

## Learning Objectives

- Students will identify and classify at least three different types of relations and functions by the end of the 2-hour session. (Understand)
- Students will evaluate determinants of  $2 \times 2$  and  $3 \times 3$  square matrices using their properties with 90% accuracy within 2 hours. (Apply)
- Students will explain the relationship between differentiability and continuity of a function, providing examples within 2 hours. (Analyze)

## Introduction: Engage with Real-life Scenarios about Functions

15 Minutes

### Implementation Script:

- Begin by greeting students and introducing the concept of functions through everyday examples to establish relevance. Start with simple relatable questions like "How do you decide the amount to pay at a store based on items bought?" or "Can you think about how temperature changes during a day?" Use these scenarios to explain that these situations involve relationships between two quantities, which can be understood as functions. Display several simple graphs or tables representing these scenarios. Encourage students to share any other examples from their daily life where an input corresponds to an output. Use this interactive exchange to lead them toward understanding the idea of relations and functions, emphasizing how functions model daily-life phenomena (aligning with competency CG-8). Use accessible language such as, "Imagine if every hour of the day you check the temperature — the time is like the input, and the temperature is the output." Pace this activity actively by circulating around the room, listening to student input, and prompting deeper thinking by asking open-ended questions like, "What do you notice about how inputs relate to outputs in your examples?" Formative Questions: 1. Can you give an example from your daily life where one thing depends on another? 2. What do you think makes a relationship between two quantities a 'function'? Expected Responses: 1. Examples such as how the number of hours studied affects marks, how speed affects travel time, etc. 2. A function is a relation where every input has exactly one output. Common misconceptions may include thinking a function can assign multiple outputs to one input; clarify that does not qualify as a function. Teacher Notes: Emphasize clear communication with relatable examples. Use questioning to scaffold understanding and keep students engaged by relating content to their experiences. Adjust pacing by monitoring student participation, allowing more time for discussion if needed.

### Formative Questions:

Q1. Can you give an example from your daily life where one thing depends on another?

Q2. What do you think makes a relationship between two quantities a 'function'?

### Expected Responses:

Ans 1. Examples such as how the number of hours studied affects marks, how speed affects travel time, etc.

Ans 2. A function is a relation where every input has exactly one output.



### Teacher Notes:

Emphasize clear communication with relatable examples. Use questioning to scaffold understanding and keep students engaged by relating content to their experiences. Adjust pacing by monitoring student participation, allowing more time for discussion if needed.

**Implementation Script:**

- Transition to revisiting foundational knowledge with a brief review of integers and the number line. Begin by asking students to recall what integers are and how they can be positioned on a number line. Show a large number line display, and invite volunteers to plot sample points, including positive, negative integers and zero. Next, connect this to the concept of intervals by highlighting segments on the number line and naming intervals such as open, closed, and disjoint intervals. Use clear diagrams and explain the differences with examples. Pose formative diagnostic questions throughout to check understanding, such as asking students to identify whether a given interval is open or closed. Utilize real-life linkages, for example, explain how temperature ranges or time intervals represent types of intervals on the real line. Maintain active student engagement by encouraging group discussions or think-pair-share on given interval examples. Formative Questions: 1. How would you represent the number  $-3$  on the number line? 2. What is the difference between an open interval and a closed interval? Expected Responses: 1. A point to the left of zero, three units away. 2. Closed intervals include their endpoints; open intervals do not include the endpoints. Misconceptions might include confusion about whether endpoints are included in open intervals. Teacher Notes: Circulate to monitor student responses and clarify misunderstandings immediately. Use strategic questioning to promote deeper thinking and connect with previous knowledge. Ensure language is accessible and provide multiple representations for clarity.

**Formative Questions:**

- Q1. How would you represent the number  $-3$  on the number line?  
Q2. What is the difference between an open interval and a closed interval?

**Expected Responses:**

- Ans 1. A point to the left of zero, three units away.  
Ans 2. Closed intervals include their endpoints; open intervals do not include the endpoints.

**Teacher Notes:**

Circulate to monitor student responses and clarify misunderstandings immediately. Use strategic questioning to promote deeper thinking and connect with previous knowledge. Ensure language is accessible and provide multiple representations for clarity.

