EM

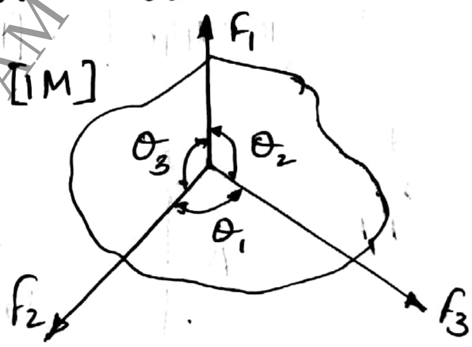
IAT-I SOLUTION

Q1] (a)

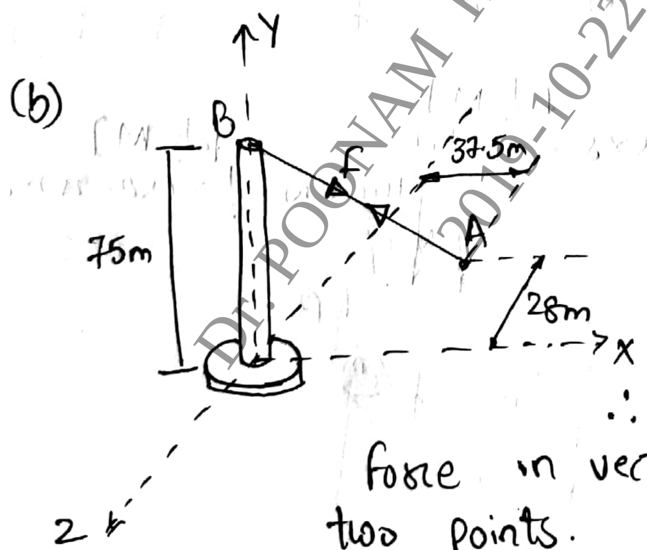
<u>Varignon's Theorem</u> :- "It states that the moment of resultant of all the forces in a plane about any point is equal to the algebraic sum of moment of all the forces about the same point." [1M]

<u>Lami's Theorem</u> :- "If three concurrent coplanar forces acting on a body having same nature (i.e. pulling or pushing) ~~are~~ keep the body in equilibrium, then each force is proportional to the ~~sum~~ sine of angle included between the other two forces."

i.e. $\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$ —[1M]



(b)



F = 650N (passes from B to A)

A (37.5, 0, −28) .. from fig

B (0, 75, 0) .. from fig

$\therefore r_{BA} = +37.5\,i + 75\,j + 28\,k$

Force in vector form passing through two points.

$|r_{BA}| = 88.4$ ~~units~~ m.

$\therefore \hat{e}_{BA} = 0.424\,i - 0.848\,j - 0.317\,j$ [1M]

$$\bar{F} = F \cdot \hat{e}_{BA}$$

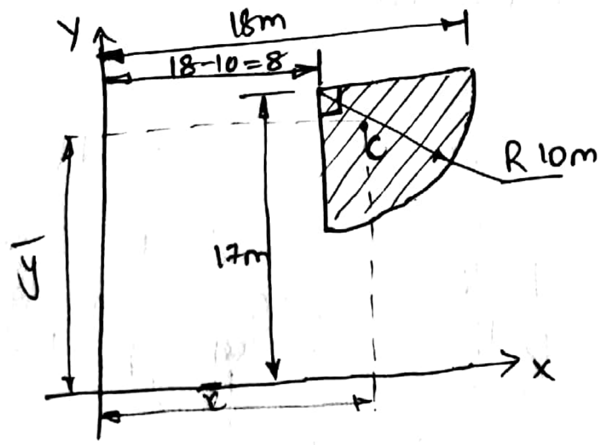
$$= F\left[\frac{(x_2-x_1)\,i + (y_2-y_1)\,j + (z_2-z_1)\,k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

$$= 650\left[\frac{37.5\,i + (-75)\,j + (-28)\,k}{\sqrt{(37.5)^2 + (-75)^2 + (28)^2}}\right]$$

$$\boxed{\bar{F} = 275.72\,i - 551.44\,j + 205.87\,k}$$ — [1M]

1.

