



**IEE 578: Regression Analysis  
Final Project Report**

**Regression Analysis of World Happiness Score 2018**

**Submitted to Dr. Douglas C. Montgomery  
on December 4, 2019**

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## 1. Introduction

This report is a landmark survey of the state of global happiness for the year 2018 that ranks 156 countries by how happy their citizens perceive themselves to be. It focuses on how happiness has evolved over the past dozen years, with a focus on the technologies, social norms, conflicts, and government policies that have driven those changes. Leading experts describe how measurements of well-being can be used effectively to assess the progress of nations.

For the regression analysis, the response variable is the **Happiness Score**. The regressors (attributes) or the predictor variables are –

- (1) Economy or GDP per capita
- (2) Social Support
- (3) Healthy Life Expectancy
- (4) Freedom
- (5) Generosity
- (6) Corruption

The multilinear regression analysis is performed on the data and to do the model validation, the data set for the year 2019 is used and validated from the prediction equation received from the fitted model of the analysis.

## 2. Description of Data

The data set consists of 156 rows and seven columns in which the happiness score corresponds to the response column and the rest six are the regressor columns. The source of the data is – [\(View data\)](#)

### 2.1 Missing value

There is one value that is missing in the data set that corresponds to the corruption for row 21 (with the happiness score of 6.774). There are various ways to take care for this missing value such as calculating parameters associated with measure of central tendency like mean, median, or mode for that column and replace that missing value with one of them or the deletion of the entire row and analyzing the remaining data or look the other sources to get that data.

In this case, the mean of the remaining 155 values for the corruption was taken and that value was filled in place of the missing value.

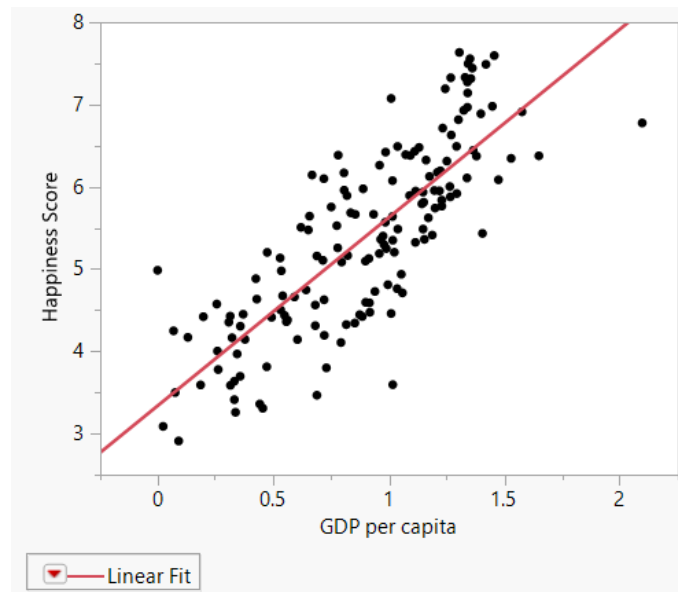
### 3. Analysis

The entire regression analysis was performed in the JMP Pro 14 software. The data set was exported from [Excel](#) to JMP. The following procedure is followed to perform the analysis:

#### 3.1 Response with each regressor

The response (happiness score) was analyzed with each regressor to see their significance in the further regression analysis. Following results are obtained for each attribute –

##### 3.1.1 Happiness Score vs GDP per capita



**Figure 1** Happiness Score vs GDP per capita (Scatter Plot)

**Table 1** Summary Statistics (Score vs GDP)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.802124	0.738006	0.851887	<.0001*
Covariance	0.351938			
Count	156			
Variable	Mean	Std Dev		
GDP per capita	0.891449	0.391921		
Happiness Score	5.375917	1.119506		

Linear Fit

$$\text{Happiness Score} = 3.3333994 + 2.2912336 * \text{GDP per capita}$$

**Table 2** Summary of Fit (Score vs GDP)

Summary of Fit	
RSquare	0.643403
RSquare Adj	0.641087
Root Mean Square Error	0.670688
Mean of Response	5.375917
Observations (or Sum Wgts)	156

Lack of Fit

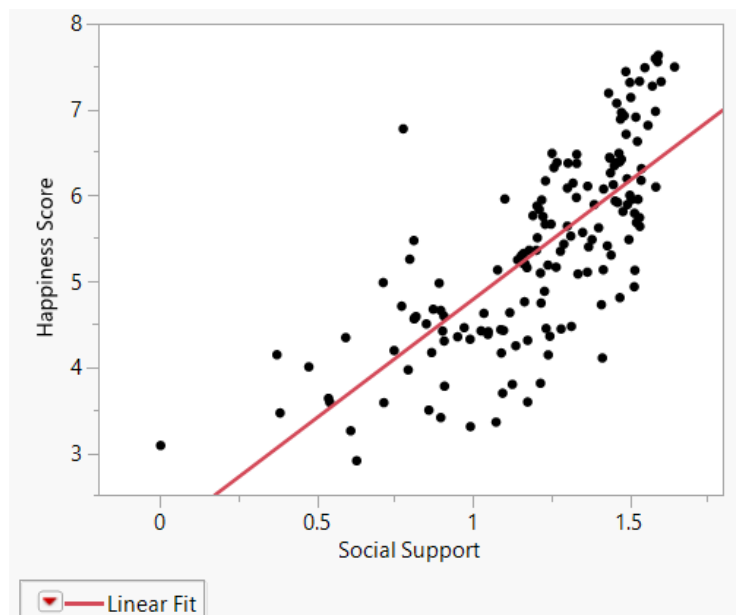
**Table 3** Analysis of Variance (Score vs GDP)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	124.98774	124.988	277.8598
Error	154	69.27273	0.450	<b>Prob &gt; F</b>
C. Total	155	194.26047		<.0001*

**Table 4** Parameter Estimate (Score vs GDP)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.3333994	0.133783	24.92	<.0001*
GDP per capita	2.2912336	0.137454	16.67	<.0001*

### 3.1.2 Happiness Score vs Social Support



**Figure 2** Happiness Score vs Social Support (Scatter Plot)

**Table 5** Summary Statistics (Score vs Social Support)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.74576	0.666756	0.80819	<.0001*
Covariance	0.252445			
Count	156			
Variable	Mean	Std Dev		
Social Support	1.213237	0.302372		
Happiness Score	5.375917	1.119506		

Linear Fit

$$\text{Happiness Score} = 2.0260319 + 2.7611129 * \text{Social Support}$$

**Table 6** Summary of Fit (Score vs Social Support)

Summary of Fit	
RSquare	0.556158
RSquare Adj	0.553276
Root Mean Square Error	0.748249
Mean of Response	5.375917
Observations (or Sum Wgts)	156

Lack of Fit

**Table 7** Analysis of Variance (Score vs Social Support)

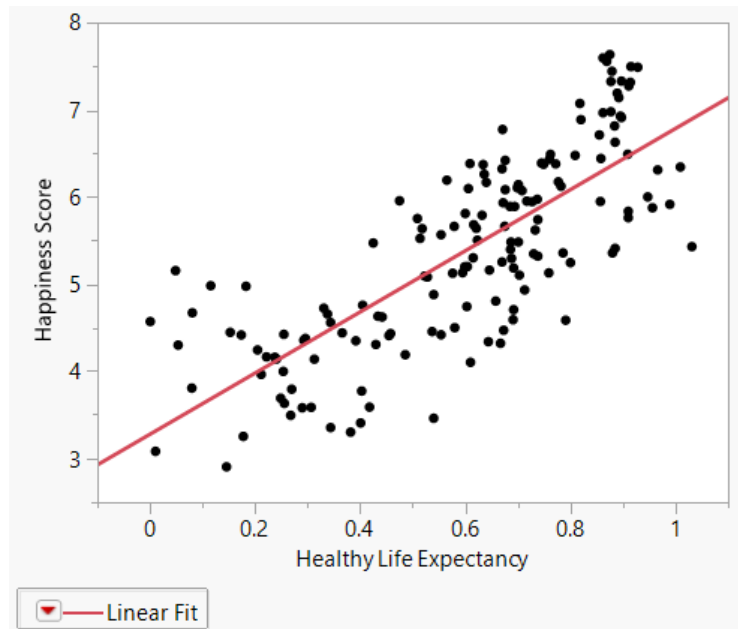
Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	108.03956	108.040	192.9705
Error	154	86.22091	0.560	Prob > F
C. Total	155	194.26047		<.0001*