**Implementation Script:**

- Begin the session by introducing the Real Number System, highlighting its various types such as natural numbers, whole numbers, integers, rational and irrational numbers. Use a large number line diagram displayed visually to plot examples illustrating these types.
- Engage students with a brief hands-on activity: distribute number cards and ask them to come and place their cards on the number line, identifying which category each number belongs to. During this interaction, circulate the room to prompt students with clarifying questions such as 'Why is this number rational?' or 'Can this number be placed exactly on the number line?'.
- Next, explain the key properties of real numbers including closure, commutativity, associativity, distributivity, identity, and inverse elements with relatable real-life examples (e.g., combining quantities, ordering of ingredients).
- Transition into intervals on the real line: open intervals, closed intervals, half-open intervals, and disjoint intervals. Use visual diagrams projected or drawn on the board to show each type. Invite students to sketch examples on mini whiteboards and identify endpoints and interval types.
- Facilitate a guided discussion prompting students to differentiate intervals based on inclusion or exclusion of endpoints, asking formative questions like, 'What happens to the interval if we include or exclude an endpoint?' and 'Can intervals overlap, and how do we describe those?'. Observe and respond to student reasoning, providing further examples or corrections as needed.
- Conclude by summarizing the importance of understanding real number properties and intervals as foundational for functions and calculus concepts to be explored later.

**Formative Questions:**

Q1. What type of number is  $-3.5$  and where would it lie on the number line?

Q2. Can you describe the difference between an open and closed interval?

Q3. If an interval is  $(2, 5]$ , what numbers are included or excluded?

**Expected Responses:**

Ans 1.  $-3.5$  is a rational number and lies between  $-4$  and  $-3$  on the number line.

Ans 2. An open interval does not include its endpoints, while a closed interval does.

Ans 3. Numbers greater than  $2$  up to  $5$ , excluding  $2$  but including  $5$ .

**Teacher Notes:**

Ensure the use of graphical visuals and real-life examples aligns with Danielson 3a for clear communication. Use strategic questioning (3b) to stimulate discussion and differentiate understanding. Circulate to check individual student understanding during activity (3d). Be ready to adjust grouping if some students struggle with concepts (3e). Emphasize conceptual understanding relevant to competencies CG-1 and CG-8 with interactive participation.

**Implementation Script:**

- Introduce the concept of continuity and limits by first recalling the properties of functions on real numbers, such as constant and identity functions. Use graphical illustrations to exhibit these functions and discuss their behavior visually.
- Utilize GeoGebra software or pre-prepared graph projections to display graphs of continuous functions like  $f(x) = x$  and constant functions, emphasizing their unbroken nature.
- Introduce an example of a function with a discontinuity, such as  $f(x) = 1$  if  $x \leq 0$  and  $f(x) = 2$  if  $x > 0$ , displaying its graph. Foster a discussion on why this function is discontinuous at  $x=0$ .
- Guide students in small groups to analyze different function graphs provided via handouts or digital resources, identifying points of continuity and discontinuity.
- Employ strategic formative assessment by asking, 'What does it mean for a function to approach a limit at a point?' and 'How do continuity and limits connect?' Gather various student explanations and clarify misconceptions.
- Demonstrate the formal definition of limit with examples, using graphical aids to show left-hand and right-hand limits.
- Summarize by linking continuity to the equality of function value and limit at a point, giving examples where this holds and where it doesn't.
- Encourage students to create at least one real-life example demonstrating properties of real numbers and their functions, facilitating transfer and higher-order understanding.

**Formative Questions:**

- Q1. Can you explain why the graph of  $f(x) = x$  is continuous?
- Q2. Identify the point(s) where this given function is discontinuous.
- Q3. How does the left-hand limit compare to the right-hand limit at the point of discontinuity?

**Expected Responses:**

Ans 1. The graph of  $f(x) = x$  is a straight line without breaks, so it is continuous everywhere.

Ans 2. The function is discontinuous at  $x=0$  where the graph has a jump.