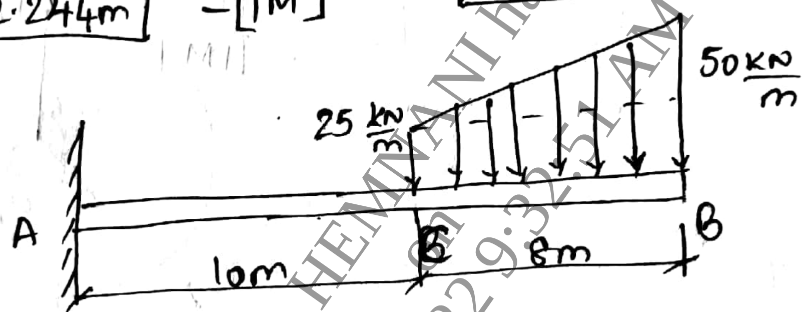
(c)



$$\bar{x} = 8 + \frac{4R}{3\pi}$$
$$= 8 + \frac{4(10)}{3\pi}$$
$$\boxed{\bar{x} = 12.244\,m} \quad -[1M]$$

$$\bar{y} = 17 - \frac{4R}{3\pi}$$
$$\bar{y} = 17 - \frac{4\times(10)}{3\pi}$$
$$\boxed{\bar{y} = 12.756\,m} \quad -[1M]$$

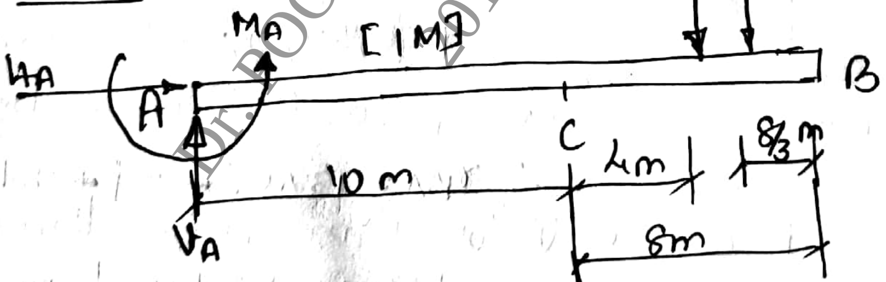
(d)



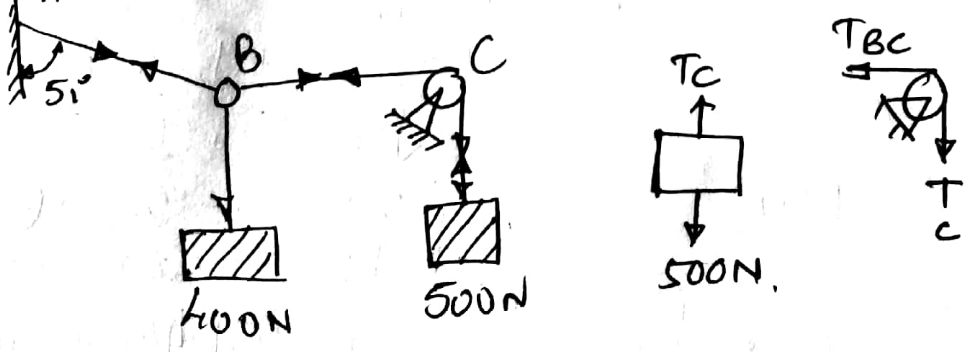
[1/2 M]
$25\times8 = 200\,kN$

[1/2 M]
$100\,kN = \frac{1}{2}\times(50-25)\times8$

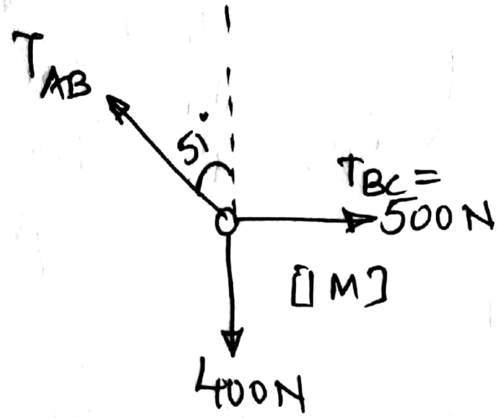
FBD



(e)



FBD at B

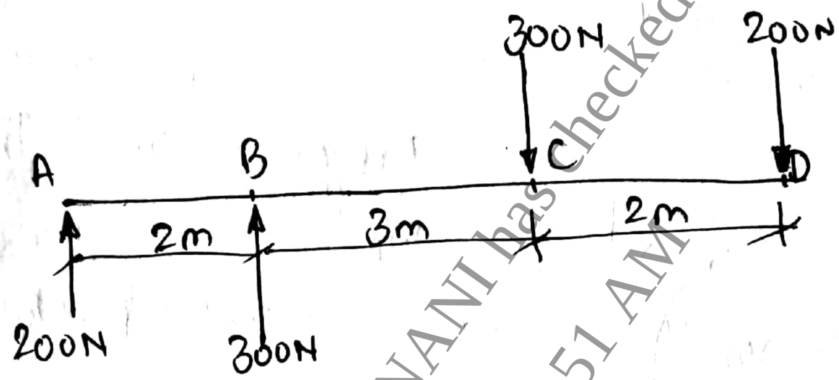


Applying Lami's Theorem,

$$\frac{T_{AB}}{\sin 90} = \frac{400}{\sin(90+51)} = \frac{500}{\sin(180-51)}$$

$\boxed{T_{AB} = 635.6 \text{ N}}$ [1M]