**Table 8** Parameter Estimates (Score vs Social Support)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	2.0260319	0.248478	8.15	<.0001*
Social Support	2.7611129	0.198764	13.89	<.0001*



### 3.1.3 Happiness Score vs Health Life Expectancy



**Figure 3** Happiness Score vs Health Life Expectancy (Scatter Plot)

**Table 9** Summary Statistics (Score vs Health)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.775814	0.704568	0.831575	<.0001*
Covariance	0.215029			
Count	156			
Variable	Mean	Std Dev		
Healthy Life Expectancy	0.597346	0.247579		
Happiness Score	5.375917	1.119506		

Linear Fit

$$\text{Happiness Score} = 3.2803744 + 3.508087 * \text{Healthy Life Expectancy}$$

**Table 10** Summary of Fit (Score vs Health)

Summary of Fit	
RSquare	0.601887
RSquare Adj	0.599302
Root Mean Square Error	0.708656
Mean of Response	5.375917
Observations (or Sum Wgts)	156

Lack of Fit

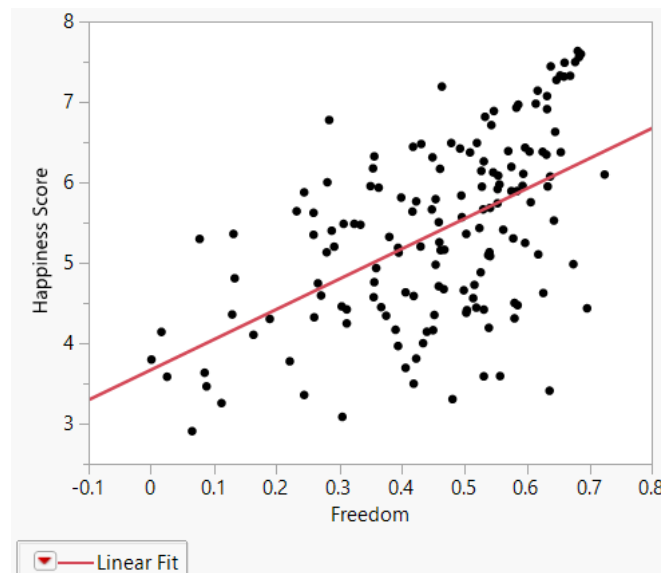
**Table 11** Analysis of Variance (Score vs Health)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	116.92280	116.923	232.8246
Error	154	77.33767	0.502	<b>Prob &gt; F</b>
C. Total	155	194.26047		<.0001*

**Table 12** Parameter Estimates (Score vs Health)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.2803744	0.148594	22.08	<.0001*
Healthy Life Expectancy	3.508087	0.229909	15.26	<.0001*

### 3.1.4 Happiness Score vs Freedom



**Figure 4** Happiness Score vs Freedom (Scatter Plot)

**Table 13** Summary Statistics (Score vs Freedom)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.54428	0.423347	0.646156	<.0001*
Covariance	0.098969			
Count	156			
Variable	Mean	Std Dev		
Freedom	0.454506	0.162424		
Happiness Score	5.375917	1.119506		

## Linear Fit

$$\text{Happiness Score} = 3.6708645 + 3.7514369 \cdot \text{Freedom}$$

**Table 14** Summary of Fit (Score vs Freedom)

Summary of Fit	
RSquare	0.296241
RSquare Adj	0.291671
Root Mean Square Error	0.942202
Mean of Response	5.375917
Observations (or Sum Wgts)	156

## Lack of Fit

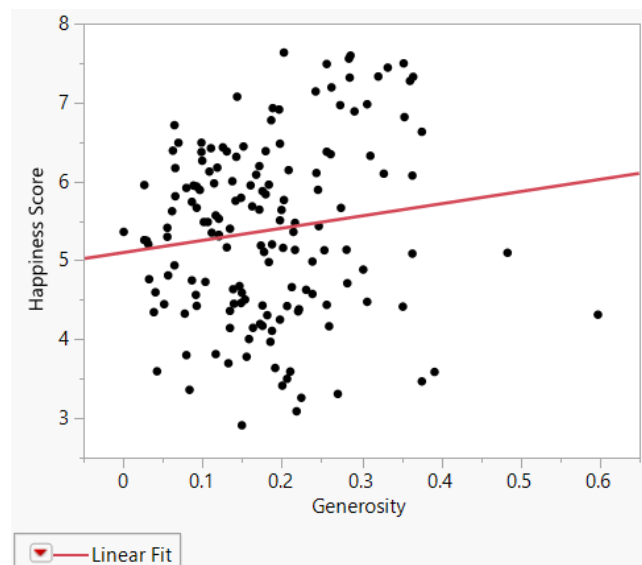
**Table 15** Analysis of Variance (Score vs Freedom)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	57.54785	57.5478	64.8248
Error	154	136.71262	0.8877	<b>Prob &gt; F</b>
C. Total	155	194.26047		<.0001*

**Table 16** Parameter Estimates (Score vs Freedom)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.6708645	0.224806	16.33	<.0001*
Freedom	3.7514369	0.465937	8.05	<.0001*

## 3.1.5 Happiness Score vs Generosity



**Figure 5** Happiness Score vs Generosity (Scatter Plot)

**Table 17** Summary Statistics (Score vs Generosity)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.135825	-0.02178	0.286843	0.0909
Covariance	0.014973			
Count	156			
Variable	Mean	Std Dev		
Generosity	0.181006	0.098471		
Happiness Score	5.375917	1.119506		

Linear Fit

$$\text{Happiness Score} = 5.0964097 + 1.5441828 * \text{Generosity}$$

**Table 18** Summary of Fit (Score vs Generosity)

Summary of Fit	
RSquare	0.018448
RSquare Adj	0.012075
Root Mean Square Error	1.112727
Mean of Response	5.375917
Observations (or Sum Wgts)	156

Lack of Fit

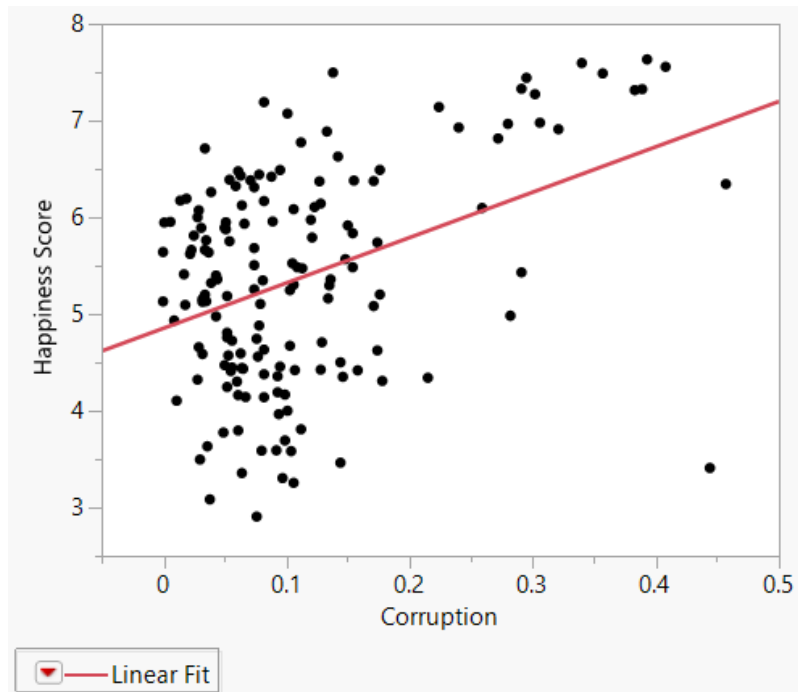
**Table 19** Analysis of Variance (Score vs Generosity)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	3.58379	3.58379	2.8944
Error	154	190.67668	1.23816	<b>Prob &gt; F</b>
C. Total	155	194.26047		0.0909

**Table 20** Parameter Estimates (Score vs Generosity)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	5.0964097	0.18689	27.27	<.0001*
Generosity	1.5441828	0.907645	1.70	0.0909

### 3.1.6 Happiness Score vs Corruption



**Figure 6** Happiness Score vs Corruption (Scatter Plot)

**Table 21** Summary Statistics (Score vs Corruption)

Summary Statistics				
	Value	Lower 95%	Upper 95%	Signif. Prob
Correlation	0.403234	0.262742	0.526983	<.0001*
Covariance	0.043418			
Count	156			
Variable	Mean	Std Dev		
Corruption	0.112	0.09618		
Happiness Score	5.375917	1.119506		

