*#@author: Goian Tudor George, IE1, gr912|2*
*#1.Hyperbolic equilibria*
*#1.1* $x'=2x-x^2-xy$, $y'=-y+xy$
*#first we find the equilibrias of the non-linear system*
$solve(\{2\cdot x-x^2-x\cdot y=0,-y+x\cdot y=0\});$

$$\{x=0, y=0\}, \{x=2, y=0\}, \{x=1, y=1\} \tag{1}$$

*#after we found the 3 equilibrias, due to it being a non linear syste, we must use a linearization method*
*#we choose to form the Jacobian matrix of the system*
$with(linalg) : with(DEtools) : with(VectorCalculus) :$
$Jm1 := Jacobian([2\cdot x-x^2-x\cdot y, -y+x\cdot y], [x, y]);$

$$Jm1 := \begin{bmatrix} -2x-y+2 & -x \\ y & x-1 \end{bmatrix} \tag{2}$$

*#now we substitute each equilibria in the found matrix one at a time*
*#first equilibria (0,0)*
$A1 := subs([x=0, y=0], Jm1);$

$$A1 := \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \tag{3}$$

*#finding eigenvalues...*
$eigenvalues(A1);$

$$2, -1 \tag{4}$$

*#both eigenvalues are real and non-zero, so (0,0) is hyperbolic; the system X'=A1·X has a saddle, so (0, 0) of the initial system is unstable*

*#second equilibria (2,0)*
$A2 := subs([x=2, y=0], Jm1);$

$$A2 := \begin{bmatrix} -2 & -2 \\ 0 & 1 \end{bmatrix} \tag{5}$$

*#finding eigenvalues*
$eigenvalues(A2);$

$$-2, 1 \tag{6}$$

*#the argument is the same as for (0,0)*

*#third equilibria (1,1)*
$A1 := subs([x=1, y=1], Jm1);$

$$A1 := \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \tag{7}$$

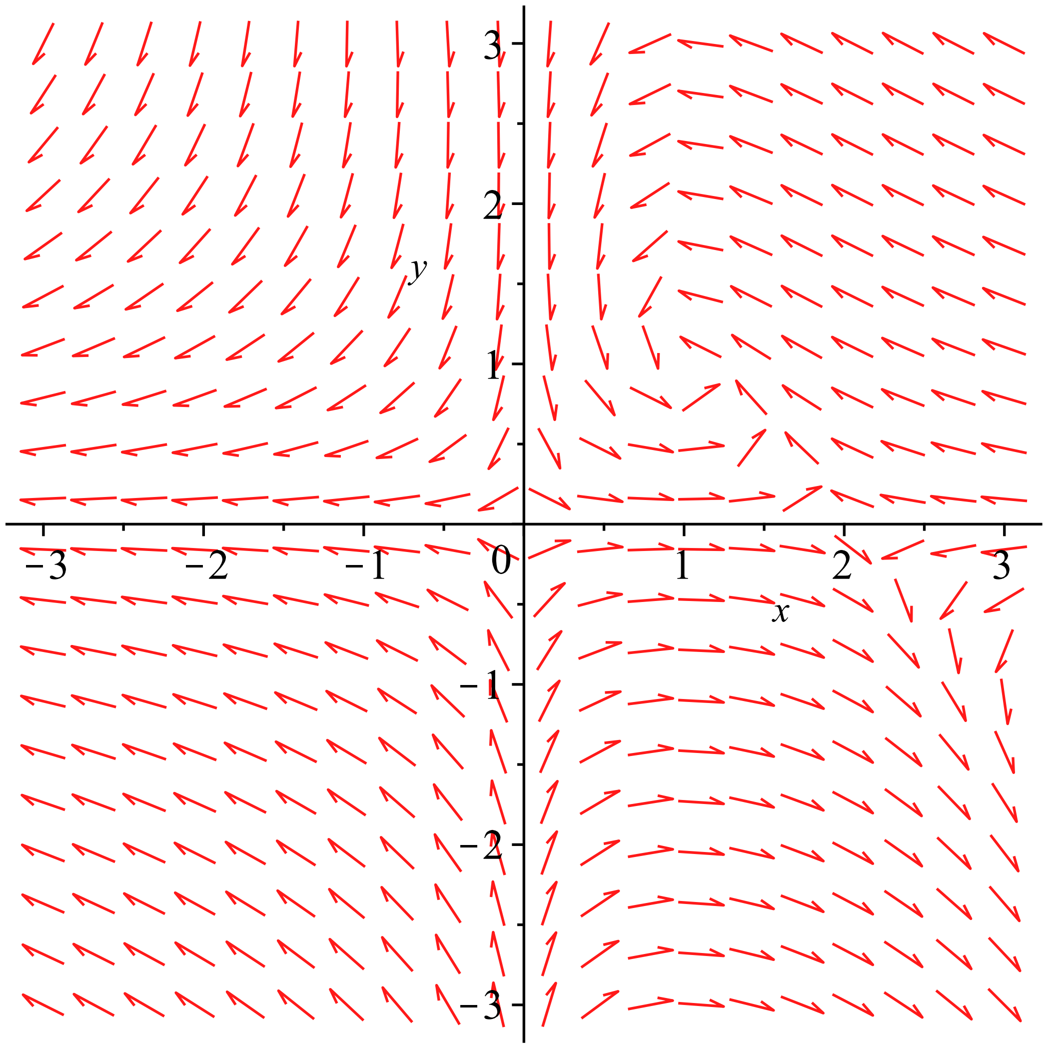
*#finding eigenvalues...*
$eigenvalues(A1);$

$$-\frac{1}{2}+\frac{I\sqrt{3}}{2}, -\frac{1}{2}-\frac{I\sqrt{3}}{2} \tag{8}$$

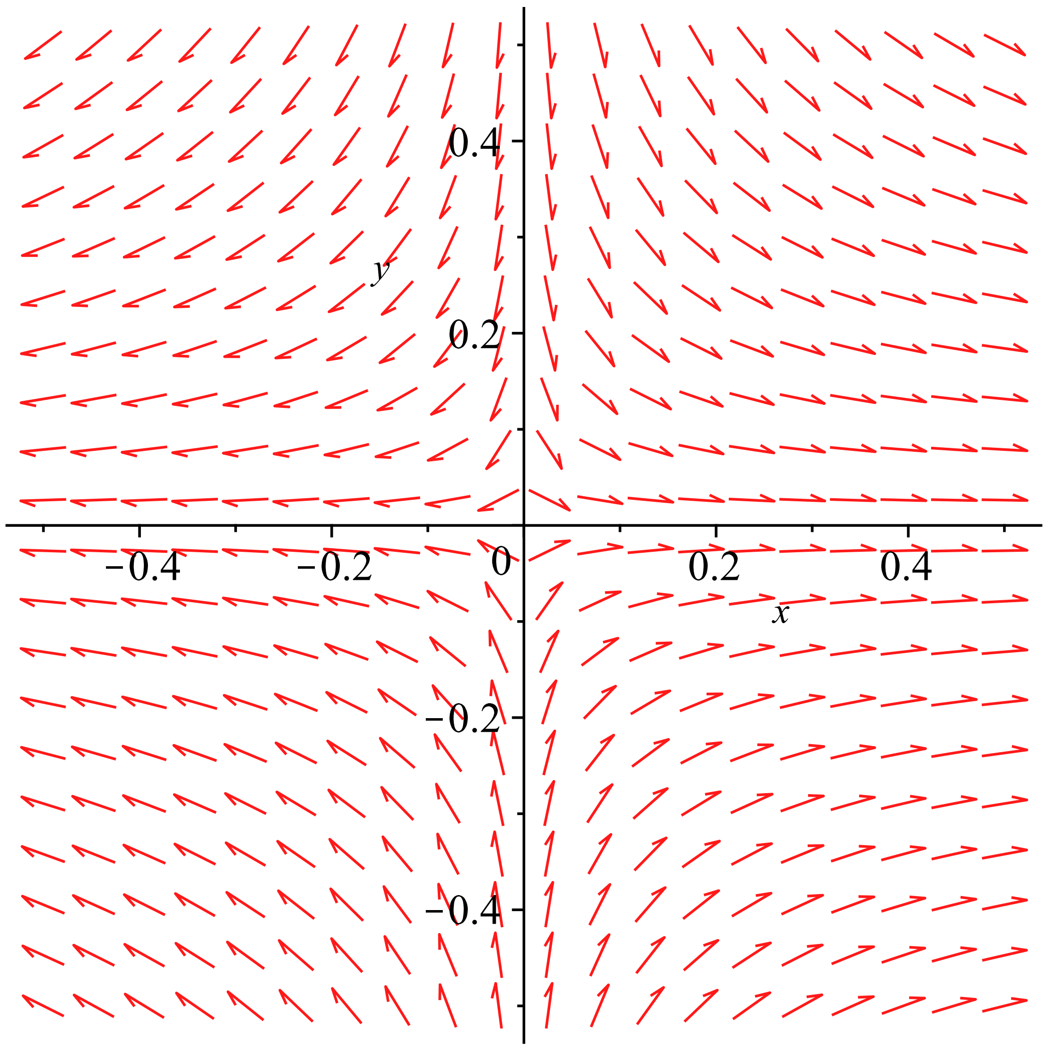
*#this time we notice that both eigenvalues are complex and have a real part. Therefore (1,1) is hyperbolic and X'=A3·X has a focus; (1,) of the initial system is an attractor*