(f)

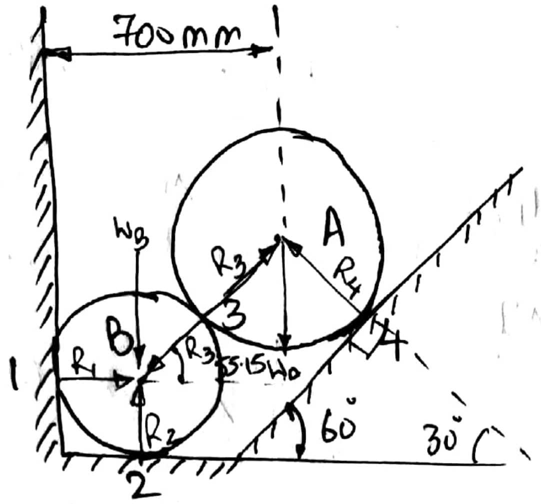


- $\Sigma F_x = 0$
- $\Sigma F_y = 200 + 300 - 300 - 200$
  $= 0$
- $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\boxed{R = 0 \text{ N}}$ —[1M]

$$\Sigma M_A = -300 \times 2 + 300 \times 5 + 200 \times 7$$
$$= 2300 \text{ Nm } \circlearrowright$$ —[1M]

Q2] (a)



$W_A = 1000\ N$

$W_B = 750\ N$

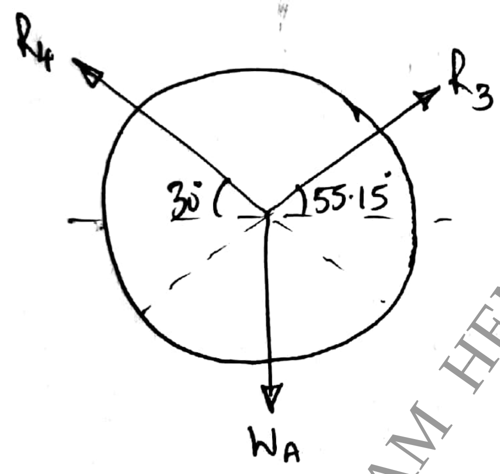
$R_A = 400\ mm$

$R_B = 300\ mm.$

$$\theta = \cos^{-1}\left(\frac{700-300}{300+400}\right)$$

$$\boxed{\theta = 55.15^\circ} \text{ — [1M]}$$

FBD of - A



Applying lami's Theorem,

$$\frac{W_A}{\sin(180-30-55.15)} = \frac{R_4}{\sin(90+55.15)} = \frac{R_3}{\sin(90+30)}$$

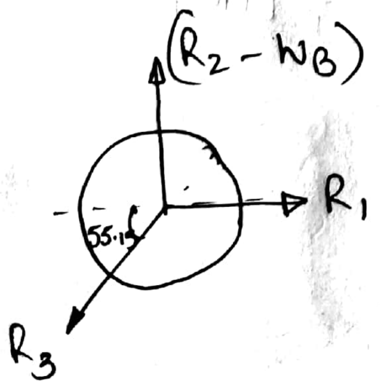
Considering, $\frac{1000}{\sin(94.85)} = \frac{R_4}{\sin(145.15)}$

$$\boxed{R_4 = 573.483\ N} \text{ [1M]}$$

Considering, $\frac{1000}{\sin(94.85)} = \frac{R_3}{\sin(120)}$

$$\boxed{R_3 = 869.137\ N} \text{ [1M]}$$

FBD of B



Applying Lami's Theorem,

$$\frac{R_3}{\sin(90)} = \frac{R_2 - W_B}{\sin(180-55.15)} = \frac{R_1}{\sin(90+55.15)}$$