Linear Fit

$$\text{Happiness Score} = 4.8502422 + 4.693522 * \text{Corruption}$$

**Table 22** Summary of Fit (Score vs Corruption)

Summary of Fit	
RSquare	0.162598
RSquare Adj	0.15716
Root Mean Square Error	1.027777
Mean of Response	5.375917
Observations (or Sum Wgts)	156

Lack of Fit

**Table 23** Analysis of Variance (Score vs Corruption)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	31.58632	31.5863	29.9021
Error	154	162.67415	1.0563	<b>Prob &gt; F</b>
C. Total	155	194.26047		<b>&lt;.0001*</b>

**Table 24** Parameter Estimates (Score vs Corruption)

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	4.8502422	0.126541	38.33	<b>&lt;.0001*</b>
Corruption	4.693522	0.858318	5.47	<b>&lt;.0001*</b>

## 3.2 Model Analysis

In the model analysis, the standard least-squares method is used to predict the regression equation. The following procedure is followed:

### 3.2.1 Check for Multicollinearity

To check for multicollinearity, the model is analyzed with all the regressors and the variance inflation factor (VIF) is computed for every regressor. VIF is a measure of how much the standard error of the estimate of the coefficient is inflated due to multicollinearity. Following results are obtained from the JMP:

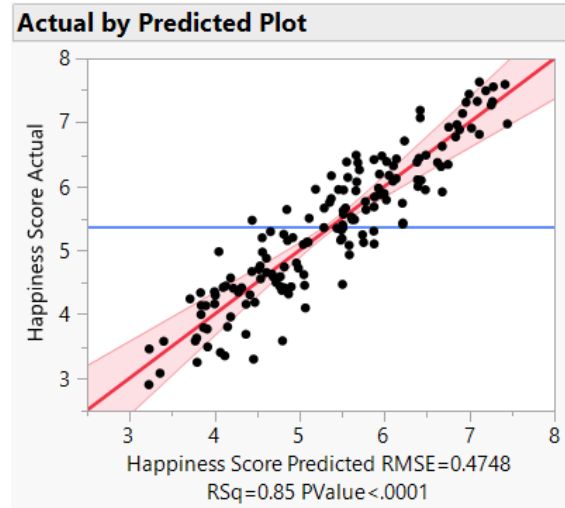
**Table 25** Parameter Estimates (Multicollinearity)

Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	VIF
Intercept	1.8849523	0.194758	9.68	<b>&lt;.0001*</b>	.
GDP per capita	1.0942376	0.210199	5.21	<b>&lt;.0001*</b>	3.8269757
Social Support	1.009819	0.201858	5.00	<b>&lt;.0001*</b>	2.1007494
Healthy Life Expectancy	0.8150497	0.330185	2.47	<b>0.0147*</b>	3.7682481
Freedom	1.3687377	0.319551	4.28	<b>&lt;.0001*</b>	1.5190808
Generosity	0.5798965	0.47271	1.23	0.2219	1.2218032
Corruption	0.6823737	0.529596	1.29	0.1996	1.4630467

The value of VIFs is well under 5 which shows there is no collinearity (or multicollinearity) in the regressors.

### 3.2.2 Full-Model Analysis

For all the data points (156 data sets), the two-level interaction is considered (square terms and individual factor interactions) and the model is analyzed. Following results are obtained:



**Figure 7** Actual by Predicted Plot (Full-Model)

**Table 26** Effect Summary (Full-Model Analysis)

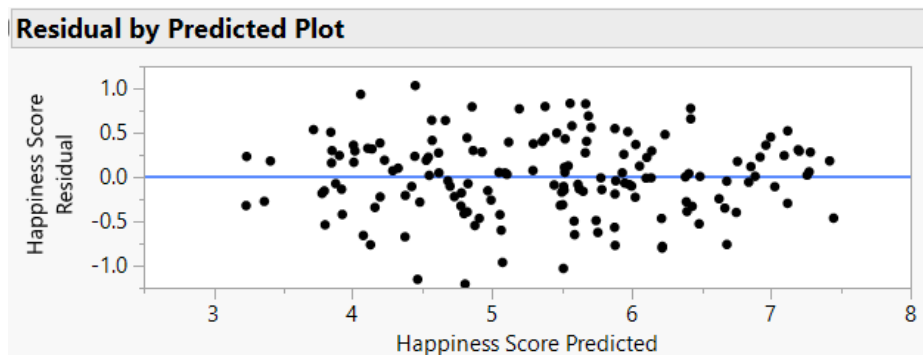
Effect Summary			
Source	LogWorth		PValue
Social Support	6.814		0.00000
GDP per capita	3.568		0.00027
Healthy Life Expectancy*Generosity	2.461		0.00346
GDP per capita*Generosity	2.372		0.00425
Freedom	1.294		0.05085
Social Support*Healthy Life Expectancy	1.162		0.06884
GDP per capita*Freedom	1.128		0.07445
Freedom*Generosity	1.102		0.07912
Generosity*Generosity	0.979		0.10501
Social Support*Freedom	0.967		0.10788
Generosity	0.862		0.13732 ^
Healthy Life Expectancy	0.817		0.15252 ^
Social Support*Generosity	0.753		0.17663
Corruption*Corruption	0.725		0.18841
Social Support*Social Support	0.626		0.23664
Generosity*Corruption	0.411		0.38849
Social Support*Corruption	0.389		0.40795
Freedom*Corruption	0.345		0.45232
GDP per capita*GDP per capita	0.246		0.56695
Healthy Life Expectancy*Freedom	0.180		0.66083
GDP per capita*Healthy Life Expectancy	0.168		0.67856
Corruption	0.091		0.81090 ^
GDP per capita*Social Support	0.084		0.82465
GDP per capita*Corruption	0.052		0.88779
Freedom*Freedom	0.040		0.91279
Healthy Life Expectancy*Corruption	0.034		0.92393
Healthy Life Expectancy*Healthy Life Expectancy	0.001		0.99745

**Table 27** Summary of Fit (Full-Model Analysis)

Summary of Fit	
RSquare	0.851431
RSquare Adj	0.820092
Root Mean Square Error	0.474844
Mean of Response	5.375917
Observations (or Sum Wgts)	156

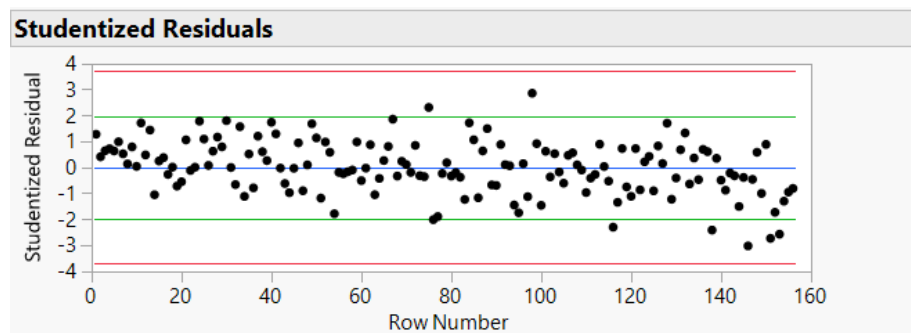
**Table 28** Analysis of Variance (Full-Model Analysis)

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	27	165.39938	6.12590	27.1686
Error	128	28.86109	0.22548	<b>Prob &gt; F</b>
C. Total	155	194.26047		<b>&lt;.0001*</b>



**Figure 8** Residual by Predicted Plot (Full-Model)

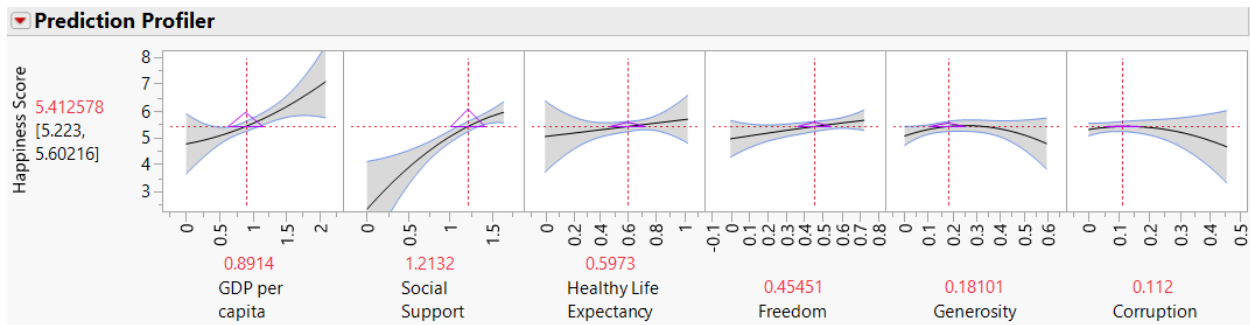
It can be observed from the residual plot that there is no pattern in the plot. All the points are distributed properly about the zero value.