*#plots:*

*dfieldplot*( [ *diff* ( $x(t)$, $t$ ) = 2 * $x(t)$ - $x(t)$ ^2 - $x(t)$ * $y(t)$, *diff* ( $y(t)$, $t$ ) = - $y(t)$ + $x(t)$ * $y(t)$ ], [ $x(t)$, $y(t)$ ], $t$ = 0 ..1, $x$ = -3 ..3, $y$ = -3 ..3 );




*dfieldplot*( [ *diff* ( $x(t)$, $t$ ) = 2 * $x(t)$ - $x(t)$ ^2 - $x(t)$ * $y(t)$, *diff* ( $y(t)$, $t$ ) = - $y(t)$ + $x(t)$ * $y(t)$ ], [ $x(t)$, $y(t)$ ], $t$ = 0 ..1, $x$ = -0.5 ..0.5, $y$ = -0.5 ..0.5 );




```
dfieldplot([diff(x(t), t) = 2*x(t), diff(y(t), t) = -y(t) ], [x(t), y(t)
 ], t = 0 ..1, x =-5 ..5, y =-5 ..5);
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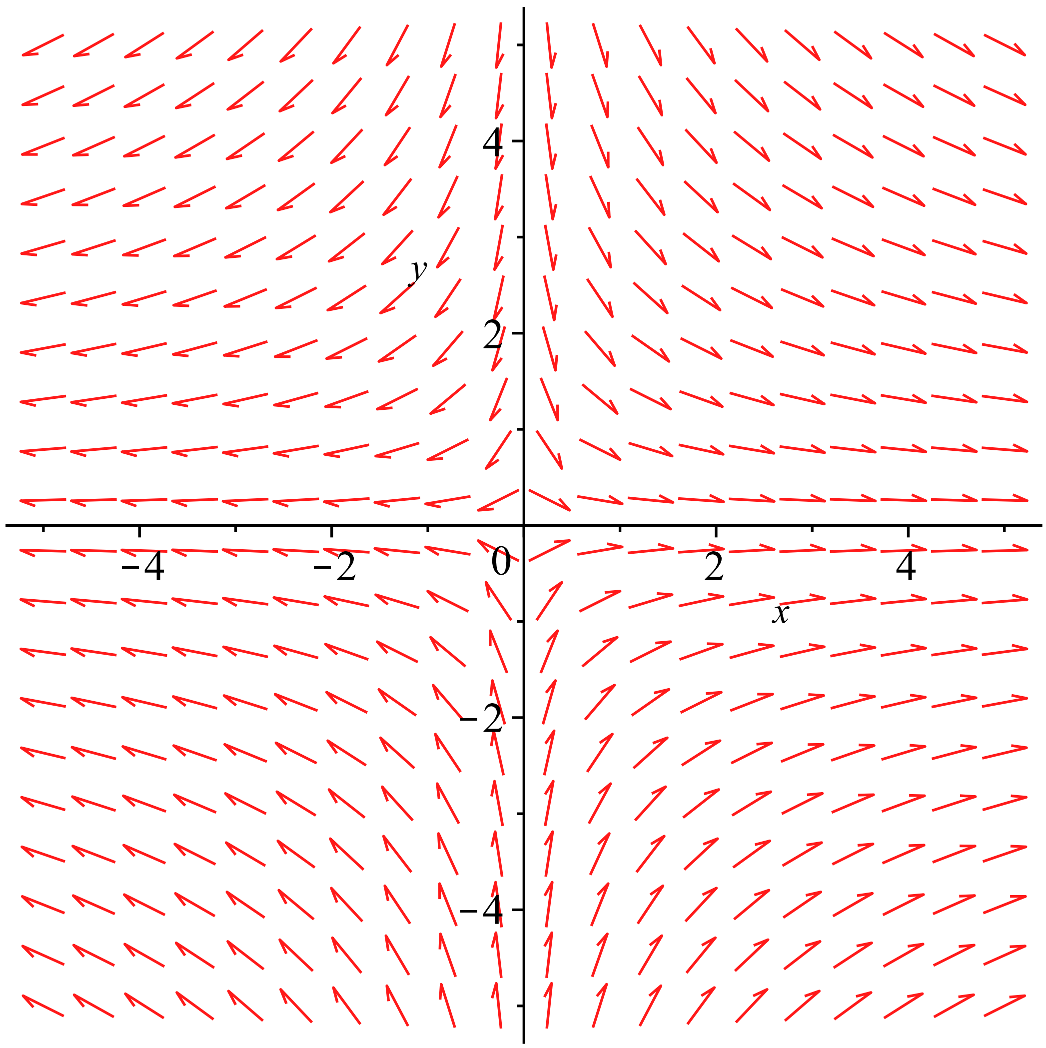
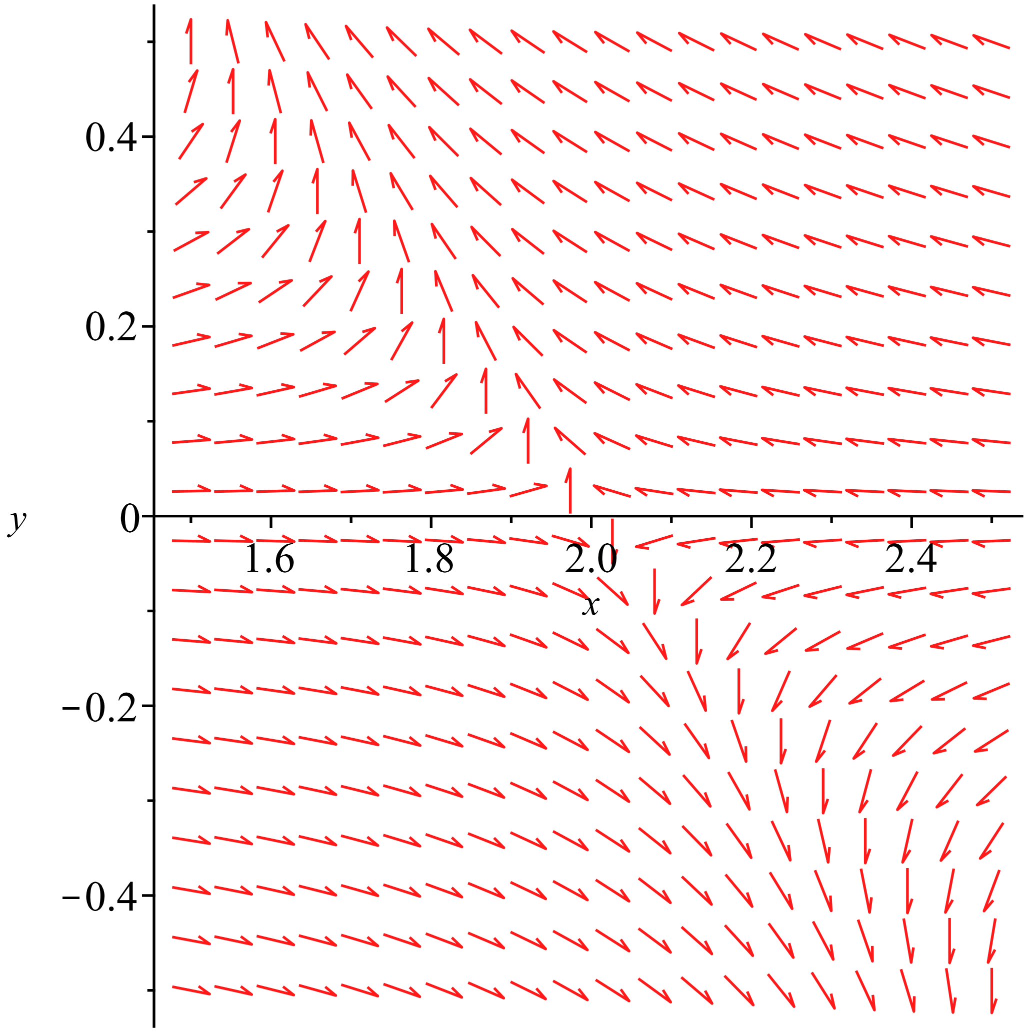
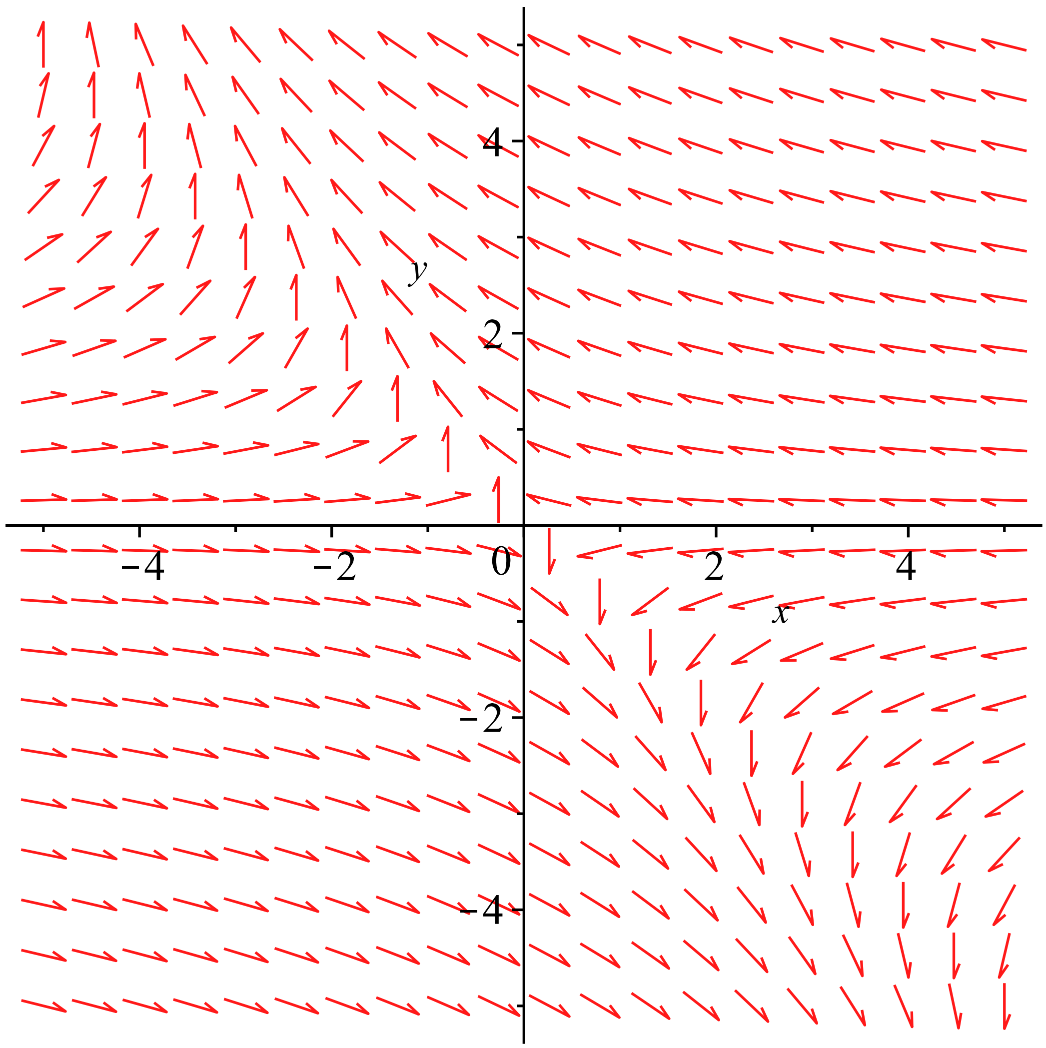


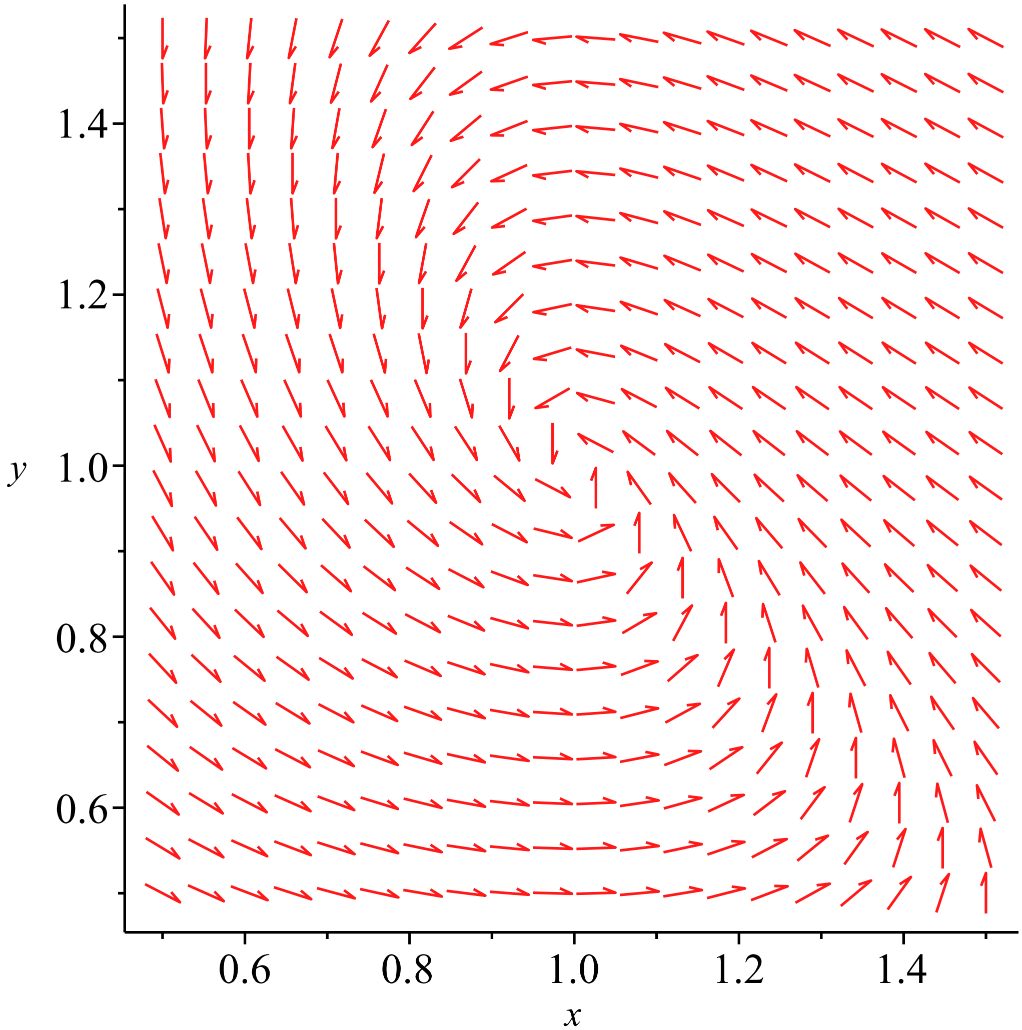

*dfieldplot*( [ *diff* (*x*(*t*), *t*) = 2 * *x*(*t*) −*x*(*t*) ^2 − *x*(*t*) * *y*(*t*), *diff* (*y*(*t*), *t*) =
−*y*(*t*) + *x*(*t*) * *y*(*t*) ], [*x*(*t*), *y*(*t*) ], *t* = 0 ..1, *x* = 1.5 ..2.5, *y* =−0.5 ..0.5 );



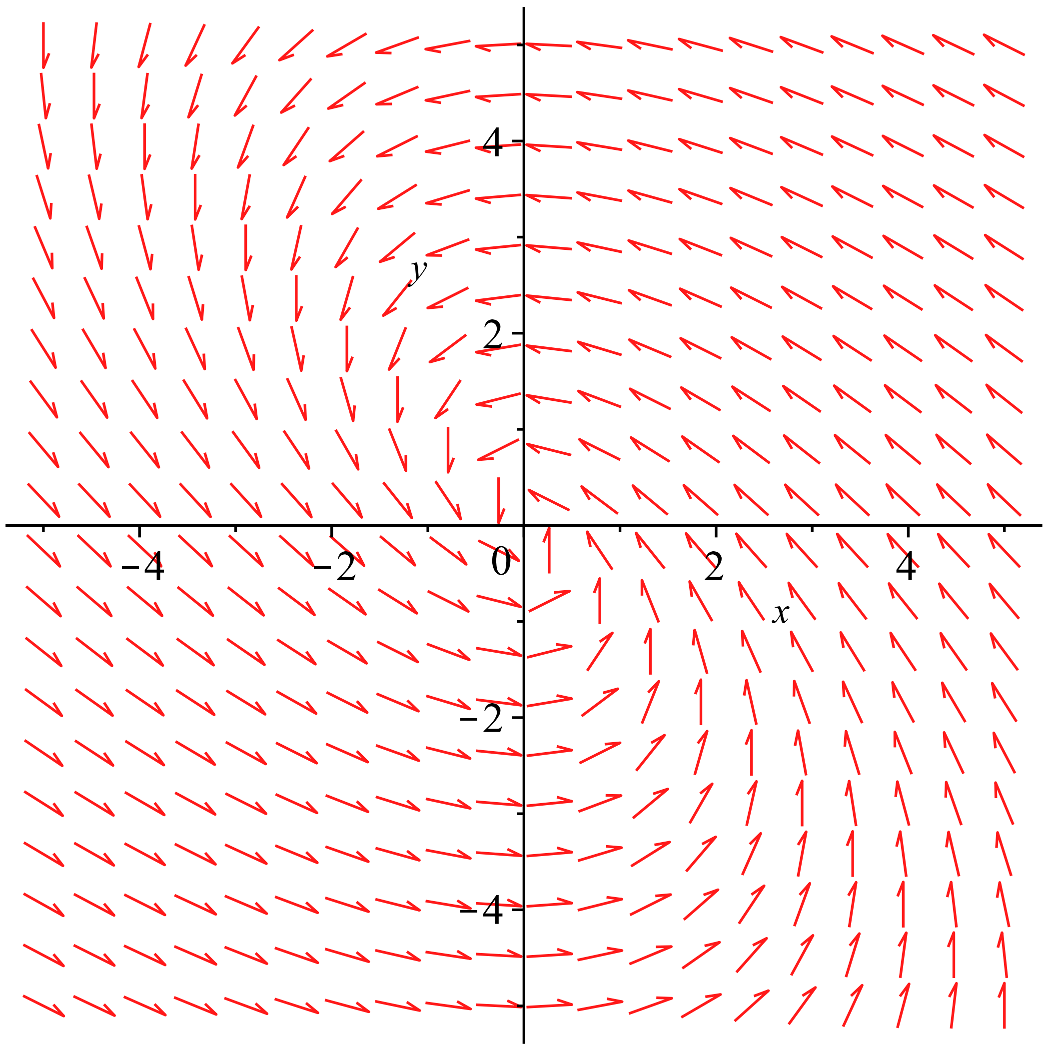

