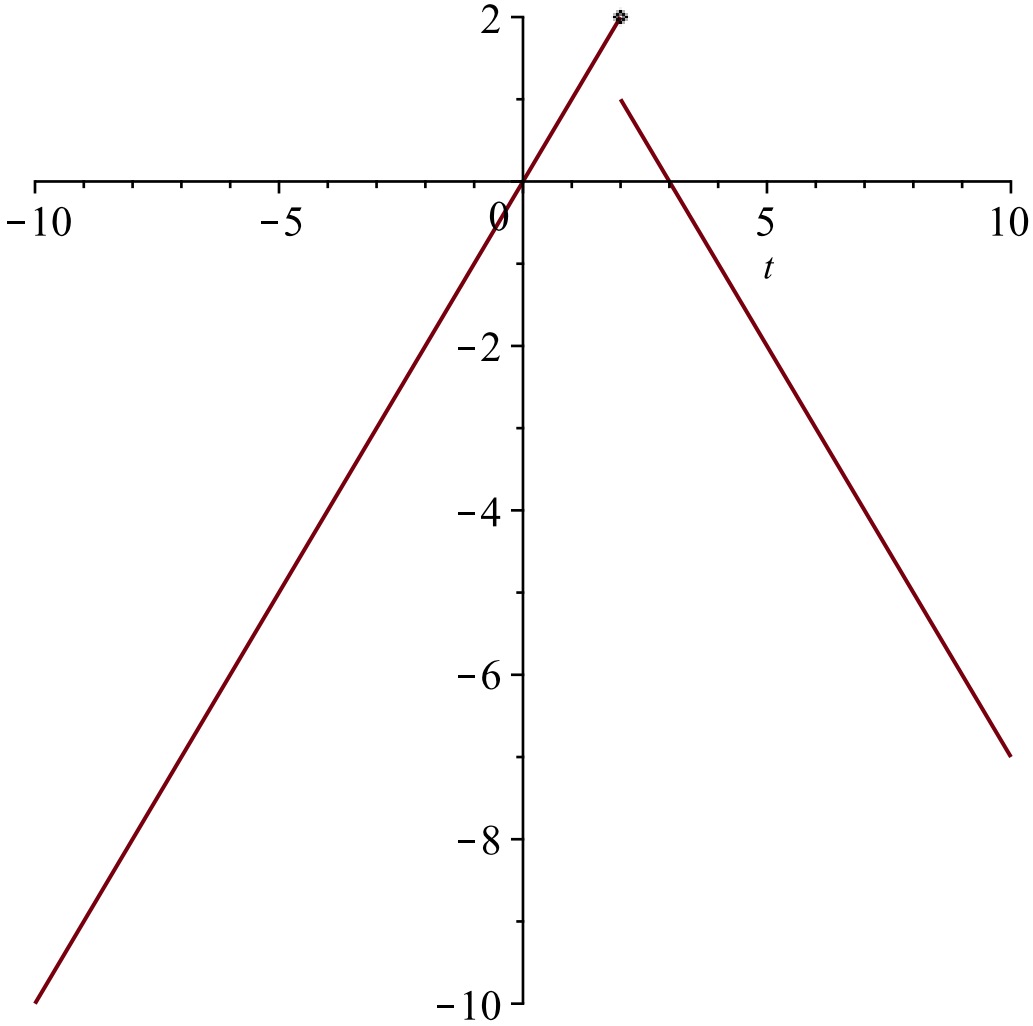
$$fc23 := \begin{cases} t & t \le 2 \\ 3 - t & 2 < t \end{cases} \tag{18}$$