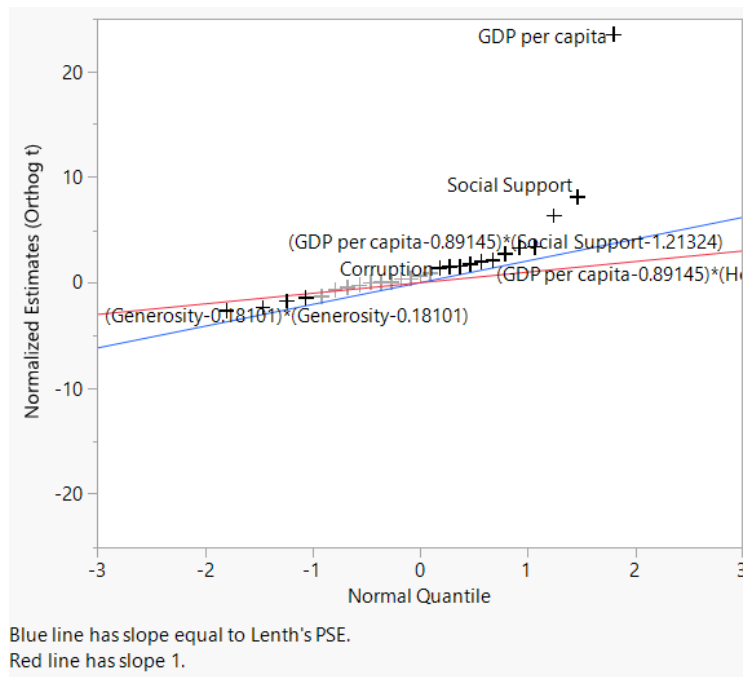
**Figure 9** Studentized Residual Plot (Full-Model)



From figure 9, it can be observed that there is no outlier in the data set.



**Figure 10** Prediction Profiler (Full-Model)



**Figure 11** Normal Plot (Full-Model)

**Table 29** Parameter Estimates (Full-Model Analysis)

<b>Parameter Estimates</b>				
<b>Term</b>	<b>Estimate</b>	<b>Std Error</b>	<b>t Ratio</b>	<b>Prob&gt; t </b>
Intercept	1.6244498	0.317265	5.12	<.0001*
GDP per capita	1.0089269	0.269308	3.75	0.0003*
Social Support	1.5747722	0.283462	5.56	<.0001*
Healthy Life Expectancy	0.6289583	0.437003	1.44	0.1525
Freedom	0.9191695	0.466278	1.97	0.0508
Generosity	0.8898182	0.595106	1.50	0.1373
Corruption	0.2119885	0.884151	0.24	0.8109
(GDP per capita-0.89145)*(Social Support-1.21324)	0.2511561	1.131188	0.22	0.8246
(GDP per capita-0.89145)*(Healthy Life Expectancy-0.59735)	-0.964234	2.321341	-0.42	0.6786
(GDP per capita-0.89145)*(Freedom-0.45451)	-3.85382	2.142769	-1.80	0.0745
(GDP per capita-0.89145)*(Generosity-0.18101)	9.6452751	3.313075	2.91	0.0042*
(GDP per capita-0.89145)*(Corruption-0.112)	-0.610144	4.315657	-0.14	0.8878
(Social Support-1.21324)*(Healthy Life Expectancy-0.59735)	3.0912197	1.684669	1.83	0.0688
(Social Support-1.21324)*(Freedom-0.45451)	2.5703252	1.587464	1.62	0.1079
(Social Support-1.21324)*(Generosity-0.18101)	3.5503653	2.613043	1.36	0.1766
(Social Support-1.21324)*(Corruption-0.112)	2.9260903	3.524376	0.83	0.4079
(Healthy Life Expectancy-0.59735)*(Freedom-0.45451)	1.1894092	2.704495	0.44	0.6608
(Healthy Life Expectancy-0.59735)*(Generosity-0.18101)	-13.61276	4.569412	-2.98	0.0035*
(Healthy Life Expectancy-0.59735)*(Corruption-0.112)	0.6849222	7.159366	0.10	0.9239
(Freedom-0.45451)*(Generosity-0.18101)	-7.338582	4.146282	-1.77	0.0791
(Freedom-0.45451)*(Corruption-0.112)	5.3688327	7.121755	0.75	0.4523
(Generosity-0.18101)*(Corruption-0.112)	5.6736082	6.556717	0.87	0.3885
(GDP per capita-0.89145)*(GDP per capita-0.89145)	0.31575	0.550049	0.57	0.5669
(Social Support-1.21324)*(Social Support-1.21324)	-0.797915	0.671075	-1.19	0.2366
(Healthy Life Expectancy-0.59735)*(Healthy Life Expectancy-0.59735)	0.0064845	2.028107	0.00	0.9975
(Freedom-0.45451)*(Freedom-0.45451)	-0.203873	1.857852	-0.11	0.9128
(Generosity-0.18101)*(Generosity-0.18101)	-5.809434	3.558424	-1.63	0.1050
(Corruption-0.112)*(Corruption-0.112)	-6.977637	5.276729	-1.32	0.1884

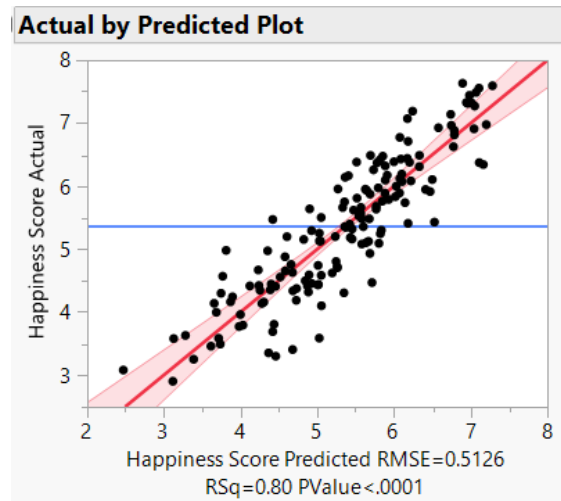
From the effect summary (table 26), none of the square terms is significant in this model and only 8 terms (considering individual regressors and 2 interaction terms with a p-value less than 5 %) are significant. So, the next step is to analyze the reduced model with only significant terms and get the prediction equation which can be later used for the model validation.

### 3.2.3 Reduced-Model Analysis

The terms which are considered in the reduced model analysis are:

GDP per capita, Social Support, Health Life Expectancy, Freedom, Generosity, Corruption, Health Life Expectancy\*Generosity, and GDP per capita\*Generosity.

