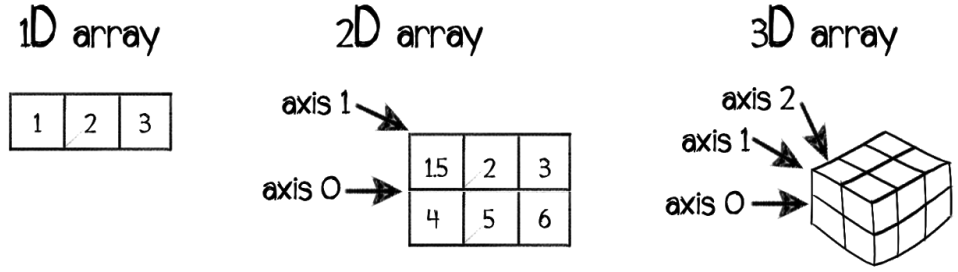
What is NumPy?

It stands for "Numerical Python". `NumPy` is a Python module that provides fast and efficient array operations of homogeneous data. It is the core library for scientific computing in Python providing a high-performance multidimensional array object, and tools for working with arrays.

NumPy is one of the many packages that are extremely essential in your data science journey because this library equips you with an array data structure that offers some benefits over the traditional data structures of Python like lists.

NumPy Arrays

The central feature of NumPy is the array object class, also called the ndarray. Arrays are very similar to lists in Python, except that every element of an array must be of the same type (in lists you can hold data which have different types), typically a numeric type like `float` or `int`. It is very much similar to an n-dimensional matrix which looks like:



Arrays make operations with large amounts of numeric data very fast and are generally much more efficient than lists. You can chose to create arrays of *n* dimensions (Python list is an array of pointers to Python objects, at least 4 bytes per pointer plus 16 bytes for even the smallest Python object; 4 for type pointer, 4 for reference count, 4 for value and the memory allocators rounds up to 16. A NumPy array is an array of uniform values -- single-precision numbers takes 4 bytes each, double-precision ones, 8 bytes).

Creating NumPy arrays

The syntax of creating a NumPy array is:

```
numpy.array(object, dtype = None, copy = True, order = None, subok = False, ndmin = 0)
```

Here, the arguments

- `object`: Any object exposing the array interface
- `dtype`: Desired data type of array, optional
- `copy`: Optional. By default (true), the object is copied
- `order`: C (row-major) or F (column-major) or A (any) (default)
- `subok`: By default, returned array forced to be a base class array. If true, sub-classes passed through
- `ndim`: Specifies minimum dimensions of the resultant array

Let's see how you can create a simple array using NumPy by first importing the package `numpy as np`

```
import numpy as np
a = np.array([1,2,3,4])                # creates a 1-dimensional array
b = np.array([[1,2,3,4], [5,6,7,8]])    # creates a 2-dimensional array
print(a)
print('----')
print(b)
```

Its output will be

```
[1 2 3 4]
----
[[1 2 3 4]
 [5 6 7 8]]
```

Advantages of using NumPy

- Absolutely free since open-sourced
- Faster access in reading and writing items
- Time and space complexity of tasks is much lower when compared with traditional data structures
- Has a lot of built-in functions for linear algebra

Attributes of NumPy arrays

Now that you know how to create a NumPy array, let us look at the most essential features of one and discuss them in details. We will be taking two arrays to illustrate the features

```
a = np.array([1,2,3,4])
b = np.array([[1,2,3,4], [5,6,7,8]])
```

The attributes of both the arrays `a` and `b` are discussed below:

Shape

It returns a tuple consisting of array dimensions i.e. tells us how many items are present in each dimension and can be found using the `.shape` the attribute of the `ndarray` object.

```
print('The shape of the array a is ', a.shape)
print('The shape of the array b is ', b.shape)
```

Output

```
'The shape of the array a is (4,)'
'The shape of the array b is (2,4)'
```

Note that the 1-D array `a` has a shape of (4,) and not (4, 1)

Dimensions

It gives the number of dimensions and can be found using the `.ndim` the attribute of `ndarray` object.

```
print('The dimensions of array a is ', a.ndim)
print('The dimensions of array b is ', b.ndim)
```

Output

```
'The dimensions of array a is 1'
'The dimensions of array b is 2'
```

Size

It tells the total number of items in the array as a whole. More precisely it is the product of the elements of the `.shape` the attribute of the array.

```
print('The size of the array a is ', a.size)
print('The size of the array b is ', b.size)
```

Output

```
'The shape of the array a is 4'
'The shape of the array b is 8'
```

Datatype

As the name suggests, it informs about the type of data in the array. Since a NumPy array consists of homogeneous data only, you will get only a single `dtype`.

```
print('The datatype of the array a is ', a.dtype)
print('The datatype of the array b is ', b.dtype)
```

Output

```
'The datatype of the array a is int64'
'The datatype of the array b is int64'
```

NumPy offers support to a much greater variety of numerical types than base Python does like `int8`, `int16`, `float32`, `float16`, `bool_`, `complex_` etc.