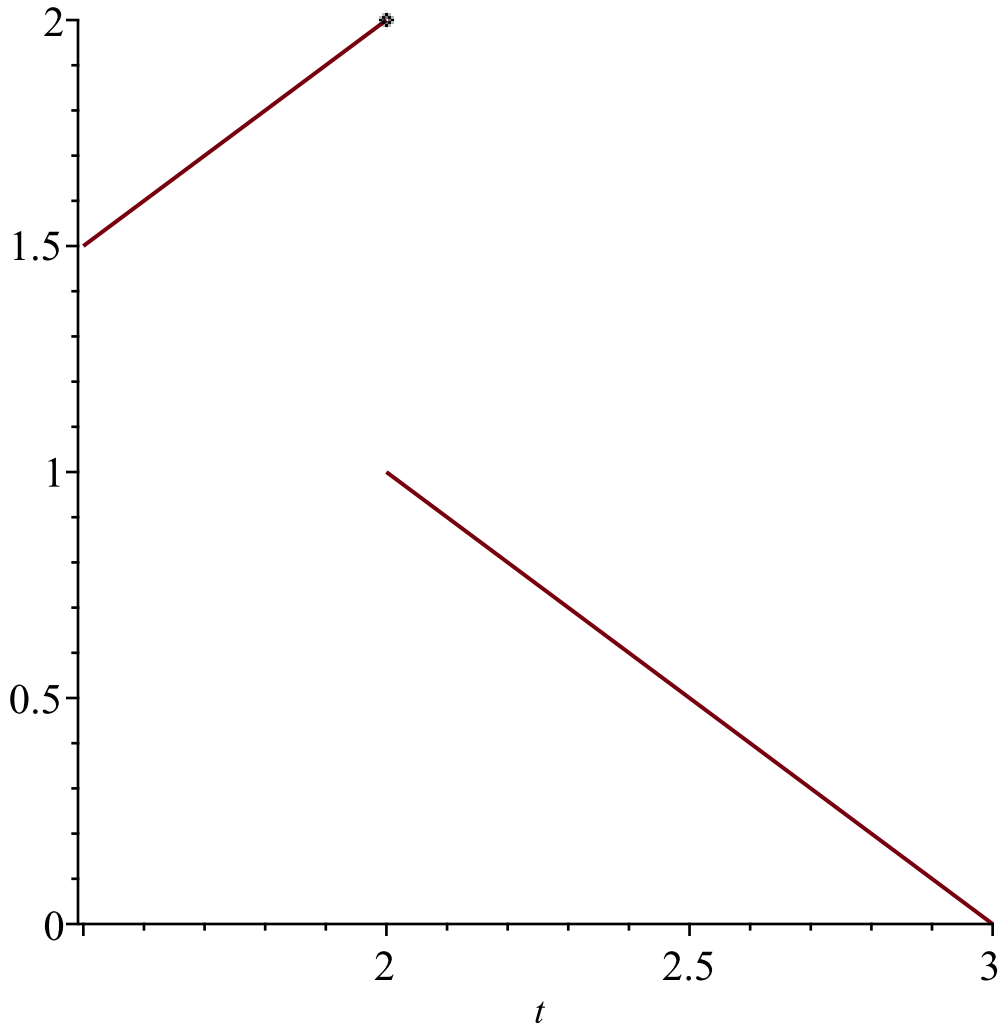
```
plot(fc23);
```



plot(*fc23*, *discont* = *true*);



$plot(fc23, t = 1.5..3, discont = true);$



#execise 27

ec27 := *diff*(*x*(*t*), *t*$2) − 4·*x*(*t*) = exp(alpha·*t*);

$$ec27 := \frac{d^2}{dt^2} x(t) - 4\,x(t) = e^{\alpha t} \tag{19}$$

ic27 := *x*(0) = 0, D(*x*)(0) = 0

$$ic27 := x(0) = 0,\ D(x)(0) = 0 \tag{20}$$

sol27 := *dsolve*({*ec27*, *ic27*}, *x*(*t*));

$$sol27 := x(t) = -\frac{e^{2t}}{4(\alpha-2)} + \frac{e^{-2t}}{4(\alpha+2)} + \frac{e^{\alpha t}}{\alpha^2-4} \tag{21}$$

#why is unapply useful?

exprsol27 := *unapply*(*rhs*(*sol27*), *t*, alpha);

$$exprsol27 := (t, \alpha) \mapsto -\frac{e^{2t}}{4(\alpha-2)} + \frac{e^{-2t}}{4(\alpha+2)} + \frac{e^{\alpha t}}{\alpha^2-4} \tag{22}$$

limit(*exprsol27*(*t*, alpha), alpha = 2);

$$\frac{4\,t\,(e^t)^4 - (e^t)^4 + 1}{16\,(e^t)^2} \tag{23}$$

#!!this exercise gives me an error and I don't really understand how should I solve it
#exercise 31

$$eq31 := piecewise\left(t = \left[0, \frac{Pi}{2}\right], t, t = \left[\frac{Pi}{2}, Pi\right], Pi, t = [Pi, infinity], 0\right);$$

$$eq31 := 0 \tag{24}$$

$$ic311 := diff(x(t), t\$2) + x(t) = eq1;$$

$$ic311 := \frac{d^2}{dt^2} x(t) + x(t) = \left(\frac{d^2}{dt^2} x(t) + x(t) = 0\right) \tag{25}$$

$$ic312 := x(0) = 5;$$

$$ic312 := x(0) = 5 \tag{26}$$

$$ic313 := D(x)(0) = 0$$

$$ic313 := D(x)(0) = 0 \tag{27}$$

$$sol := dsolve(\{eq31, ic311, ic312, ic313\}, x(t))$$

Error, (in dsolve) invalid input: `PDEtools/NumerDenom` expects its 1st argument, ee, to be of type algebraic, but received diff(diff(x(t), t), t)+x(t) = 0

