EM

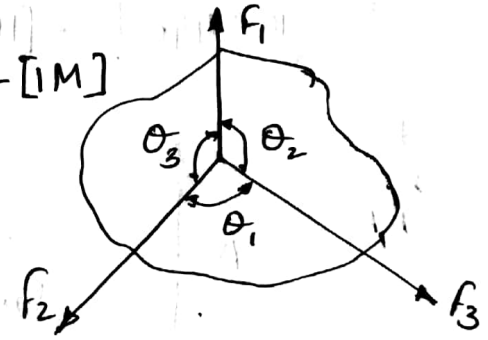
IAT-I SOLUTION

(21)(a)

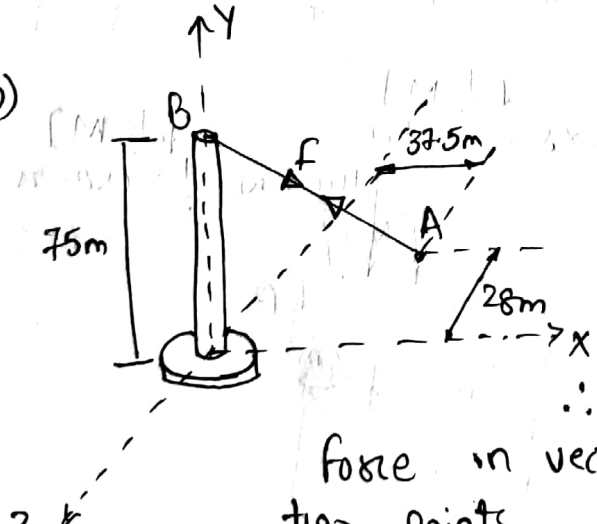
Varignon's Theorem :- "It States That the moment of resultant of all the forces in a plane about any point is equal to the algebraic sum of moment of all the forces about the same point." [1M]

Lamis Theorem :- "If Three concurrent coplanar forces acting on a body having same nature (ie. pulling or pushing) keep the body in equilibrium, then each force is proportional to the sine of angle included between the other two forces."

i.e. $\frac{f_1}{\sin\theta_1} = \frac{f_2}{\sin\theta_2} = \frac{f_3}{\sin\theta_3}$ — [1M]



(b)



F = 650N (passes from B to A)

A (37.5, 0, -28) .. from Fig

B (0, 75, 0) .. from Fig

$\therefore r_{BA} = +37.5\,i + 75\,j + 28\,k$

$|r_{BA}| = 88.4$ m.

$\therefore \hat{e}_{BA} = 0.428\,i - 0.848\,j - 0.317\,j$ — [1M]

force in vector form passing through two points.

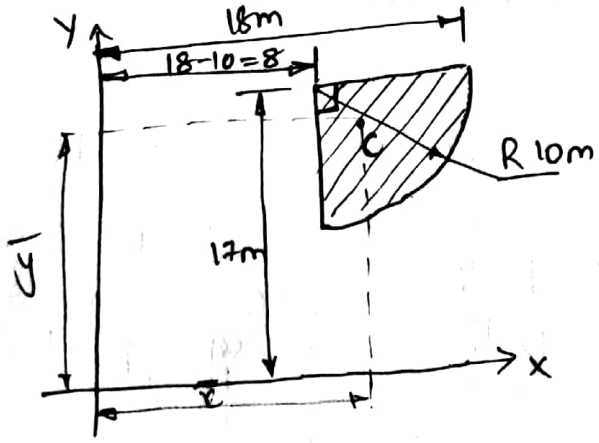
$\bar{F} = F \cdot \hat{e}_{BA}$

$= F\left[\frac{(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$

$= 650\left[\frac{37.5i + (-75)j + (-28)k}{\sqrt{(37.5)^2 + (-75)^2 + (28)^2}}\right]$

$\boxed{\bar{F} = 275.72\,i - 551.44\,j + 205.87\,k}$ — [1M]