Itemsize

It represents the number of bytes in each element of the array.

```
print('The number of bytes in each element of the array a is  ', a.itemsize)
print('The number of bytes in each element of the array b is ', b.itemsize)
```

Output

```
'The number of bytes in each element of the array a is 8'
'The number of bytes in each element of the array b is 8'
```

With the help of the following attributes, you can get the necessary information of the NumPy array as a whole along with its elements.

Creating New NumPy Arrays (Low Level ndarray Constructor)

You already know to create a NumPy array using the `numpy.array()` command. Now, let's look at some of the other ways to make NumPy arrays taking the help of the low-level `ndarray` constructor (we will be using `np` as an alias for `numpy`).

`np.empty()`

Creates an uninitialized (arbitrary) array of specified shape and dtype

```
np.empty((3,4),dtype='int8')
```

Output

```
array([[  0,   0,   0,   0],
       [  0,   0,   0,  96],
       [124,   0, -65,  21]], dtype=int8)
```

`np.zeros()`

Creates a new array of specified size, filled with zeros

```
np.zeros((3,4),dtype='int8')
```

Output

```
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)
```

`np.ones()`

Creates a new array of specified size and type, filled with ones

```
np.ones((3,4),dtype='int8')
```

Output

```
array([[1, 1, 1, 1],
       [1, 1, 1, 1],
       [1, 1, 1, 1]], dtype=int8)
```

`np.full()`

Creates a new array of given shape and type, filled with a constant value

```
np.full((2,2),7)
```

Output

```
array([[7, 7],
       [7, 7]])
```

`np.eye()`

Creates a 2-D array with ones on the diagonal and zeros elsewhere

```
np.eye(3)
```

Output

```
array([[1., 0., 0.],
       [0., 1., 0.],
       [0., 0., 1.]])
```

Example:

- Create an array out a 4x4 identity matrix with dtype `float32` and save it in a variable named `array`

```
import numpy as np
# identity 4x4 matrix
array = np.eye(4,dtype='float32')
print(array)
# display
```

Arrays with Existing Data

New NumPy arrays can also be created from already existing ones. Let us look at some ways of doing so.

`np.asarray( )`

This command is very similar to the np.array() command. Some examples are:

- Converting a list to array

```
#python list
a=[1,2,3]
#convert to NumPy array
b=np.asarray(a)
print(b)
```

Output

```
[1 2 3]
```

- Converting tuples into array

```
#python tuples
a=((1,2),(3,4))

#convery to NumPy array
b=np.asarray(a)
print(b)
```

Output

```
[[1 2]
 [3 4]]
```

`np.fromiter( )`

This function creates a NumPy ndarray from an iterable. For example:

```
a=np.fromiter([1,2,3,4],dtype='int8')
b=np.fromiter((1,2,3,4),dtype='int8')
c=np.fromiter(range(1,5),dtype='int8')
d=np.fromiter('string',dtype='S50')

print("Array a is ",a)
print("Array b is ",b)
print("Array c is ",c)
print("Array d is ",d)
```

Output

```
Array a is  [1 2 3 4]
Array b is  [1 2 3 4]
Array c is  [1 2 3 4]
Array d is  [b's' b't' b'r' b'i' b'n' b'g']
```

Here we are creating arrays from list `a`, tuple `b`, `range( )` and finally a string `d`.

Creating new arrays

In vector mathematics, it is necessary to generate a set of numbers within some predefined range. You can create them easily with the help of some NumPy functions. Let's look at some of them.

`np.arange( )`

It returns an array containing evenly spaced values within a given range.

Syntax :`numpy.arange(start, stop, step, dtype).`

Here, numbers are created in the range of `[start, stop-1]`. We can also specify steps and data type using the `step` and `dtype` hyperparameters.

```
# NumPy array from 1 to 19
print(np.arange(1,20,dtype='int32'))

# NumPy array from 1 to 19 with step size 2
print(np.arange(1,20,2,dtype='int8'))
```

Output

```
[ 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19]
[ 1  3  5  7  9 11 13 15 17 19]
```

In the above example, we first created an array from 1 to 19 with step size 1 and dtype `int32` and in the second one we created one from 1 to 19 with step size 2 and dtype `int8`.