$$plot([sol);$$

Error, unable to match delimiters

$$plot([sol);$$

#exercise 35

$$eq351 := diff(x(t), t) + x(t) - 3 \cdot y(t) = 0;$$

$$eq351 := \frac{d}{dt} x(t) + x(t) - 3\, y(t) = 0 \tag{28}$$

$$eq352 := diff(y(t), t) + 3 \cdot x(t) + y(t) = 0;$$

$$eq352 := \frac{d}{dt} y(t) + 3\, x(t) + y(t) = 0 \tag{29}$$

$$ic351 := x(0) = 1;$$

$$ic351 := x(0) = 1 \tag{30}$$

$$ic352 := y(0) = 1;$$

$$ic352 := y(0) = 1 \tag{31}$$

$$sol35 := dsolve(\{eq351, eq352, ic351, ic352\}, \{x(t), y(t)\})$$

$$sol35 := \left\{x(t) = e^{-t}(\cos(3\,t) + \sin(3\,t)), y(t) = e^{-t}(\cos(3\,t) - \sin(3\,t))\right\} \tag{32}$$

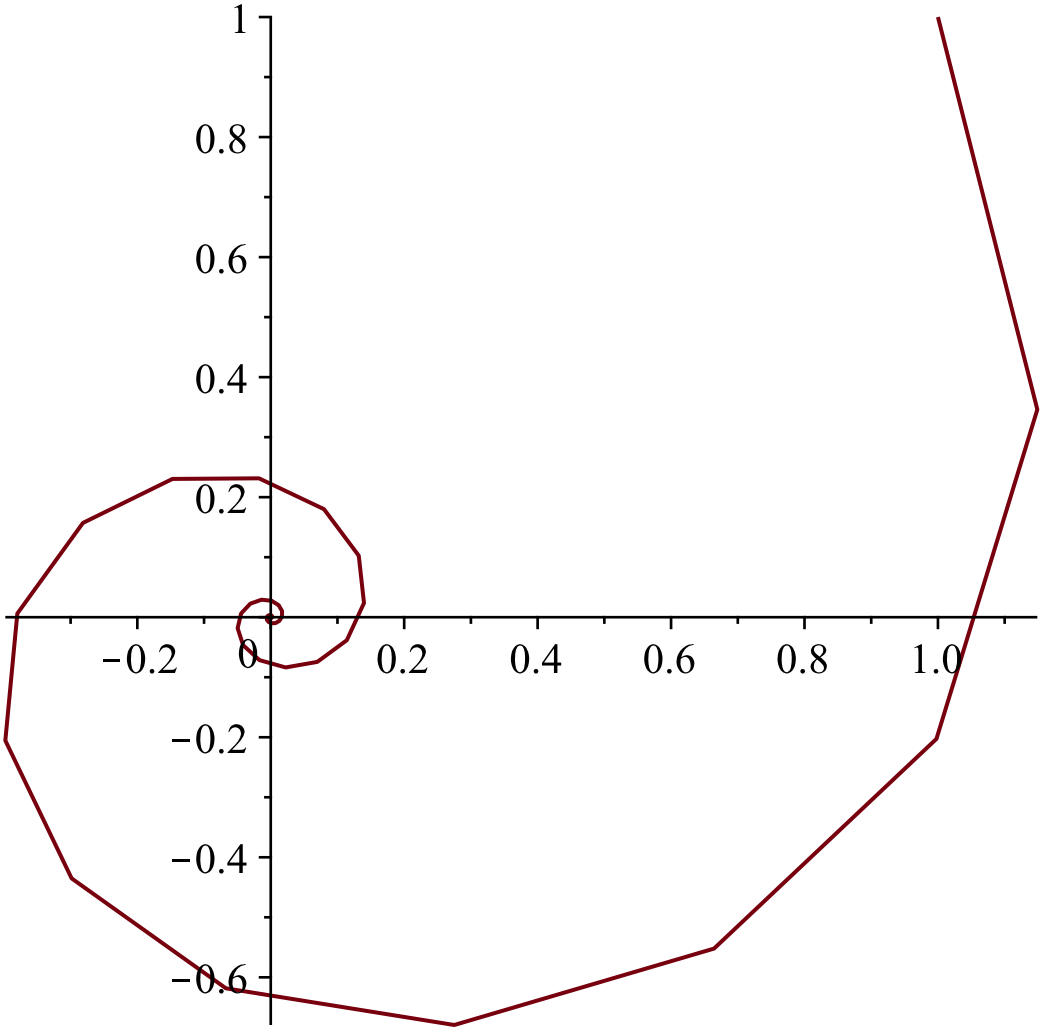
$$exprsol35x := rhs(sol35[1]);$$

$$exprsol35x := e^{-t}(\cos(3\,t) + \sin(3\,t)) \tag{33}$$

$$exprsol35y := rhs(sol35[2]);$$

$$exprsol35y := e^{-t}(\cos(3\,t) - \sin(3\,t)) \tag{34}$$

$$plot([exprsol35x, exprsol35y, t = 0..1000])$$


1
0.8
0.6
0.4
0.2
−0.2
0
0.2
0.4
0.6
0.8
1.0
−0.2
−0.4
−0.6