$$f(x) = \frac{3}{2}x_1^2 + x_2^2 + 2x_1x_2 + 2x_1^3 + \frac{1}{2}x_1^4$$

~~$\frac{\partial f}{\partial x_1} = 0 + 2x_2 - 2x_2$~~

$$\frac{\partial f}{\partial x_1} = 3x_1 + 0 - 2x_2 + 6x_1^2 + 2x_1^3$$

$$\frac{\partial f}{\partial x_2} = 0 + 2x_2 - 2x_1 + 0$$

$$\nabla f = \begin{bmatrix} 3x_1 - 2x_2 + 6x_1^2 + 2x_1^3 \\ 2x_2 - 2x_1 \end{bmatrix}$$

Stationary Points: $\nabla f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$3x_1 - 2x_2 + 6x_1^2 + 2x_1^3 = 0$$

$$2x_2 - 2x_1 = 0 \Rightarrow x_2 = x_1$$

$$3x_1 - 2x_1 + 6x_1^2 + 2x_1^3 = 0$$

$$2x_1^3 + 6x_1^2 + x_1 = 0$$

$$x_1(2x_1^2 + 6x_1 + 1) = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-6 \pm \sqrt{6^2 - 4x2}}{2x2}$$

$$\frac{-6 \pm \sqrt{36 - 8}}{4}$$

$$\frac{-6 \pm 2\sqrt{7}}{4} = \frac{-3 \pm \sqrt{7}}{2}$$

Stationary points:

$$\left\{(0,0), \left(\frac{-3}{2} + \sqrt{7}, \frac{-3}{2} + \sqrt{7}\right), \left(\frac{-3}{2} - \sqrt{7}, \frac{-3}{2} - \sqrt{7}\right)\right\}$$