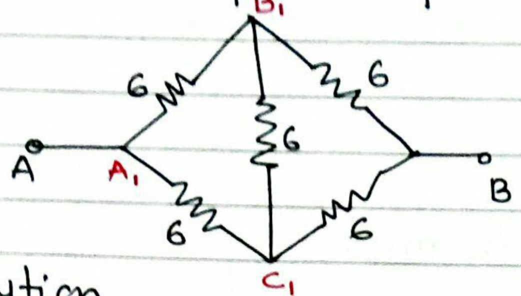
Subject: BEE
Class : FE-AL

Academic year - 2017-18.

IAT-1 SOLUTION SET

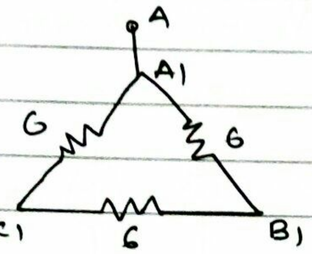
Q1.

(a) Find R_{AB} for the following n/ω.



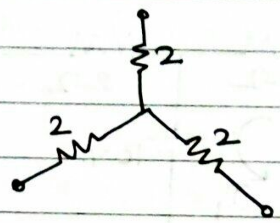
Solution

convert the delta $A_1B_1C_1$ to star n/ω.

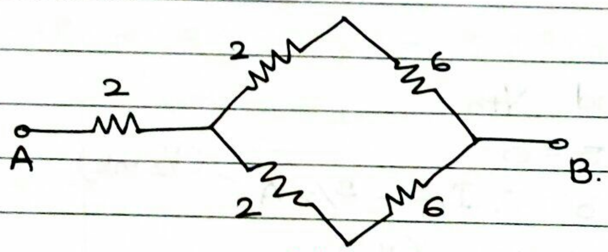


$\Rightarrow$

$R_\Delta = 3R_Y$ (½ mk)

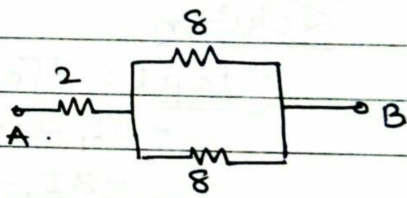


∴ The ckt reduces to



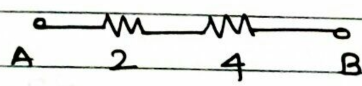
(½ mk)

$\Rightarrow$

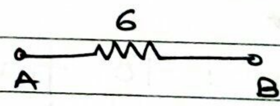


(½ mk)

$\Rightarrow$

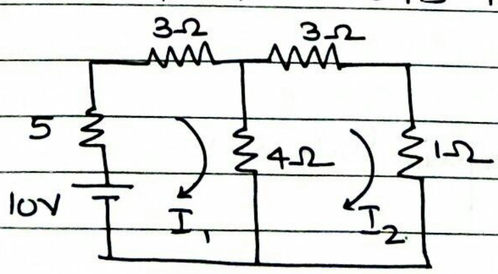


$\Rightarrow$



$\therefore R_{AB} = 6\,\Omega$ (½ mk)

(b) Find Mesh currents in the ckts



Mesh 1

$-3I_1 - 4(I_1 - I_2) + 10 - 5I_1 = 0$

$\Rightarrow -3I_1 - 4I_1 + 4I_2 - 5I_1 + 10 = 0$

$\Rightarrow -12I_1 + 4I_2 + 10 = 0$

$\Rightarrow 12I_1 - 4I_2 = 10$ — (I)

(½ mk)

Mesh-2

$-3I_2 - 1I_2 - 4(I_2 - I_1) = 0$

$\therefore -3I_2 - I_2 - 4I_2 + 4I_1 = 0$

$\therefore 4I_1 - 8I_2 = 0$

$\therefore I_1 - 2I_2 = 0$ — (II)

(½ mk)

× Eq (II) by 12

$\cancel{12}I_1 - 24I_2 = 0$

$\cancel{12}I_1 - 4I_2 = 10$

$-\quad + \quad -$

$-20I_2 = -10$

$\therefore I_2 =$

(P.T.O.)

