Reliability Analysis of a Semi-Elliptical Leaf Spring using Probabilistic Methods

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1. Abstract

Leaf spring is a very important part of the automobile industry. It plays a very important role in the suspension system of a vehicle. In this research work, we present the probability of failure of a semi-elliptical leaf spring using the concepts of probabilistic methods. The purpose of this study is to propose the dimensions of leaf spring such that it can withstand the applied load without failure. We considered the span length, the width of the leaf, thickness and the load as the random variables and the bending stress as the output parameter. The material of the leaf spring we considered is structural steel. The other-dimensional parameters and the number of leaves are assumed to be constant. The failure criteria are defined as the bending stress with the considered safety factor should be well within the tensile yield stress of the material. We implemented the First-Order Reliability Method (FORM) and the Monte Carlo Simulation to perform the reliability analysis. The sensitivity analysis was performed to determine the sensitiveness of the output parameter due to the variation in the input parameters. We finally compared the reliability index and the probability of failure for both the methods and proposed the safe values of the input parameters. This paper gives detailed information and the methodology used with all the results.

Keywords: Leaf spring, bending stress, yield stress, reliability analysis, FORM, Monte Carlo Simulation, MATLAB

2. Introduction

There are various parts in the automobile industry and every part has some important characteristics. The suspension system in a vehicle prevents the body and other vehicle parts from the higher shock vibrations and minimizes the jerk effect that occurs and impacts due to the irregularities of the roads. Leaf spring is used for the purpose of the suspension system. Spring is a coil or an elastic body that distorts when a load (either compression or tension) is applied on it and stores the kinetic energy in the form of strain energy, which later releases this energy after the removal of load and retains its original shape without producing any kinds of effects. To fulfill this purpose, leaf spring is used as it has various characteristics such as uniform load distribution, rough use, lower manufacturing & maintenance cost, and it can be firmly attached to the working frame. Different types of leaf springs used are parabolic leaf spring, elliptical leaf spring, semi-elliptical leaf spring, quarter leaf spring, and three-quarter leaf spring. In our study, the focus is on the semi-elliptical leaf spring and its general model is shown in figure 1.

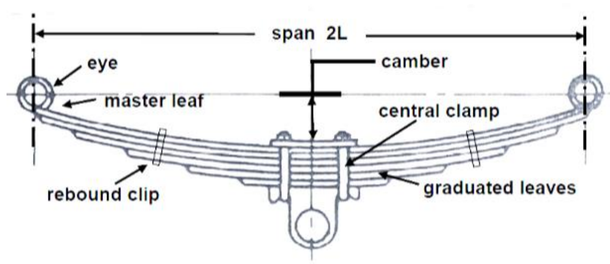

Figure 1 Semi-elliptical leaf spring [12]

2.1 Nomenclature

The following terms are frequently used in this research work:

L length/span of the spring (eye to eye) (mm)

t thickness of the leaf (mm)

b width of the leaf (mm)

n total number of leaves

W load acting on the spring (N)

I moment of inertia of the leaf (mm^4)

σ maximum bending stress developed (MPa)

M maximum bending moment (N-mm)

β reliability Index

p_f probability of failure

α sensitivity index

g limit state function (MPa)

2.2 Literature Review

Jei, Y.-G., et al., [1] introduced a new lateral vibration damper using leaf springs and oil. The stiffness of the leaf spring damper depends on the stiffness of the leaf spring which was varied with the thickness, effective length, and the number of leaves in spring groups. It was found out that the damping was viscous due to oil pressure build-up in the oil spaces and was frictional due to O-rings and small slips between leaf springs.

Shiva Shankar G.S., et al., [2] designed and manufactured a unidirectional E-Glass fiber/Epoxy mono composite leaf spring without end joints and composite leaf spring using bonded end joints using hand lay-up technique. Composite leaf spring weight was reduced by 85% compared to steel leaf spring. Adhesively bonded end joints enhance the performance of composite leaf spring for delamination and stress concentration when compared with bolted joints.

Sorathiya M., et al., [3] compared load carrying capacity, stiffness and weight reduction of composite leaf spring with that of steel leaf spring. They modeled three different composite mono leaf springs by considering uniform cross-section & unidirectional fiber orientation angle for each lamina of a laminate and considered stress & deflection as the design constraints. A weight reduction of 79.617% was achieved for composite leaf spring and 90.09% for mono leaf spring.

Ishtiaque M.T., et al., [4], designed the conventional flat profile leaf spring and a parabolic shape leaf spring. They modified the number of leaves for the same simulation conditions and obtained the optimized design which incorporates the better fatigue life, reduced deformation, reduced weight of the spring, and increased factor of safety.

Qureshi H.A. [5], designed, analyzed and performed experimental investigations of composite leaf spring. It was concluded that the GFRP leaf springs have more flexibility and hardness with the reduction in noise parameters as compared to conventional steel leaf springs. Also, the natural frequency and weight were reduced up to 80%.

Gebremeskel S.A. [6], designed, simulated and manufactured a single composite leaf spring for light-weight vehicles. E-Glass/Epoxy material was used for manufacturing the leaf spring. The maximum stress failure criterion was considered for analysis and the hand layup technique was considered for manufacturing. It was found out that the fatigue life of E-Glass/Epoxy leaf spring is 221.16x103 cycles.

Ghag M.D., et al., [7] performed static and dynamic analysis of steel and laminated composite leaf spring using ANSYS 14.5. The aluminum alloy unidirectional laminates leaf spring was analyzed by using the layer stacking method for composites by changing reinforcement angles from 3 layers, 5 layers, and 11 layers. A weight reduction of 27.5% was achieved by using composite leaf spring.

Solanki P., et al., [8] performed the design and computational analysis of semi-elliptical and parabolic leaf spring and compared the deflection, stresses, and safety factor. They concluded that by varying the span length and camber, the semi-elliptical leaf spring produced the maximum deflection and maximum stresses than the parabolic leaf spring under the same boundary conditions.

Ekbote T., et al., [9] implemented finite element analysis to suggest an optimal design of a leaf spring. The comparison was performed between steel leaf spring and E-glass fiber with epoxy resin matrix leaf spring and concluded that later has much lower stress and its weight without eye units is nearly 65% lower than the former one.

Kueh, J.-T. J. et al., [10] compared the mechanical properties of steel leaf spring and composite leaf spring using finite element analysis. They further made a comparison between E-glass epoxy and E-glass/vinyl ester leaf spring. The static and fatigue performances were improved with composite material and the different strength and stiffness composite spring were attainable through the arrangement of fiber lay-up and orientation.

Zheng Yinhuan, Xue Ka, et al., [11] analyzed the mechanics characteristic of a composite leaf spring made from glass fiber reinforced plastics and performed the comparison between the performance of the GFRP and the steel spring. It was observed that the stress of composite leaf spring is lower than steel spring and its resistance to fatigue ability is much stronger.

