Adams - Bashforth Method
(Predictor - Corrector Formula)

Predictor Formula - Adam Bashforth

$$y_4^p = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$$

Adam Bashforth - Corrector Formula

$$y_4^c = y_3 + \frac{h}{24}\left[9f_4^p + 19f_3 - 5f_2 + f_1\right]$$

Predictor Formula - Adam Bashforth

$$y_1 = y_0 + \frac{h}{24}\left[55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3} \ldots\right]$$

Corrector Formula - Adam Bashforth

$$y_1^{(1)} = y_0 + \frac{h}{24}\left[9f_1 + 19f_0 - 5f_{-1} + f_{-2} \ldots\right]$$

Note :- Jo ~~sbse~~ badi value hoti hai use ko lena hai and so on.

For applying Adams-Bashforth method we need 4 starting values of y which can be calculated by means of Taylor series, Euler's method or Runga-Kutta method. In Runga-Kutta method, Adam Bashforth Predictor and Corrector formula are now useful.

Eg. 5 (Pg 4.48) Given

$$\frac{dy}{dx} = x^2(1+y) \text{ and}$$

$y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$

$y(1.3) = 1.979$

Evaluate $y(1.4)$ by Adams-Bashforth Method

Sol^n $\frac{dy}{dx} = x^2(1+y) = f(x,y)$

$h = 0.1$

$x_0 = 1$ $y_0 = 1$

$x_1 = 1.1$ $y_1 = 1.233$

$x_2 = 1.2$ $y_2 = 1.548$

$x_3 = 1.3$ $y_3 = 1.979$

$$f_0 = x_0^2(1+y_0)$$
$$= 1^2(1+1)$$

$$\boxed{f_0 = 2}$$

$$f_1 = (x_1)^2 (1 + y_1)$$

$$= (1.1)^2 [1 + 1.233]$$

$$= 1.21 \times 2.233$$

$$= 2.70193$$

$$\boxed{f_1 = 2.702}$$

$$f_2 = (x_2)^2 (1 + y_2)$$

$$= (1.2)^2 (1 + 1.548)$$

$$= 1.44 \times 2.548$$

$$= 3.66912$$

$$\boxed{f_2 = 3.6691}$$

$$f_3 = (x_3)^2 (1 + y_3)$$

$$= (1.3)^2 (1 + 1.979)$$

$$1.69 \times 2.979$$

$$= 5.03451$$

$$\boxed{f_3 = 5.0345}$$

Using Adam Bashforth Predictor Formula - ③

$$y_4 = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$$

$$= 1.979 + \frac{0.1}{24}\left[55(5.0345) - 59(3.6691) + 37(2.702) - 9\times 2\right]$$

$$= 1.979 + \frac{0.1}{24}\left[276.8975 - 216.4769 + 99.974 - 18\right]$$

$$= 1.979 + \frac{0.1}{24}\left[376.8715 - 234.4769\right]$$

$$= 1.979 + \frac{0.1}{24}\left[142.3946\right]$$

$$= 1.979 + \frac{14.23946}{24}$$

$$= 1.979 + 0.59331$$

$$= 2.57231$$

$$\boxed{y_4 = 2.5723}$$

Thus, we have,

$x_4 = 1.4 \qquad y_4 = 2.5723$

$f_4 = (1.4)^2 (1 + 2.5723)$

$= 1.96 \times 3.5723$

$= 7.001708$

$\boxed{f_4 = 7.0017}$

Using Adam Bash fouth corrector formula

$y_4^{(c)} = y_3 + \frac{h}{24}\left[9f_4 + 19f_3 - 5f_2 + f_1\right]$

$= 1.979 + \frac{0.1}{24}\left[9 \times 7.0017 + 19(5.0345) - 5(3.6691) + 2.702\right]$

$= 1.979 + \frac{0.1}{24}\left[63.0153 + 95.6555 - 18.3455 + 2.702\right]$

$= 1.979 + \frac{0.1}{24}\left[161.3728 - 18.3455\right]$

$$= 1.979 + \frac{0.1}{24}\left[143.0273\right]$$

$$= 1.979 + \frac{14.30273}{24}$$

$$= 1.979 + 0.59595$$

$$= 2.57495$$

$$\boxed{y_4^{(1)} = 2.5750}$$

Thus at $x_4 = 1.4$, we have

$$y_4 = 2.5750$$

Hence

$$\boxed{y(1.4) = 2.5750}$$

ty-6 Pg 4.50

Using Adams-Bashforth method, obtain the solution of $\frac{dy}{dx} = x - y^2$, at $x = 0.8$ given the values

x	0	0.2	0.4	0.6
y	0	0.0200	0.0795	0.1762

Sol: Here $\frac{dy}{dx} = x - y^2 = f(x, y)$

Taking $h = 0.2$, starting values are

$x_0 = 0$ $\quad y_0 = 0$

$x_1 = 0.2$ $\quad y_1 = 0.0200$

$x_2 = 0.4$ $\quad y_2 = 0.0795$

$x_3 = 0.6$ $\quad y_3 = 0.1762$

$x_4 = 0.8$ $\quad y_4 = ?$

$f_0 = 0 - (y_0)^2$

$\boxed{f_0 = 0}$

(3)

$$f_1 = x_1 - (y_1)^2$$
$$= 0.2 - (0.0200)^2$$
$$= 0.2 - 0.0004$$
$$\boxed{f_1 = 0.1996}$$

$$f_2 = x_2 - (y_2)^2$$
$$= 0.4 - (0.0795)^2$$
$$= 0.4 - 0.0063$$
$$\boxed{f_2 = 0.3937}$$

$$f_3 = x_3 - (y_3)^2$$
$$= 0.6 - (0.1762)^2$$
$$= 0.6 - 0.031005$$
$$= 0.6 - 0.0311$$
$$\boxed{f_3 = 0.5689}$$