Following results are obtained for the reduced model –



**Figure 12** Actual by Predicted Plot (Reduced-Model)

**Table 30** Effect Summary (Reduced-Model Analysis)

Effect Summary				
Source	LogWorth			PValue
Social Support	6.484			0.00000
GDP per capita	6.084			0.00000
Freedom	4.532			0.00003
GDP per capita*Generosity	2.392			0.00405
Healthy Life Expectancy*Generosity	1.511			0.03085
Healthy Life Expectancy	1.199			0.06317 ^
Generosity	0.543			0.28637 ^
Corruption	0.452			0.35301

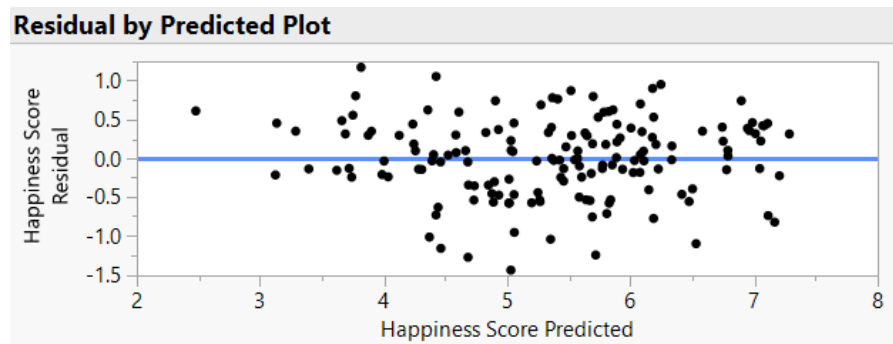
**Table 31** Summary of Fit (Reduced-Model Analysis)

Summary of Fit	
RSquare	0.801174
RSquare Adj	0.790354
Root Mean Square Error	0.51259
Mean of Response	5.375917
Observations (or Sum Wgts)	156

**Table 32** Analysis of Variance (Reduced-Model Analysis)

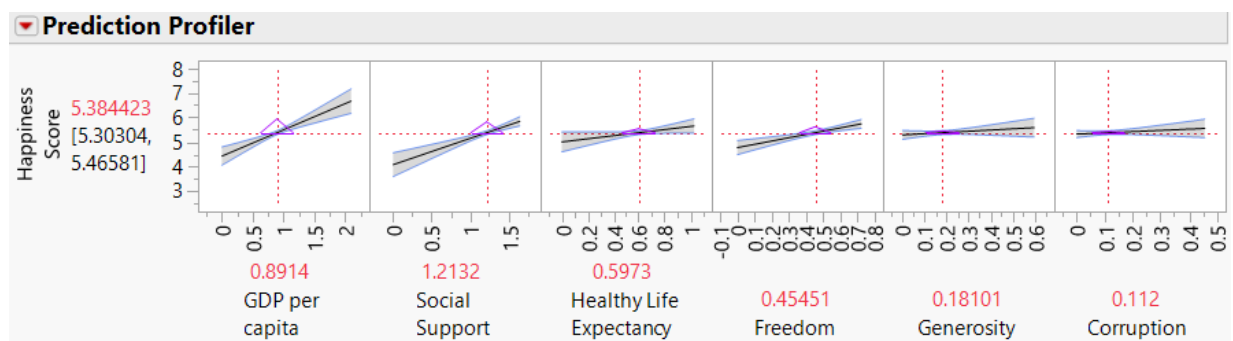
Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	8	155.63651	19.4546	74.0427
Error	147	38.62396	0.2627	<b>Prob &gt; F</b>
C. Total	155	194.26047		<.0001*

From the summary of the fit of the reduced model, it can be observed that there is not much significant difference in the RSquare and RSquare Adj value.

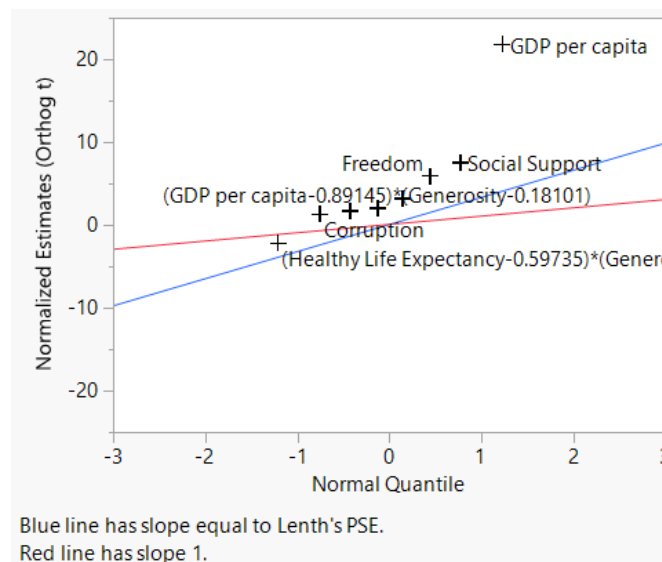


**Figure 13** Residual by Predicted Plot (Reduced-Model)

In the residual plot above, all the points are widely spread and there is no pattern in the data points.



**Figure 14** Prediction Profiler (Reduced-Model)



**Figure 15** Normal Plot (Reduced-Model)

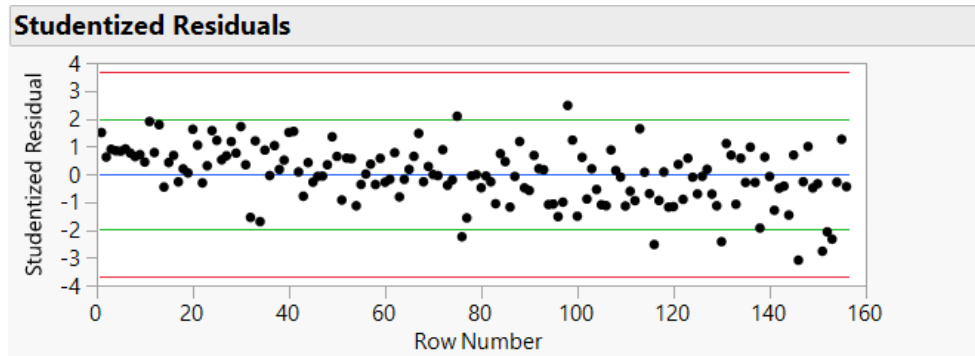
Therefore, from the analysis above, the prediction equation is given as:

*Happiness Score*

$$\begin{aligned}
 &= 1.9701022982 + (1.0780197168 * GDP \text{ per capita}) \\
 &+ (1.083974302 * Social \text{ Support}) \\
 &+ (0.6342282528 * Health \text{ Life Expectancy}) + (1.3479943329 * Freedom) \\
 &+ (0.4963592909 * Generosity) + (0.5084737571 * Corruption) \\
 &+ (GDP \text{ per capita} - 0.8914487179) \\
 &* ((Generosity - 0.1810064103) * 8.6397046272) \\
 &+ (Health \text{ Life Expectancy} - 0.5973461538) \\
 &* ((Generosity - 0.1810064103) * (-9.744276207))
 \end{aligned}$$

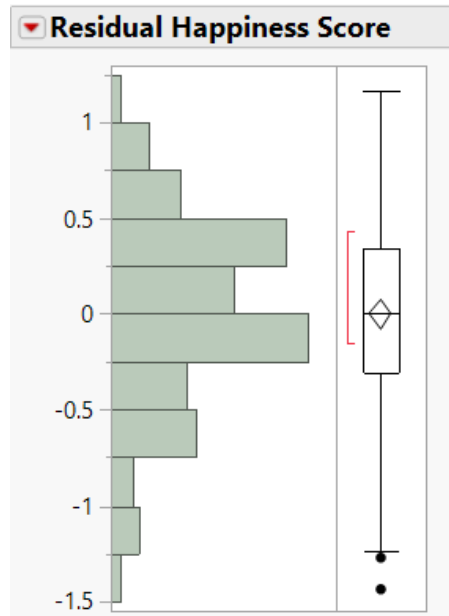
### 3.3 Model Adequacy Checking

To check the model adequacy, two plots are considered. One is the studentized residual plot and the second is the distribution of residuals or error in the response variable.



**Figure 16** Studentized Residuals (Reduced-Model)

From figure 16, it can be observed that there is no outlier in the model and all the values are under the limits.



**Figure 17** Residual Happiness Score (Model Adequacy)

**Table 33** Summary Statistics (Model Adequacy)

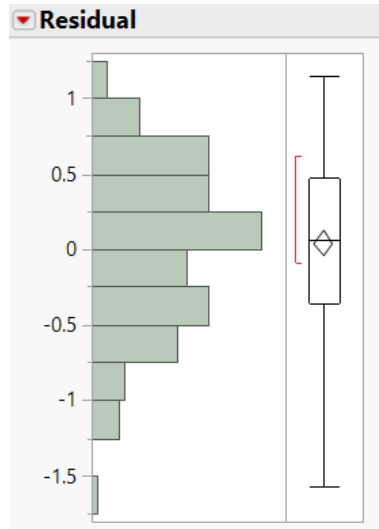
Summary Statistics	
Mean	-1.08e-16
Std Dev	0.4991862
Std Err Mean	0.0399669
Upper 95% Mean	0.0789501
Lower 95% Mean	-0.07895
N	156

In figure 17, the error is normally distributed, and the mean value is very close to zero (Table 32).

Therefore, it can be concluded that the assumptions in regression analysis are satisfied, and the reduced model is adequate.

