Preliminary Spectral Analysis of Spectroscopic Binary HIP-61732

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1 Analysis of Radial Velocity Data: Spectroscopic Binaries

The spectral data available from the spectroscopic binaries can be used to obtain the radial velocities of the components using Doppler red-shift. Analysis of the radial velocity data reveals many parameters describing the system. One major drawback is that unless the spectral data is complemented with photometric data, the inclination and hence a few properties of the system remain undetermined.

1.1 The Data

We use the system HIP61732 (RA:189.80°, Dec:+16.51°) for analysis. the observations were performed at the T193 telescope of the Haute-Provence Observatory, with the SOPHIE spectrograph. The data was used for analysis in Halbwachs et al. (2020) and we use their results to compare my results.

1.2 The Equations

We have the radial velocities of the binary components as a function of their true anomalies. The observational data obtained is the velocity at certain time instants. Hence, for the analysis we use the following equations connecting the radial velocity with time:

$$\frac{2\pi}{P}(t-T) = E - e\sin E$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$v_{LOS} = K[\cos(\theta+\omega) + e\cos\omega] + V_\gamma$$

The above equations are the modelling equations using which we try to fit the data. Once these parameters are obtained, we can go on to determine the minimum masses and projected semi major axes. As obtained in Celestial Mechanics by J.B. Tatum,

$$K = \frac{2\pi a\sin i}{P\sqrt{1-e^2}} \tag{1}$$

Thus, we get projected semi major axis of each component as:

$$a_{1,2}\sin i = \frac{K_{1,2}P\sqrt{1-e^2}}{2\pi} \tag{2}$$

According to the Kepler's third law we have:

$$\begin{aligned} P^2 &= \frac{4\pi^2(a_1+a_2)^3}{G(m_1+m_2)} \\ \Rightarrow m_1+m_2 &= \frac{4\pi^2(a_1+a_2)^3}{GP^2} \end{aligned} \tag{3}$$

We know that the masses are inversely proportional to the semi-major axes, thus we get:

$$\begin{aligned} m_1 + \frac{m_1K_1}{K_2} &= \frac{4\pi^2 \times \frac{P^3(1-e^2)^{\frac{3}{2}}}{8\pi^3\sin^3 i}(K_1+K_2)^3}{GP^2} \\ \Rightarrow \frac{m_1(K_1+K_2)}{K_2} &= \frac{P(1-e^2)^{\frac{3}{2}}(K_1+K_2)^3}{2\pi G\sin^3 i} \end{aligned}$$

And thus we have the minimum masses:

$$m_{1,2}\sin^3 i = \frac{P(1-e^2)^{\frac{3}{2}}K_{2,1}(K_1+K_2)^3}{2\pi G} \tag{4}$$

Since, radial velocity curves provide no information regarding the inclination, we obtain just the lower limit of the masses, that is, $m \sin i^3$.

1.3 Algorithm

We employ Nonlinear Least Squares methodology to fit the radial velocity data and obtain the best fit parameters. Let the modelling equation be defined as $v_{cal}(t)$ defined by the parameters $(P, T, e, K, V_\gamma, \omega)$ and v_i be the data point obtained via observations at time t_i. The task is to minimise the sum of square of differences to obtain the best fit parameters. The sum S is defined as:

$$S = \sum_{i=1} [v_i - v_{cal}(t_i)]^2 \tag{5}$$

The minima occurs when,

$$\frac{\partial S}{\partial P} = \frac{\partial S}{\partial T} = \frac{\partial S}{\partial e} = \frac{\partial S}{\partial K} = \frac{\partial S}{\partial V_\gamma} = \frac{\partial S}{\partial \omega} = 0 \tag{6}$$

Since these partial derivatives are functions of both independent variable and parameters, a closed form solution does not exist. Instead, initial values must be chosen for the parameters. Then, the parameters are refined by iterating, that is, the values are obtained by successive approximation.

The above algorithm is implemented by `scipy.optimize.curve_fit` function of Python3 which we used in analysing the radial velocity data and obtain the best fit parameters. We used the Lomb-Scargle Periodogram (VanderPlas 2018), a classic method for finding periodicity in irregularly-sampled data, to obtain the initial guess for P, which gave a fairly good value, usually within 5% error range on providing a rough range in which P would be lying. The periodogram can be implemented through `astropy.timeseries.LombScargle` class in Python3. Our curve fit code was sensitive to the initial guess value of eccentricity hence we iterated from 0 to 1 at the step of 0.1, providing at each iteration the respective value as the guess for eccentricity and finally obtain the best fit parameters.