In this research work, the leaf spring model is used to inquire bending stress reliability by considering the variation in the span length, the width of the leaf, the thickness of the leaf, and the variability in the load acting on the spring, and according to that, the calculations are performed. To simplify the analysis, several assumptions are made. Based on the research data, it was found out that there is a large variation in the values of all the random variables based on the different geometry of leaf spring, and to cater to these variations, the probabilistic approach is used. Also, the failure ability of the leaf spring is greatly affected by the variation in all the above parameters. Hence it is ideal to use the probabilistic approach rather than the deterministic approach. This study includes the calculation of reliability index, probability of failure, and sensitivity index by implementing the First-Order Reliability Method (FORM) (using Excel and MATLAB) and sensitivity analysis. The results generated by FORM are then compared with the results from the Monte Carlo Simulation.

2.3 Determination of Bending Stress

The cross-section of the leaf is rectangular. For the calculations, the effective length of the semi-elliptic leaf spring is determined by finding the distance between two eyes. But practically, the distance between the eye and the center bolt is calculated to get the exact length of the spring. Based on the labeled diagram shown in figure 2, the following formula is derived to calculate the bending stress in the leaf spring [13] and used to make the limit state function later.

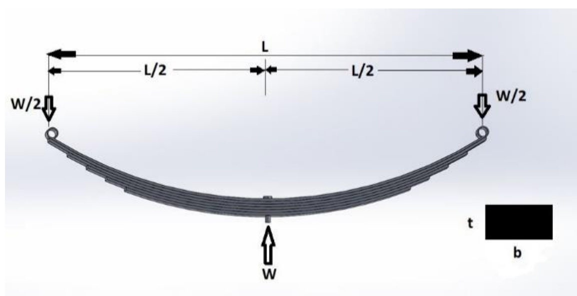

Figure 2 Labeled Leaf Spring

Maximum bending moment,

$$M = \frac{WL}{4}$$

Using the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$y = \frac{t}{2} \quad \text{and} \quad I = \frac{bt^3}{12}$$

which will give,

$$M = \frac{\sigma b t^2}{6}$$

The total bending moment for ‘n’ plates,

$$M = \frac{\sigma b t^2 n}{6}$$

After calculations,

$$\boldsymbol{\sigma = \frac{3WL}{2bt^2 n}}$$

3. Methodology

This section explains the methodology used to perform the analysis using both the probabilistic methods. The assumptions made in the study are detailed first.

3.1 Assumptions

- The material considered is structural steel and its properties remain the same under different loads acting on the spring. It is assumed to have a linear stress-strain relationship and follow Hooke’s law.
- Other than span, width, and thickness, all the parameters associated with the geometry of spring such as camber, the radius of the spring are assumed to be constant.
- The number of leaves is assumed to be constant. We considered two full-length leaves and four graduated leaves.
- The load acting on the spring is assumed to follow a normal distribution and is distributed uniformly in the middle of the spring.
- The leaves are considered to have a rectangular cross-section throughout the length.

- The width and thickness of all the leaves are assumed to be the same and they follow a semi-elliptical pattern for the entire length.
- Effects of other parts that are linked or attached to the leaf spring in a vehicle and the environmental effects are neglected.
- All the non-linear effects are not considered.
- There is no correlation between all the random variables.
- The strain energy, deflection, and the stiffness of the spring are not considered in the calculations and analysis process.

3.2 Data Set

To perform the probabilistic methods, twenty-five sets of values for the length, width, and thickness of the spring are collected from various research papers which are shown in table 1.

Table 1: Data Set

S. No.	Length	Width	Thickness
1	1245	50	7
2	1150	34	5.5
3	1165	38.1	6.1
4	940	60	8
5	1150	34	5.5
6	1151	50	6
7	1220	70	7
8	1100	56	6
9	1220	60	7
10	1000	40	5
11	1100	70	11
12	1120	50	6
13	1120	50	5
14	1130	45	10
15	1019	60	10.6
16	1170	63	8

17	1250	60	7
18	1150	70	8
19	1150	50	6
20	1045	60	5
21	1200	50	6
22	1320	63.5	9.525
23	1125	50	8
24	1340	60	8
25	1150	70	9

As mentioned in the assumptions, the load acting on the spring follows the normal distribution. Based on the different research papers, we found that there is a very wide range of values of the load depending on the type of vehicle and its static and dynamic behavior. Thus, we have assumed the mean value of load to be 6000 N with the standard deviation of 500 N. These values can be different as per the requirements.

4. Numerical Results

It is important to understand that the variation in the results arises due to the uncertainty in the input parameters and the applied system. To quantify the uncertainty and precise clarification and justification of the relationship between the parameters, there is a need to establish the basis for predicting the results. There is no correlation between the random variables.

4.1 Chi-Square Test

For the given data set, to determine the proper distribution for all the variables, the histograms are to be plotted. Based on the number of samples, the number of bins is calculated using the following empirical rule:

$$k = 1 + 3.3log_{10}n$$

where n = 25, which gives

$$k = 1 + 3.3log_{10}25$$

$$k = 5.61 \approx 6$$

Thus, 6 intervals or bins are calculated. Using excel, the following histograms are plotted for the length, width, and thickness of the leaf.