(c)



$$\bar{x} = 8 + \frac{4R}{3\pi}$$

$$= 8 + \frac{4(10)}{3\pi}$$

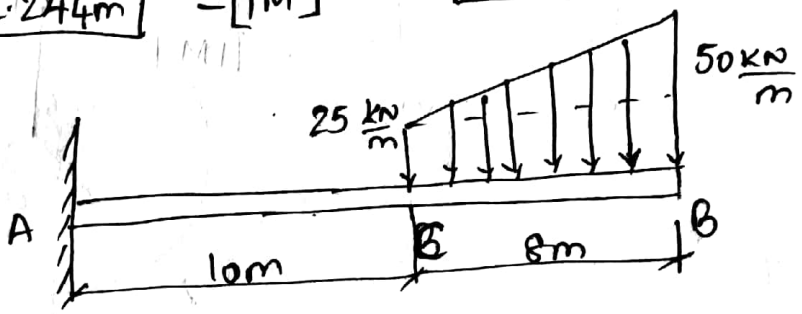
$$\boxed{\bar{x} = 12.244\,m} \quad -[1M]$$

$$\bar{y} = 17 - \frac{4R}{3\pi}$$

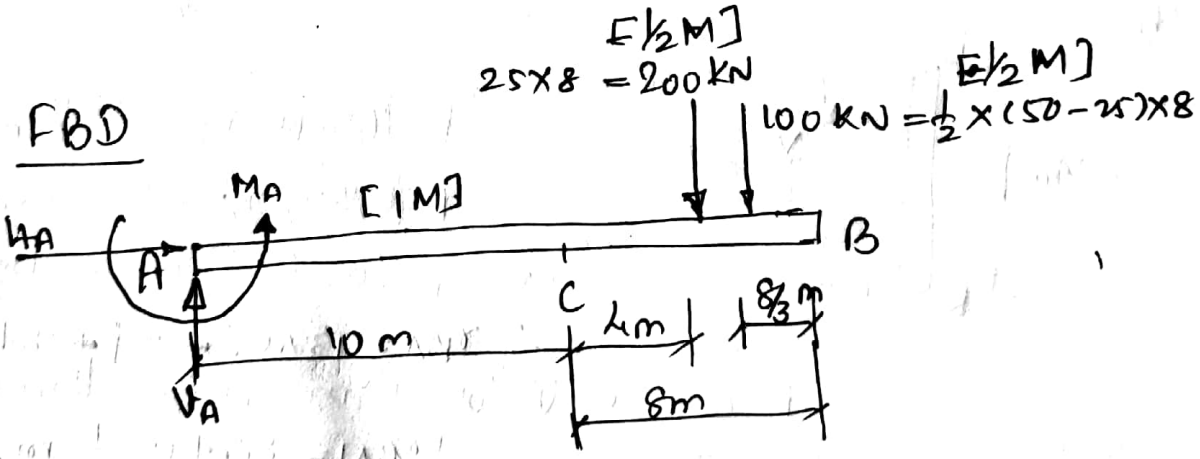
$$\bar{y} = 17 - \frac{4\times(10)}{3\pi}$$

$$\boxed{\bar{y} = 12.756\,m} \quad -[1M]$$

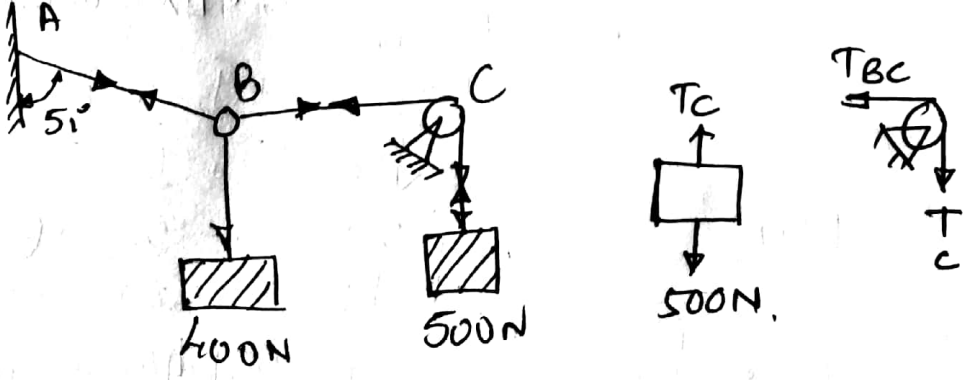
(d)