*dfieldplot*( [ *diff* (x(t), t) =-2 * x(t) - 2 * y(t), *diff* (y(t), t) = y(t) ], [x(t), y(t) ], t = 0 ..1, x =-5 ..5, y =-5 ..5);



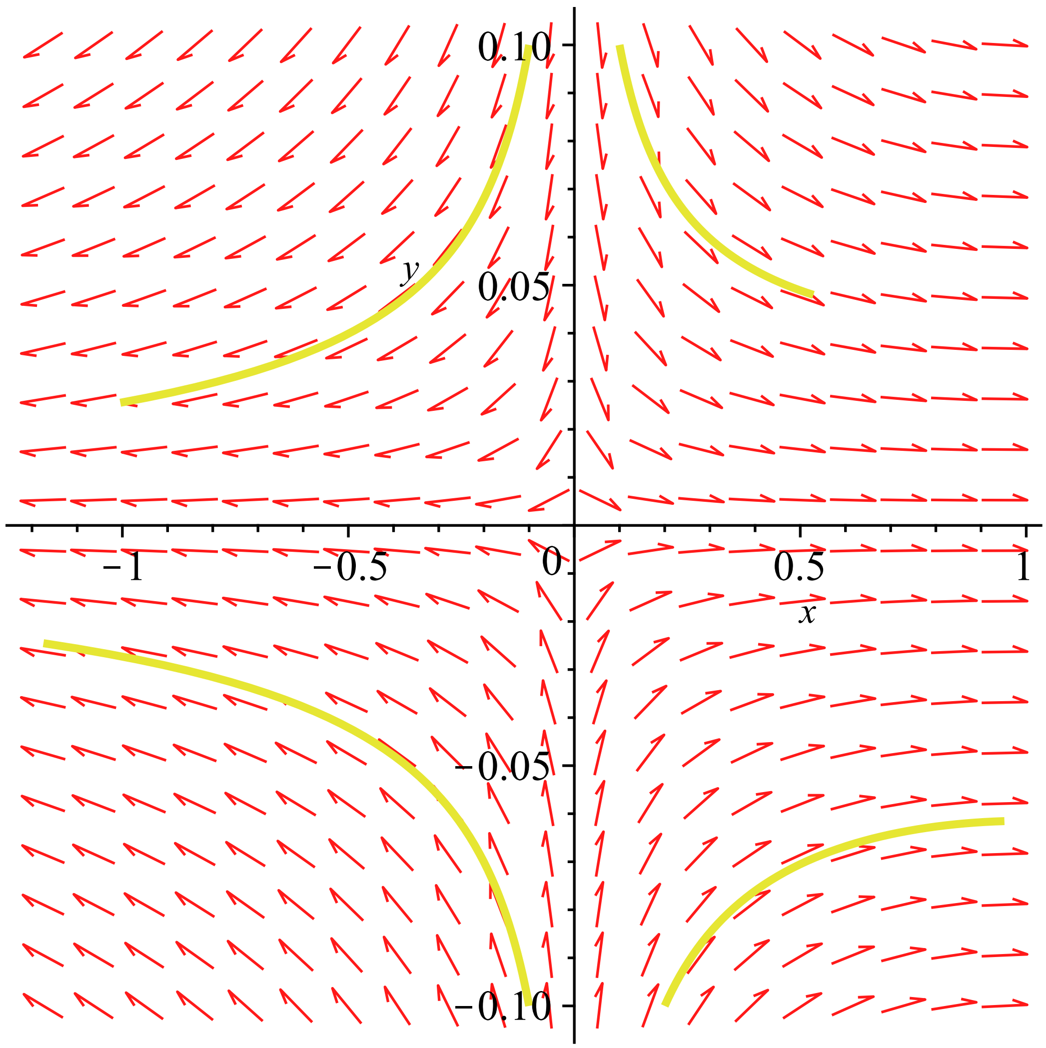

*dfieldplot*( [ *diff* ( $x(t)$, $t$ ) = 2 * $x(t)$ − $x(t)$^2 − $x(t)$ * $y(t)$, *diff* ( $y(t)$, $t$ ) = − $y(t)$ + $x(t)$ * $y(t)$ ], [ $x(t)$, $y(t)$ ], $t$ = 0 ..1, $x$ = 0.5 ..1.5, $y$ = 0.5 ..1.5 );



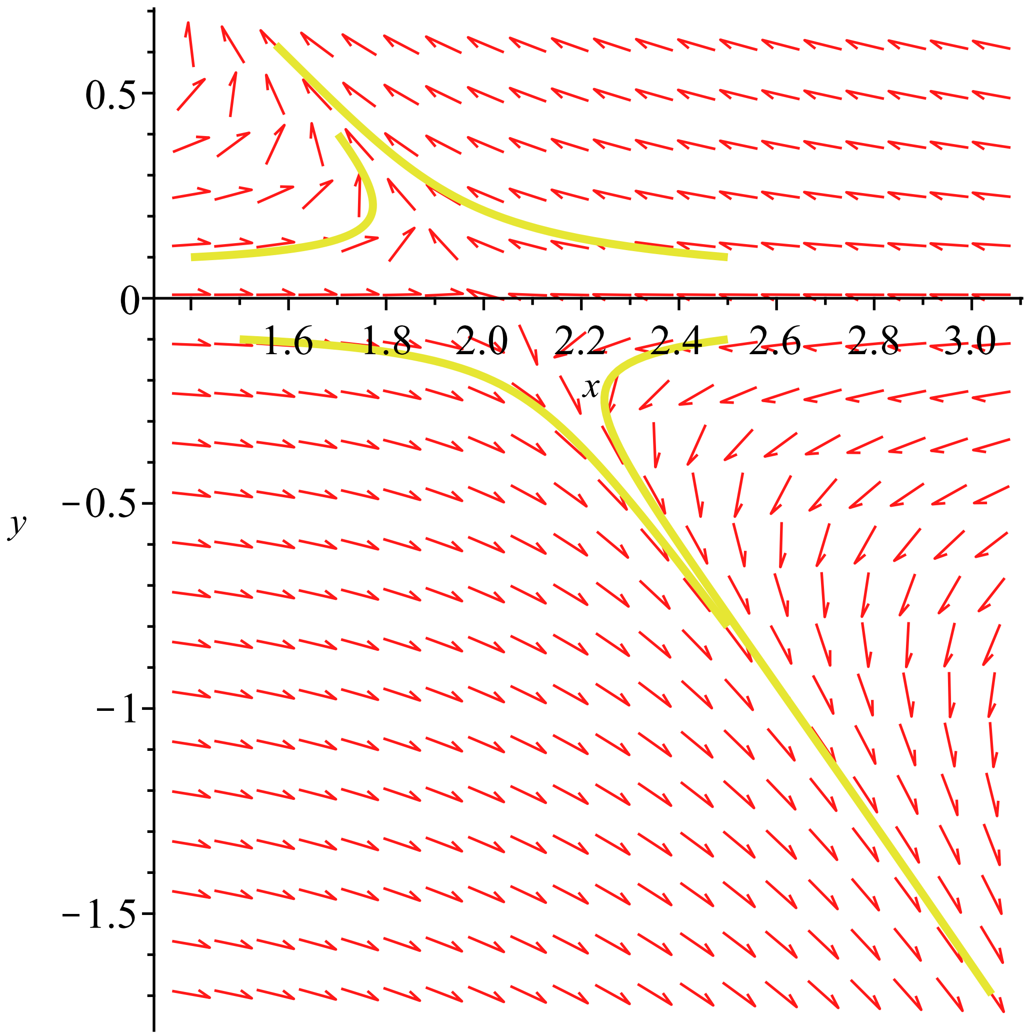

*dfieldplot*( [ *diff* ( $x(t)$, $t$ ) =− $x(t)$ − $y(t)$, *diff* ( $y(t)$, $t$ ) = $x(t)$ ], [ $x(t)$, $y(t)$ ], $t$ = 0 ..1, $x$ =− 5 ..5, $y$ =− 5 ..5 );