Ans 3. The left-hand and right-hand limits at the discontinuity differ; they are not equal.

Ans 4.

**Teacher Notes:**

Models clear communication of abstract concepts with visual and real-life illustrations (3a). Use open-ended questions promoting strategic discussion (3b). Engage students actively in group analysis of graphs (3c). Use formative assessment by circulating and probing understanding (3d) and be responsive to different learning paces, providing additional examples or scaffolding (3e). Aligned well with SMART objectives targeting analysis of differentiability and continuity, and competencies CG-1 and CG-8.

**Implementation Script:**

- Step 1: Begin by presenting various examples of relations and functions using charts and diagrams. Include examples such as even integers related to zero, odd integers to one, and simple function graphs (constant, identity, reciprocal). Use visual aids like number lines and plotted points to illustrate these.
- Step 2: Facilitate a group activity where learners classify these relations into categories (one-to-one, onto, both, or neither). Students will discuss in small groups their observations about which relations fit which type, justifying their reasoning based on properties.
- Step 3: Teacher circulates around groups, asking strategic questions such as, "Why do you think this relation is one-to-one? Can you provide an example?" or "Can this function be onto? How can you tell?" Use students' responses to clarify misconceptions and deepen understanding.
- Step 4: Bring the class together for a guided discussion. Invite groups to share their classifications and reasoning. Highlight correct classifications and address any errors or confusion.
- Step 5: Show graphs of trigonometric functions on domains like  $(0, \pi)$  or  $(-\pi, \pi)$  using software like GeoGebra. Discuss one-oneness and onto-ness in these contexts, introducing the idea of inverse functions.
- Formative Checkpoint 1: After group classification, ask students to individually write down one example of a one-to-one function and explain why. Teacher reviews responses to identify students needing more support.
- Formative Checkpoint 2: After discussion on inverse trigonometric functions, pose the question: "Can all functions have an inverse? Why or why not?" Collect answers to gauge understanding and adjust follow-up instruction accordingly.

**Formative Questions:**

- Q1. What makes a function one-to-one?
- Q2. Can a function be onto but not one-to-one? Give an example.
- Q3. Why does the domain affect whether a trigonometric function is invertible?
- Q4. Explain why some functions do not have inverses.

**Expected Responses:**

- Ans 1. A function is one-to-one if each element in domain maps to a unique element in codomain.
- Ans 2. Yes, for example, a constant function is onto but not one-to-one if codomain equals range.
- Ans 3. Because the function's output repeats over some domains, limiting invertibility; restricting domain can make it one-to-one.
- Ans 4. Functions that map multiple inputs to the same output do not have inverses because inverses require unique mappings.

**Teacher Notes:**

During group discussions, prompt students to justify their reasoning and guide them towards using definitions of one-to-one and onto functions. Use the GeoGebra demonstrations to make abstract ideas visual and concrete. Monitor student responses at formative checkpoints to provide targeted support. Encourage peer explanation to promote engagement and deeper understanding.

### Implementation Script:

- Step 1: Begin with a brief review of the definition and properties of determinants for  $2 \times 2$  and  $3 \times 3$  matrices using a prepared chart.
- Step 2: Distribute worksheets with practice problems on finding determinants of various  $2 \times 2$  and  $3 \times 3$  matrices.
- Step 3: Organize students into pairs to solve these problems collaboratively, encouraging discussion about each calculation step.
- Step 4: Teacher circulates, providing feedback, asking probing questions such as "How did you determine the minor for this element?" and "Can you explain the sign pattern in the  $3 \times 3$  determinant calculation?"
- Step 5: Conduct a whole-class review of select problems, modeling the determinant calculation step-by-step on the board or using projection.
- Formative Checkpoint 1: Mid-activity, ask students to explain the process for finding the determinant of one  $2 \times 2$  matrix they solved. Collect responses to identify misconceptions.
- Formative Checkpoint 2: After completing initial problems, provide a more challenging  $3 \times 3$  matrix and ask students to predict the effect of certain row operations on the determinant before computing it.
- Materials should include worksheets, calculators for verification, and visual aids illustrating minors and cofactors.

