Gauss Seidel Method – Working Rule

i) Consider the system of linear equations

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

ii) Solve given equations for x, y, z

$$x = \frac{1}{a_1}\left[d_1 - b_1y - c_1z\right] \quad — ①$$

$$y = \frac{1}{b_2}\left[d_2 - a_2x - c_2z\right] \quad — ②$$

$$z = \frac{1}{c_3}\left[d_3 - a_3x - b_3y\right] \quad — ③$$

iii) First Approximation

Put $y = z = 0$ into ① and find $x = x_1$

Put $x = x_1$ and $z = 0$ into ② and find $y = y_1$

Put $x = x_1$ and $y = y_1$ into ③ and find $z = z_1$

Second approximation

Put $y = y_1$ and $z = z_1$ into (1) and find $x = x_2$

Put $x = x_2$ and $z = z_1$ in (2) and find $y = y_2$

Put $x = x_2$ and $y = y_2$ in (3) and find $z = z_2$

iv) Repeat above steps until the required solution is obtained

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(2)

Apply Gauss-Seidel Iteration Method to solve the following equations :-

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Solⁿ Solving each equation for the unknown having larger coefficients, we have,

$$x = \frac{1}{20}[17 - y + 2z] \quad \text{— (1)}$$

$$y = \frac{1}{20}[-18 - 3x + z] \quad \text{— (2)}$$

$$z = \frac{1}{20}[25 - 2x + 3y] \quad \text{— (3)}$$

For first Iteration -

Let initial approximation be $y = 0$ and $z = 0$.

Substitute values in (1), we have

$$x^{(1)} = \frac{1}{20}[17 - 0 + 2(0)] = \frac{17}{20}$$

$$= 0.8500$$

Now Putting $x = x^{(1)}$, $z = 0$ in ②, we get

$$y^{(1)} = \frac{1}{20}[-18 - 3 \times 0.8500 + 0]$$

$$= \frac{1}{20}[-18 - 2.55]$$

$$= \frac{-20.55}{20} = -1.0275$$

Putting $x = x^{(1)}$ and $y = y^{(1)}$ in ③, we get

$$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)]$$

$$= \frac{1}{20}[25 - 1.7 - 3.0825]$$

$$= \frac{1}{20}[20.2175]$$

$$\boxed{z^{(1)} = 1.0109}$$

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for Second Iteration :-

Put $y = y^{(1)}$ and $z = z^{(1)}$ in (1),

$$x^{(2)} = \frac{1}{20}\left[17 - (-1.0275) + 2(1.0109)\right]$$

$$= \frac{1}{20}\left[17 + 1.0275 + 2.0218\right]$$

$$= \frac{1}{20}\left[18.0275 + 2.0218\right]$$

$$= \cancel{\frac{1}{20}\left[16.0057\right]} \Rightarrow \frac{1}{20} \times 20.0493$$

$$\boxed{x^{(2)} = 1.0025}$$

Put $x = x^{(2)}$ and $z = z^{(1)}$ in (2)

$$y^{(2)} = \frac{1}{20}\left[-18 - 3(1.0025) + 1.0109\right]$$

$$= \frac{1}{20}\left[-18 - 3.0075 + 1.0109\right]$$

$$= \frac{1}{20}\left[-21.0075 + 1.0109\right]$$

$$= \frac{(-19.9966)}{20} = -0.9998$$

Put $x = x^{(2)}$ and $y = y^{(2)}$ in (3)

$$z^{(2)} = \frac{1}{20}\left[25 - 2 \times 1.0025 + 3(-0.9998)\right]$$

$$= \frac{1}{20}\left[25 - 2.005 - 2.9994\right]$$

$$= \frac{1}{20}\left[19.9956\right]$$

$$\boxed{z^{(2)} = 0.9998}$$

For third iteration,

Put $y = y^{(2)}$ and $z = z^{(2)}$ in (1)

$$x^{(3)} = \frac{1}{20}\left[17 + 0.9998 + 2 \times 0.9998\right]$$

$$= \frac{1}{20}\left[17 + 0.9998 + 1.9996\right]$$

$$= \frac{1}{20} \times 19.9994$$

$$= 0.99997$$

$$x^{(3)} \Rightarrow 1.0000 \text{ (approx)}$$

(4)

Put $x = x^{(3)}$ and $z = z^{(2)}$ in (1)

$$y^{(3)} = \frac{1}{20}\left[-18 - 3(1.0000) + 0.9998\right]$$

$$= \frac{1}{20}\left[-18 - 3.0000 + 0.9998\right]$$

$$= \frac{1}{20}\left[-21.0000 + 0.9998\right]$$

$$= -\frac{20.0002}{20} = -1.0000$$

$$\boxed{y^{(3)} = -1.0000}$$

Put $x = x^{(3)}$ and $y = y^{(3)}$ in (2)

$$z^{(3)} = \frac{1}{20}\left[25 - 2(1.0000) + 3(-1.0000)\right]$$

$$= \frac{1}{20}\left[25 - 2.0000 - 3.0000\right]$$

$$= \frac{1}{20}\left[25 - 5.0000\right]$$

$$= \frac{20.0000}{20} = 1.0000$$

$$\boxed{z^{(3)} = 1.0000}$$

Since the values obtained in second and third iterations are very close, we can stop the process and hence the solution is $\boxed{x=1}$, $\boxed{y=1}$, $\boxed{z=1}$

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