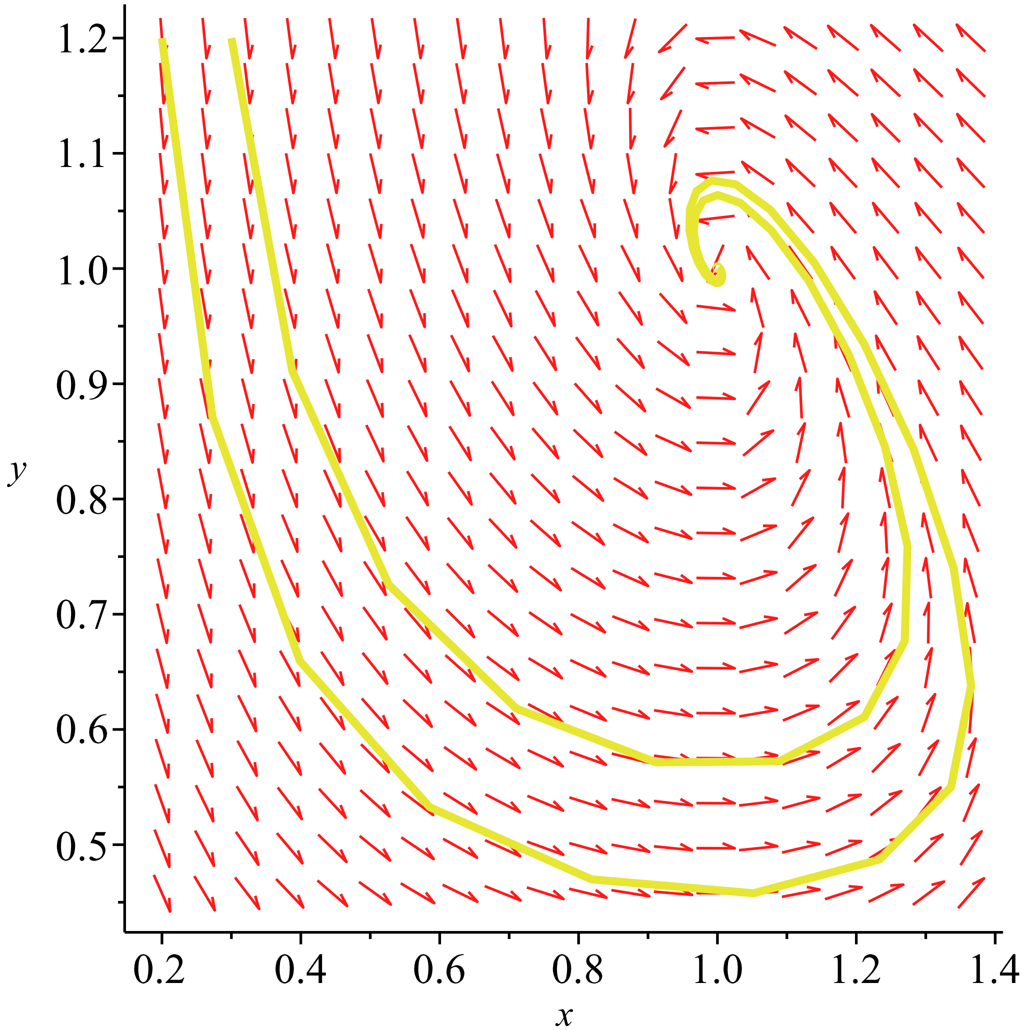

*DEplot*( [ *diff* ( *x*( *t* ), *t* ) = 2 * *x*( *t* ) - *x*( *t* ) ^2 - *x*( *t* ) * *y*( *t* ), *diff* ( *y*( *t* ), *t* ) = - *y*( *t* ) + *x*( *t* ) * *y*( *t* ) ], [ *x*( *t* ), *y*( *t* ) ], *t* = 0 ..1, [ [ *x*( 0 ) = 0.1, *y*( 0 ) = 0.1 ], [ *x*( 0 ) = -0.1, *y*( 0 ) = 0.1 ], [ *x*( 0 ) = 0.2, *y*( 0 ) =- 0.1 ], [ *x*( 0 ) =- 0.1, *y*( 0 ) =- 0.1 ] ] );



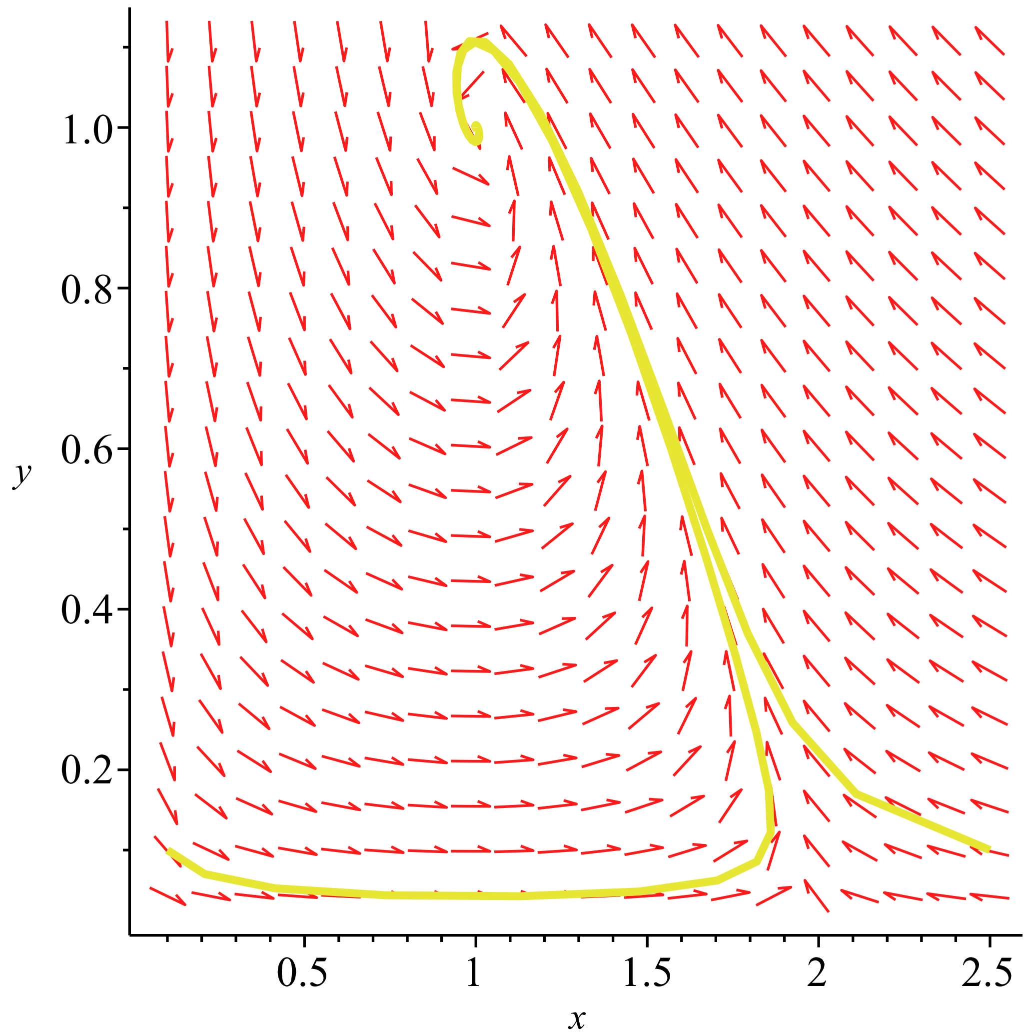