### 3.4 Model Validation

To validate the model, the 156 data points for the year 2019 ([View data](#)) are taken and the value of happiness score is predicted for all the data points (using excel) from the regression (prediction) equation obtained above. The table is imported in the JMP and the distribution of regression is plotted.



**Figure 18** Residual Plot (Model Validation)

**Table 34** Summary Statistics (Model Validation)

Summary Statistics	
Mean	0.0374544
Std Dev	0.5357657
Std Err Mean	0.0428956
Upper 95% Mean	0.1221898
Lower 95% Mean	-0.047281
N	156

The residual is normally distributed, and the mean value is approximately equal to zero.

Also, from the new prediction data, the average squared prediction error is calculated using the formula:

$$\frac{\sum_{i=1}^{156} (y_i - \hat{y}_i)^2}{156}$$

$$= 0.286608$$

where,  $y_i$  is the actual value and  $\hat{y}_i$  is the predicted value.

The mean square residual from the fitted model (reduced model) (from table 32),

$$MS_{res} = 0.2627$$

The excel sheet for the above calculations can be found [here](#).

The  $MS_{res}$  is smaller but very close to the average squared prediction error. Therefore, the least square model is a successful predictor for the new data as well as it fits the existing data.

## 4. Conclusion

In the regression analysis above, the happiness score for 156 countries is considered as a response variable and the six regressors namely GDO per capita, Social Support, Health Life Expectancy, Generosity, Freedom, and Corruption. The response variable is analyzed individually with all the regressors. Then, after that to check the multicollinearity, the model is analyzed with all the regressors together without any interaction terms and the variance inflation factor is computed. It is found that there is no multicollinearity between regressors as the value of VIFs is under 5. After that, the full model is analyzed with all the regressors using the standard least-squares method. From all the 27 terms, 8 terms were considered to analyze the reduced model and get the prediction equation of the analysis. The model adequacy test is performed and the studentized residual graph and the distribution of residuals are plotted. There is no outlier and the residuals are normally distributed. To validate the data, model validation is performed and do that, the data set (156 points) for the year 2019 is considered and the value of response (happiness score) is calculated. It is concluded that the value of the residual mean square of the fitted model (reduced model) and the averaged squared prediction error from the new prediction data is almost the same which validates the prediction equation.



## 5. References

<https://www.kaggle.com/unsdsn/world-happiness>

Montgomery, D. C., Peck, E. A., & Vining, G. G. (2013). *Introduction to linear regression analysis*. Oxford: Wiley-Blackwell.

Montgomery, D. C. (2020). *Design and analysis of experiments*. Hoboken, NJ: Wiley.

The excel sheets for the data set can be found here:

[https://docs.google.com/spreadsheets/d/1g\\_6nXti0BpWBW8MpE1BnMgYcFJTtNKQxgLxK5IP1V6k/edit?usp=sharing](https://docs.google.com/spreadsheets/d/1g_6nXti0BpWBW8MpE1BnMgYcFJTtNKQxgLxK5IP1V6k/edit?usp=sharing)

[https://docs.google.com/spreadsheets/d/1Ek4\\_CTUSrsRAAkZBR7wkNpI1IrPYdoTTLQFQ-IEbJPY/edit?usp=sharing](https://docs.google.com/spreadsheets/d/1Ek4_CTUSrsRAAkZBR7wkNpI1IrPYdoTTLQFQ-IEbJPY/edit?usp=sharing)

## Data Set for the year 2018

Happiness Score	GDP per capita	Social Support	Healthy Life Expectancy	Freedom	Generosity	Corruption
7.632	1.305	1.592	0.874	0.681	0.202	0.393
7.594	1.456	1.582	0.861	0.686	0.286	0.34
7.555	1.351	1.59	0.868	0.683	0.284	0.408
7.495	1.343	1.644	0.914	0.677	0.353	0.138
7.487	1.42	1.549	0.927	0.66	0.256	0.357
7.441	1.361	1.488	0.878	0.638	0.333	0.295
7.328	1.33	1.532	0.896	0.653	0.321	0.291
7.324	1.268	1.601	0.876	0.669	0.365	0.389
7.314	1.355	1.501	0.913	0.659	0.285	0.383
7.272	1.34	1.573	0.91	0.647	0.361	0.302
7.19	1.244	1.433	0.888	0.464	0.262	0.082
7.139	1.341	1.504	0.891	0.617	0.242	0.224
7.072	1.01	1.459	0.817	0.632	0.143	0.101
6.977	1.448	1.583	0.876	0.614	0.307	0.306
6.965	1.34	1.474	0.861	0.586	0.273	0.28
6.927	1.324	1.483	0.894	0.583	0.188	0.24
6.91	1.576	1.52	0.896	0.632	0.196	0.321
6.886	1.398	1.471	0.819	0.547	0.291	0.133
6.814	1.301	1.559	0.883	0.533	0.354	0.272
6.774	2.096	0.776	0.67	0.284	0.186	0.112
6.711	1.233	1.489	0.854	0.543	0.064	0.034
6.627	1.27	1.525	0.884	0.645	0.376	0.142
6.489	1.293	1.466	0.908	0.52	0.098	0.176
6.488	1.038	1.252	0.761	0.479	0.069	0.095
6.476	1.131	1.331	0.808	0.431	0.197	0.061
6.441	1.365	1.436	0.857	0.418	0.151	0.078
6.43	1.112	1.438	0.759	0.597	0.125	0.063
6.419	0.986	1.474	0.675	0.493	0.11	0.088
6.388	1.073	1.468	0.744	0.57	0.062	0.054
6.382	0.781	1.268	0.608	0.604	0.179	0.071
6.379	1.093	1.459	0.771	0.625	0.13	0.155
6.374	1.649	1.303	0.748	0.654	0.256	0.171
6.371	1.379	1.331	0.633	0.509	0.098	0.127
6.343	1.529	1.451	1.008	0.631	0.261	0.457
6.322	1.161	1.258	0.669	0.356	0.311	0.059
6.31	1.251	1.538	0.965	0.449	0.142	0.074
6.26	0.96	1.439	0.635	0.531	0.099	0.039
6.192	1.223	1.492	0.564	0.575	0.171	0.019
6.173	1.21	1.537	0.776	0.354	0.118	0.014

6.167	0.806	1.231	0.639	0.461	0.065	0.082
6.141	0.668	1.319	0.7	0.527	0.208	0.128
6.123	1.176	1.448	0.781	0.546	0.108	0.064
6.105	1.338	1.366	0.698	0.594	0.243	0.123
6.096	0.719	1.584	0.605	0.724	0.328	0.259
6.083	1.474	1.301	0.675	0.554	0.167	0.106
6.072	1.016	1.417	0.707	0.637	0.364	0.029
6	1.264	1.501	0.946	0.281	0.137	0.028
5.973	0.889	1.33	0.736	0.556	0.114	0.12
5.956	0.807	1.101	0.474	0.593	0.183	0.089
5.952	1.197	1.527	0.716	0.35	0.026	0.006
5.948	1.219	1.506	0.856	0.633	0.16	0.051
5.945	1.116	1.219	0.726	0.528	0.088	0.001
5.933	1.148	1.454	0.671	0.363	0.092	0.066
5.915	1.294	1.462	0.988	0.553	0.079	0.15
5.891	1.09	1.387	0.684	0.584	0.245	0.05
5.89	0.819	1.493	0.693	0.575	0.096	0.031
5.875	1.266	1.204	0.955	0.244	0.175	0.051
5.835	1.229	1.211	0.909	0.495	0.179	0.154
5.81	1.151	1.479	0.599	0.399	0.065	0.025
5.79	1.143	1.516	0.631	0.454	0.148	0.121
5.762	1.229	1.191	0.909	0.423	0.202	0.035
5.752	0.751	1.223	0.508	0.606	0.141	0.054
5.739	1.2	1.532	0.737	0.553	0.086	0.174
5.681	0.835	1.522	0.615	0.541	0.162	0.074
5.663	0.934	1.249	0.674	0.53	0.092	0.034
5.662	0.855	1.23	0.578	0.448	0.274	0.023
5.64	0.657	1.301	0.62	0.232	0.171	0
5.636	1.016	1.533	0.517	0.417	0.199	0.037
5.62	1.171	1.401	0.732	0.259	0.061	0.022
5.566	0.985	1.35	0.553	0.496	0.116	0.148
5.524	0.775	1.312	0.513	0.643	0.12	0.105
5.504	0.62	1.205	0.622	0.459	0.197	0.074
5.483	1.039	1.498	0.7	0.307	0.101	0.154
5.483	1.148	1.38	0.686	0.324	0.106	0.109
5.472	0.652	0.81	0.424	0.334	0.216	0.113
5.43	1.405	1.29	1.03	0.524	0.246	0.291
5.41	1.188	1.429	0.884	0.562	0.055	0.017
5.398	0.975	1.369	0.685	0.288	0.134	0.043
5.358	1.154	1.202	0.879	0.131	0	0.044
5.358	0.965	1.179	0.785	0.503	0.214	0.136
5.347	1.017	1.279	0.729	0.259	0.111	0.081
5.321	1.115	1.161	0.737	0.38	0.12	0.039