1.4 Best Fit Results

On giving the range for period P as 300 to 900 days, the LombScargle Periodogram gave the approximate value of period as 583.828 days. Using that, the following best-fit parameters were obtained:

Parameter		Our Result	Halbwachs et al. (2020)
Period($days$)		595.214	595.18 ± 0.20
Systemic Velocity(km/s)		-15.774	-15.956 ± 0.032
Eccentricity		0.3394	0.3393 ± 0.0019
Argument of Periastron($degs$)		64.584	64.85 ± 0.67
Time of Periastron Passage($days$)		117.581	118.62
Semi-Amplitudes(km/s)	K_1	9.207	9.197 ± 0.021
	K_2	13.057	13.068 ± 0.034
Projected axes(Gm)	$a_1 \sin i$	70.895	70.81 ± 0.17
	$a_2 \sin i$	100.540	100.61 ± 0.27
Minimum Masses($M_\odot$)	$m_1 \sin^3 i$	0.3323	0.3326 ± 0.0022
	$m_2 \sin^3 i$	0.2343	0.2341 ± 0.0014

Table 1: Comparison of our results with the values given in Halbwachs et al. (2020)

Note: Due to periodic motion, several values of time of periastron passage(T) will give the same RV values. This happens for values of $T-P$, T, $T+P$, $T+2P$ and so on. Therefore, we mention here the $TmodP$ value which tallies and provides confirmation that our result is correct.

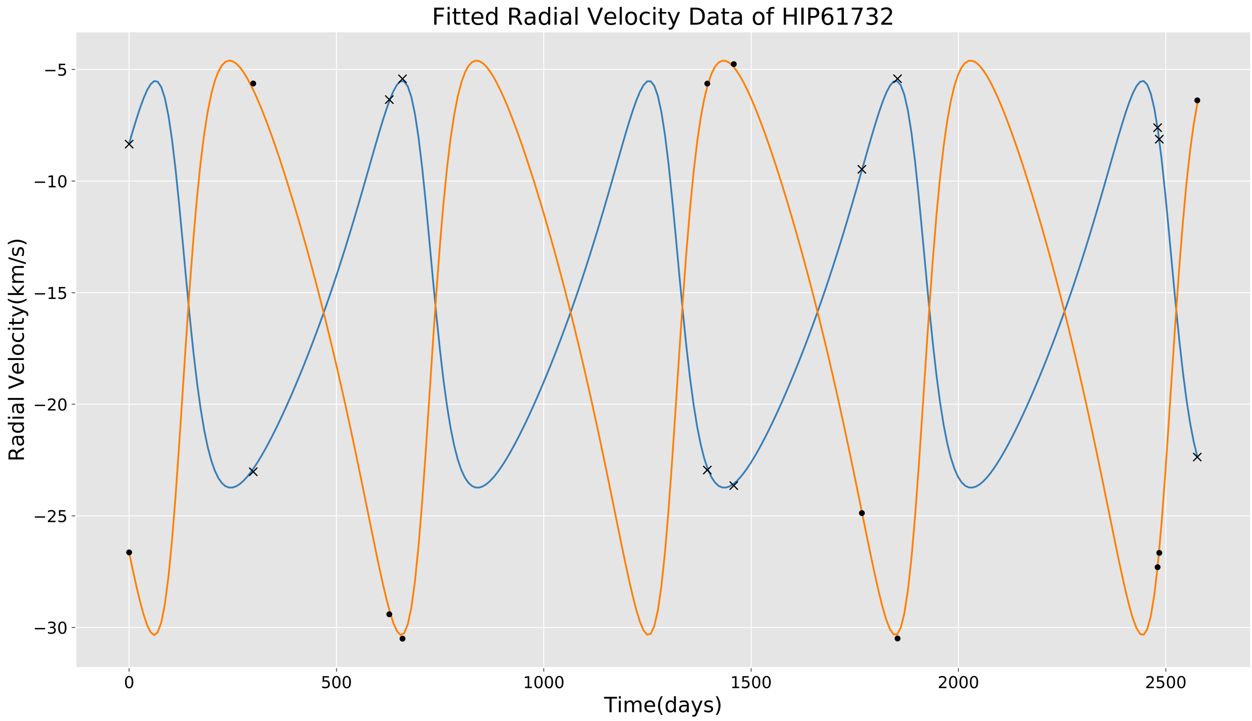

Figure 1: The radial velocity curve plotted using obtained parameters. The crosses and dots represent the observed data of individual component of the system.