### Formative Questions:

- Q1. How do you find the minor of an element in a matrix?
- Q2. What is the sign pattern used in the cofactor expansion?
- Q3. How does swapping two rows affect the determinant?
- Q4. Can you calculate the determinant of this matrix step-by-step?

### Expected Responses:

- Ans 1. The minor is the determinant of the matrix formed by removing the element's row and column.
- Ans 2. Signs alternate starting with plus in the top-left corner.
- Ans 3. Swapping two rows changes the determinant's sign.
- Ans 4. (Students provide stepwise calculations showing understanding.)



### Teacher Notes:

Circulate actively to provide timely feedback and clarify misconceptions, especially about sign patterns and cofactor expansions. Encourage pair discussion for peer learning. Use formative checkpoints to adjust pacing and provide additional examples or explanations as needed. Reinforce the practical relevance of determinants in solving linear systems and real-life applications.

## **Independent Practice: Create Real-life Examples Demonstrating Function Properties**

**30 Minutes**

### **Implementation Script:**

- Instructions:
  - 1. Recall the different types of functions studied: constant, identity, reciprocal, and trigonometric functions.
  - 2. Think of real-life contexts where these function types apply (e.g., constant temperature in a room, identity function in measuring distance, reciprocal function in speed-distance problems).
  - 3. Independently, write down at least two distinct real-life examples demonstrating properties of these functions.
  - 4. For each example, specify the function type involved and explain why it fits that category.
  - 5. Sketch a simple graph or diagram representing your example wherever possible.
  - 6. Submit your written examples and sketches for self-review.
- Tools/Resources:
  - - Notebook or worksheet for writing and sketching.
  - - Graph paper or graphing app (optional).
- Expected outputs:
  - - At least two real-life contextual examples linked to function properties.
  - - Clear explanations connecting example to function type.
  - - Simple graphs/diagrams illustrating examples.

### **Formative Questions:**

Q1. Can you identify the function type in your first real-life example?

Q2. How does the graph you sketched reflect the properties of the function?

### **Expected Responses:**

Ans 1. Yes, the first example is a constant function because the output value does not change regardless of input.

Ans 2. The graph is a horizontal line indicating the constant output value, matching the property of a constant function.

### **Teacher Notes:**

Observe students as they write their examples and ask clarifying questions to check understanding (Danielson 3a). Circulate to offer feedback and encourage deeper thinking (3e). Use their explanations to assess conceptual grasp and provide formative feedback (3d). Encourage sharing examples in pairs to stimulate discussion (3c). Adjust prompts for students who struggle, offering guided hints.

## Solve Graph-based Problems on Continuity and Limits

40 Minutes

### Implementation Script:

- Instructions:
  - 1. Study provided graphs of different functions showing various types of continuity and limits.
  - 2. Analyze each graph and answer questions related to continuity at specific points and the limit values when approaching from left and right.
  - 3. Work independently to solve the problem set, showing your reasoning step-by-step.
  - 4. Check your answers using graph interpretations and provided solution hints.
- Tools/Resources:
  - - Printed or digital graphs of functions.
  - - Worksheet with problem sets related to continuity and limits.
  - - Calculator (optional).
- Expected outputs:
  - - Written solutions identifying points of continuity/discontinuity.
  - - Correct evaluation of left-hand and right-hand limits where applicable.
  - - Reasoned explanations referencing graph shapes.

### Formative Questions:

Q1. At point c on the graph, is the function continuous? Why or why not?

Q2. What is the left-hand limit of the function at  $x = a$ ? Does it equal the function's value at  $a$ ?

### Expected Responses:

Ans 1. The function is continuous at point c because the limit from both sides equals the function value there.

Ans 2. The left-hand limit at  $x = a$  is 3, but since the function value at  $a$  is 2, the function is not continuous at  $a$ .



### Teacher Notes:

Circulate among students, asking probing questions to assess understanding and clarify misconceptions (Danielson 3a, 3b). Monitor engagement and allow students to explain reasoning aloud when possible (3c). Use student responses for formative assessment and tailor further support accordingly (3d, 3e). Provide encouragement to students who hesitate, and prompt more detailed explanations from those who answer quickly.