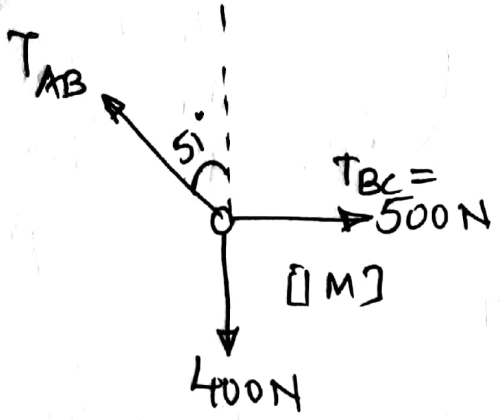
FBD



(e)



FBD at B

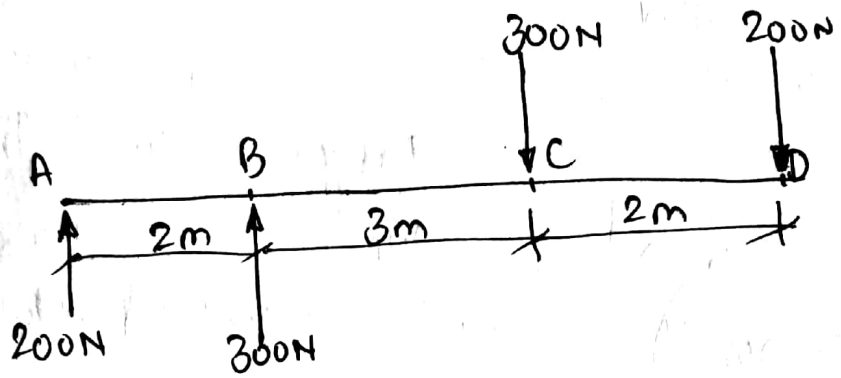


Applying Lami's Theorem,

$$\frac{T_{AB}}{\sin 90} = \frac{400}{\sin(90+51)} = \frac{500}{\sin(180-51)}$$

$$\boxed{T_{AB} = 635.6 \text{ N}} \quad [1M]$$

(f)



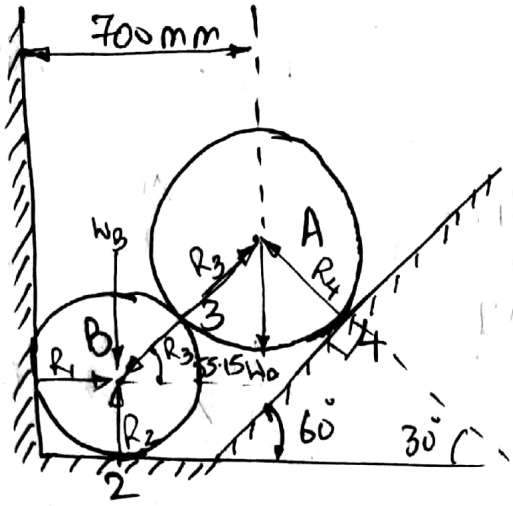
- $\Sigma F_x = 0$
- $\Sigma F_y = 200 + 300 - 300 - 200$

  $= 0$
- $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\boxed{R = 0 \text{ N}}$ —[1M]

$$\Sigma M_A = -300 \times 2 + 300 \times 5 + 200 \times 7$$
$$= 2300 \text{ Nm } \circlearrowleft \quad —[1M]$$

Q2](a)



