FE AM- 1 IAT 1 SOLUTION

FE SEM I IAT 1 SOLUTION

Q.1(a) Let $L = \lim_{x \to y} \dfrac{x^y - y^x}{x^x - y^y}$ $\left(\frac{0}{0} \text{ form}\right)$

Applying L'Hospital's rule

$$L = \lim_{x \to y} \frac{y x^{y-1} - y^x \cdot \log y}{x^x(1+\log x) - 0}$$ 1m

$$L = \lim_{x \to y} \frac{y x^{y-1} - y^x \log y}{x^x(1+\log x)}$$

$$= \frac{y y^{y-1} - y^y \log y}{y^y(1+\log y)}$$

$$L = \frac{1-\log y}{1+\log y}$$ 2m

Q.1(b) Given message 'MOVE' and encoding matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\therefore$ M O V E

13 15 22 5

$$\therefore B = \begin{bmatrix} 13 & 22 \\ 15 & 5 \end{bmatrix}$$ 1m

Encoding given message as

$$C = AB = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 13 & 22 \\ 15 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 27 \\ 15 & 5 \end{bmatrix}$$

$\therefore$ Encoded message is 28, 15, 27, 5. 2m

Q.1(C) Given system of equations

$2x - 3y + 7z = 5$

$3x + y - 3z = 13$

$2x + 19y - 47z = 32$

writing in matrix form

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$ 1m

$-R_1 + R_2$

$$\begin{bmatrix} 1 & 4 & -10 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 32 \end{bmatrix}$$

$R_2 - 3R_1,\ R_3 - 2R_1$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 11 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 16 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 5 \end{bmatrix}$$

$\therefore$ rank$(A) = 3$, rank$([A, B]) = 3$

$\therefore$ rank$(A) \neq$ rank$[A, B]$

$\therefore$ Given system is inconsistent. 2m

Q1(d) $z = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}}$

$\therefore |z| = 1$

$\overset{\theta}{\cancel{\arg(z)}} = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ 1m

$\therefore \arg(z) = \frac{\pi}{\cancel{3}} - \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Exponential form of z is

$z = 1 \times e^{i\pi/3}$

$z = e^{i\frac{2\pi}{3}}$

2m

Q.1(e)

$$A = \begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$$ 1m

Now $AA^{\theta} = \begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix} \begin{bmatrix} 2-i & -2i \\ -2i & 2+i \end{bmatrix}$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$\therefore AA^{\theta} \neq I$

$\therefore$ A is not unitary matrix 2m

Q.1(f)

(i) Formula for Regula-Falsi method

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$ 1m

(ii) Formula for Newton-Raphson method

$$c = a - \frac{f(a)}{f'(a)}$$ 2m

Q.2). let $K_1X_1 + K_2X_2 + K_3X_3 = 0$ — (*) 1m

$K_1(1,3,4,2) + K_2(3,-5,2,6) + K_3(2,-1,3,4) = 0$

$K_1 + 3K_2 + 2K_3 = 0$

$3K_1 - 5K_2 - K_3 = 0$

$4K_1 + 2K_2 + 3K_3 = 0$

$2K_1 + 6K_2 + 4K_3 = 0$

The matrix form, $AX = 0$.

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & -5 & -1 \\ 4 & 2 & 3 \\ 2 & 6 & 4 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ 2m

$$\begin{array}{l} R_2-3R_1 \\ R_3-4R_1 \\ R_4-2R_1 \end{array} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -14 & -7 \\ 0 & -10 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} -\frac{1}{7}R_2, \\ -\frac{1}{5}R_3 \end{array} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3-R_2, \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ 3m

$\therefore$ $K_1 + 3K_2 + 2K_3 = 0$

$2K_2 + K_3 = 0$

Put $K_2 = t$, $K_3 = -2t$

$K_1 = -3K_2 - 2K_3 = -3t + 4t = t$ Since K_1, K_2 & K_3 are not all Zero.

Put K_1, K_2 & K_3 in $(*)$. $\therefore$ vectors are L.D. 4m

$tX_1 + tX_2 - 2tX_3 = 0$

$X_1 + X_2 - 2X_3 = 0$

$$\boxed{X_1 = 2X_3 - X_2}$$

5m

b] Let $A = I_3 A I_4$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 1m

$$\begin{array}{l} R_2-2R_1 \\ R_3-3R_1 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 2m

$$\begin{array}{l} C_2-2C_1 \\ C_3-3C_1 \\ C_4-4C_1 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 3m

$-\frac{C_2}{3}$, $-C_3/2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 2 & 2 & -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2/3 & 3/2 & -4 \\ 0 & -1/3 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~R_3-R_2~~ $R_3 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & \cancel{-6}\ -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2/3 & 3/2 & -4 \\ 0 & -1/3 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 - C_2$, $C_4 + 5C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2/3 & 5/6 & -2/3 \\ 0 & -1/3 & 1/3 & -5/3 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$-\frac{R_3}{12}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1/6 & 1/3 & -1/6 \end{bmatrix} A \begin{bmatrix} 1 & 2/3 & 5/6 & -2/3 \\ 0 & -1/3 & 1/3 & -5/3 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \leftrightarrow C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1/6 & 1/3 & -1/6 \end{bmatrix} A \begin{bmatrix} 1 & 2/3 & -2/3 & 5/6 \\ 0 & -1/3 & -5/3 & 1/3 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4m

$$[I_3 \quad 0] = PAQ.$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1/6 & 1/3 & -1/6 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 2/3 & -2/3 & 5/6 \\ 0 & -1/3 & -5/3 & 1/3 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$|P| = -1/6 \neq 0$ $\qquad$ $|Q| \neq 0$

Rank$(A) = 3$.

5m

Q3

a. $\dfrac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$

By De Moivre's Theorem,

$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ —— (1) 1m

by Binomial Expansion,

$(\cos\theta + i\sin\theta)^5 = {}^5C_0 \cos^5\theta + {}^5C_1 \cos^4\theta \cdot i\sin\theta + {}^5C_2 \cos^3\theta (i\sin\theta)^2 + {}^5C_3 \cos^2\theta (i\sin$

$+ {}^5C_4 \cos\theta \,(i\sin\theta)^4 + {}^5C_5 \,(i\sin\theta)^5$

$= \cos^5\theta + 5\cos^4\theta \cdot i\sin\theta - 10\cos^3\theta\sin^2\theta - 10\cos^2\theta\sin^3\theta\, i$
$+ 5\cos\theta\sin^4\theta + \sin^5\theta\, i$ 2m

$= (\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) +$
$i\,(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ — (2)

from (1), (2)

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ 3m

$\therefore \frac{\sin 5\theta}{\sin\theta} = 5\cos^4\theta - 10\cos^2\theta\sin^2\theta + \sin^4\theta$ 4m

$= 5\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= 5\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

$= 16\cos^4\theta - 12\cos^2\theta + 1$ 5m

b. $x = \frac{7.85 + 0.1y + 0.2z}{3}$

$y = \frac{19.3 - 0.1x + 0.3z}{7}$

$z = \frac{71.4 - 0.3x + 0.2y}{10}$

Iteration No.	$x = \frac{7.85+0.1y+0.2z}{3}$	$y = \frac{19.3-0.1x+0.3z}{7}$	$z = \frac{71.4-0.3x+0.2y}{10}$	
				1m
	0	0	0	2m
1.	2.6167	2.7198	7.1159	3m
2.	3.1817	3.0167	7.1049	
3.	3.1909	3.0161	7.1046	
4.	3.1908	3.016	7.1046	4m

$\therefore x = 3.191,\ y = 3.016,\ z = 7.105$ 5m