5.302	0.982	1.441	0.614	0.578	0.12	0.106
5.295	0.979	1.154	0.687	0.077	0.055	0.135
5.254	0.779	0.797	0.669	0.46	0.026	0.074
5.246	0.989	1.142	0.799	0.597	0.029	0.103
5.201	1.024	1.161	0.603	0.43	0.031	0.176
5.199	0.474	1.166	0.598	0.292	0.187	0.034
5.185	0.959	1.239	0.691	0.394	0.173	0.052
5.161	0.822	1.265	0.645	0.468	0.13	0.134
5.155	0.689	1.172	0.048	0.462	0.201	0.032
5.131	0.53	1.416	0.594	0.54	0.281	0.035
5.129	0.915	1.078	0.758	0.28	0.216	0
5.125	0.914	1.517	0.575	0.395	0.253	0.032
5.103	0.715	1.365	0.702	0.618	0.177	0.079
5.093	0.899	1.215	0.522	0.538	0.484	0.018
5.082	0.796	1.335	0.527	0.541	0.364	0.171
4.982	0	0.712	0.115	0.674	0.238	0.282
4.975	0.535	0.891	0.182	0.454	0.183	0.043
4.933	1.054	1.515	0.712	0.359	0.064	0.009
4.88	0.425	1.228	0.539	0.526	0.302	0.078
4.806	0.996	1.469	0.657	0.133	0.056	0.052
4.758	1.036	1.164	0.404	0.356	0.032	0.052
4.743	0.642	1.217	0.602	0.266	0.086	0.076
4.724	0.94	1.41	0.33	0.516	0.103	0.056
4.707	1.059	0.771	0.691	0.459	0.282	0.129
4.671	0.541	0.872	0.08	0.467	0.146	0.103
4.657	0.592	0.896	0.337	0.499	0.212	0.029
4.631	0.429	1.117	0.433	0.406	0.138	0.082
4.623	0.72	1.034	0.441	0.626	0.23	0.174
4.592	0.9	0.906	0.69	0.271	0.04	0.063
4.586	0.916	0.817	0.79	0.419	0.149	0.032
4.571	0.256	0.813	0	0.355	0.238	0.053
4.559	0.682	0.811	0.343	0.514	0.091	0.077
4.5	0.532	0.85	0.579	0.58	0.153	0.144
4.471	0.918	1.314	0.672	0.585	0.307	0.05
4.456	1.01	0.971	0.536	0.304	0.148	0.095
4.447	0.37	1.233	0.152	0.367	0.139	0.056
4.441	0.874	1.281	0.365	0.519	0.051	0.064
4.433	0.549	1.088	0.457	0.696	0.256	0.065
4.424	0.314	1.097	0.254	0.312	0.175	0.128
4.419	0.885	1.025	0.553	0.312	0.092	0.107
4.417	0.198	0.902	0.173	0.531	0.206	0.158
4.41	0.493	1.048	0.454	0.504	0.352	0.055
4.377	0.562	1.047	0.295	0.503	0.221	0.082

4.356	0.557	1.245	0.292	0.129	0.134	0.093
4.35	0.308	0.95	0.391	0.452	0.22	0.146
4.34	0.853	0.592	0.643	0.375	0.038	0.215
4.321	0.816	0.99	0.666	0.26	0.077	0.028
4.308	0.682	1.174	0.429	0.58	0.598	0.178
4.301	0.358	0.907	0.053	0.189	0.181	0.06
4.245	0.069	1.136	0.204	0.312	0.197	0.052
4.19	0.721	0.747	0.485	0.539	0.172	0.093
4.166	0.131	0.867	0.221	0.39	0.175	0.099
4.161	0.322	1.09	0.237	0.45	0.259	0.061
4.141	0.378	0.372	0.24	0.44	0.163	0.067
4.139	0.605	1.24	0.312	0.016	0.134	0.082
4.103	0.793	1.413	0.609	0.163	0.187	0.011
3.999	0.259	0.474	0.253	0.434	0.158	0.101
3.964	0.344	0.792	0.211	0.394	0.185	0.094
3.808	0.472	1.215	0.079	0.423	0.116	0.112
3.795	0.73	1.125	0.269	0	0.079	0.061
3.774	0.262	0.908	0.402	0.221	0.155	0.049
3.692	0.357	1.094	0.248	0.406	0.132	0.099
3.632	0.332	0.537	0.255	0.085	0.191	0.036
3.59	1.017	1.174	0.417	0.557	0.042	0.092
3.587	0.186	0.541	0.306	0.531	0.21	0.08
3.582	0.315	0.714	0.289	0.025	0.392	0.104
3.495	0.076	0.858	0.267	0.419	0.206	0.03
3.462	0.689	0.382	0.539	0.088	0.376	0.144
3.408	0.332	0.896	0.4	0.636	0.2	0.444
3.355	0.442	1.073	0.343	0.244	0.083	0.064
3.303	0.455	0.991	0.381	0.481	0.27	0.097
3.254	0.337	0.608	0.177	0.112	0.224	0.106
3.083	0.024	0	0.01	0.305	0.218	0.038
2.905	0.091	0.627	0.145	0.065	0.149	0.076

### Data Set for the year 2019 (Validation Data)

Happiness Score	GDP per capita	Social Support	Healthy Life Expectancy	Freedom	Generosity	Corruption
7.769	1.34	1.587	0.986	0.596	0.153	0.393
7.6	1.383	1.573	0.996	0.592	0.252	0.41
7.554	1.488	1.582	1.028	0.603	0.271	0.341
7.494	1.38	1.624	1.026	0.591	0.354	0.118
7.488	1.396	1.522	0.999	0.557	0.322	0.298
7.48	1.452	1.526	1.052	0.572	0.263	0.343
7.343	1.387	1.487	1.009	0.574	0.267	0.373
7.307	1.303	1.557	1.026	0.585	0.33	0.38
7.278	1.365	1.505	1.039	0.584	0.285	0.308
7.246	1.376	1.475	1.016	0.532	0.244	0.226
7.228	1.372	1.548	1.036	0.557	0.332	0.29
7.167	1.034	1.441	0.963	0.558	0.144	0.093
7.139	1.276	1.455	1.029	0.371	0.261	0.082
7.09	1.609	1.479	1.012	0.526	0.194	0.316
7.054	1.333	1.538	0.996	0.45	0.348	0.278
7.021	1.499	1.553	0.999	0.516	0.298	0.31
6.985	1.373	1.454	0.987	0.495	0.261	0.265
6.923	1.356	1.504	0.986	0.473	0.16	0.21
6.892	1.433	1.457	0.874	0.454	0.28	0.128
6.852	1.269	1.487	0.92	0.457	0.046	0.036
6.825	1.503	1.31	0.825	0.598	0.262	0.182
6.726	1.3	1.52	0.999	0.564	0.375	0.151
6.595	1.07	1.323	0.861	0.433	0.074	0.073
6.592	1.324	1.472	1.045	0.436	0.111	0.183
6.446	1.368	1.43	0.914	0.351	0.242	0.097
6.444	1.159	1.369	0.92	0.357	0.187	0.056
6.436	0.8	1.269	0.746	0.535	0.175	0.078
6.375	1.403	1.357	0.795	0.439	0.08	0.132
6.374	1.684	1.313	0.871	0.555	0.22	0.167
6.354	1.286	1.484	1.062	0.362	0.153	0.079
6.321	1.149	1.442	0.91	0.516	0.109	0.054
6.3	1.004	1.439	0.802	0.39	0.099	0.086
6.293	1.124	1.465	0.891	0.523	0.127	0.15
6.262	1.572	1.463	1.141	0.556	0.271	0.453
6.253	0.794	1.242	0.789	0.43	0.093	0.074
6.223	1.294	1.488	1.039	0.231	0.158	0.03
6.199	1.362	1.368	0.871	0.536	0.255	0.11
6.198	1.246	1.504	0.881	0.334	0.121	0.014
6.192	1.231	1.477	0.713	0.489	0.185	0.016