```
DEplot([diff(x(t), t) = 2*x(t)-x(t)^2 - x(t)*y(t), diff(y(t), t) = -y
(t) + x(t)*y(t)], [x(t), y(t)], t=0..2, [ [x(0) = 1.4, y(0) = 0.1], [x(0) =
2.5, y(0) = 0.1], [x(0) = 2.5, y(0) =-0.1], [x(0) = 1.5, y(0) =-0.1]]);
```




$DEplot([diff(x(t), t) = 2*x(t) - x(t)^2 - x(t)*y(t), diff(y(t), t) = -y(t) + x(t)*y(t)], [x(t), y(t)], t = 0..20, [[x(0) = 0.3, y(0) = 1.2], [x(0) = 0.2, y(0) = 1.2]]);$




$DEplot([diff(x(t), t) = 2*x(t) - x(t)^2 - x(t)*y(t), diff(y(t), t) = -y(t) + x(t)*y(t)], [x(t), y(t)], t = 0..20, [[x(0) = 0.1, y(0) = 0.1], [x(0) = 2.5, y(0) = 0.1]]);$




*#1.2*

$$solve\left(\left\{x-2\,x\cdot y=0,\ \frac{x^2}{2}-y=0\right\}\right);$$

$$\{x=0,y=0\},\ \left\{x=1,y=\frac{1}{2}\right\},\ \left\{x=-1,y=\frac{1}{2}\right\} \quad \textbf{(9)}$$

*with(linalg) : with(DEtools) : with(VectorCalculus) :*

$$Jm2 := Jacobian\left(\left[x-2\cdot x\cdot y,\ \frac{x^2}{2}-y\right],\ [x,y]\right);$$

$$Jm2 := \begin{bmatrix} -2\,y+1 & -2\,x \\ x & -1 \end{bmatrix} \quad \textbf{(10)}$$

*#again, having 3 equilibrias for the non-linear initial syste, we will find the the eqigenvalues for each matrix found using the Jacobian matrix method as linearization method*

$$B1 := subs([x=0,y=0],Jm2);$$

$$B1 := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \textbf{(11)}$$

*eigenvalues(B1)*

$$1,\ -1 \quad \textbf{(12)}$$

# the same argument as for A1

$B2 := subs\left(\left[x=1, y=\frac{1}{2}\right], Jm2\right);$

$$B2 := \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \tag{13}$$

$eigenvalues(B2);$

