Algebraic solution

$$x_T = L_1 c_1 + L_2 c_{12} + L_3 c_{123}$$

$$y_T = L_1 s_1 + L_2 s_{12} + L_3 s_{123}$$

$$q_t = q_1 + q_2 + q_3$$

$$c_{123} = c_{q_t}$$

$$s_{123} = s_{q_t}$$

$$x_T - L_3 \cos(q_t) = A = L_1 c_1 + L_2 c_{12}$$

$$y_T - L_3 \sin(q_t) = B = L_1 s_1 + L_2 s_{12}$$

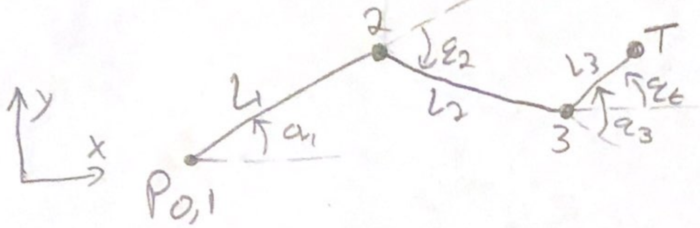
$$A^2 + B^2 = L_1^2 + L_2^2 + \underbrace{2L_1 L_2 c_1 c_{12} + 2L_1 L_2 s_1 s_{12}}_{2L_1 L_2 \cos(q_1 + q_2 \cancel{- q_1}) = 2L_1 L_2 \cos q_2}$$

$$q_2 = \pm \cos^{-1}\left(\frac{A^2 + B^2 - L_1^2 - L_2^2}{2L_1 L_2}\right)$$

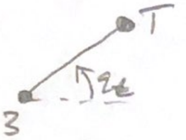
$$\begin{bmatrix} P_{3x} \\ P_{3y} \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 \\ \sin q_1 & \cos q_1 \end{bmatrix} \begin{bmatrix} L_1 + L_2 \cos q_2 & -L_2 \sin q_2 \\ L_1 + L_2 \cos q_2 & L_2 \sin q_2 \end{bmatrix}$$

Solve for q_1

$$\boxed{q_3 = q_t - q_2 - q_1}$$



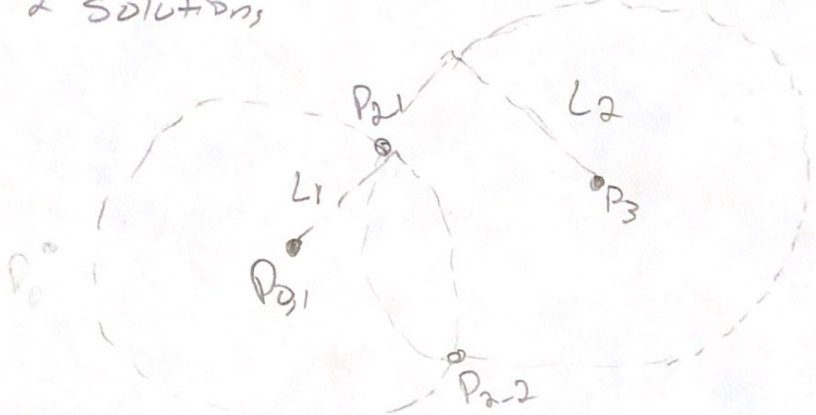
Geometric Solution



From a known q_t & T:

$$P_{3x} = T_x - \cos(q_t)$$
$$P_{3y} = T_x - \sin(q_t)$$

Once P_3 is known & P_0 is known: P2 can only have 2 solutions



We know P_2 will be L_2 away from P_3 & P_2 will be L_1 away from $P_{0,1}$ so the intersection of 2 circles can be found.

Once 2 solutions for P_2 are found, the vector from P_2 to P_1 is found & the vector from P_3 to P_2 is found this is then input into subproblem 0, which computes the ∡ between 2 vectors. q_2 has 2 solutions $\pm q_2$.

q_3 is then $\underbrace{q_t - q_1 - q_2 = q_3}$

Iterative solution

To find the solution iteratively we need forward kinematics & the Jacobian.

The algo will keep trying to find a solution until one of 2 conditions is meet

1) The linear & angular error is sufficiently small, it will never reach 0 in a decent time.
2) max-iterations are reached.
   - This is to ensure the simulation does not go take infite time.

The control system looks like this