$\therefore I_2 = \frac{10}{20} = 0.5\,A$. (½ mk)

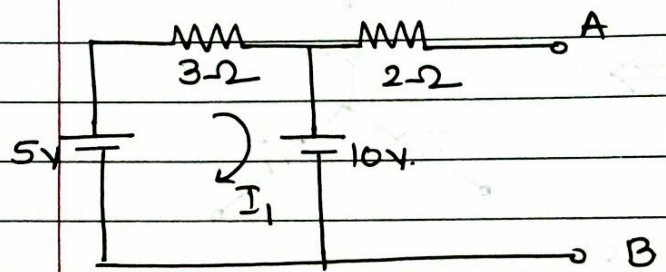
put I_2 in Eq (ii)

$\therefore I_1 = 2I_2$

$= 2(0.5)$

$\underline{I_1 = 1\,A}$. (½ mk).

(c) Draw theyenin's equivalent ckt across A and B.



Solution

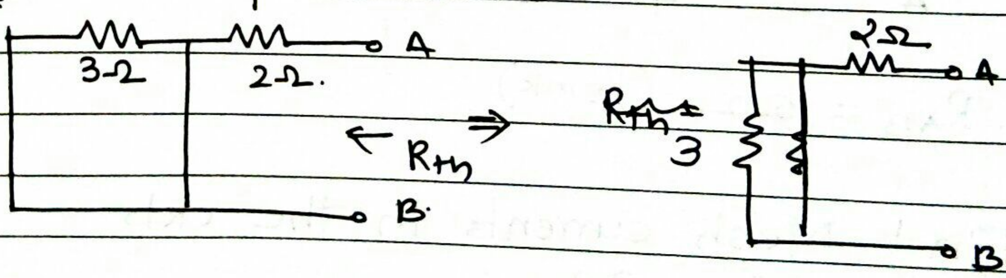
Step 1 : To find V_{th}

$-3I_1 - 10 + 5 = 0$

$\therefore -3I_1 - 5 = 0 \quad \therefore I_1 = -5/3\,A$. – (½ mk)

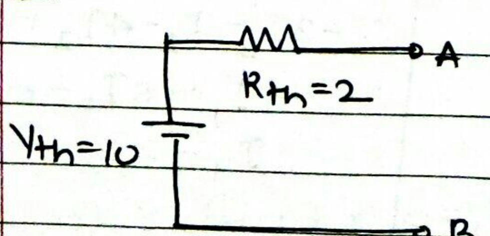
$V_{AB} = V_{th} = 10\,V$ – (½)

Step 2 : To find R_{th}



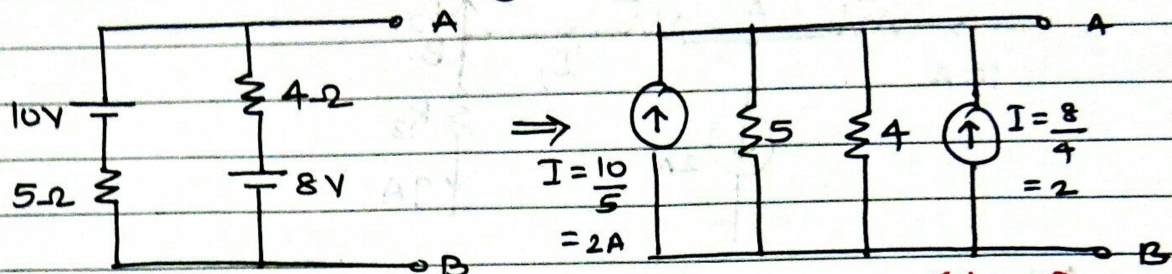
$\therefore R_{th} = 2\Omega$. (½ mk)

Step 3 Theyenin's equivalent ckt.



