Inverse Kinematics → Sub problem decomposition:

POE Parameters:

$h_1 = e_z \qquad P_{01} = L_1 e_z$

$h_2 = e_y \qquad P_{12} = 0$

$h_3 = e_y \qquad P_{23} = L_2 e_z$

$h_4 = e_x \qquad P_{34} = L_3 e_z + L_4 e_x$

$h_5 = e_y \qquad P_{45} = 0$

$h_6 = e_y \qquad P_{56} = 0$

$\qquad\qquad P_{6T} = L_5 e_z$

$$R_{0T} = R_z(q_1) R_y(q_2) R_y(q_3) R_x(q_4) R_y(q_5) R_x(q_6)$$

$$\begin{aligned} P_{0T} = {} & P_{01} + \cancel{R_{01} P_{12}} \to 0 + R_{01} R_{12} P_{23} \\ & + R_{01} R_{12} R_{23} P_{34} \\ & + \cancel{R_{01} R_{12} R_{23} R_{34} P_{45}} \to 0 \\ & + \cancel{R_{01} R_{12} R_{23} R_{34} R_{45} P_{56}} \to 0 \\ & + \underbrace{R_{01} R_{12} R_{23} R_{34} R_{45} R_{56}}_{R_{0T}} P_{6T} \end{aligned}$$

$$\begin{aligned} P_{0T} = {} & P_{01} + R_{01} R_{12} P_{23} \\ & + R_{01} R_{12} R_{23} P_{34} + R_{0T} P_{6T} \end{aligned}$$

$$\underset{\text{known}}{P_{0T}} = \underset{\text{known}}{P_{01}} + R_{01} R_{12} R_{23} + R_{01} R_{12} R_{23} P_{34} + \underset{\text{known}}{\underline{R_{0T} P_{6T}}}$$

$$\underbrace{P_{0T} - P_{01} - R_{0T} P_{6T}}_{(P_{16})_0} = \underbrace{\underline{R_{01} R_{12}}}_{R_z(q_1)R_y(q_2)} P_{23} + \underbrace{\underline{R_{01} R_{12} R_{23}}}_{R_z(q_1)R_y(q_2)R_y(q_3)} P_{34}$$

$$\|P_{0T} - P_{01} - R_{0T} P_{6T}\| = \|\cancel{R_{01} R_{12}}\,(P_{23} + R_{23} P_{34})\|$$ 1 ← since rotation does not effect the norm; R_{01} & R_{12} go away

Subproblem 3:

$\|P_2 - R(k, q) P_1\| = d$

$\left.\begin{aligned} & d = \|P_{0T} - P_{01} - R_{0T} P_{6T}\| \\ & P_2 = P_{23} \\ & P_1 = -P_{34} \\ & k = e_y \end{aligned}\right\}$ q_3 will have 2 solutions.

$$R_z(-q_1)(P_{16})_0 = R_y(q_2)(P_{23} + R_y(q_3) P_{34})$$

Subproblem 2: Parameters:

$k_1 = e_z;\ P_1 = (P_{16})_0 \qquad k_2 = e_y;\ P_2 = P_{23} + R_y(q_3) P_{34}$

2 solutions of q_1 & q_2 for each solution of q_3

so 4 solutions of $[q_1, q_2, q_3]$

$Rot = R_z(q_1) R_y(q_2+q_3) R_z(q_4) R_y(q_5) R_x(q_6)$

$(Rot\, R_y(-q_2-q_3) R_z(-q_1) = R_x(q_4) R_y(q_5) R_x(q_6))\, e_x$

$Rot\, R_y(-q_2-q_3) R_z(-q_1) e_x = R_x(q_4) R_y(q_5) e_x \cdot \cancel{R_x(q_6) e_x}$

$R_x(-q_4) R_y(-q_2-q_3) R_z(-q_1) Rot\, e_x = R_y(q_5) e_x$

1
a rotation about the X axis multiplied by the x-axis cancels out

Sub Problem 2:

$R(k_1, q_1)\, P_1 = R(k_2, q_2)\, P_2$

$k_1 = -e_x$

$q_1 = q_4$

$P_1 = R_y(-(q_2+q_3)) R_z(-q_1) Rot\, e$

$k_2 = e_y$

$q_2 = q_5$

$P_2 = e_x$

2 solutions for q_4 & q_5

so 8 sols for $[q_1, q_2, q_3, q_4, q_5]$

$\underbrace{R_y(-q_5) R_x(-q_4) R_y(-(q_2+q_3)) R_z(-q_1) Rot\, e_y = R_x(q_6) e_y}_{\text{Sub problem 1}}$

1 solution for q_6 $\therefore$ 8 total solutions

for $[q_1, q_2, q_3, q_4, q_5, q_6]$