$$-\frac{1}{2} + \frac{I\sqrt{7}}{2}, -\frac{1}{2} - \frac{I\sqrt{7}}{2} \tag{14}$$

# the same argument as for A3

$B3 := subs\left(\left[x=-1, y=\frac{1}{2}\right], Jm2\right);$

$$B3 := \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \tag{15}$$

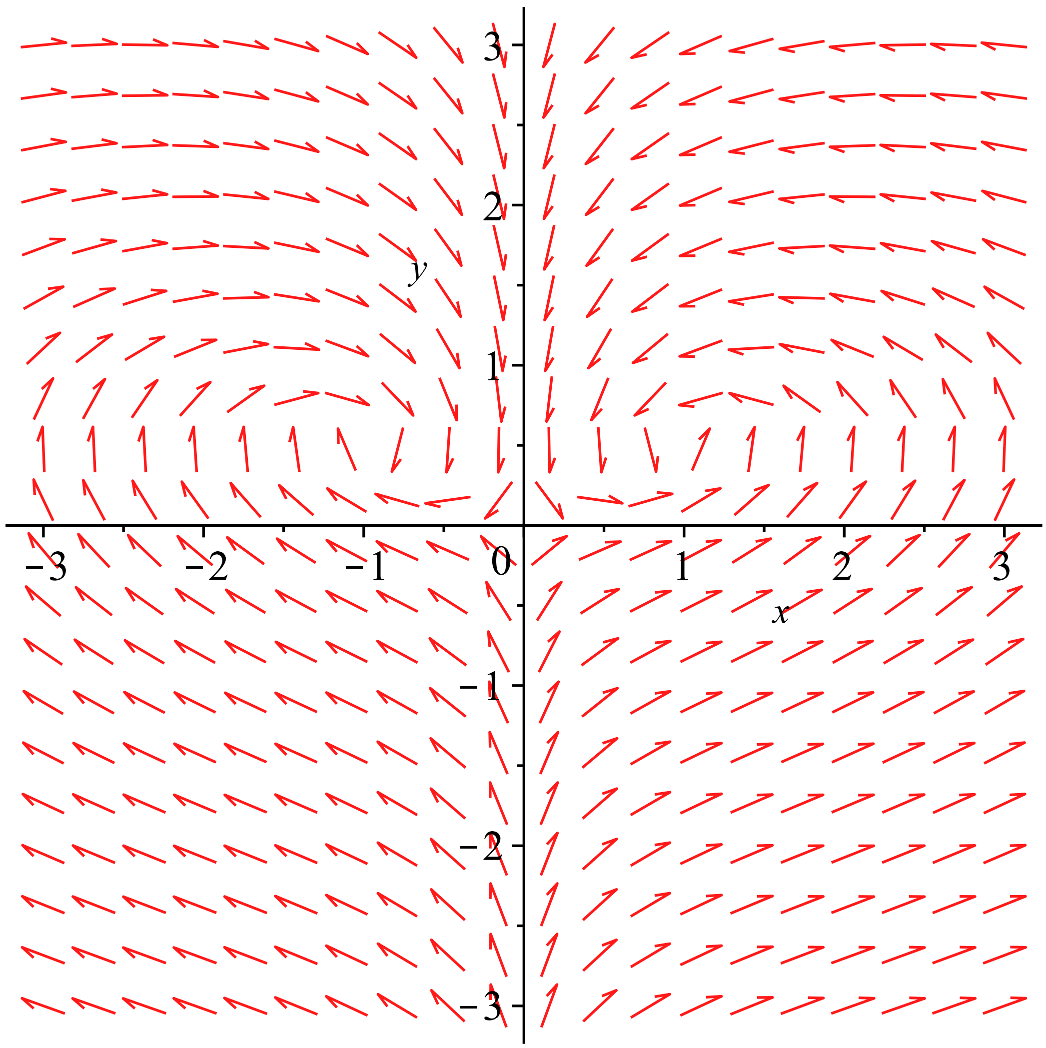
$eigenvalues(B3);$

$$-\frac{1}{2} + \frac{I\sqrt{7}}{2}, -\frac{1}{2} - \frac{I\sqrt{7}}{2} \tag{16}$$

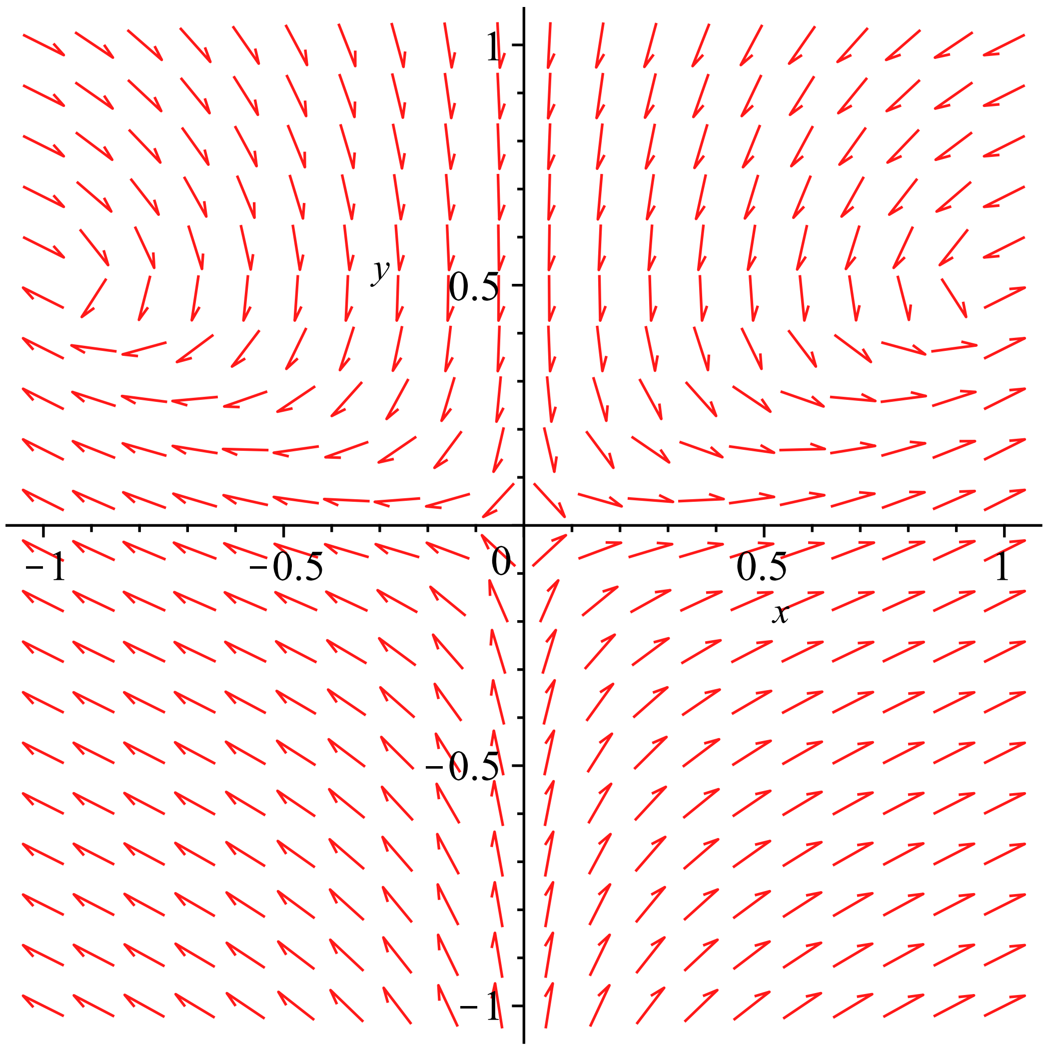
# the same argument as for A3

#a few plots:

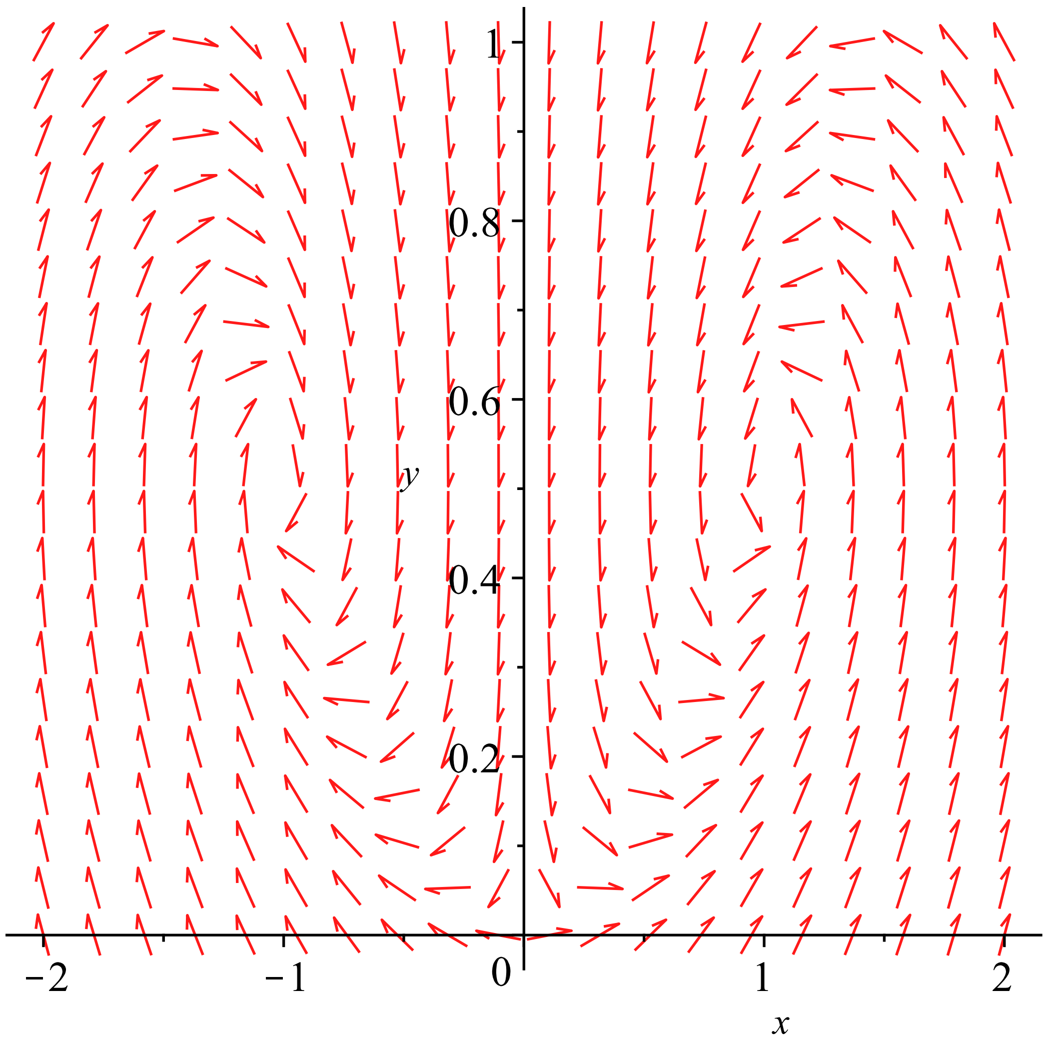
$dfieldplot\left(\left[diff(x(t), t) = x(t) - 2\cdot x(t)\cdot y(t), diff(y(t), t) = \frac{x(t)^2}{2} - y(t)\right], [x(t), y(t)], t=0..1, x=-3..3, y=-3..3\right);$




$$dfieldplot\left(\left[diff(x(t),t)=x(t)-2\cdot x(t)\cdot y(t),\ diff(y(t),t)=\frac{x(t)^2}{2}-y(t)\right],[x(t),y(t)],t=0..1,x=-1..1,y=-1..1\right);$$




$$dfieldplot\left(\left[diff(x(t),t)=x(t)-2\cdot x(t)\cdot y(t),\ diff(y(t),t)=\frac{x(t)^2}{2}-y(t)\right],\ [x(t),y(t)],\ t=0\,..1,\ x=-2\,..2,\ y=0\,..1\right);$$




*#2 Non-hyperbolic equilibria.*
*#2.3 conservative pendulum system x'= y ; y' = −4·sin x*
*with*(*linalg*) : *with*(*DEtools*) : *with*(*VectorCalculus*) :
*subs*([$x=0, y=0$], [$y$, $-4*\sin(x)$]); *eval*([$y$, $-4*\sin(x)$], [$x=0, y=0$]);

$$[0, -4\sin(0)]$$

$$[0, 0] \tag{17}$$

*Jm3* := *Jacobian*([ $y$, $-4*\sin(x)$ ], [$x, y$]);

$$Jm3 := \begin{bmatrix} 0 & 1 \\ -4\cos(x) & 0 \end{bmatrix} \tag{18}$$

*#we substitute with (0,0)*
*A* := *subs*([$x=0, y=0$], *Jm3*);

$$A := \begin{bmatrix} 0 & 1 \\ -4\cos(0) & 0 \end{bmatrix} \tag{19}$$

*#finding eigenvalues*
