# Secant Method



The Regula Falsi method is based on the condition $f(x_1) \cdot f(x_0) < 0$ i.e root of eqn $f(x) = 0$ lies in the interval $(x_0, x_1)$.

If we omit the condition $f(x_1) \cdot f(x_0) < 0$, the improved method is known as

Secant Method.

Formula

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Secant Method is improved form of Regula Falsi Method

1. Find a real root of eqn $x^4 - x - 10 = 0$ using Secant Meth upto 3 decimal places

Sol$^n$ Let $f(x) = x^4 - x - 10$

$f(0) = -10$

$f(1) = 1 - 1 - 10 = -10$

$f(2) = (2)^4 - 2 - 10$

$= 4$

Since $f(1)$ and $f(2)$ are of opposite sign. $\therefore$ Root lies b/w 1 & 2

1st Iteration

$x_0 = 1 \quad f(x_0) = -10$

$x_1 = 2 \quad f(x_1) = 4$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{1(4) + 2(10)}{4 + 10}$$

$$= \frac{4 + 20}{14} = \frac{24}{14} = 1.7142\,8$$

$$= 1.7143$$

$f(1.7143) = (1.7143)^4 - 1.7143 - 10$

$= 8.6367 - 1.7143 - 10$

$= -3.0776$

2nd iteration

$x_1 = 2 \quad f(x_1) = 4$

$x_2 = 1.7143 \quad f(x_2) = -3.0776$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{2(-3.0776) - 4(1.7143)}{-3.0776 - 4}$$

$$= \frac{-6.1552 - 6.8572}{-7.0776}$$

$$= \frac{-13.0124}{-7.0776} = 1.8385$$

$f(1.8385) = (1.8385)^4 - 1.8385 - 10$

$= 11.4250 - 1.8385 - 10$

$= -0.4135$

3rd iteration

$x_2 = 1.7143 \quad f(x_2) = -3.0776$

$x_3 = 1.8385 \quad f(x_3) = -0.4135$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1.7143(-0.4135) - 1.8385(-3.0776}{-0.4135 + 0.30776}$$

$$= \frac{-0.7089 + 5.6582}{2.1641}$$

$$= \frac{4.9493}{2.6641} = 1.8578$$

$$f(1.8578) = (1.8578)^4 - 1.8578 - 10$$

$$= 11.91230 - 1.8578 - 10$$

$$= 0.0545$$

4th iteration

$x_3 = 1.8385 \quad f(x_3) = -0.4135$

$x_4 = 1.8578 \quad f(x_4) = 0.0545$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{1.8385 \times 0.0545 + 1.8578 \times 0.4135}{0.0545 + 0.4135}$$

$$= \frac{0.1002 + 0.7682}{0.468}$$

$$= \frac{0.8684}{0.468} = 1.8556$$

$f(1.8556) = (1.8556)^4 - 1.8556 - 10$

$= 11.8560 - 1.8556 - 10$

$= 0.0004$

5th iteration

$x_4 = 1.8578 \quad f(x_4) = 0.0545$

$x_5 = 1.8556 \quad f(x_5) = 0.0001$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.8578(0.0004) - 1.8556(0.0545)}{0.0004 - 0.0545}$$

$$= \frac{0.0007 - 0.1011}{-0.0541}$$

$$= \frac{-0.1004}{-0.0541} = 1.8558$$

∴ The approximate root of eqn $f(x)$ is 1.8558

i.e 1.856 (approx).