## Closure: Summarize Key Concepts Using Student-generated Examples

15 Minutes

### Implementation Script:

- To conclude the lesson, facilitate a reflective activity where students share their own examples illustrating key concepts such as types of relations, properties of real number functions, and determinants of matrices. Prompt students verbally: 'Can someone share an example of a function that is continuous on the real line? How does its graph help you understand continuity?' Use an interactive checklist projected on the board to tick off examples that cover different concept areas. Circulate around the room to listen and provide affirming feedback, asking clarifying questions such as 'How does your example illustrate one-to-one or onto properties?' and 'What real-life context does your example connect to?'. This activity engages students in peer learning and consolidates understanding by articulating ideas in their own words. It also aligns with Danielson Framework indicators 3a (clear communication), 3b (strategic questioning), and 3c (engagement), as students actively participate and explain concepts. Connect this activity to future learning by encouraging students to identify functions and relations in real-world data as homework, preparing them to apply these concepts in advanced studies.

### Formative Questions:

Q1. Can you explain how your example shows a function is continuous?

Q2. What makes your example a one-to-one or onto function?

Q3. How does your example relate to real-life situations involving functions or matrices?

### Expected Responses:

Ans 1. The example function's graph has no breaks, so it is continuous.

Ans 2. The function pairs each input with a unique output, showing one-to-one property.

Ans 3. This real-life example, like converting currencies, shows how functions relate inputs to outputs practically.



### Teacher Notes:

Encourage multiple students to share distinct examples covering varied concepts. Use their responses to highlight important properties and gently correct misconceptions. Reinforce connections to real-life examples, emphasizing the value of applying theoretical knowledge practically.

## Discuss Relationship between Differentiability and Continuity

15 Minutes

### Implementation Script:

- Arrange a guided discussion focusing on how differentiability of a function implies continuity, while the converse is not always true. Use clear examples such as the identity function  $f(x) = x$ , which is both continuous and differentiable everywhere, and the absolute value function  $f(x) = |x|$ , which is continuous everywhere but not differentiable at  $x=0$ . Begin by posing an open-ended question: 'What do you think happens to the graph of a function at points where it is not differentiable but continuous?' Facilitate student responses, prompting deeper thinking with follow-ups like 'How does the sharp corner in the absolute value function affect differentiability?' and 'Can a function be differentiable but not continuous? Why or why not?' Use graphical illustrations or applets (e.g., GeoGebra) for visual reinforcement. This activity addresses Danielson Framework 3b (strategic questioning), 3d (formative assessment), and 3e (responsiveness) by adapting discussion based on student inputs. For closure, relate this discussion to future topics such as advanced calculus concepts and real-world modeling of smooth vs. cornered phenomena.

### Formative Questions:

Q1. Can you identify points where a function is continuous but not differentiable?

Q2. Why does the absolute value function fail to be differentiable at zero?

Q3. Is it possible for a function to be differentiable but not continuous? Explain.

### Expected Responses:

Ans 1. At points like  $x=0$  in  $f(x)=|x|$ , function is continuous but not differentiable.

Ans 2. Because there is a sharp corner at zero, the derivative from the left and right differ, so not differentiable.

Ans 3. No, differentiability requires continuity, so a function cannot be differentiable without being continuous.

### Teacher Notes:

Guide students toward precise definitions and help them clarify misunderstandings. Use visuals extensively to make abstract concepts concrete. Ensure students can connect concepts of differentiability and continuity to function behavior on graphs. Prepare to assign homework that includes finding points of continuity and differentiability in given functions and explaining their reasoning.

## Assessment: Multiple Choice Questions on Types of Relations and Functions

30 Minutes

### Implementation Script:

- Students will complete a set of MCQs designed to assess their ability to identify and classify different types of relations and functions, including understanding one-to-one, onto, and bijective properties. The questions will include examples drawn from real numbers, with some graphical interpretations.

### Formative Questions:

Q1. Which of the following is a one-to-one function?

Q2. Identify the domain and range for the given relation.

Q3. Is the given function onto? Provide reasoning.

