Computer Oriented Numerical Methods

Numerical Analysis :-

The branch of mathematics that deals with the development and use of numerical methods for solving problems.

Numerical Analysis is concerned with all aspects of the numerical solution of a problem, from understanding of numerical methods to their practical implementation as reliable and effecient computer programs.

These include the following :-

i) When presented with a problem that cannot be solved directly, they try to replace it with a 'nearby problem' that can be solved more easily.

Examples:

Use of Interpolation in developing numerical integration methods and root finding methods.

ii) There is a fundamental concern with error, its size, and its analytic form. When approximating a problem, it is prudent to understand the nature of the error in the computed solution.

Numerical Method :-

We use numerical method to find approxima solution of problems by numerical calculations with the use of calculator. For better accuracy, we have to minimize the error.

Error = Exact Value - Approximate Value

Why are Numerical Methods necessary? If we can't get exact solutions, then how do we know when our approximate solutions are any good?

⇒ When a problem can be solved exactly and in less time than forever, then it is "analytically solvable".

For Example:

Jack has 2 apples and Jill has 3 apples, how many apples do they have together?", is analytically solvable. It is 5, exactly 5.

In reality "doing math" generally involves finding an answer, rather than the answer. While you may not be able to find the exact answer, you can often find answers with "arbitrary precision". In other words, you can find an approximate

answer and the closer that approximation
will be to the correct answer.

A trick that lets you get closer and closer to an exact answer is a "numerical method". Numerical Methods find solutions close to the answer without even knowing what that answer is.

We need Numerical Methods because a lot of problems are not analytically solvable and we know they work because each seperate method comes packed with a proof that it works.

A major advantage of numerical method is that a numerical solution can be obtained for problems, where an <u>analytical solution</u> does not exist.

Ch-2 Solution of Algebraic and Transcendental ①
equations

Introduction :-

The equations of the form $f(x)=0$ are called Algebraic or Transcendental according as $f(x)$ is purely a polynomial in x or contains some other functions such as trigonometric, logarithmic and exponential functions.

For Example:

The equations

$$3x^5+4x^4+7x^3+9=0$$

and

$$1+2\cos x-3x=0$$

are algebraic and transcendental equations respectively.

If $f(x)$ is a quadratic, cubic or a biquadratic expression, then algebraic formulae are available for expressing the roots in terms of the coefficients. On the other hand, when $f(x)$ is a polynomial of higher degree or an expression involving transcendental functions, then algebraic methods are not available and recourse must be taken to find the roots by approximate methods.

Continuation or Permanence of Sign :-

If terms of a polynomial are written in descending or ascending order and if a positive sign is followed by a positive sign or a negative sign is followed by negative sign, then the continuation or a permanence of signs is said to occur.

For Example:

In the polynomial

$$x^5 - 6x^4 - 2x^3 + 7x^2 + 9x + 5 \quad — (1)$$

Continuation of sign occurs at $-2x^3$, $9x$ and 5.

Hence, there are 3 continuations of signs,

Variation or Change of Sign :-

If in a polynomial, a positive sign is followed by a negative sign, or a negative sign is followed by a positive sign, then a variation or change of sign is said to occur.

For Example:

In Polynomial — (1), the variation of sign occurs at $-6x^4$, $7x$

...ould be noted that in case of complete ...nomial, sum of continuation and variations signs is equal to degree of polynomial.

In the above polynomial, the number of continuations of signs is 3 and number of variations of signs is 2. Now, sum of continuation and variations of signs is 5, which is equal to degree of polynomial