6.182	1.206	1.438	0.884	0.483	0.117	0.05
6.174	0.745	1.529	0.756	0.631	0.322	0.24
6.149	1.238	1.515	0.818	0.291	0.043	0.042
6.125	0.985	1.41	0.841	0.47	0.099	0.034
6.118	1.258	1.523	0.953	0.564	0.144	0.057
6.105	0.694	1.325	0.835	0.435	0.2	0.127
6.1	0.882	1.232	0.758	0.489	0.262	0.006
6.086	1.092	1.432	0.881	0.471	0.066	0.05
6.07	1.162	1.232	0.825	0.462	0.083	0.005
6.046	1.263	1.223	1.042	0.406	0.19	0.041
6.028	0.912	1.312	0.868	0.498	0.126	0.087
6.021	1.5	1.319	0.808	0.493	0.142	0.097
6.008	1.05	1.409	0.828	0.557	0.359	0.028
5.94	1.187	1.465	0.812	0.264	0.075	0.064
5.895	1.301	1.219	1.036	0.159	0.175	0.056
5.893	1.237	1.528	0.874	0.495	0.103	0.161
5.89	0.831	1.478	0.831	0.49	0.107	0.028
5.888	1.12	1.402	0.798	0.498	0.215	0.06
5.886	1.327	1.419	1.088	0.445	0.069	0.14
5.86	0.642	1.236	0.828	0.507	0.246	0.078
5.809	1.173	1.508	0.729	0.41	0.146	0.096
5.779	0.776	1.209	0.706	0.511	0.137	0.064
5.758	1.201	1.41	0.828	0.199	0.081	0.02
5.743	0.855	1.475	0.777	0.514	0.184	0.08
5.718	1.263	1.252	1.042	0.417	0.191	0.162
5.697	0.96	1.274	0.854	0.455	0.083	0.027
5.693	1.221	1.431	0.999	0.508	0.047	0.025
5.653	0.677	0.886	0.535	0.313	0.22	0.098
5.648	1.183	1.452	0.726	0.334	0.082	0.031
5.631	0.807	1.293	0.657	0.558	0.117	0.107
5.603	1.004	1.383	0.854	0.282	0.137	0.039
5.529	0.685	1.328	0.739	0.245	0.181	0
5.525	1.044	1.303	0.673	0.416	0.133	0.152
5.523	1.051	1.361	0.871	0.197	0.142	0.08
5.467	0.493	1.098	0.718	0.389	0.23	0.144
5.432	1.155	1.266	0.914	0.296	0.119	0.022
5.43	1.438	1.277	1.122	0.44	0.258	0.287
5.425	1.015	1.401	0.779	0.497	0.113	0.101
5.386	0.945	1.212	0.845	0.212	0.263	0.006
5.373	1.183	1.36	0.808	0.195	0.083	0.106
5.339	1.221	1.171	0.828	0.508	0.26	0.024
5.323	1.067	1.465	0.789	0.235	0.094	0.142
5.287	1.181	1.156	0.999	0.067	0	0.034

5.285	0.948	1.531	0.667	0.317	0.235	0.038
5.274	0.983	1.294	0.838	0.345	0.185	0.034
5.265	0.696	1.111	0.245	0.426	0.215	0.041
5.261	0.551	1.438	0.723	0.508	0.3	0.023
5.247	1.052	1.538	0.657	0.394	0.244	0.028
5.211	1.002	1.16	0.785	0.086	0.073	0.114
5.208	0.801	0.782	0.782	0.418	0.036	0.076
5.208	1.043	1.147	0.769	0.351	0.035	0.182
5.197	0.987	1.224	0.815	0.216	0.166	0.027
5.192	0.931	1.203	0.66	0.491	0.498	0.028
5.191	1.029	1.125	0.893	0.521	0.058	0.1
5.175	0.741	1.346	0.851	0.543	0.147	0.073
5.082	0.813	1.321	0.604	0.457	0.37	0.167
5.044	0.549	0.91	0.331	0.381	0.187	0.037
5.011	1.092	1.513	0.815	0.311	0.081	0.004
4.996	0.611	0.868	0.486	0.381	0.245	0.04
4.944	0.569	0.808	0.232	0.352	0.154	0.09
4.913	0.446	1.226	0.677	0.439	0.285	0.089
4.906	0.837	1.225	0.815	0.383	0.11	0.13
4.883	0.393	0.437	0.397	0.349	0.175	0.082
4.812	0.673	0.799	0.508	0.372	0.105	0.093
4.799	1.057	1.183	0.571	0.295	0.043	0.055
4.796	0.764	1.03	0.551	0.547	0.266	0.164
4.722	0.96	1.351	0.469	0.389	0.13	0.055
4.719	0.947	0.848	0.874	0.383	0.178	0.027
4.707	0.96	1.427	0.805	0.154	0.064	0.047
4.7	0.574	1.122	0.637	0.609	0.232	0.062
4.696	0.657	1.247	0.672	0.225	0.103	0.066
4.681	0.45	1.134	0.571	0.292	0.153	0.072
4.668	0	0.698	0.268	0.559	0.243	0.27
4.639	0.879	1.313	0.477	0.401	0.07	0.056
4.628	0.138	0.774	0.366	0.318	0.188	0.102
4.587	0.331	1.056	0.38	0.255	0.177	0.113
4.559	0.85	1.055	0.815	0.283	0.095	0.064
4.548	1.1	0.842	0.785	0.305	0.27	0.125
4.534	0.38	0.829	0.375	0.332	0.207	0.086
4.519	0.886	0.666	0.752	0.346	0.043	0.164
4.516	0.308	0.939	0.428	0.382	0.269	0.167
4.509	0.512	0.983	0.581	0.431	0.372	0.053
4.49	0.57	1.167	0.489	0.066	0.106	0.088
4.466	0.204	0.986	0.39	0.494	0.197	0.138
4.461	0.921	1	0.815	0.167	0.059	0.055
4.456	0.562	0.928	0.723	0.527	0.166	0.143



4.437	1.043	0.98	0.574	0.241	0.148	0.089
4.418	0.094	1.125	0.357	0.269	0.212	0.053
4.39	0.385	1.105	0.308	0.327	0.153	0.052
4.374	0.268	0.841	0.242	0.309	0.252	0.045
4.366	0.949	1.265	0.831	0.47	0.244	0.047
4.36	0.71	1.181	0.555	0.525	0.566	0.172
4.35	0.35	0.766	0.192	0.174	0.198	0.078
4.332	0.82	1.39	0.739	0.178	0.187	0.01
4.286	0.336	1.033	0.532	0.344	0.209	0.1
4.212	0.811	1.149	0	0.313	0.074	0.135
4.189	0.332	1.069	0.443	0.356	0.252	0.06
4.166	0.913	1.039	0.644	0.241	0.076	0.067
4.107	0.578	1.058	0.426	0.431	0.247	0.087
4.085	0.275	0.572	0.41	0.293	0.177	0.085
4.015	0.755	0.765	0.588	0.498	0.2	0.085
3.975	0.073	0.922	0.443	0.37	0.233	0.033
3.973	0.274	0.757	0.505	0.142	0.275	0.078
3.933	0.274	0.916	0.555	0.148	0.169	0.041
3.802	0.489	1.169	0.168	0.359	0.107	0.093
3.775	0.046	0.447	0.38	0.22	0.176	0.18
3.663	0.366	1.114	0.433	0.361	0.151	0.089
3.597	0.323	0.688	0.449	0.026	0.419	0.11
3.488	1.041	1.145	0.538	0.455	0.025	0.1
3.462	0.619	0.378	0.44	0.013	0.331	0.141
3.41	0.191	0.56	0.495	0.443	0.218	0.089
3.38	0.287	1.163	0.463	0.143	0.108	0.077
3.334	0.359	0.711	0.614	0.555	0.217	0.411
3.231	0.476	0.885	0.499	0.417	0.276	0.147
3.203	0.35	0.517	0.361	0	0.158	0.025
3.083	0.026	0	0.105	0.225	0.235	0.035
2.853	0.306	0.575	0.295	0.01	0.202	0.091