Closure Properties of Asymptotic Notations

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Reflexive Property

- **Reflexivity:**
  If f(n) is given then **f(n) = O(f(n))**

Example:

If $f(n) = n^3 \Rightarrow O(n^3)$

Similarly,

- $f(n) = \Omega(f(n))$
- $f(n) = \Theta(f(n))$

Symmetric Property

- **$f(n) = \Theta(g(n))$**

  if and only if $g(n) = \Theta(f(n))$

Example:

If $f(n) = n^2$ and $g(n) = n^2$

then $f(n) = \Theta(n^2)$ and $g(n) = \Theta(n^2)$

Symmetric Property (Cont'd)

- **If $f(n) = O(g(n))$**

  then $g(n) \neq O(f(n))$

- **If $f(n) = \Omega(g(n))$**

  then $g(n) \neq \Omega(f(n))$

Transitive Property

- **f(n) = O(g(n)) and g(n) = O(h(n))**
  ⇒ f(n) = O(h(n))

Example:

If $f(n) = n$, $g(n) = n^2$ and $h(n) = n^3$

⇒ n is $O(n^2)$ and n^2 is $O(n^3)$

then n is $O(n^3)$

Transitive Property (Cont'd)

- $f(n) = \Omega(g(n))$ and $g(n) = \Omega(h(n))$

  $\Rightarrow f(n) = \Omega(h(n))$

- $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$

  $\Rightarrow f(n) = \Theta(h(n))$

Other Properties

- **f(n) = O(f(n)^2)**
- **f(n) = O(f(n)/2)**
- **if f(n) = O(g(n))**
  then h(n).f(n) = O(h(n).g(n))

Other Properties (Cont'd)

- **if f(n) = O(g(n)) and d(n) = O(h(n)) then**

  i) f(n) + g(n) = max(f(n), g(n))
  = O(f(n)+ g(n))

  ii) f(n) * g(n) = O(f(n)*g(n))

  iii) f(n) / g(n) = Not Possible

  iv) f(n) - d(n) = Not Possible