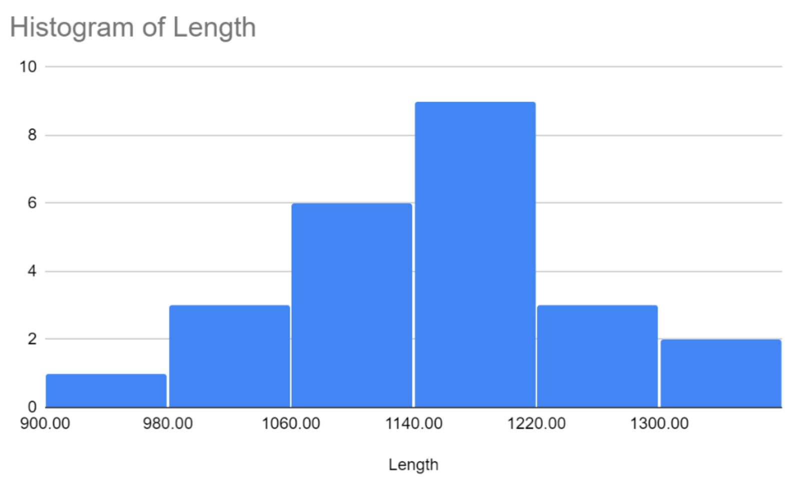

Figure 3 Histogram of Length

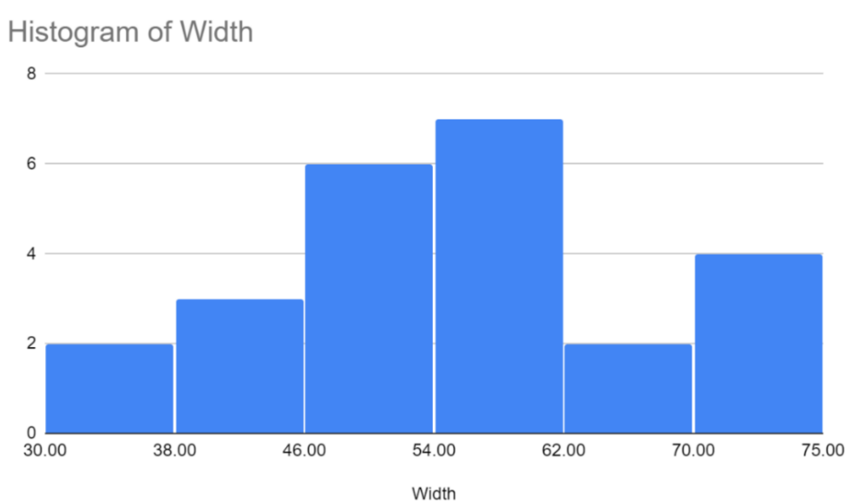

Figure 4 Histogram of Width

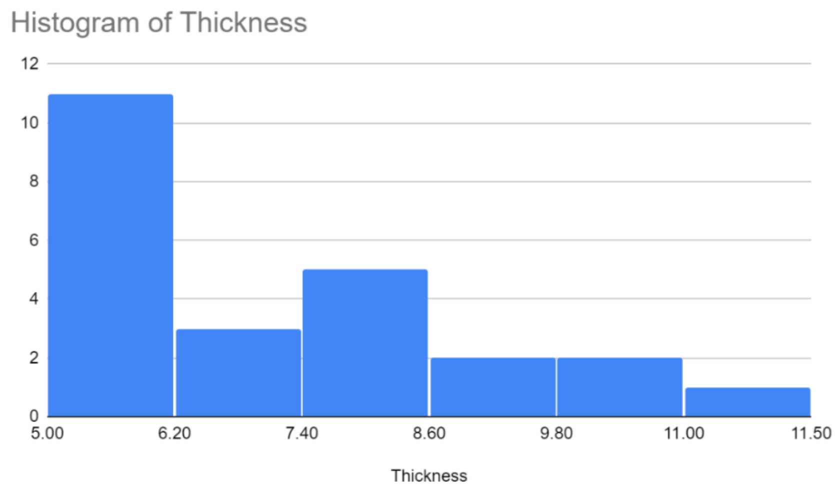

Figure 5 Histogram of Thickness

Observing the histograms with the profile of the bell curve, the distribution of the three parameters can be taken as normal or lognormal.

To select one type of distribution for all the random parameters to perform further analysis, the Chi-square tests are performed for the normal and lognormal distribution. The test has been performed as per the procedure in the book [14].

From table 1, values for the parameters associated with both the distributions are calculated and shown in table 2.

The two parameters for the lognormal distribution are calculated using the following formulas:

$$\zeta = \sqrt{1 + (\frac{\sigma}{\mu})^2}$$

and,

$$\lambda = \ln(\mu) - \frac{1}{2}\zeta^2$$

where μ and σ are a mean and standard deviation respectively.

Table 2: Statistical Parameters

Parameters	**Length**	**Width**	**Thickness**
μ	1149.2	54.544	7.209
Σ	91.1834232	10.89660039	1.762498227
ζ	0.07922067162	0.1978250044	0.2409461131
λ	7.04368337	3.979440349	1.946302731

Table 3, 4, and 5 shows the Chi-Square test performed for length, width, and thickness of the spring.

Table 3: Chi-Square Test for Length

Length	**Observed Frequency, n_i**	**Theoretical Frequency, e_i**		$\frac{(n_i - e_i)^2}{e_i}$	
		Normal	**Lognormal**	**Normal**	**Lognormal**
<975	1	0.70175	0.5295	0.1267589063	0.4180741265
975<l<1050	3	2.74475	2.86225	0.0237371573	0.006629421784
1050<l<1125	4	6.48925	6.8345	0.9548662114	1.175563721
1125<l1200	10	7.87075	7.83375	0.5760195105	0.5990284426
1200<l<1275	5	5.09875	4.72725	0.001912539838	0.01573696388
>1275	2	2.09475	2.21275	0.004285744122	0.02045534403
Total	25	25	25	1.68758007	2.23548802

Table 4: Chi-Square Test for Width

Width	**Observed Frequency, n_i**	**Theoretical Frequency, e_i**		$\frac{(n_i - e_i)^2}{e_i}$	
		Normal	**Lognormal**	**Normal**	**Lognormal**
<38	2	1.64	1.0455	0.07902439024	0.8714206121
38<w<46	3	3.8025	4.54525	0.1693639053	0.525339104
46<w<54	7	6.559	7.40775	0.02965101387	0.0224440704
54<w<62	7	6.79225	6.33575	0.006354310059	0.06964101527
62<w<70	2	4.2245	3.49275	1.171357616	0.6379794038
>70	4	1.98175	2.173	2.055422259	1.536092499
Total	25	25	25	3.511173495	3.662916704

Table 5: Chi-Square Test for Thickness

Thickness	Observed Frequency, n_i	Theoretical Frequency, e_i		$\frac{(n_i - e_i)^2}{e_i}$	
		Normal	**Lognormal**	**Normal**	**Lognormal**
<6	5	6.1275	6.52725	0.2074673603	0.3573469015
6<t<7	6	5.1785	5.97275	0.1303200251	0.0001243250596
7<t<8	4	5.535	5.221	0.4256955736	0.2855470216
8<t<9	5	4.3125	3.54975	0.1096014493	0.5924994894
9<t<10	2	2.42025	1.99325	0.07297182626	0.00002285839709
t>10	3	1.42625	1.736	1.736504163	0.9203317972
Total	25	25	25	2.682560398	2.155872393

For both the distributions, the number of parameters associated is two and based on this, the degrees of freedom, f = m – 1 – k = 6 – 1 – 2 = 3, where 'm' is the number of intervals and 'k' is the number of distribution parameters estimated from the data, which are sample mean and variance here. Assuming the significance level, α = 5 %, the corresponding value of $c_{0.95,3}$ is found to be 7.815 (from Appendix 3 in the book [14]), which is much greater than the calculated errors for both the distributions. Therefore, both distributions are acceptable.

For the length, the error value for the normal distribution is less than that of the lognormal distribution, so the distribution for the length is normal. Similarly, for the width and thickness, the error value is less for normal and lognormal distribution respectively. Therefore, width follows normal distribution and thickness follows a lognormal distribution.

The Probability density function (PDF) and the Cumulative density function (CDF) for all the four parameters are shown in the figures below. The MATLAB code used to plot these graphs is attached in Appendix 1.