*eigenvalues*(*A*);

$$2\,\mathrm{I}, -2\,\mathrm{I} \tag{20}$$

$dsolve\left(diff(y(x), x) = -\dfrac{4\cdot\sin(x)}{y(x)}\right)$

$$y(x) = \sqrt{8\cos(x) + \_C1},\ y(x) = -\sqrt{8\cos(x) + \_C1} \tag{21}$$

$H := y^\wedge 2 - 8 * \cos(x);\ y * diff(H, x) - 4 * \sin(x) * diff(H, y);$

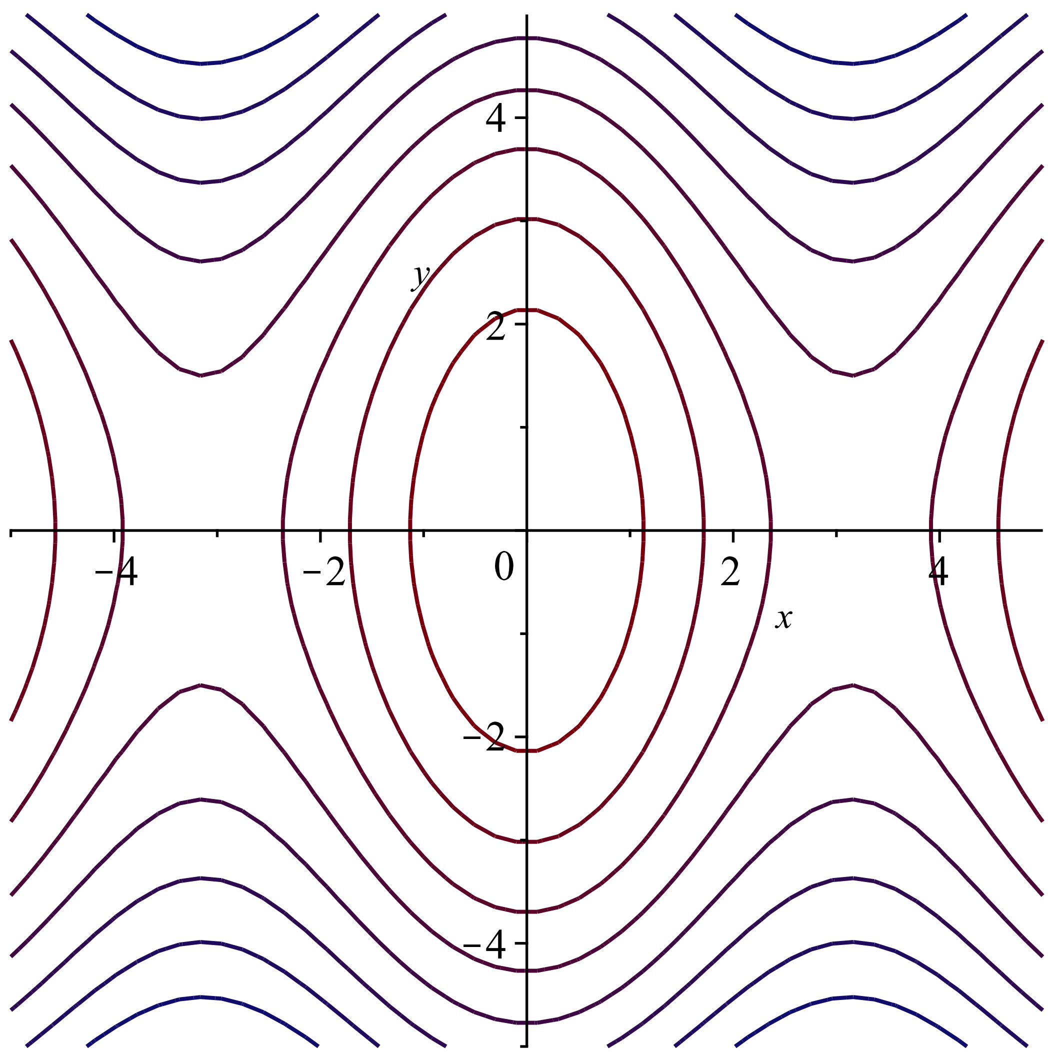
$$H := y^2 - 8\cos(x)$$

$$0 \tag{22}$$

$with(plots);$

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(23)**

$contourplot(y^\wedge 2 - 8 * \cos(x), x = -5..5, y = -5..5);$



#2.4 x'= x-x·y, y'= -0.3·y + 0.3·x·y

$with(linalg) : with(DEtools) : with(VectorCalculus) :$

$subs([x=1, y=1], [y, -0.3\cdot y + 0.3\cdot x\cdot y]); eval([y, -0.3\cdot y + 0.3\cdot x\cdot y], [x=1, y=1]);$

$$[1, 0.]$$

$$[1, 0.] \tag{24}$$

$Jm4 := Jacobian([y, -0.3\cdot y + 0.3\cdot x\cdot y], [x, y]);$

$$Jm4 := \begin{bmatrix} 0 & 1 \\ 0.3\, y & -0.3 + 0.3\, x \end{bmatrix} \tag{25}$$

$B := subs([x=0, y=0], Jm4);$

$$B := \begin{bmatrix} 0 & 1 \\ 0. & -0.3 \end{bmatrix} \tag{26}$$

$eigenvalues(B)$

$$0., -0.300000000000000 \tag{27}$$

$H2 := y - \ln(y) + 0.3\cdot(x - \ln(x))$

$$H2 := y - \ln(y) + 0.3\, x - 0.3\, \ln(x) \tag{28}$$

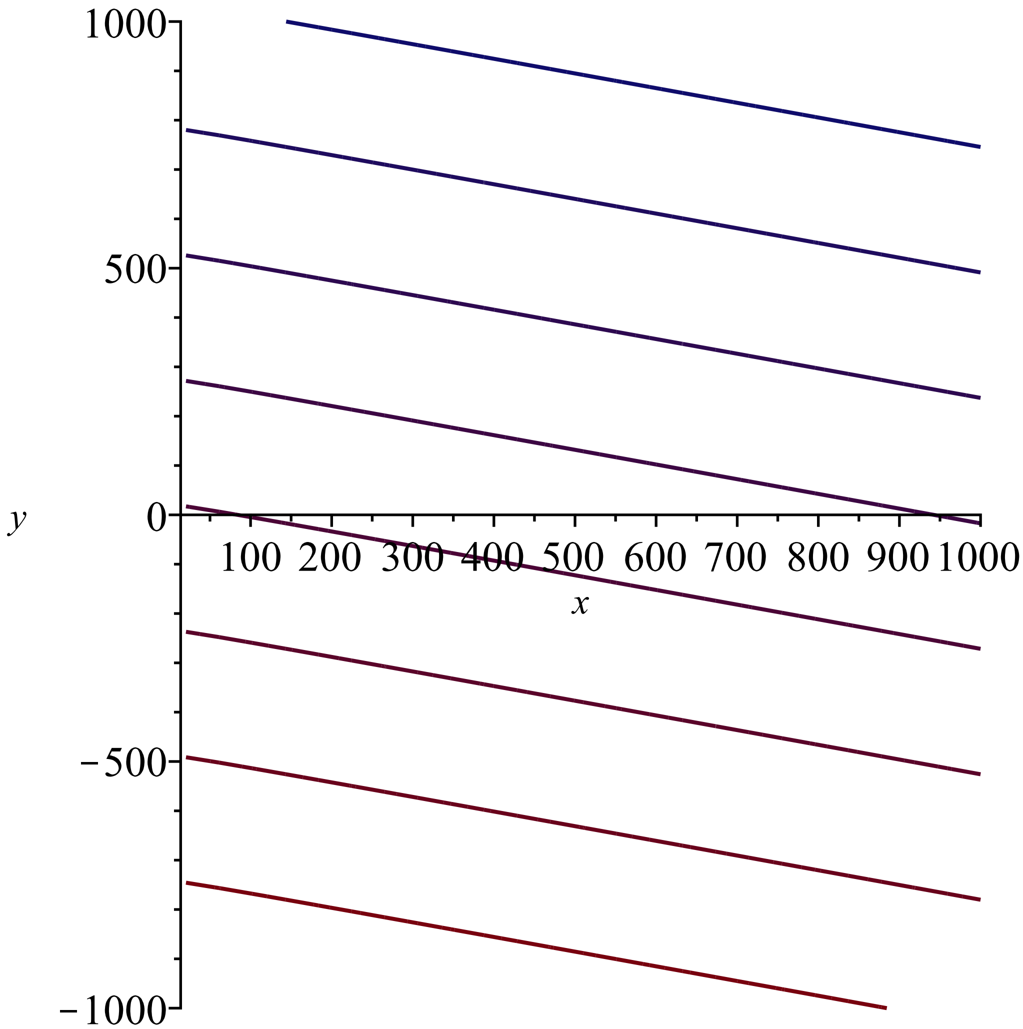
$y\cdot diff(H, x) + (0.3\cdot y + 0.3\cdot x\cdot y)\cdot diff(H, y);$

$$8\, y \sin(x) + 2\, (0.3\, y + 0.3\, x\, y)\, y \tag{29}$$

$with(plots);$

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(30)**

$contourplot(y - \ln(x) + 0.3\cdot x - 0.3\cdot\ln(x), x=-1000..1000, y=-1000..1000)$


1000
500
0
−500
−1000
y
100 200 300 400 500 600 700 800 900 1000
x