`np.linspace( )`

It also returns an array within a range but not according to the step size as in the case of `.arange()` but according to the number of values we want within that range.

Syntax: `numpy.linspace(start, stop, num, endpoint, retstep, dtype)`

Here, `start` and `stop` means the same as in `.arange( )` but the difference lies in `num`, which gives us the number of equally spaced numbers you want to insert within the range `[start, stop-1]`. The `endpoint` argument generally is by default set to the `stop` value (try changing it for some interesting results)

```
# NumPy array from 1 to 20 with 100 numbers in between
print(np.linspace(1,20,100))
```

Output

```
[ 1.          1.19191919  1.38383838  1.57575758  1.76767677  1.95959596
  2.15151515  2.34343434  2.53535354  2.72727273  2.91919192  3.11111111
  3.3030303   3.49494949  3.68686869  3.87878788  4.07070707  4.26262626
  4.45454545  4.64646465  4.83838384  5.03030303  5.22222222  5.41414141
  5.60606061  5.7979798   5.98989899  6.18181818  6.37373737  6.56565657
  6.75757576  6.94949495  7.14141414  7.33333333  7.52525253  7.71717172
  7.90909091  8.1010101   8.29292929  8.48484848  8.67676768  8.86868687
  9.06060606  9.25252525  9.44444444  9.63636364  9.82828283 10.02020202
 10.21212121 10.4040404  10.5959596  10.78787879 10.97979798 11.17171717
 11.36363636 11.55555556 11.74747475 11.93939394 12.13131313 12.32323232
 12.51515152 12.70707071 12.8989899  13.09090909 13.28282828 13.47474747
 13.66666667 13.85858586 14.05050505 14.24242424 14.43434343 14.62626263
 14.81818182 15.01010101 15.2020202  15.39393939 15.58585859 15.77777778
 15.96969697 16.16161616 16.35353535 16.54545455 16.73737374 16.92929293
 17.12121212 17.31313131 17.50505051 17.6969697  17.88888889 18.08080808
 18.27272727 18.46464646 18.65656566 18.84848485 19.04040404 19.23232323
 19.42424242 19.61616162 19.80808081 20.        ]
```

In the above example, we have created 100 evenly spaced numbers in the range `[1, 20]`

`np.logspace( )`

This function returns an array containing numbers that are evenly spaced on a log scale. Start and stop endpoints of the scale are indices of the base, usually 10.

Syntax: `numpy.logspace(start, stop, num, endpoint, base, dtype)`

Here, the range of values is

$$[{base}^{start}, {base}^{stop}]$$

$$[base$$

$$start$$

$$,base$$

$$stop$$

] with `num` being the number of equally spaced values on log scale within the range.

```
# NumPy array from 10^0 to 10^2 with 100 numbers in log scale
print(np.logspace(0,2,100))
```

Output

```
[  1.          1.04761575   1.09749877   1.149757     1.20450354
   1.26185688   1.32194115   1.38488637   1.45082878   1.51991108
   1.59228279   1.66810054   1.7475284    1.83073828   1.91791026
   2.009233     2.10490414   2.20513074   2.3101297    2.42012826
   2.53536449   2.65608778   2.7825594    2.91505306   3.05385551
```

```
   3.19926714   3.35160265   3.51119173   3.67837977   3.85352859
   4.03701726   4.22924287   4.43062146   4.64158883   4.86260158
   5.09413801   5.33669923   5.59081018   5.85702082   6.13590727
   6.42807312   6.73415066   7.05480231   7.39072203   7.74263683
   8.11130831   8.49753436   8.90215085   9.32603347   9.77009957
  10.23531022  10.72267222  11.23324033  11.76811952  12.32846739
  12.91549665  13.53047775  14.17474163  14.84968262  15.55676144
  16.29750835  17.07352647  17.88649529  18.73817423  19.6304065
  20.56512308  21.5443469   22.5701972   23.64489413  24.77076356
  25.95024211  27.18588243  28.48035868  29.8364724   31.2571585
  32.74549163  34.30469286  35.93813664  37.64935807  39.44206059
  41.320124    43.28761281  45.34878508  47.50810162  49.77023564
  52.14008288  54.62277218  57.22367659  59.94842503  62.80291442
  65.79332247  68.92612104  72.20809018  75.64633276  79.24828984
  83.02175681  86.97490026  91.11627561  95.45484567 100.        ]
```

In the example above, there are 100 values in the range of `[10^0, 10^2]`.

Indexing

You now know how to create different types of NumPy arrays and check their features. But how about accessing a particular value or taking a chunk of values from the array itself? In this topic, we are going to discuss exactly that. Like Python lists, the index starts at 0 for arrays as well.