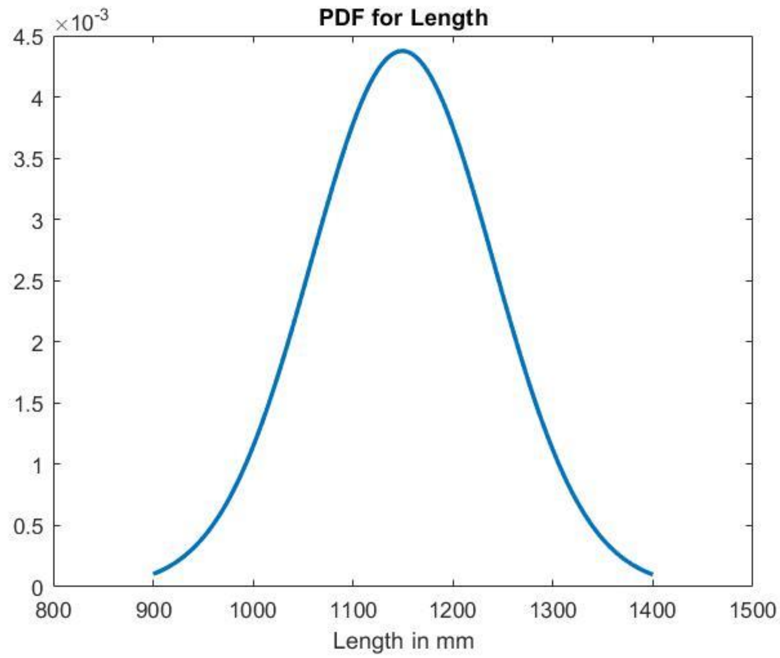

Figure 6 PDF for Length

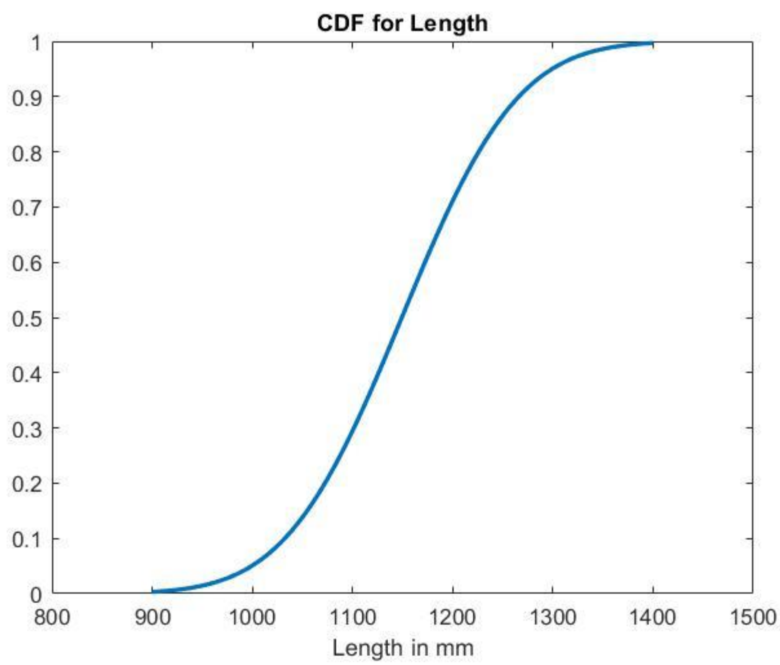

Figure 7 CDF for Length

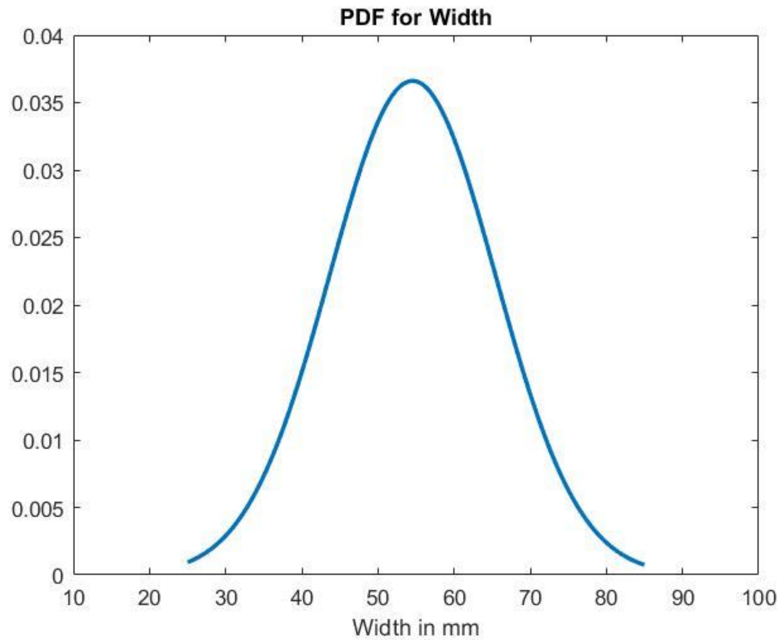

Figure 8 PDF for Width

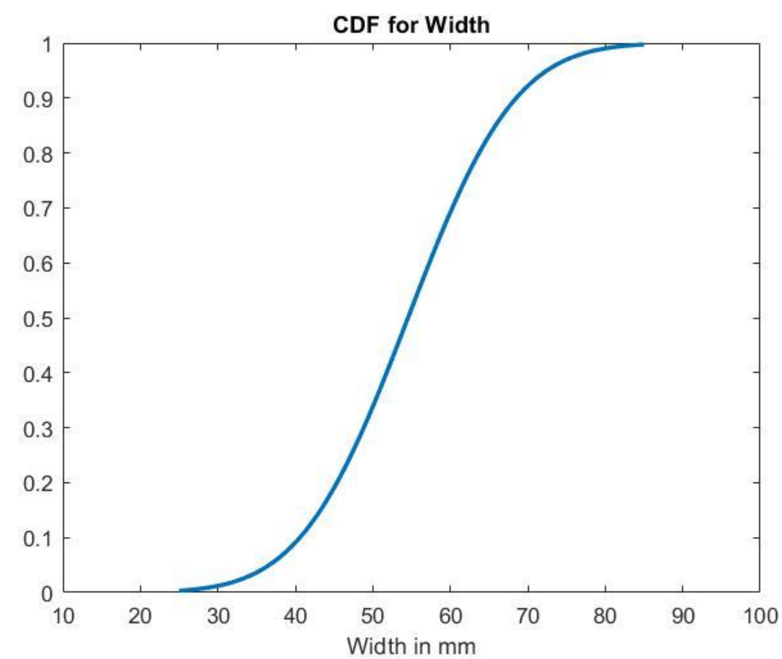

Figure 9 CDF for Width

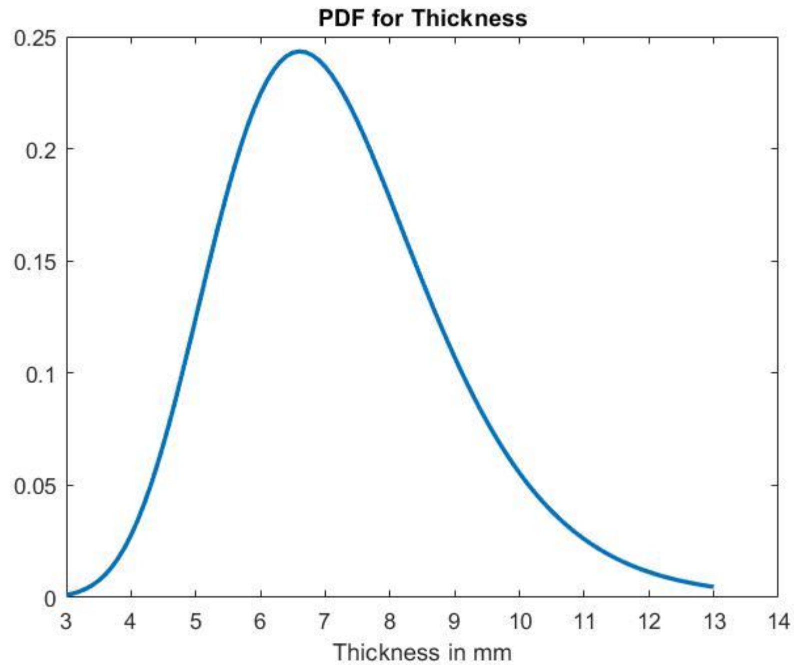

Figure 10 PDF for Thickness

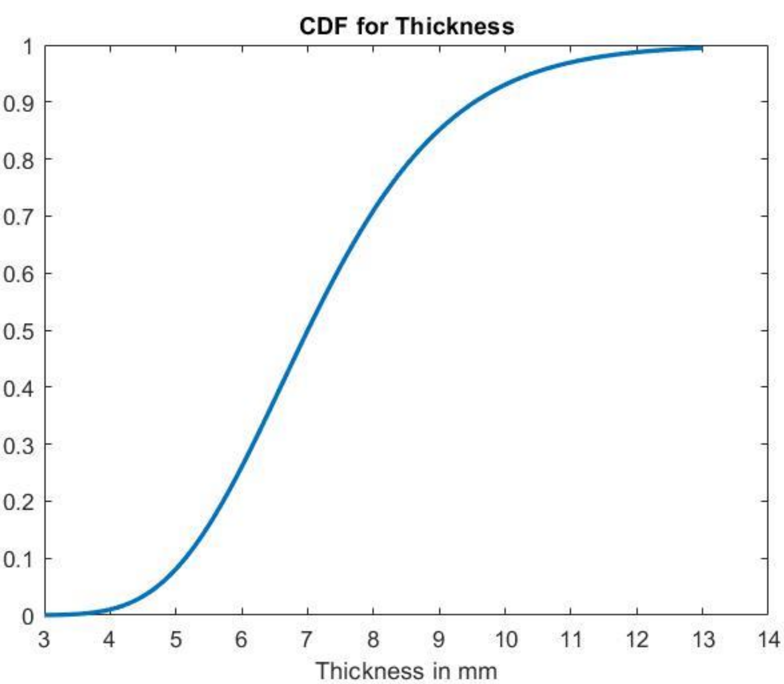

Figure 11 CDF for Thickness

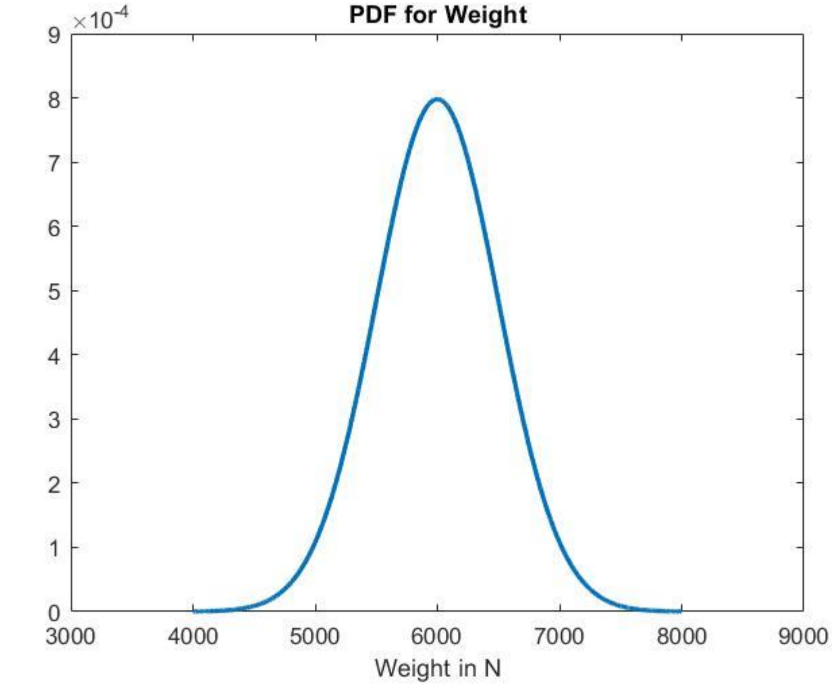

Figure 12 PDF for Weight

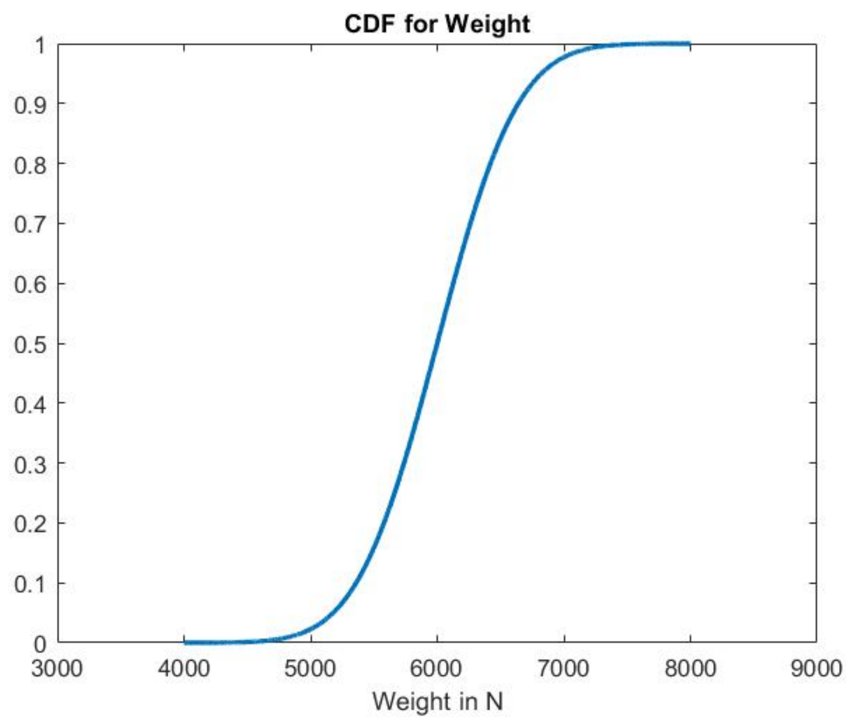

Figure 13 CDF for Weight

To plot the smooth graphs, the interval for the plot is assumed to be very less and the range of plot is also considered to be different to make the complete curves. All the values of ranges are written in the MATLAB code.

All the input parameters with their distributions are summarized in table 6.

Table 6: Input Parameters with their distributions

Length	Normal Distribution
Width	Normal Distribution
Thickness	Lognormal Distribution
Weight	Normal Distribution

4.2 Reliability Analysis

The First-Order Reliability Method (FORM), also referred to as Mean Value First Order Second Moment (MVFOSM), Second-Order Reliability Method (SORM), and Monte Carlo (MC) Simulation provides an analytical solution to predict the probability of failure for an engineering problem.

FORM uses the first-order Taylor series to expand the limit state function at the checking point and linearizes the limit state function. SORM uses second-order Taylor series expansion to expand the limit state function and evaluate it by the parabolic approximation.

The reliability index used in FORM and SORM, which indicates the shortest distance from the origin to the surface of a limit state function, can be used to predict the probability of failure by solving an optimization problem. The major difference between FORM and SORM is that the SORM may provide more accurate results comparing with FORM if the limit state function is nonlinear and more complex to handle. In our study, although the limit state function is non-linear, it is not at all complex to handle. So, we used the FORM. MC simulation is used to justify the validity of the FORM method to evaluate the failure.