$W_A = 1000\ N$

$W_B = 750\ N$

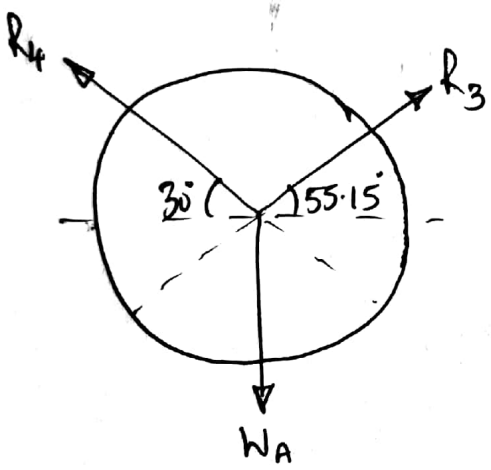
$R_A = 400\ mm$

$R_B = 300\ mm.$

$$\theta = \cos^{-1}\left(\frac{700-300}{300+400}\right)$$

$$\boxed{\theta = 55.15^\circ} \quad \text{—[1M]}$$

FBD of - A



Applying Lami's Theorem,

$$\frac{W_A}{\sin(180-30-55.15)} = \frac{R_4}{\sin(90+55.15)} = \frac{R_3}{\sin(90+30)}$$

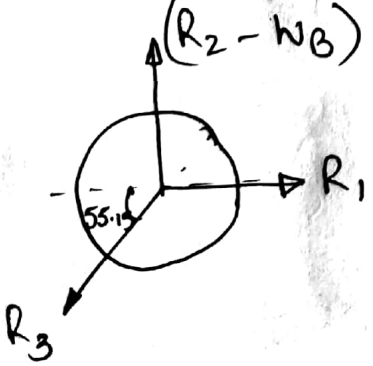
considering, $\frac{1000}{\sin(94.85)} = \frac{R_4}{\sin(145.15)}$

$$\boxed{R_4 = 573.483\ N} \quad [1M]$$

considering, $\frac{1000}{\sin(94.85)} = \frac{R_3}{\sin(120)}$

$$\boxed{R_3 = 869.137\ N} \quad [1M]$$

FBD of B



Applying Lami's Theorem,

$$\frac{R_3}{\sin(90)} = \frac{R_2 - W_B}{\sin(180-55.15)} = \frac{R_1}{\sin(90+55.15)}$$

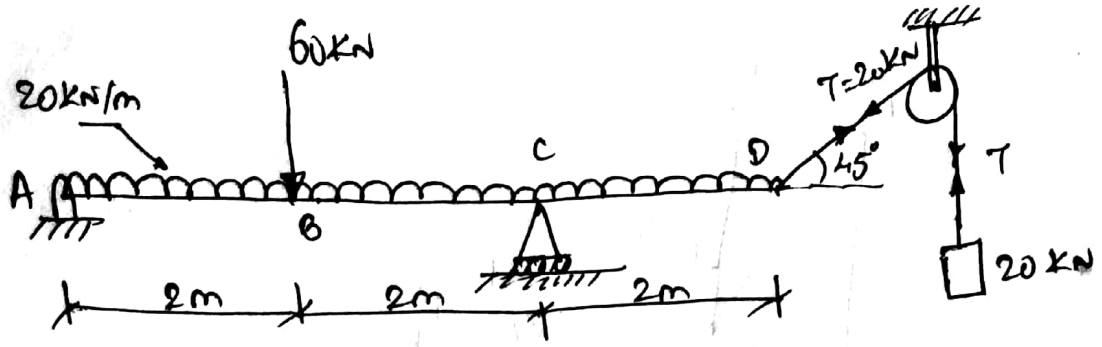
considering, $\frac{869.137}{\sin 90} = \frac{R_2 - 750}{\sin(124.85)}$