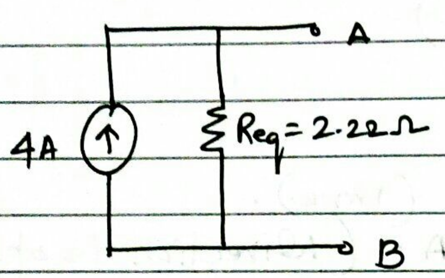
(½ mk)

(d) Convert the given circuit to single vtg. souxe in series with a single resistor



(1mk)

Currents in //al can be added.

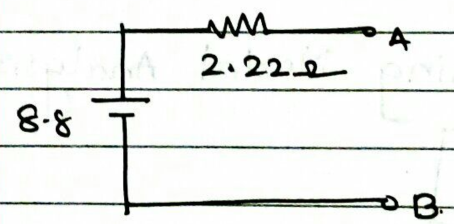


$R_{eq} = (5 // 4)$

$R_{eq} = \frac{5 \times 4}{5+4} = \frac{20}{9} = 2.22\Omega$

(½ mk)

$V = IR = 4 \times 2.22 = 8.88$ Volts.



(½ mk).

(e) An alternating current is given by $i(t) = 141.4 \sin(314t)$. Find the maximum value of current and frequency.

Solution

$i(t) = 141.4 \sin(314t)$ —① ... given

$i(t) = I_m \sin(\omega t)$ —② ... standard form.

Compare ① & ②

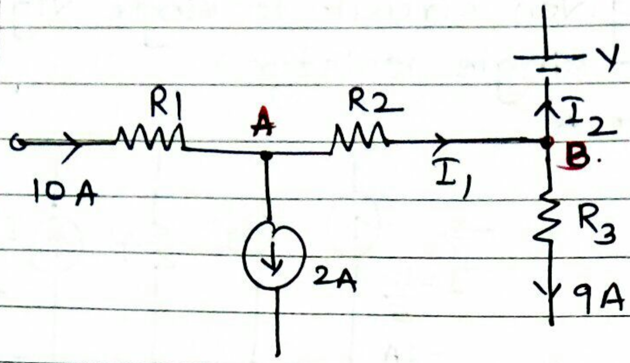
(a) $I_m = 141.4$ Amp. (1mk)

(b) $\omega = 314$

$2\pi f = 314$

$\therefore f = \frac{314}{2 \times \pi} = \frac{314}{2 \times 3.14} = 50 Hz$ (½mk)

(f) Find the unknown currents using KCL.



Apply KCL at node A. (incoming I = outgoing I)

$$10 = 2 + I_1$$

$$\therefore I_1 = 8A. \quad \text{(1mk)}$$

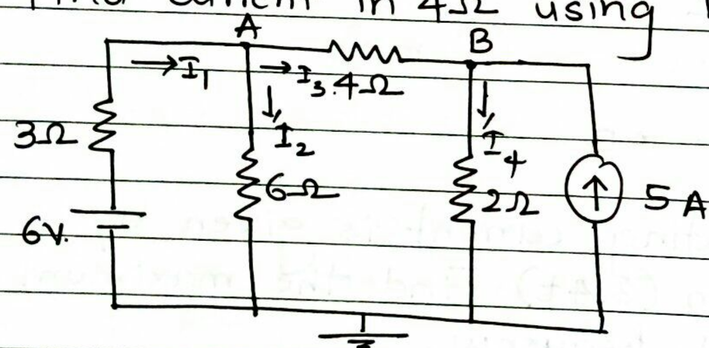
Apply KCL at node B.

$$I_1 = I_2 + 9$$

$$8 = I_2 + 9 \quad \text{(1mk).}$$

$$\therefore I_2 = 8 - 9 = -1\,A.$$ (Direction is opposite to the marked)

2. a) Find current in 4Ω using Nodal Analysis.