Array indexing and slicing are exactly similar like Python indexing and slicing. It follows the same pattern of `array[start:stop: step]`. Let us look at an example to observe this behaviour.

```
a = np.array([[1,2,3],[4,5,6],[7,8,9]])



# Pull out second element of third row

print(a[2][1])

print('==========')

# Pull out first two rows and columns

print(a[:2,:2])

print('==========')

# Pull all elements of the third row

print(a[2,:])
```

Output

```
8

==========
```

```
[[1 2]

 [4 5]]

==========

[7 8 9]
```

Integer array indexing

Integer array indexing allows you to construct arbitrary arrays using the data from another array. Let us understand from the example

```
# An example of integer array indexing

a=np.array([[1,2],[3,4],[5,6]])



print(a[[0,1,2],[0,1,0]])

print('==========')



print(np.array([a[0,0],a[1,1],a[2,0]]))

print('==========')



print(a[[0,0],[1,1]])

print('==========')



print(np.array([a[0,1],a[0,1]]))
```

Output

```
[1 4 5]

==========

[1 4 5]

==========

[2 2]

==========

[2 2]
```

Explanation: The `print` statements in line numbers 4 and 7 yields the same result, likewise in lines 10 and 13. In the first case, `a[[0, 1, 2], [0, 1, 0]]` essentially means we are indexing the value in first row-first column, second row-second column and third row-first column, which is the same as `a[0, 0], a[1, 1], a[2, 0]]`. Similarly, you should be able to deduce the logic behind the second case.

Boolean indexing

This type is generally used for comparison purposes. For ex: How about checking if how many numbers in the array are greater than 50(say)? It can be performed using a simple comparison operator (>=, >, ==, <, <=)

A boolean index array is of the same shape as the array-to-be-filtered and it contains only `True` and `False` values. You can filter those you want using the concept of masking. For ex: If for some array `a` and boolean condition `condition = a > 2`, `a[condition]` will result in an array that contains only the numbers in array `a` that is greater than 2.

Let us look at an example below:

```
a = np.array([[4,7,1],[2,5,7],[7,1,1]])



# Boolean condition for values greater than 3

mask = a > 3

print(mask)



# Masking for the above boolean condition in the array

print(a[mask])
```

Output

```
[[ True  True False]

 [False  True  True]

 [ True False False]]

[4 7 5 7 7]
```

Example:

- Create a (4,) array with values 3, 4.5, 3 + 5j and 0 using "`np.array()`". Save it to a variable `array`

- Create a boolean condition `real` to retain only a real number using `.isreal(array)`. (Note: `.isreal(array)` returns a Boolean value which is `True` if the number inside the array is a real number and `False` otherwise)
- Now apply this Boolean condition i.e. `real` on `array` using Boolean indexing (explained in the topic) by `array[real]` and store it in variable `real_array`.
- Similarly, create a Boolean condition `imag` to retain only complex numbers which you can do it using `.iscomplex(array)`. This time create an array `imag_array` which contains only complex numbers using the Boolean condition `array[imag]`

```
# Code starts here
import numpy as np
# initialize array
array = np.array([3,4.5,3+5j,0])
real =  np.isreal(array)
real_array = array[real]
imag =  np.iscomplex(array)
imag_array = array[imag]
```

What is vectorization?

Vectorization is the ability of NumPy by which we can perform operations on entire arrays rather than on a single element. When looping over an array or any data structure in Python, there's a lot of overhead involved. Vectorized operations in NumPy delegate the looping internally to highly optimized C and Fortran functions, making for a cleaner and faster Python code.

Examples

You have already come across such an example in Boolean indexing.

```
import numpy as np

a = np.array([1,2,3,4,5,6,7])

print(a[a > 2])
```

The above codeblock will output the array `[3, 4, 5, 6, 7]` as it compares each element being greater than or less than 2.

Vectorized operations

No, let us look at how you can do some elementary vectorized operations like addition, subtraction, multiplication etc. Images below depict the type of operations and their corresponding output.