$$\boxed{R_2 = 1463.25\ N} \quad [1M]$$

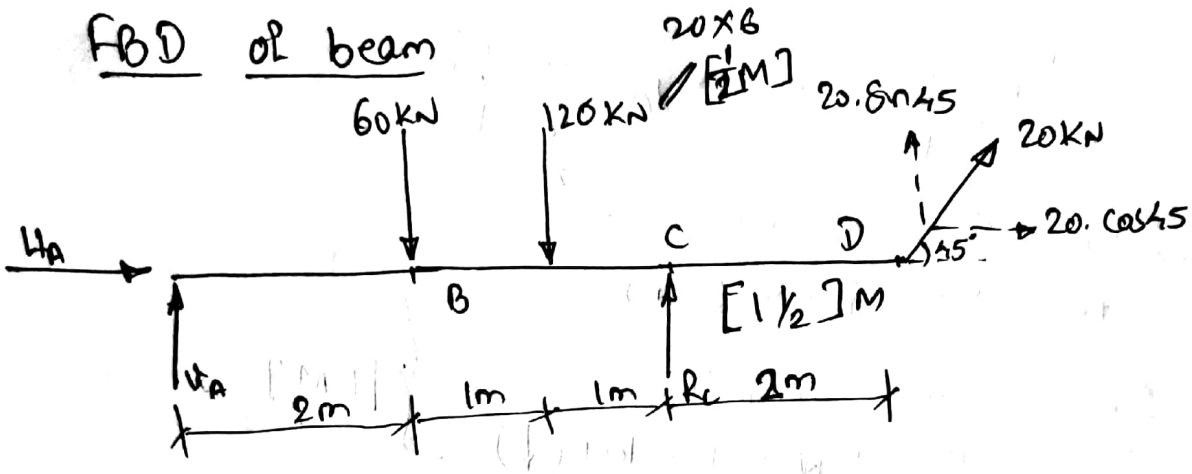
considering, $\frac{869.137}{\sin 90} = \frac{R_1}{\sin(145.15)} \Rightarrow \boxed{R_1 = 496.64N}$

[1M]

(b)



FBD of beam



As Beam is in equilibrium,

- $\Sigma F_x = 0$, $20\cos(45) + H_A = 0$

$$\therefore \boxed{H_A = -14.14\ KN} \quad \text{— [1M]}$$

$$H_A = 14.14\ KN \ (\leftarrow)$$

- $\Sigma F_y = 0$,

$$V_A + R_c - 60 - 120 + 20.\sin 45 = 0$$

$$V_A + R_c - 165.858 = 0 \quad \cdots ①$$

- $\Sigma M_D = 0$,

$$-R_c \times 2 + 120 \times 3 + 60 \times 4 - V_A \times 6 = 0$$

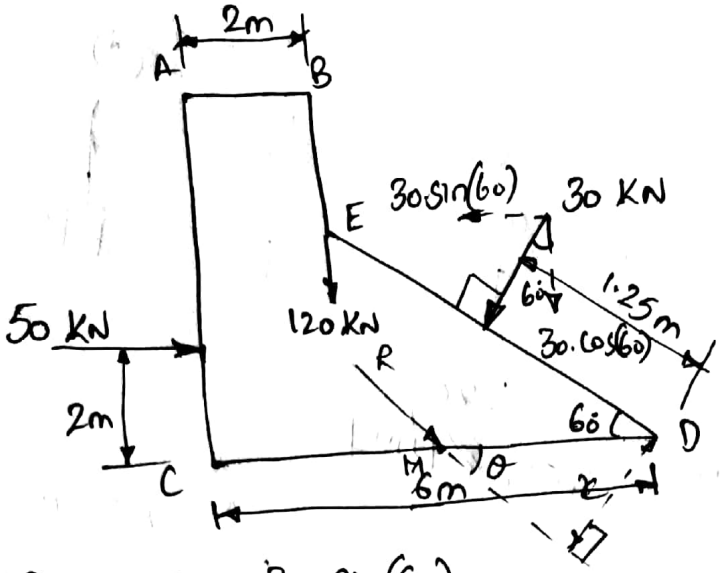
$$-6V_A - 2R_c + 600 = 0 \quad \cdots ②$$

on solving eqn ① & ②,

$$\boxed{V_A = 67.071\ KN}\ (\uparrow) \text{ — [1M]}$$

$$\boxed{R_c = 98.787\ KN}\ (\uparrow) \text{ — [1M]}$$

Q3](a)