Apply KCL at node A.

$$\frac{6 - V_A}{3} = \frac{V_A}{6} + \frac{V_A - V_B}{4}$$

$$\Rightarrow 2 - \frac{V_A}{3} - \frac{V_A}{6} - \frac{V_A}{4} + \frac{V_B}{4} = 0$$

$$\Rightarrow 2 - V_A\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right) + \frac{V_B}{4} = 0$$

$$\Rightarrow 2 - V_A\left(\frac{4+2+3}{12}\right) + \frac{V_B}{4} = 0$$

$$\Rightarrow 2 \cancel{- \frac{V_A}{9} + \frac{V_B}{4}} = 0$$

$$\therefore \frac{4}{9}V_A - \frac{1}{4}V_B = 2$$

$$\Rightarrow 2 - V_A \times \frac{9}{12} + \frac{V_B}{4} = 0$$

$$2 = \frac{3}{4}V_A - \frac{1}{4}V_B \quad \text{--- (1)}$$

$-\frac{1}{4}+\frac{1}{4}=\frac{\cancel{2}}{\cancel{4}}$

my companion

Apply KCL at node B.

$$\frac{V_A - V_B}{4} + 5 = \frac{V_B}{2}$$

$$\Rightarrow \frac{V_A}{4} - \frac{V_B}{4} - \frac{V_B}{2} = -5$$

$$\Rightarrow \frac{V_A}{4} - V_B\left(\frac{1}{4}+\frac{1}{2}\right) = -5$$

$$\Rightarrow \frac{V_A}{4} - V_B\left(\frac{1+2}{4}\right) = -5$$

$$\Rightarrow \frac{V_A}{4} - \frac{3}{4}V_B = -5 \quad \text{— (II)}$$

$V_A = 1/2$

$\frac{9}{4}-\frac{1}{4}=\frac{\cancel{8}^{2}}{\cancel{4}}$

$V_B =$

$$\frac{3}{4}V_A - \frac{1}{4}V_B = 2 \quad \text{— (I)}$$

$$\frac{1}{4}V_A - \frac{3}{4}V_B = -5 \quad \text{— (II)} \times 3$$

$$\frac{3V_A}{4} - \frac{1}{4}V_B = 2$$

$$\frac{3}{4}V_A - \frac{9}{4}V_B = -15$$

$$2V_B = +17$$

$$V_B = 17/2$$

put in Eq (I)

$$\frac{3}{4}V_A - \frac{1}{4}V_B = 2$$

$$\frac{3}{4}V_A - \frac{1}{4}\times\frac{17}{2} = 2$$

$$\frac{3}{4}V_A = 2 + \frac{17}{8}$$

$$= \frac{16+17}{8}$$

$$\frac{3}{4}V_A = \frac{33}{8}$$

$$\therefore V_A = \frac{35}{8} \times \frac{4}{3}$$

$$V_A = \frac{35}{6} \text{ volts}$$

$$I_{4\Omega} = \frac{V_A - V_B}{4}$$

$$= \frac{\frac{11}{2} - \frac{17}{2}}{4}$$

~~$I_{4\Omega} = 0.66\,A$~~

$$I_{4\Omega} = -0.75\,A$$

$$I_{4\Omega} = 0.75\,A\,(\leftarrow)$$

$$V_A = \frac{33 \times 4}{8 \times 3} = \frac{11}{2}\ V$$

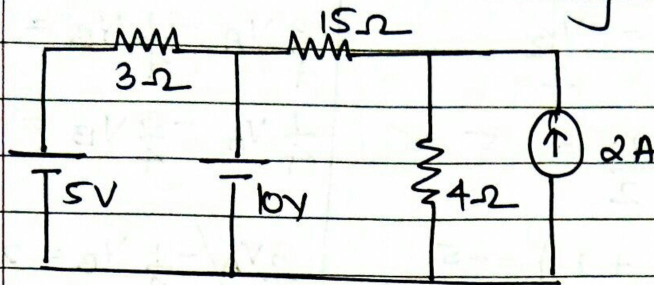
V_A cal. → 2 mks.