Using Predictor formula,

$$y_4 = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[55 \times 0.5689 - 59 \times 0.3937 + 37 \times 0.1996 - 9 \times 0\right]$$

$$y_4 = 0.1762 + \frac{0.2}{24}\left[31.2895 - 23.2283 + 7.3852 - 0\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[38.6747 - 23.2283\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[15.4464\right]$$

$$0.1762 + \frac{3.08928}{24}$$

$$0.1762 + 0.12872$$

$$y_4 = 0.30492$$

$$\boxed{y_4 = 0.3049}$$

Thus, we have

$$x_4 = 0.8 \qquad y_4 = 0.3049$$

$$f_4 = x_4 - (y_4)^2$$

$$= 0.8 - (0.3049)^2$$

$$= 0.8 - 0.0930$$

$$\boxed{f_4 = 0.707}$$

Using the corrector formula, we have (8)

$$y_4^{(1)} = y_3 + \frac{h}{24}\left[9f_4 + 19f_3 - 5f_2 + f_1\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[9 \times 0.1707 + 19 \times 0.5689 - 5 \times 0.3937 + 0.1996\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[6.1363 + 10.8091 - 1.9685 + 0.1996\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[17.3717 - 1.9685\right]$$

$$= 0.1762 + \frac{0.2}{24}\left[15.4032\right]$$

$$= 0.1762 + \frac{3.08064}{24}$$

$$= 0.1762 + 0.12836$$

$$= 0.30456$$

$$\boxed{y_4^{(1)} = 0.3046}$$

Thus we have at

$x_4 = 0.8$, $y_4 = 0.3046$

$$\boxed{y(0.8) = 0.3046}$$

Eg. 7 (Pg 451) Given $\frac{dy}{dx} = x^2 - y$; $y(0) = 1$

and starting values $y(0.1) = 0.90516$,

$y(0.2) = 0.82127$, $y(0.3) = 0.74918$.

Evaluate $y(0.4)$ using Adams Bashforth Method

Sol^n Here $\frac{dy}{dx} = x^2 - y = f(x,y)$

Taking $h = 0.1$, starting values are

$x_0 = 0$ $\quad$ $y_0 = 1$

$x_1 = 0.1$ $\quad$ $y_1 = 0.90516$

$x_2 = 0.2$ $\quad$ $y_2 = 0.82127$

$x_3 = 0.3$ $\quad$ $y_3 = 0.74918$

$x_4 = ?$ $\quad$ $y_4 = ?$

$$f_0 = x_0^2 - y_0$$
$$= 0 - 1$$
$$\boxed{f_0 = -1}$$

$$f_1 = x_1^2 - y_1$$
$$= (0.1)^2 - 0.90516$$
$$= 0.01 - 0.90516$$
$$\boxed{f_1 = -0.89516}$$

$$f_2 = x_2^2 - y_2$$
$$= (0.2)^2 - 0.82127$$
$$= 0.04 - 0.82127$$
$$\boxed{f_2 = -0.78127}$$

$$f_3 = x_3^2 - y_3$$
$$= (0.3)^2 - 0.74918$$
$$= 0.09 - 0.74918$$
$$\boxed{f_3 = -0.65918}$$

Using Predictor formula, we have:

$$y_4 = y_3 + \frac{h}{24}\left[55 f_3 - 59 f_2 + 37 f_1 - 9 f_0\right]$$

$$\cancel{= 0.1762 + \frac{0.2}{24}\left[55 \times 0.5689 - 59 \times 0.3937 + 37 \times 0.1996 - 9 \times 0\right]}$$

$$\cancel{= 0.1762 + \frac{0.2}{24}\left[\right.}$$

$$= 0.74918 + \frac{0.1}{24}\left[55 \times (-0.65918) - 59 \times (-0.78127) + 37 \times (-0.89516) - 9(-1)\right]$$

$$= 0.74918 + \frac{0.1}{24}\left[-36.2549 + 46.09493 - 33.12092 + 9\right]$$

$$= 0.74918 + \frac{0.1}{24}\left[-69.37582 + 55.09493\right]$$

$\therefore 0.74918 + \frac{0.1}{24}\left[-14.28089\right]$ (8)

$= 0.74918 - \frac{1.428089}{24}$

$0.74918 - 0.05950$

$\boxed{y_4 = 0.68968}$

Thus we have

$x_4 = 0.4 \quad y_4 = 0.68968$

$f_4 = (0.4)^2 - 0.68968$

$= 0.16 - 0.68968$

$\boxed{f_4 = -0.52968}$

Using the Corrector Formula,

$y_4^{(1)} = y_3 + \frac{h}{24}\left[9f_4 + 19f_3 - 5f_2 + f_1\right]$

$$= 0.74918 + \frac{0.1}{24}\left[9(-0.52968) + 19(-0.65918) - 5(-0.78127) + (-0.89516)\right]$$

$$= 0.74918 + \frac{0.1}{24}\left[-4.76712 - 12.52442 + 3.90635 - 0.89516\right]$$

$$= 0.74918 + \frac{0.1}{24}\left[3.90635 - 18.18670\right]$$

$$= 0.74918 + \frac{0.1}{24}\left[-14.28035\right]$$

$$= 0.74918 - \frac{1.428035}{24}$$

$$= 0.74918 - 0.05950$$

$$\boxed{y_4^{(1)} = 0.68968}$$

Thus, at $x = 0.4$, $y = 0.68968$

$\Rightarrow \boxed{y(0.4) = 0.68968}$