Addition

Two ways to go about it, using either + or `np.add()`

```
a = np.array([[1,2,3],[4,5,6],[7,8,9]])

b=np.array([[10,11,12],[13,14,15],[16,17,18]])


# element wise addition

print(a+b)

print('==========')

print(np.add(a,b))
```

Output

```
[[11 13 15]

 [17 19 21]

 [23 25 27]]

==========

[[11 13 15]

 [17 19 21]

 [23 25 27]]
```

Subtraction

Two ways, - or `np.subtract()`

```
# element wise subtractions

print(a-b)

print('==========')

print(np.subtract(a,b))
```

Output

```
[[-9 -9 -9]

 [-9 -9 -9]

 [-9 -9 -9]]
```

```
==========
[[-9 -9 -9]
 [-9 -9 -9]
 [-9 -9 -9]]
```

Multiplication

Two ways, `*` or `np.multiply()`

```
# element wise multiplication
print(a*b)
print('==========')
print(np.multiply(a,b))
```

Output

```
[[ 10  22  36]
 [ 52  70  90]
 [112 136 162]]
==========
[[ 10  22  36]
 [ 52  70  90]
 [112 136 162]]
```

Division

Two ways, `/` or `np.divide()`

```
# element wise division
print(a/b)
print('==========')
print(np.divide(a,b))
```

Output

```
[[0.1        0.18181818 0.25      ]
 [0.30769231 0.35714286 0.4       ]
 [0.4375     0.47058824 0.5       ]]
==========
[[0.1        0.18181818 0.25      ]
 [0.30769231 0.35714286 0.4       ]
 [0.4375     0.47058824 0.5       ]]
```

Square root transform

Use `np.sqrt()`

```
# element wise square root transform
a = np.array([[1,4,9],[16,25,36]])
print(np.sqrt(a))
```

Output

```
[[1. 2. 3.]
 [4. 5. 6.]]
```

Log transform

Use `np.log()`

```
# element wise square root transform
a = np.array([[1,4,9],[16,25,36]])
print(np.log(a))
```

Output

```
[[0.         1.38629436 2.19722458]
 [2.77258872 3.21887582 3.58351894]]
```

Aggregrate operations

Aggregration operations are those where we perform some operation on the entire array. Some commonly used aggregrate operations are listed below:

Command	Description
a.sum()	Array-wise sum
a.min()	Array-wise minimum value
a.max(axis=0)	Maximum value of an array row
a.cumsum(axis=1)	Cumulative sum of the elements
a.mean()	Mean
np.median(a)	Median
np.corrcoef(a)	Correlation coefficient
np.std(a)	Standard deviation

Array comparison

You already saw how you can perform element-wise comparison of array elements. With NumPy you also perform entire array comparisons. Use the command `np.array_equal()` for array comparison. It is illustrated with examples below:

```
a = np.array([1,2,3,4])

b = np.array([1,2,3,4])

print(np.array_equal(a,b))
```

Output

```
True
```

```
a = np.array([1,2,3,4])

b = np.array([5,6,7,8])
```

```
print(np.array_equal(a,b))
```

Output

```
False
```

Understanding Axes notation

In NumPy, an axis refers to a single dimension of a multidimensional array. By changing `axis` you can compute across dimensions, whereas not specifying `axis` will result in computation over the entire array.

```
a = np.array([[1,4,9],[16,25,36]])


# computes sum over columns

print(a.sum(Axis=0))

print('==========')


# computes sum over rows

print(a.sum(axis=1))

print('==========')


# computes total sum

print(a.sum())
```

Output

```
[17 29 45]

==========

[14 77]

==========

91
```

In the image above you can calculate the sum over the rows, columns and the entire array just by playing around the parameter `axis`. Try it more on arrays with 3 or more dimensions!

Example:

Buy/ Sell for maximum profit

In this task, you will combine vectorization and aggregation methods to solve a BUY-SELL problem. You have a range of prices at 3 intervals of a day for 2 consecutive days and you can buy and sell only once. First, you will buy and then sell for the maximum profit.

- Initialize an array `prices = [[40, 35, 20], [21, 48, 70]]`. Here, 40 is the market price during the first interval of the first day and 35 during the second interval and so on. Similarly, 21 is the price during the first half of the second day and 48 during the second half
- Flatten the array with the `.flatten()` method. This method will convert your 2-D array into a 1-D array and store it in variable `prices`.
- Find the minimum over the flattened array using `np.min()`, this will be your `buying_price`
- Also find the index of buying price by first converting the array into a list using `list(array)` and then use the `.index(buying_price)` attribute to pick the index of the buying price and store it in a variable `index`
- Create a new subset starting from the index (created in the previous step) till the end of the array and find the maximum value using `np.max()` in it. This will be your `selling_price`
- Find the difference between buying and selling price and save the same in variable `profit`

```
import numpy as np
# initialize array for prices
prices = np.array([[40, 35, 20], [21, 48, 70]])
# Code starts here
# flatten the array
prices = prices.flatten()
print(prices)
# minimum price
buying_price = np.min(prices)
print(buying_price)
# index of buying price
index = list(prices).index(buying_price)
print(index)
# create subset
prices1 =prices[index:]
print(prices1)
# selling price
selling_price = np.max(prices1)
print(selling_price)
# profit
profit = selling_price - buying_price
print(profit)
# display
```

