Iterative method of Jacobian finding:

For $i = 1, 2, 3 \ldots$ # of joints (n)

$$H_i = \begin{bmatrix} R_{0i} h_i \\ 0 \end{bmatrix} \text{ if ith joint is Revolute} \quad \text{or} \quad H_i = \begin{bmatrix} 0 \\ R_{0i} h_i \end{bmatrix} \text{ if ith joint is Prismatic}$$

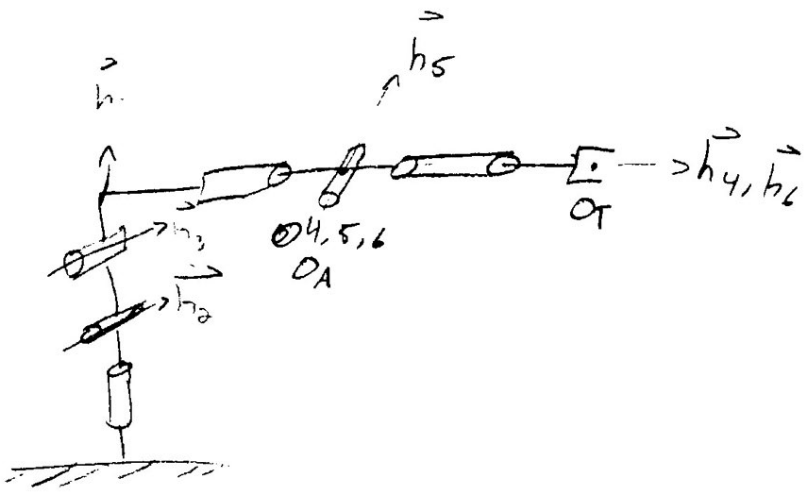
$$\phi_{ij} = \begin{bmatrix} R_{ij} & 0 \\ -(R_{ij} P_{ij})^{\times} & R_{ij} \end{bmatrix}$$

ϕ_{ij} is the Adjoint operator,

$$\phi_{i,i-1} = \begin{bmatrix} I & 0 \\ -(R_{0,i-1} P_{i-1,i} & I \end{bmatrix}$$

$$J_i = \phi_{i,i-1} J_{i-1} + \begin{bmatrix} 0 \ldots 0 \; \overset{\text{the ith column}}{H_i} \; 0 \ldots 0 \end{bmatrix}$$

$$\boxed{[J_T]_0 = \phi_{T,n} J_n}$$



$$(J_4)_3 = \begin{bmatrix} -\sin(q_2+q_3) & 0 & 0 & 1 & 0 & \cos(q_5) \\ 0 & 1 & 1 & 0 & \cos(q_4) & \sin(q_5)\cdot\sin(q_4) \\ \cos(q_2+q_3) & 0 & 0 & 0 & \sin(q_4) & -\cos(q_4)\sin(q_5) \\ 0 & L_3+L_2\cos(q_3) & L_3 & 0 & 0 & 0 \\ L_4\cos(q_2+q_3)+L_3\sin(q_2+q_3)+L_2\sin(q_3) & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2\sin(q_3)-L_4 & -L_4 & 0 & 0 & 0 \end{bmatrix}$$

$$\det(J_4)_3 = L_2\sin(q_5)\left(L_4\cos(q_3)+L_3\sin q_3\right)\left(L_4\cos(q_2+q_3)+L_3\sin(q_2+q_3)+L_2\sin(q_2)\right)$$

Singularity when $\det(J_4)_3 = 0$:

1) $L_2 \sin(q_5) = 0$

2) $L_4 \cos q_3 + L_3 \sin q_3 = 0$

3) $L_4\cos(q_2+q_3) + L_3\sin(q_2+q_3) + L_2\sin(q_2) = 0$

eq. 1)

$$\left.\begin{array}{l} L_2 \sin(q_5) = 0 \\ q_5 = 0, -\pi, \pi \end{array}\right\}$$ Since $q_5 = -\pi, \pi$ is not Physically Possible due to joint limits & body collision $q_5 = 0$ for singularity.

$$L_4 \cos q_3 + L_3 \sin(q_3) = 0 \quad \underline{eq. 2}$$

$$-L_4 \cos(q_3) = L_3 \sin(q_3)$$

$$\frac{-L_4}{L_3} = \tan(q_3)$$

$$\tan^{-1}\left(\frac{-L_4}{L_3}\right) = q_3$$

$\underline{eq. 3}$ $$L_4 \cos(q_2 + q_3) + L_3 \sin(q_2 + q_3) + L_2 \sin(q_2) = 0$$

$$L_4 \cos q_2 \cos q_3 - L_4 \sin q_2 \sin q_3 + L_3 \sin q_2 \cos q_3 + L_3 \cos q_2 \sin q_3 + L_2 \sin q_2 = 0$$

$$\cos q_2 (L_4 \cos q_3 + L_3 \sin q_3) + \sin q_2 (-L_4 \sin q_3 + L_3 \cos q_3 + L_2) = 0$$

$$\cos q_2 (L_4 \cos q_3 + L_3 \sin q_3) = -\sin q_2 (-L_4 \sin q_3 + L_3 \cos q_3 + L_2)$$

$$-\tan(q_2) = \frac{L_4 \cos q_3 + L_3 \sin q_3}{-L_4 \sin q_3 + L_3 \cos q_3 + L_2}$$

$$q_2 = \operatorname{atan}\left[\frac{-L_4 \cos q_3 - L_3 \sin q_3}{-L_4 \sin q_3 + L_3 \cos q_3 + L_2}\right]$$