- $\Sigma F_x = 50 - 30.\sin(60)$

  $\underline{\Sigma F_x = 24.019 \text{ KN } (\rightarrow)}$

- $\Sigma F_y = -120 - 30.\cos(60)$

  $\underline{\Sigma F_y = -135 \text{ KN} = 135 \text{ kN } (\downarrow)}$ } [1 M]

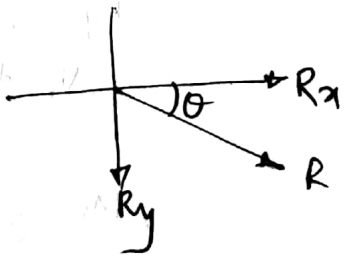
- $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

  $= \sqrt{24.019^2 + (-135)^2}$

  $\boxed{R = 137.12 \text{ KN}}$ — [1 M]

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{135}{24.019}\right)$

$\underline{\theta = 79.91^\circ}$ — [1 M]



To find position of Resultant,
Applying Varignons Theorem,

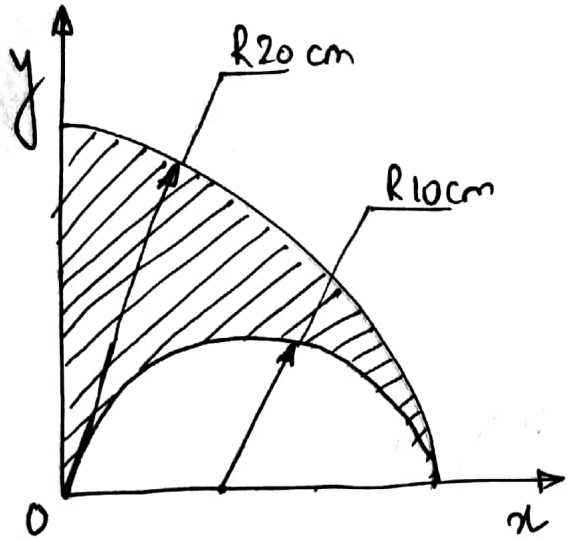
$\Sigma M_D^F = M_D^R$

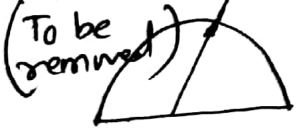
$-(50 \times 2) + (120 \times 4) + (30 \times 1.25) = 137.12 \times x$

$\boxed{x = 3.0447 \text{ m}}$ — [1 M]

$\Sigma M_D^F = 417.5 \text{ kNm. } \circlearrowleft$ — [1 M]

(b)



	SHAPE	Area (cm^2)	$\bar{x}$ (cm)	$\bar{y}$ (cm)	$A_i x_i$ (cm^3)	$A_i \cdot y_i$ (cm^3)
1.		$\frac{\pi(20)^2}{4}$ $= 314.16$	$\frac{4\times(20)}{3\pi}$ $= 8.488$	$\frac{4\times(20)}{3\pi}$ $= 8.488$	2666.59	2666.59
2. (To be removed)		$-\frac{\pi(10)^2}{2}$ $= -157.08$	10	$\frac{4\times(10)}{3\pi}$ $= 4.244$	−1570.8	−666.65

$\Sigma A = 157.08$ [1M]

$\Sigma A x_i = 1095.79$ [1M]

$\Sigma A_i y_i = 1999.94$ [1M]

$$\bar{x} = \frac{\Sigma A_i \cdot x_i}{\Sigma A_i} = \frac{1095.79}{157.08} = \boxed{6.98 \text{ cm}}$$ [1M]

$$\bar{y} = \frac{\Sigma A_i \cdot y_i}{\Sigma A_i} = \frac{1999.94}{157.08} = \boxed{12.732 \text{ cm}}$$ [1M]