Q4. Classify the function as constant, identity, or reciprocal.

Q5. Determine if the relation is bijective and justify your answer.

### Expected Responses:

Ans 1. Option B shows a one-to-one function because each element of the domain maps to a unique range value.

Ans 2. Domain is all real numbers; range is from -1 to 1.

Ans 3. Yes, the function covers all values in the codomain, making it onto.

Ans 4.  $f(x) = k$  is constant;  $f(x) = x$  is identity;  $f(x) = 1/x$  is reciprocal.

Ans 5. The function is bijective if it is both one-to-one and onto as shown by mapping all domain elements uniquely to the range.

### Teacher Notes:

To implement, read questions aloud for clarity; circulate to observe student responses; provide hints as needed; encourage peer discussion for justification; collect worksheets for formative feedback.

## Worksheet Task on Matrix Determinants and Properties

30 Minutes

### Implementation Script:

- Provide students with a problem set that includes calculating determinants of  $2 \times 2$  and  $3 \times 3$  matrices, applying properties like switching rows, scalar multiplication, and addition of rows. Tasks include evaluating determinant values and explaining the steps.

### Formative Questions:

- Compute the determinant of the given  $2 \times 2$  matrix.
- Explain what happens to the determinant when two rows are swapped.
- Calculate determinant using cofactor expansion for the  $3 \times 3$  matrix.
- Identify if the determinant equals zero and what it implies.
- Verify determinant properties with given examples.

### Expected Responses:

- Ans 1. Determinant of matrix  $[[a,b],[c,d]]$  is  $ad - bc$ .
- Ans 2. Swapping rows changes the sign of the determinant.
- Ans 3. Determinant calculated through minors equals 5.
- Ans 4. Determinant zero implies matrix is singular and not invertible.
- Ans 5. Properties verified as stated with calculations matching expected results.



#### Teacher Notes:

Guide students by modeling one example; circulate to check computation; emphasize steps rather than just answers; address misconceptions on determinant properties; allow calculators for tedious arithmetic if necessary.

## Short Conceptual Essay on Differentiability and Continuity

20 Minutes

### Implementation Script:

- Students will write a short answer explaining the relationship between differentiability and continuity of functions, supported by examples from real functions such as constant, identity, and piecewise functions.

### Formative Questions:

- Define continuity for a real function at a point.
- State if differentiability implies continuity with justification.
- Provide example of a function continuous but not differentiable at a point.
- Explain with an example when differentiability fails.
- Describe the practical significance of this relationship.

### Expected Responses:

- Ans 1. A function is continuous at a point if limit of function equals the function value at that point.
- Ans 2. Differentiability implies continuity, but not vice versa.
- Ans 3. Example: Absolute value function is continuous everywhere but not differentiable at 0.
- Ans 4. Differentiability fails if function graph has a sharp corner or cusp at point.
- Ans 5. It helps understand function behavior in calculus and modeling real-world changes.



#### Teacher Notes:

Encourage use of formal definitions and graphical illustrations; prompt students to explain in their own words; review essays for conceptual clarity; provide constructive feedback emphasizing reasoning.

**Implementation Script:**

- In small groups, students will create two examples demonstrating real-life applications of properties of real numbers and functions (e.g., constant function for speed regulation, reciprocal function in physics). They will prepare a brief presentation explaining the math concepts used.

**Formative Questions:**

- Q1. What property of real numbers is used in your example?
- Q2. Explain the function type and its graph.
- Q3. How does this function model the real-life scenario?
- Q4. What conclusions can be drawn from the function's behavior?
- Q5. Describe any limitations of your example.

**Expected Responses:**

- Ans 1. Example 1 uses constant functions to model fixed speed limits.
- Ans 2. Function type identified with graph showing behavior.
- Ans 3. Models how speed remains constant over time.
- Ans 4. Concludes steady state behavior in system.
- Ans 5. Limitations include ignoring acceleration or external forces.

**Teacher Notes:**

Facilitate group collaboration; circulate to guide relevance and connection to math concepts; ask clarifying questions to deepen analysis; encourage creativity; allow sufficient time for presentations and peer feedback.