V_B cal → 2 mks

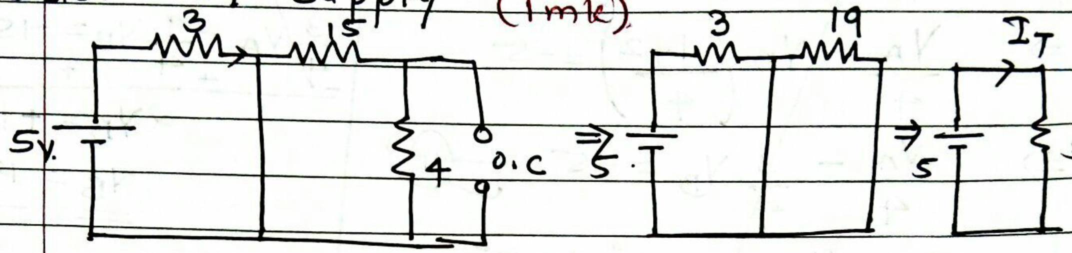
$I_{4\Omega} =$ 1 mk.

(P.T.O).

(b) Find current in 15Ω using S.T.



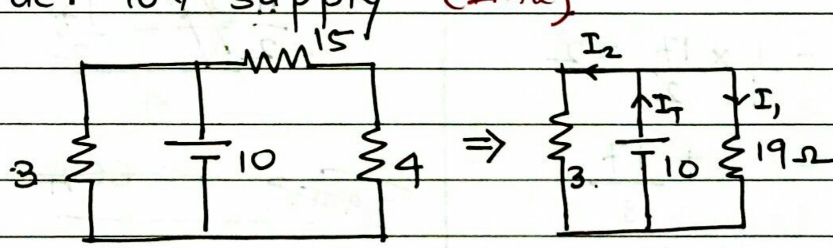
consider 5V supply (1mk)



$$I_T = \frac{V}{R_{eq}} = \frac{5}{3} \text{ Amp} = 1.66\Omega$$

$\therefore I_{15\Omega} = 0 \text{ Amp} (\leftrightarrow)$ (because of s.c).

consider 10V supply (2mk)

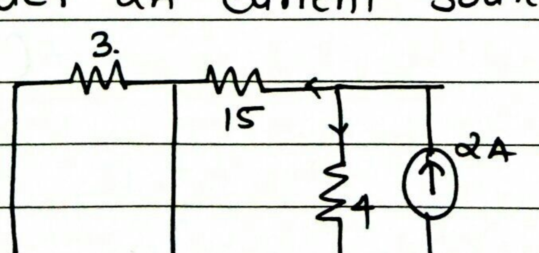


$$R_{eq} = \frac{19\times 3}{19+3} = 2.59 \quad \therefore I_T = \frac{10}{2.59} = 3.85\text{ A}.$$

$$\therefore I_1 = 3.85 \times \frac{3}{19+3} = 0.526$$

$$\therefore (I_{15}) = 0.526 (\rightarrow) \text{ Amp}.$$

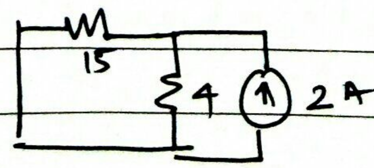
Consider 2A current source. (1mk)