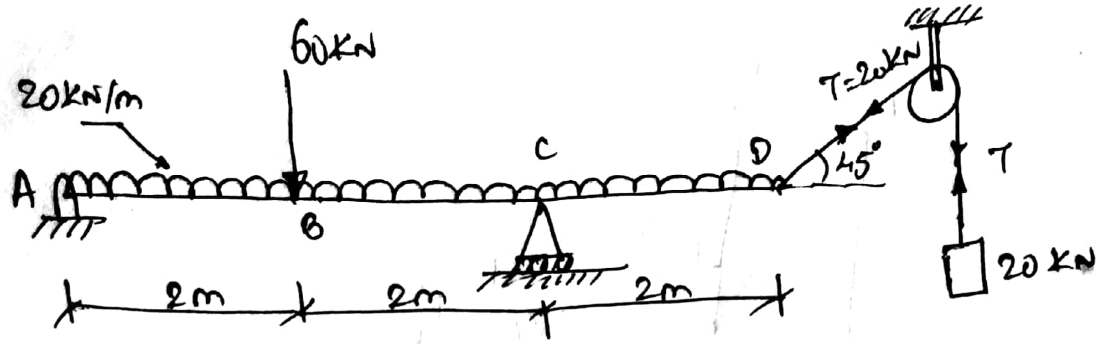
Considering, $\frac{869.137}{\sin 90} = \frac{R_2 - 750}{\sin(124.85)}$

$$\boxed{R_2 = 1463.25\ N} \text{ [1M]}$$

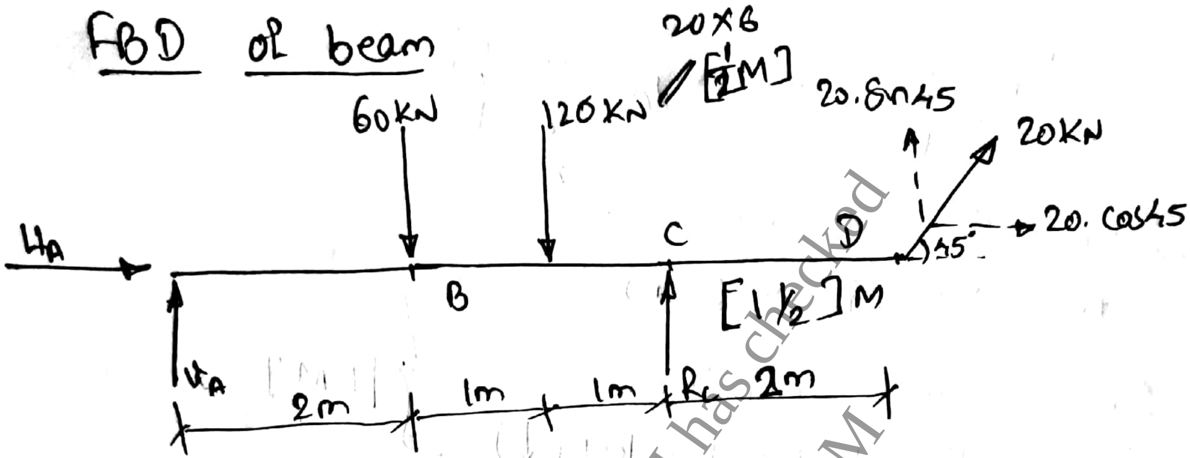
Considering, $\frac{869.137}{\sin 90} = \frac{R_1}{\sin(145.15)} \Rightarrow \boxed{R_1 = 496.64 N}$

[1M]

(b)



FBD of beam



As Beam is in equilibrium,

- $\Sigma F_x = 0, \quad 20\cos(45) + H_A = 0$

$$\boxed{H_A = -14.14 \text{ KN}} \quad \text{— [1M]}$$

$$H_A = 14.14 \text{ KN} \ (\leftarrow)$$

- $\Sigma F_y = 0,$

$$V_A + R_C - 60 - 120 + 20.\sin 45 = 0$$

$$V_A + R_C - 165.858 = 0 \quad \ldots (1)$$

- $\Sigma M_D = 0,$

$$-R_C \times 2 + 120 \times 3 + 60 \times 4 - V_A \times 6 = 0$$

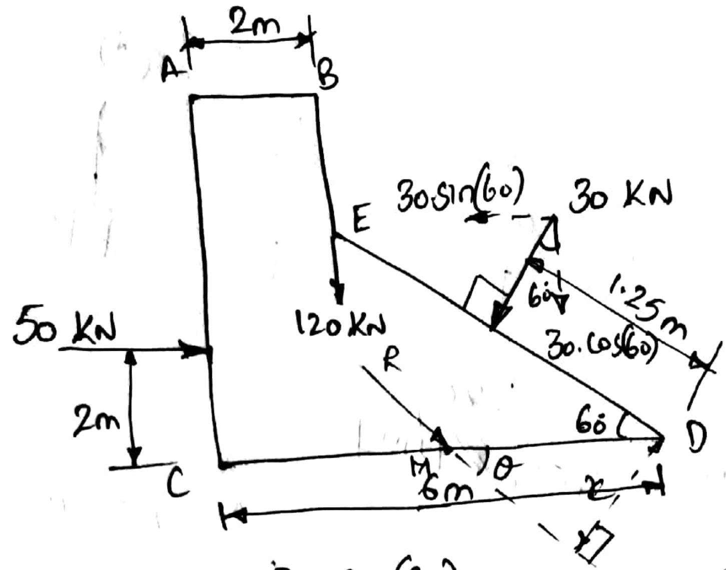
$$-6V_A - 2R_C + 600 = 0 \quad \ldots (2)$$

