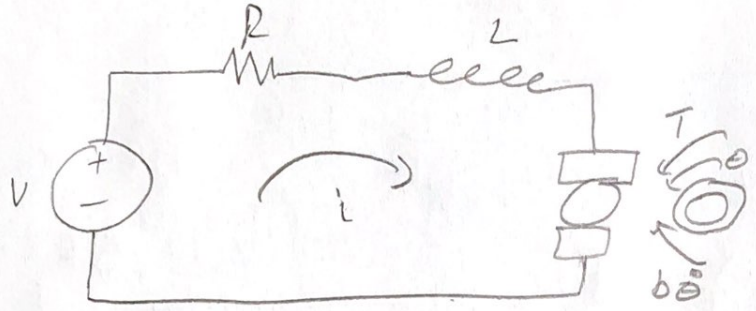

2.1) $T = k_t\, i(t)$

$e = k_e\, \dot{\theta}(t)$

mechanical $\begin{cases} J\ddot{\theta}(t) = T - b\dot{\theta}(t) \Rightarrow \\ J\ddot{\theta}(t) = k_t i(t) - b\dot{\theta}(t) \end{cases}$

electrical $[\, V(t) - R\,i(t) - L\frac{di}{dt} - k_e\dot{\theta} = 0$

2.2) Laplace Transforms

$V(s) = RI(s) + LsI(s) + k_e\Omega(s)$

$Js\,\Omega(s) = k_t I(s) - b\Omega(s)$

$I(s) = \frac{Js\,\Omega(s) + b\Omega(s)}{k_t} \Rightarrow \Omega(s)\left(\frac{Js+b}{k_t}\right)$

$V(s) = \frac{R\,\Omega(s)(Js+b)}{k_t} + \frac{Ls\,\Omega(s)(Js+b)}{k_t} + \frac{k_e\Omega(s)k_t}{k_t}$

$\frac{V(s)}{\Omega(s)} = \frac{RJs + Rb + LJs^2 + Lbs + k_e k_t}{k_t}$

$i(t) \Rightarrow I(s)$

$i'(t) \Rightarrow sI(s) + i(0)^0$

$\dot{\theta}(t) \Rightarrow \Omega(s)$

$\theta(t) = s\Omega(s) + \theta(0)^0$

Laplace transforms

$$\boxed{\frac{\Omega(s)}{V(s)} = \frac{k_t}{JLs^2 + (bL + JR)s + (bR + k_t k_e)}}$$

3.1)

$$\Omega(s) = V(s) \frac{k_t}{JLs^2 + (bL + JR)s + (bR + k_t k_e)}$$

for ramp input $V(s) = \frac{A_1}{s}$

$$\lim_{t \to \infty} w(t) = \lim_{s \to 0} s\Omega(s) = \cancel{s}\frac{A_1}{\cancel{s}} \frac{k_t}{JLs^2 + (bL + JR)s + (bR + k_t k_e)}$$

$$\lim_{t \to 0} w(t) = \frac{A_1 k_t}{bR + k_t k_e}$$

3.4) $$Gain = \frac{k_t}{bR + k_t k_e} = \frac{\Delta output}{\Delta input} = \frac{slope(output)}{slope(input)}$$