$$I_{15} = 2\times \frac{4}{19}$$

$$= \frac{8}{19} = 0.421 (\leftarrow)$$

$\Rightarrow$

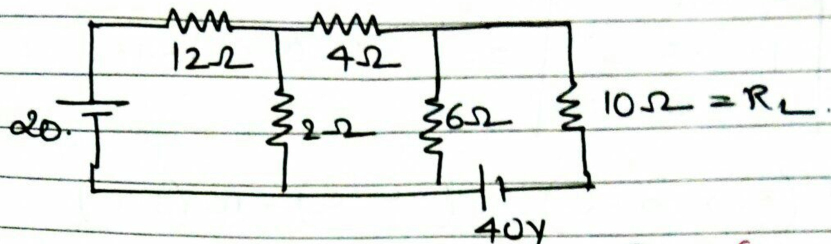


$$\therefore (I_{15\Omega}) = (I_{15})_{5V} + (I_{15})_{10V} + (I_{15})_{2A}$$

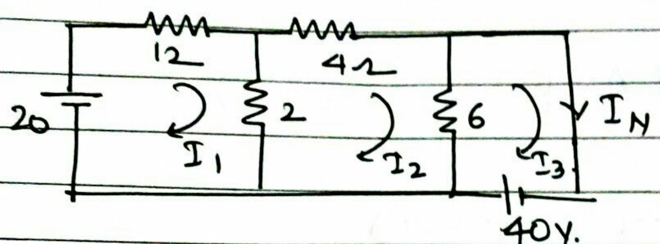
$$= 0 + 0.526(\rightarrow) + 0.421(\leftarrow)$$

$$(I_{15\Omega}) = 0.105(\rightarrow)\,\text{Amp} \quad (1\text{mk})$$

Q3 ⓐ Find current in 10Ω using Norton's Analysis.



Step 1 : Find s.c current I_N. (2mk).



$I_3 = I_N$

Mesh 1 (1mk)

$$-12I_1 - 2(I_1 - I_2) + 20 = 0$$

$$\Rightarrow -12I_1 - 2I_1 + 2I_2 + 20 = 0$$

$$\Rightarrow -14I_1 + 2I_2 = -20$$

$$\Rightarrow 7I_1 - I_2 = 10 \quad \text{—— (1)}$$

Mesh-2 (1mk)

$$-4I_2 - 6(I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$\Rightarrow -4I_2 - 6I_2 + 6I_3 - 2I_2 + 2I_1 = 0$$

$$\Rightarrow 2I_1 - 12I_2 + 6I_3 = 0$$

$$\Rightarrow I_1 - 6I_2 + 3I_3 = 0 \quad \text{—— (11)}$$

Mesh-3 (1mk)

$$40 - 6(I_3 - I_2) = 0$$

$$\Rightarrow -6I_3 + 6I_2 = -40$$

$$\Rightarrow 6I_2 - 6I_3 = -40$$

$$\Rightarrow 3I_2 - 3I_3 = -20 \quad \text{—— (111)}$$

In matrix form it can be written as -

$$\begin{bmatrix} 7 & -1 & 0 \\ 1 & -6 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

$$\Delta = 7((-6 \times -3)-9) + 1(-3-0) + 0$$

$$\Delta = 7(9) - 3 = 60$$

$$\Delta_3 = \begin{vmatrix} 7 & -1 & 10 \\ 1 & -6 & 0 \\ 0 & 3 & 20 \end{vmatrix} = \qquad \Delta, \Delta_3 - 1mk.$$

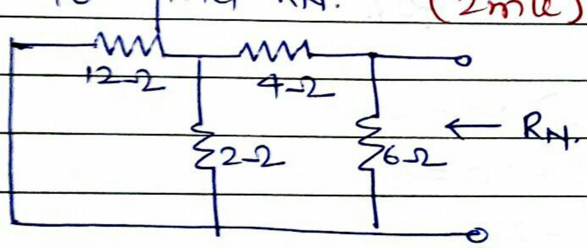
$$= 7(-6 \times 20 - 0) - 1(20-0) + 10(3-0)$$

$$= 7(-120) - 20 + 30$$

$$= -830$$

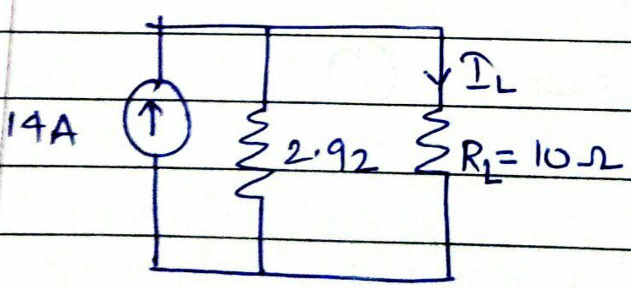
$$I_3 = I_N \approx \underline{\underline{14 \text{ Amp}}} \quad \text{— 0.5 mk)}.$$