4.3 Limit State Function

Depending on the manufacturing process, various types of steel are available. The tensile yield strength of steel varies from 200 MPa to 2100 MPa [16]. We considered the material to be structural steel which has the following properties [8]:

Table 7: Material Properties of Structural Steel

Properties	**Value**	**Unit**
Density	7850	Kg/m^3
Coefficient of thermal expansion	1.2E-05	C^{-1}
Young’s modulus	2E+11	Pa
Poison ratio	0.3	~
Bulk modulus	1.6667E+11	Pa
Shear modulus	7.6923E+10	Pa
Compressive yield strength	2.5E+08	Pa
Tensile yield strength	2.5E+08	Pa

The safety factor is the backbone of all the structures. It is implemented in case a structure experiences a heavier than expected load. The value of safety factor originates from engineers and here, following the criteria for determining the safety factor, we are assuming its value to be equal to 2.

So, according to the yield strength, the limiting value for the stress comes out to be –

$$= \frac{250}{2} = 125 \text{ MPa}$$

The limit state function becomes,

$$g = \sigma - 125 \text{ MPa}$$

$$g = \frac{3WL}{2bt^2n} - 125$$

4.4 First-Order Reliability Method

Based on the statistical parameters mentioned in table 2, the FORM is used to calculate the reliability of the leaf spring. Initially, the FORM is performed using Excel and is shown in table 8 below. Based on the limit state function, the number of iterations performed is 3. The third one is performed just to show that only after iteration 2, we got the desired value of limit state function. More iterations can be performed to get less value of bending stress but is not required here.

Table 8: FORM using Excel

Limit State Function is given as, $g = \frac{3WL}{2bt^2n}$, in MPa

Initial value	Type	u	sigma
L	Normal	1149.2	91.1834232
b	Normal	54.544	10.89660039
t	Lognormal	7.209	1.762498227
W	Normal	6000	500
n	Constant Value	6	
	#1	#2	#3
Lambda t	1.946302731	1.946302731	1.946302731
Kesi t	0.240946113	0.240946113	0.240946113
L	1149.2	1122.375299	1097.95122
b	54.544	62.61511982	67.6627567
t	7.209	10.10318241	14.51543243
W	6000	5845.514583	5704.125794
g	608.1207766	256.6287169	109.8253268
Eqv. Norm Mean L	1149.2	1149.2	1149.2
Eqv. Norm Std.Dev L	91.1834232	91.1834232	91.1834232
Eqv. Norm Mean b	54.544	54.544	54.544
Eqv. Norm Std.Dev b	10.89660039	10.89660039	10.89660039

Eqv. Norm Mean t	6.999740646	6.399883794	3.93499354
Eqv. Norm Std.Dev t	1.73698053	2.434322533	3.497437024
Eqv. Norm Mean W	6000	6000	6000
Eqv. Norm Std.Dev W	500	500	500
L'-correlated	0	-0.29418397	-0.562040534
b'-correlated	0	0.740700726	1.203931156
t'-correlated	0.120473057	1.521285109	3.025197828
W'-correlated	0	-0.308970833	-0.591748412
(dg/dL)*	0.529168793	0.228647866	0.10002751
(dg/db)*	-11.14917822	-4.098510355	-1.623128175
(dg/dt)*	-168.7115485	-50.80156063	-15.13221565
(dg/dW)*	0.101353463	0.043901818	0.019253665
(dg/dL')*	48.25142197	20.84889514	9.120850791
(dg/db')*	-121.4881397	-44.65982953	-17.68657911
(dg/dt')*	-293.0486749	-123.6673837	-52.92397125
(dg/dW')*	50.67673138	21.95090896	9.626832472
DG	324.8587766	134.9245752	57.35527557
DG2	105533.2248	18204.64101	3289.627636
L'-new-correlated	-0.29418397	-0.562040534	-0.837458704
b'-new-correlated	0.740700726	1.203931156	1.623947147
t'-new-correlated	1.786687711	3.333801714	4.859375665
W'-new-correlated	-0.308970833	-0.591748412	-0.883916954
Beta	1.980630638	3.637270934	5.266249373
Beta diff		1.656640295	1.628978439
L-new-correlated	1122.375299	1097.95122	1072.837649
b-new-correlated	62.61511982	67.6627567	72.23950312
t-new-correlated	10.10318241	14.51543243	20.9303539
W-new-correlated	5845.514583	5704.125794	5558.041523
g-new	256.6287169	109.8253268	47.10509757

The probability of failure is calculated using Appendix 1 from the book [14] by using the following formula:

$$p_f = phi\ (-\beta) = 1 - phi(\beta) = 0.00014$$

It is calculated for the of Beta from the iteration 2 only.

The FORM method is also performed using the MATLAB to validate the results from excel. The code is attached in Appendix 2.

4.5 Sensitivity Analysis

If there is a very small change in the input parameters, then the resulting change in the output parameters is given by the sensitivity analysis. It depicts the importance of uncertainty in the model inputs.

Sensitivity index for various input parameters is calculated using the formula below:

$$\alpha_i = \frac{(\frac{\partial g}{\partial X_i'})^*}{\sqrt{\sum_{i=1}^{n}(\frac{\partial g}{\partial X_i'})^{2*}}}$$

$$\text{where, } (\frac{\partial g}{\partial X_i'})^* = (\frac{\partial g}{\partial X_i})^* \sigma_{X_i}^N$$

and X_i are the input (random) parameters

Taking the data from table 8, and performing the calculations, the value for -

Sensitivity index for L = 0.154

Sensitivity index for b = - 0.331

Sensitivity index for t = - 0.917

Sensitivity index for W = 0.163

The negative valued sensitivity index implies an inverse controlling effect of the distribution parameter on the structural failure probability. This can be confirmed from the final values of the parameters mentioned later in the result section. The above values indicate that the uncertainty in the bending stress highly depends on the uncertainty in the thickness and this variable plays a significant role in the reliability analysis.

4.6 Monte Carlo Simulation

Monte Carlo simulation is a sample-based method that determines the probability of failure by knowing statistics of the sample in advance. It is adaptable for most engineering problems including non-linear state function with the advantage of dimensional independence. To obtain the results for the reliability index and the probability of failure, we need to take the number of samples. In our study, 200000 samples are considered. The MATLAB code to perform the MC simulation is attached in Appendix 3.

Based on the different number of samples, the reliability index and the number of samples failed are determined and plotted on the following graphs:

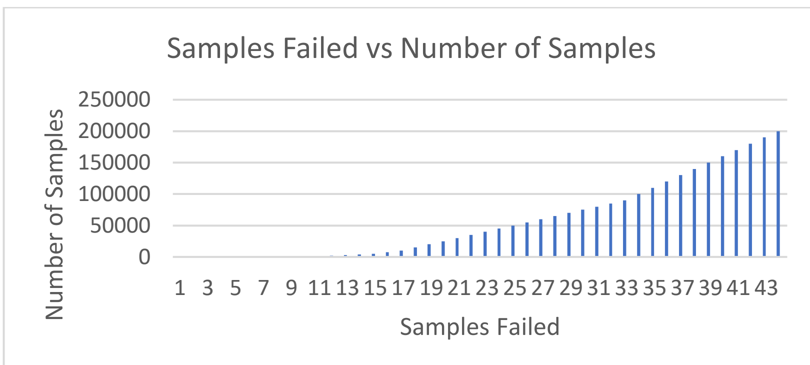

Figure 14 Samples Failed vs Number of Samples

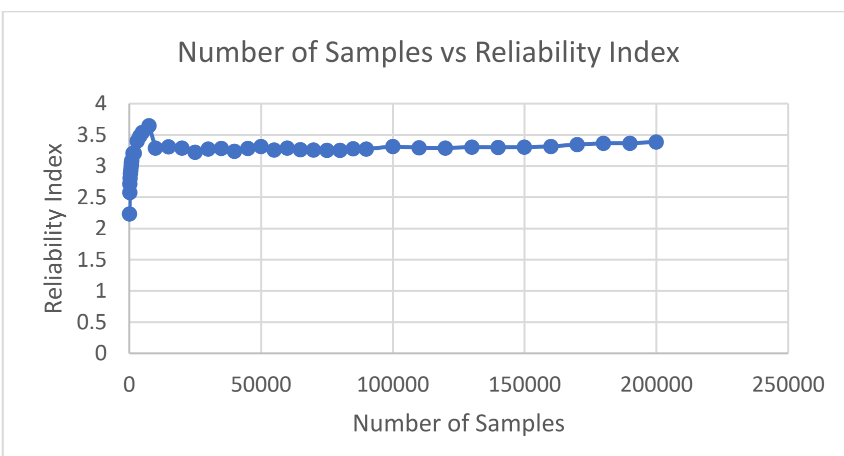

Figure 15 Number of Samples vs Reliability Index

From figure 14, it can be observed that as the number of samples increases, the samples that fail will also increase. And, from the figure 15, it can be concluded that as the number of samples increases, the reliability index becomes constant. In our case, the value of reliability index converges as the sample size become more than 50000.

5. Results

The values for reliability index and the probability of failure for FORM using Excel and MATLAB and for MC simulation are shown in table 9.

Table 9: Reliability Index and Probability of Failure for FORM and Monte Carlo Simulation

Method	First Order Reliability Method (Excel)	First Order Reliability Method (MATLAB)	Monte Carlo Simulation
Iteration	2	2	200000
Reliability Index β	3.637270934	3.63727093354750	3.385688
Probability of Failure P_f	0.00014	0.000137771026529010	0.000355000000

And the final values of input parameters from the MATLAB or excel (FORM method) that satisfy the limit state function are:

Length 1097.95122 mm

Width 67.6627567 mm

Thickness 14.51543243 mm

Weight 5704.125794 N

The value of the limit state function is 109.8253268 MPa.

The value of the probability of failure for FORM using Excel is an approximate value. The difference in the reliability index for both the methods is approximately 6.92 % with respect to the FORM method. From the Monte Carlo simulation, the number of samples that can fail out from 200000 samples is 71.

6. Discussion

- Although our limit state function is non-linear, we performed the analysis using FORM, not the SORM. The results from SORM could have been better.
- The same analysis can be performed using different materials for the component and the best material can be proposed by considering the deflection and stiffness into account.
- The weight acting on the spring is assumed to follow a normal distribution with a considered mean value and standard deviation. Different distribution and the values of these parameters can lead to different results and can give some deviations from the analysis performed.
- More accurate results can be obtained with a greater number of data points.
- Variables other than length, width, and thickness of the leaf also play an important role in its mechanical behavior which here are assumed to be constant.
- Based on the results obtained, the FORM method used here is also justifiable and we didn’t require to perform the complex calculations involved in the SORM.
- The results from the Monte Carlo simulation varies due to the different number of samples taken for the analysis.
- We cannot perform the experimental analysis to verify the results obtained.
- Environmental effects, fatigue effects, and non-linear effects are not considered in the analysis to limit the calculations. The future study could count these effects as factors in the probability analysis.

7. Conclusion

This research work involves the reliability analysis of bending stress in leaf spring by using the First-Order Reliability Method and Monte Carlo Simulation. The variables considered to be random are the length, width, and thickness of the spring, along with the varying load acting on the spring. The probability of failure from both the methods is very close, which justifies that the model and analysis performed are rational and correct. The difference obtained for the probability of failure from both the methods is 6.92 % which is very less and hence indicates a strong coherence between the two analyses. The sensitivity index for thickness is high in magnitude which indicates that it plays a vital role in the reliability analysis of this model.

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9. Justification

- Uncertainty quantification was performed on all the input (random) variables to determine the various parameters associated with them to perform further analysis. It was performed on the data that was collected from various research papers.
- Uncertainty propagation was performed using the chi-square test to determine the distributions for the input variables. For load, the distribution was assumed to be normal. FORM and Monte Carlo simulation techniques were used to perform the uncertainty propagation for the output variable.
- A sensitivity study was performed and the change in output due to the variation in the inputs was calculated and shown in the work.
- The detailed reliability analysis was performed using two methods and determined the probability distribution for the inputs and the required value of output from the results.
- Numerical verification was performed by comparing the reliability solutions from the FORM

and Monte Carlo simulations. Experimental validation would be difficult since it will be hard to prove that out of 200000 specimens, 71 will fail if the safety factor is 2.

- The assumptions in the methodology section and the model advantages, limitations, and potential improvements are mentioned in the discussion section respectively.

10. Team Organization and Management

Following table shows the distribution of work between us to complete this research work:

Section	Varun Agrawal	Sanjay Keshava Murthy	Deepak Mahabashyam
Literature Review	25 %	35 %	40 %
Data Collection	20 %	40 %	40 %
Uncertainty Quantification	35 %	35 %	30 %
Uncertainty Propagation	40 %	35 %	25 %
Mathematical Modeling	30 %	30 %	40 %
FORM	40 %	30 %	30 %
Monte Carlo Simulation	40 %	30 %	30 %
Sensitivity Analysis	60 %	20 %	20 %
Term Paper	50 %	25 %	25 %

APPENDIX 1

MATLAB code for PDF and CDF

```
%% PDF and CDF of the random variables
% t is log-normally distributed, L is normally distributed, b is normally
% distributed, and W is normally distributed
clc; clear; close all; format long;
L = [900:1:1400]; % actual range of the length is [930, 1340]
b = [25:0.1:85]; % actual range of the width is [34, 70]
t = [3:0.1:13]; %actual range of the thickness is [5, 11]
W = [4000:1:8000]; %range of the weight
mu_L = 1149.2; %mean value of length
sigma_L = 91.1834232; %standard deviation of length
mu_b = 54.544; %mean value of width
sigma_b = 10.89660039; %standard deviation of width
lambda_t = 1.946302731; %transformed mean
kesi_t = 0.2409461131; %transformed standard deviation
mu_W = 6000; %mean value of weight
sigma_W = 500; %standard deviation of weight

%PMF and CDF for Length
pd_L = pdf('Normal',L ,mu_L,sigma_L);
figure (1)
plot(L,pd_L,'LineWidth',2)
xlim([800 1500]); xlabel('Length in mm'); title('PDF for Length');
cd_L = cdf('Normal', L, mu_L,sigma_L);
figure (2)
plot(L,cd_L,'LineWidth',2)
xlim([800 1500]); xlabel('Length in mm'); title('CDF for Length');

%PDF and CDF for Width
pd_b = pdf('Normal',b ,mu_b,sigma_b);
figure (3)
plot(b,pd_b,'LineWidth',2)
xlim([10 100]); xlabel('Width in mm'); title('PDF for Width');
cd_b = cdf('Normal', b, mu_b,sigma_b);
figure (4)
plot(b,cd_b,'LineWidth',2)
xlim([10 100]); xlabel('Width in mm'); title('CDF for Width');

%PDF and CDF for Thickness
pd_t = pdf('LogNormal',t ,lambda_t,kesi_t);
figure (5)
plot(t,pd_t,'LineWidth',2)
xlim([3 14]); xlabel('Thickness in mm'); title('PDF for Thickness');
```

```
cd_t = cdf('LogNormal', t, lambda_t,kesi_t);
figure (6)
plot(t,cd_t,'LineWidth',2)
xlim([3 14]); xlabel('Thickness in mm'); title('CDF for Thickness');

%PDF and CDF for Weight
pd_W = pdf('Normal',W ,mu_W,sigma_W);
figure (7)
plot(W,pd_W,'LineWidth',2)
xlim([3000 9000]); ylim([0 9*10^-4]);
xlabel('Weight in N'); title('PDF for Weight');
cd_W = cdf('Normal', W, mu_W,sigma_W);
figure (8)
plot(W,cd_W,'LineWidth',2)
xlim([3000 9000]); xlabel('Weight in N'); title('CDF for Weight');
```