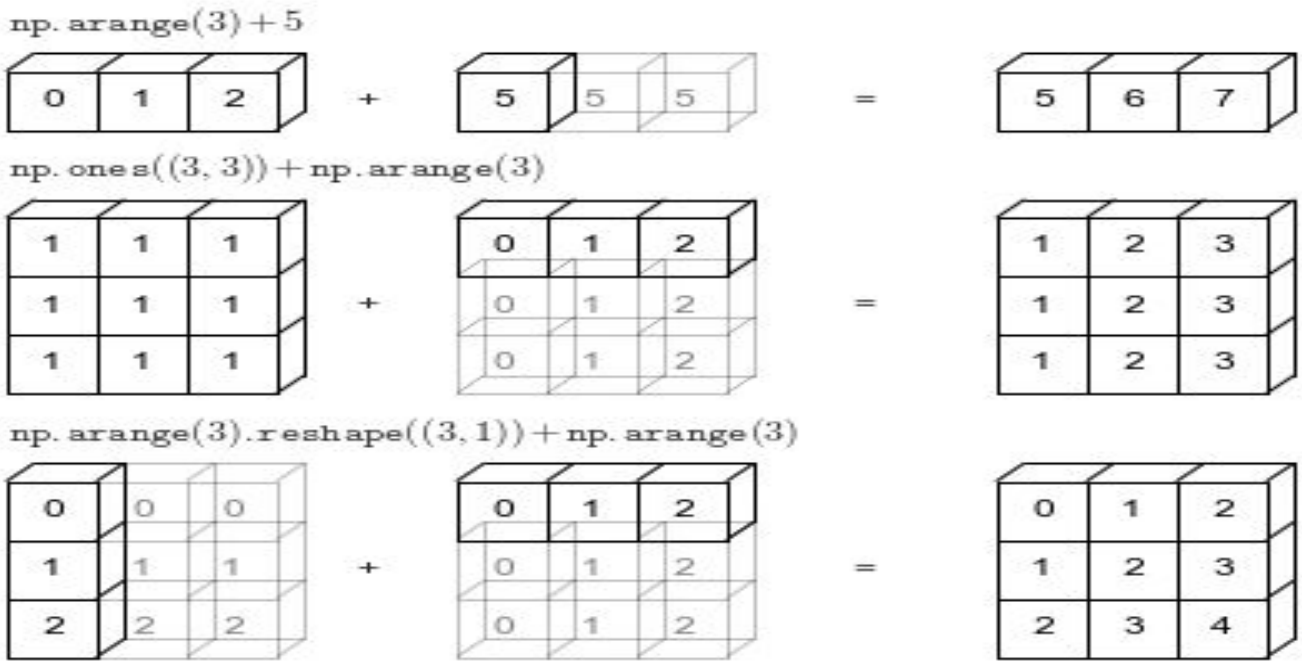
What is broadcasting?

Have you wondered how this operation `np.array([1, 2, 3]) + 4` was successfully carried out? It was all due to the broadcasting power of NumPy arrays. Let's discuss this property in details.

In NumPy you do not need arrays to be of the same shape while performing operations among them until these conditions are satisfied:

- If the arrays do not have the same rank, prepend the shape of the lower rank array with 1s until both shapes have the same length.
- The two arrays are said to be compatible in a dimension if they have the same size in the dimension, or if one of the arrays has size 1 in that dimension.
- The arrays can be broadcast together if they are compatible in all dimensions.
- After broadcasting, each array behaves as if it had shape equal to the element-wise maximum of shapes of the two input arrays.
- In any dimension where one array had size 1 and the other array had a size greater than 1, the first array behaves as if it were copied along that dimension.

Visual intuition of broadcasting



Example:
Normalize a 5x5 matrix

In this task, you will normalize i.e. subtract the minimum value and divide by the range

- Create a random 5x5 matrix with the help of `np.random.random((5,5))` and save it as `z`
- Calculate the minimum and maximum value with the help of `.min()` and `.max()` methods respectively for the 5x5 array. Save them as `zmax` and `zmin` for maximum and minimum respectively

- Now using the power of broadcasting subtract the minimum value from each element and divide by their range (maximum-minimum) to normalize. Save this normalized as `z_norm`
- Print the standardized array

```
import numpy as np
np.random.seed(21)

# Code starts here
z = np.random.random((5,5))
# create random 5x5 array
zmin = np.min(z)
zmax = np.max(z)
# minimum and maximum values
z_norm = z - zmin / (zmax - zmin)
# normalize
# display
```