Step 2 To find R_N. (2mk)



$$\therefore R_N = \{[12 \parallel 12] + 4\} \parallel 6 = 2.92\ \Omega.$$

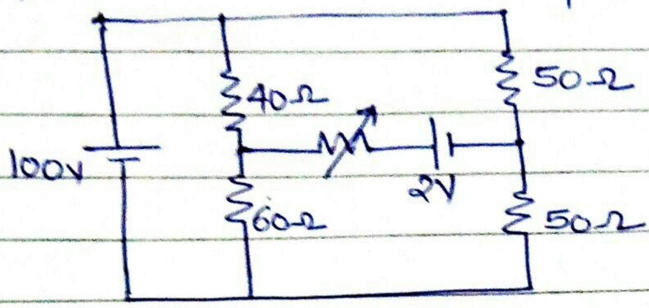
Step 3 : To draw the Norton Equivalent ckt & find $I_{10\Omega}$. (1mk)



(0.5mk)

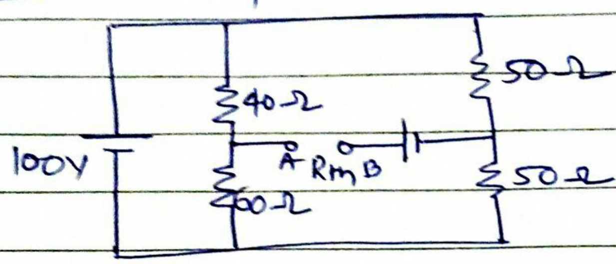
$$\therefore I_L = 14 \times \frac{2.92}{10 + 2.92} = \underline{\underline{3.18 \text{ A}}} \quad (0.5)\text{mk}.$$

Q3(b) Find maximum power delivered load R_L.

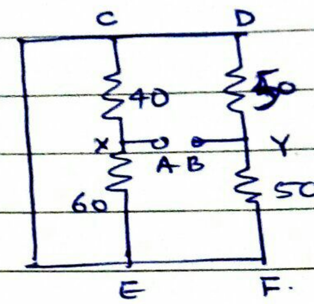


$P_L = \frac{V_{th}^2}{4R_L}$

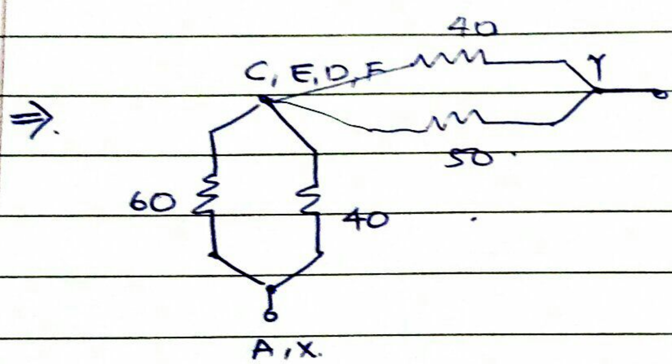
Step 1 : To find R_{th}. (2mk)



$\Rightarrow$



$\Rightarrow$



$60 \| 40 = \frac{40 \times 60}{40+60} = \frac{2400}{100}$

24Ω.

$50 \| 50 = \frac{50 \times 50}{50+50} = \frac{2500}{100} = 25$

$\therefore R_{th} = \cancel{46.22} = 49\,\Omega$

$V_{th} = 8V$. (2mk)

$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{8 \times 8}{4 \times \cancel{46.22}\ 49} = \cancel{\frac{64}{184.88}}$

$\therefore P_{max} = 0.326$ Watts (1mk)

—x— x—