APPENDIX 2

MATLAB code for First-Order Reliability Method

```
%% FORM Method
%%%% Given limit state function is G() = (3*W*L)/(2*b*t^2*n)
% L is normally distributed, b is normally distributed,
% t is log-normally distributed, W is normally distributed
clc; clear; close all; format long;
%Given values
L = [1149.2,91.1834232]; %mean and standard deviation of L respectively
b = [54.544,10.89660039]; %mean and standard deviation of b respectively
t = [7.209,1.762498227]; %mean and standard deviation of t respectively
W = [6000,500]; %mean and standard deviation of W respectively
E = 2.1e5; %Load applied at the center of the spring
n = 6; %total number of leaves
G = (3*W(1)*L(1))/(2*b(1)*t(1)^2*n); %initializing at the mean values
%Transformation from Lognormal to Normal for t
kesi_t = sqrt(log(1+(t(2)/t(1))^2));
lambda_t = log(t(1))-0.5*(kesi_t^2);

count = 1; Betai = 1;
while G>125
   count=count +1
      %transformation to equivalent normal
      mu_Ln = 1149.2;
      sigma_Ln = 91.1834232;
      mu_bn = 54.544;
      sigma_bn = 10.89660039;
      mu_tn = t(1)*(1-log(t(1))+lambda_t);
      sigma_tn = t(1)*kesi_t;
      mu_Wn = 6000;
      sigma_Wn = 500;
      %transformation to reduced space
      L_pr = (L(1)-mu_Ln)/sigma_Ln;
      b_pr = (b(1)-mu_bn)/sigma_bn;
      t_pr = (t(1)-mu_tn)/sigma_tn;
      W_pr = (W(1)-mu_Wn)/sigma_Wn;
      %calculation of derivatives
      dg_L = 3*W(1)/(2*b(1)*t(1)^2*n);
      dg_b = -3*W(1)*L(1)/(2*b(1)^2*t(1)^2*n);
      dg_t = -3*W(1)*L(1)/(b(1)*t(1)^3*n);
      dg_W = 3*L(1)/(2*b(1)*t(1)^2*n);
      dgprMM = [dg_L*sigma_Ln;dg_b*sigma_bn;dg_t*sigma_tn;dg_W*sigma_Wn];
      %Gradient vector
      DG2 = dgprMM(1)^2+dgprMM(2)^2+dgprMM(3)^2+dgprMM(4)^2;
```

```
    %Calculate the new reduced coordinates
    L_prMn = (dgprMM(1)*L_pr + dgprMM(2)*b_pr + dgprMM(3)*t_pr + dgprMM(4)*W_pr
- G)*dgprMM(1)/DG2;
    b_prMn = (dgprMM(1)*L_pr + dgprMM(2)*b_pr + dgprMM(3)*t_pr + dgprMM(4)*W_pr -
G)*dgprMM(2)/DG2;
    t_prMn = (dgprMM(1)*L_pr + dgprMM(2)*b_pr + dgprMM(3)*t_pr + dgprMM(4)*W_pr -
G)*dgprMM(3)/DG2;
    W_prMn = (dgprMM(1)*L_pr + dgprMM(2)*b_pr + dgprMM(3)*t_pr + dgprMM(4)*W_pr
- G)*dgprMM(4)/DG2;
    %Transformation to original space
    L_new = L_prMn*sigma_Ln+mu_Ln;
    b_new = b_prMn*sigma_bn+mu_bn;
    t_new = t_prMn*sigma_tn+mu_tn;
    W_new = W_prMn*sigma_Wn+mu_Wn;
    %New value of limit state function
    G = (3*W_new*L_new)/(2*b_new*t_new^2*n)
    %Calculate Beta
    Beta = sqrt(L_prMn^2+b_prMn^2+t_prMn^2+W_prMn^2);
    Betadif = Beta - Betai;
    Betai = Beta;
    L(1) = L_new;
    b(1) = b_new;
    t(1) = t_new;
    W(1) = W_new;
    pf=1-normcdf(Betai);
end
fprintf('The final value of limit state function is equal to %s \n', G)
fprintf('The Reliability Index is equal to %s \n', Betai)
fprintf('The probability of failure is equal to %s', pf)
```

APPENDIX 3

MATLAB code for Monte Carlo Simulation

```
%% Monte Carlo Simulation
function pf =MonteCarlo
L = [1149.2,91.1834232]; %mean and standard deviation of L respectively
b = [54.544,10.89660039]; %mean and standard deviation of b respectively
t = [7.209,1.762498227]; %mean and standard deviation of t respectively
W = [6000,500]; %mean and standard deviation of W respectively
E = 2.1e5; %Load applied at the center of the spring
n = 6; %total number of leaves
W = (3*W(1)*L(1))/(2*b(1)*t(1)^2*n); %initializing at the mean values
%Transformation from Lognormal to Normal for t
kesi_t = sqrt(log(1+(t(2)/t(1))^2));
lambda_t = log(t(1))-(kesi_t^2)/2;
nsamples = 200000; %number of samples
mu_Ln = 1149.2;
sigma_Ln = 91.1834232;
mu_bn = 54.544;
sigma_bn = 10.89660039;
mu_Wn = 6000;
sigma_Wn = 500;
R=rand(nsamples,1);
j=1;
for i=1:nsamples
    LS(i)=mu_Ln+sigma_Ln*norminv(R(i));
    bS(i)=mu_bn+sigma_bn*norminv(R(i));
    tS(i)=exp(lambda_t+kesi_t*norminv(R(i)));
    WS(i)=mu_Wn+sigma_Wn*norminv(R(i));
    G(i) = (3*WS(i).*LS(i))./(2.*bS(i).*tS(i).^2*n);
    if G(i)<125
        g(j)=G(i);
        j=j+1;
    else
    end
end
length(g)
pf=length(g)/nsamples; %probability of failure
Beta = abs(norminv(pf));
fprintf('The Reliability Index is equal to %s \n', Beta)
fprintf('The probability of failure is equal to %s', pf)
end
```