Similarities between NumPy arrays and lists

NumPy arrays and Python lists are in fact very similar to one another in many aspects. In fact, Numpy arrays with more than a single dimension can be thought of as nested lists. Other common similarities are:

- Both are used for storing data
- Both are mutable
- Both can be indexed and iterated through
- Both can be sliced

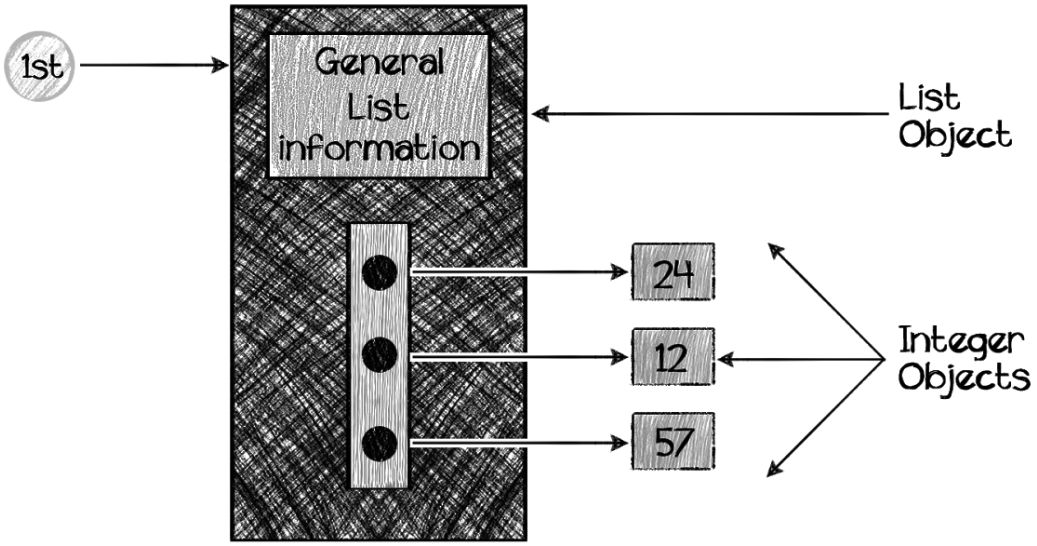
Advantages of using NumPy

Numpy data structures perform better in:

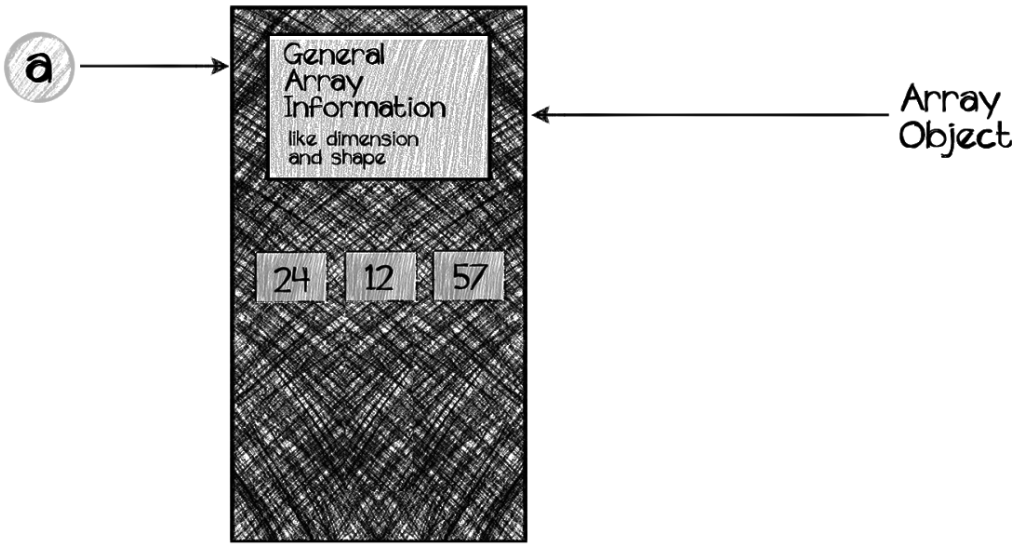
- `Size`: Numpy data structures take up less space
- `Performance`: They have a need for speed and are faster than lists
- `Functionality`: NumPy has optimized functions such as linear algebra operations built-in.