on solving eqn (1) & (2),

$$\boxed{V_A = 67.071 \text{ KN}} \ (\uparrow) \text{ — [1M]}$$

$$\boxed{R_C = 98.787 \text{ KN}} \ (\uparrow) \text{ — [1M]}$$

Q3] (a)



- $\Sigma F_x = 50 - 30 \cdot \sin(60)$

  $\underline{\Sigma F_x = 24.019 \text{ KN } (\rightarrow)}$

- $\Sigma F_y = -120 - 30 \cdot \cos(60)$

  $\underline{\Sigma F_y = -135 \text{ KN} = 135 \text{ kN } (\downarrow)}$ [1 M]

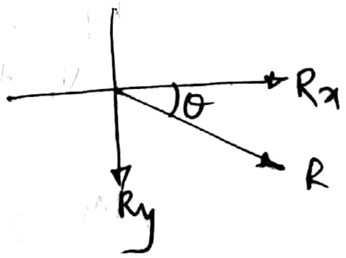
- $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

  $= \sqrt{24.019^2 + (-135)^2}$

  $\boxed{R = 137.12 \text{ KN}}$ — [1 M]

  $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{135}{24.019}\right)$

  $\underline{\theta = 79.91}$ — [1 M]



To find position of Resultant, Applying Varignon's Theorem,

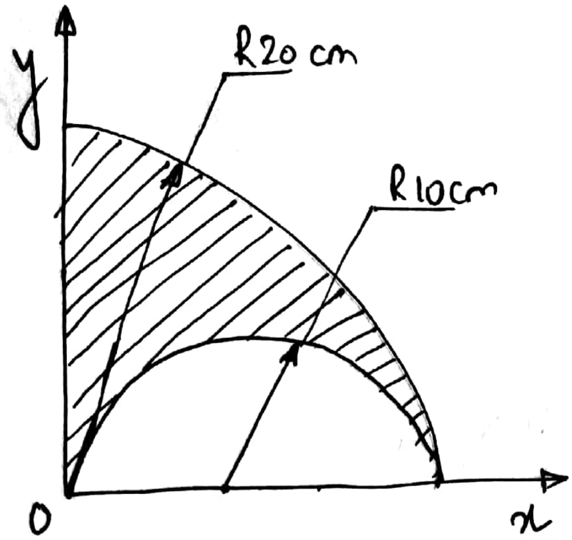
$$\Sigma M_D^F = M_D^R$$

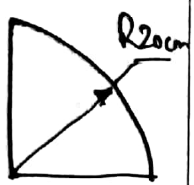
$$-(50 \times 2) + (120 \times 4) + (30 \times 1.25) = 137.12 \times x$$

$$\boxed{x = 3.0447 \text{ m}} \text{ — [1 M]}$$

$$\Sigma M_D^F = 417.5 \text{ kNm } \circlearrowright \text{ — [1 M]}$$

(b)



SHAPE	Area (cm^2)	x (cm)	y (cm)	$A_i x_i$ (cm^3)	$A_i \cdot y_i$ (cm^3)
1. 	$\frac{\pi(20)^2}{4}$ $= 314.16$	$\frac{4\times(20)}{3\pi}$ $= 8.488$	$\frac{4\times(20)}{3\pi}$ $= 8.488$	2666.59	2666.59
2. (To be removed) 	$-\frac{\pi(10)^2}{2}$ $= -157.08$	10	$\frac{4\times(10)}{3\pi}$ $= 4.244$	-1570.8	-666.65

$\Sigma A = 157.08$ [1M] $\quad \Sigma A x_i = 1095.79$ [1M] $\quad \Sigma A_i y_i = 1999.94$ [1M]

$$\bar{x} = \frac{\Sigma A_i \cdot x_i}{\Sigma A_i} = \frac{1095.79}{157.08} = \boxed{6.98\ cm} \text{ [1M]}$$

$$\bar{y} = \frac{\Sigma A_i \cdot y_i}{\Sigma A_i} = \frac{1999.94}{157.08} = \boxed{12.732\ cm} \text{ [1M]}$$