Memory power of NumPy

- `For Python Lists` Let us assume a case where we add only integers to the list. For every new element, we need another eight bytes for the reference to the new object. The new integer object itself consumes 28 bytes. The size of a list `lst` can be calculated with:
- $64 + 8\times len(lst) + \text{size of the elements} + 28\times len(lst)$
- $64+8\times len(lst)+\text{size of the elements}+28\times len(lst)$



- `For NumPy arrays`: NumPy takes up less space. This means that an arbitrary integer array of length n in NumPy needs: 96 + 8n bytes. So more the numbers you need to store; the better you do with NumPy arrays.



Speed of NumPy vs Python lists

Code snippet Speed comparison while performing element-wise addition of two Python lists and element-wise addition of two NumPy.

```
np.random.seed(21)

# import packages
import time
import numpy as np

# initialize variable
num = 10000

# initialize lists
l1, l2 = [i for i in range(num)], [i+2 for i in range(num)]

# initialize arrays
a1, a2 = np.array(l1), np.array(l2)

# start time
start_list = time.time()

# element-wise addition for both lists
sum_lists = [i+j for i, j in zip(l1, l2)]

# stop time
stop_list = time.time()

# display time
print(stop_list - start_list)


# start time
start_array = time.time()

# element-wise addition for both arrays
sum_arrays = a1 + a2

# stop time
stop_array = time.time()

# display time
print(stop_array - start_array)
```

When the above code was run, it displayed the time taken for list operation as `0.0020928382873535156` seconds against `0.00028586387634277344` seconds with NumPy arrays. So, NumPy arrays are more suited for this kind of operations.

A Parting Thought: Don’t Over-Optimize

When you are working with large data, it's important to optimize code with the help of NumPy. However, there is a subset of cases where avoiding a native Python for-loop isn’t possible. But always remember, "Premature optimization is the root of all evil." Programmers may incorrectly predict where in their code a bottleneck will

appear, spending hours trying to fully vectorize an operation that would result in a relatively insignificant improvement in runtime.

There's nothing wrong with for-loops sprinkled here and there. Often, it can be more productive to think instead of optimizing the flow and structure of the entire script at a higher level of abstraction.

Example: 1 : Find Roots of a Quadratic Equation

- The quadratic equation that you will be solving is $x^2 - 4x + 4 = 0$
- Make an array `coeff` to store the coefficients (1,-4, 4) of the quadratic equation
- Pass this array `coeff` as argument to `np.roots()` method which will return the roots for the quadratic equation $x^2 - 4x + 4 = 0$.Save it as `roots`
- Print out `roots` to see your result

```
# import packages
import numpy as np
# Code starts here
coeff = np.array([1,-4,4])
# co-efficients of x
roots = np.roots(coeff)
print(roots)
```

Example 2 : Solving System of Equations with NumPy

One of the more common problems in linear algebra is solving a matrix-vector equation. Here is an example. We seek the vector x that solves the equation Ax = b where A and b are matrices and we are required to find the matrix x which satisfies the equation.

The usual way to go forward is to first inverse matrix A, and then multiply it with the transpose of b (if it is not a column matrix). NumPy provides a simple method `.linalg.solve()` to find the matrix x.
You have a system of a linear equation

$$Ax = b$$

$Ax=b$ where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{ and } b = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$$

- Define variables `A` and `b` as shown above

- Use `.linalg.solve()` to solve for the system of linear equation. Go through the documentation for numpy.linalg.solve to better understand its behaviour. Save this result as `x`
- Check that your solution is correct with the function `np.allclose(np.dot(A,x), b)` and save it to a variable `check`. Print out `check` to see if it is `True` or else try again!

```
# Code starts here
import numpy as np
# initialize matrix A and b
A = np.array([[2,1,2],[3,0,1],[1,1,-1]])
b = np.array([[-3],[5],[2]])
print(A)
print(b)
# Solve for x
x = np.linalg.solve(A,b)
print(x)
check = np.allclose(np.dot(A,x), b)
print(check)
# Check solution
```

Example 3 : Find a 3x3 Non Singular Matrix

Take a pen and paper for this task to find a 3×3 nonsingular matrix A satisfying $3A=A^2+AB$

$$B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Find a non singular matrix

- Assume that `A` is a non-singular matrix
- Resolve the equation $3A = A^2 + AB$ until it reduces to a simpler form for you to carry out simple array addition. Array A should be coming as $A=3I-B$ where `I` is the Identity matrix
- Initialize array `B` using `np.array()` and `I` using `np.identity()`
- Save resultant array as `A` which is given mathematically by $A = 3I - B$ and Display matrix `A`

```
import numpy as np
# initialize array B and Identity matrix
B = np.array([[2,0,-1],[0,2,-1],[-1,0,1]])
I = np.identity(3)
A = 3*I - B
print(A)
# calculate result
# Code ends here
```