COSM Important Questions

Unit-1 : Important Questions

1.A random variable has following probability function

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find i) k ii)P(X<6) iii)P(0<X<5) iv)P(0≤X≤4) v)distribution function F(X) vi)mean vii)variance viii)mode

2.For continuous probability function f(x) = kx^2e^{-x} , $x \geq 0$

Find i)k ii)mean iii)variance iv)P($-\frac{1}{2} < X < \frac{1}{2}$)

3. In a bolt factory machines A,B,C manufacture 20%,30% and 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from i) Machine A ii) Machine B iii) Machine C

4. A businessman goes to hotels X, Y, Z, 20%,50%,30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X,Y,Z hotels have faulty plumbing's, what is the probability that businessman's room having faulty plumbing is assigned to hotel Z ?

5. Box 'A' contains 5 red and 3 white marbles and Box 'B' contains 2 red and 6 white marbles. If a marble is drawn from each box, what is the probability that they are both of same color?

6. If the probability density functions of a random variable X is given by

$f(x) = 2kxe^{-x^2}\ x > 0\ and\ f(x) = 0\ otherwise$. Determine $(i)k\ (ii)F(x)$

7. Two dice are thrown. Let X assign to each point (a,b) in S the maximum of its numbers. (i.e.) X (a, b) = max: (a,b). Find the probability distribution. X is a random variable with X(S) = {1, 2, 3, 4, 5,6}.Find the mean and variance and also calculate P(1< x < 6).

8. A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the Expected number E of defective items.

9. The probabilities that students A,B,C,D solve a problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}, \frac{1}{4}$ respectively. If all of them try to solve the problem, what are the probabilities that the problem is solved?

10. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X, Y). Also compute the marginal and conditional distributions of X and Y. Check whether the random variables X and Y are independent or not.

11. Given the joint density function $f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & , \quad otherwise \end{cases}$

Find the marginal densities of X and Y and the conditional density of X given Y.

12. Probability Density function of Continuous Random variable X is given by f(x)=$\begin{cases} 3x^2, 0 < x < 1 \\ 0, \quad otherwise \end{cases}$

Find a,b such that $(i) P(X \leq a) = P(X > a)$ $(ii)\ P(X > b) = 0.05$

13. Suppose a continuous random variable X has the probability density function

$f(x) = k(1 - x^2) for\ 0 < x < 1\ and\ f(x) = o\ otherwise$. Find i) K ii) Mean ii) Variance.

Unit-2 : Important Questions

1. Ten coins are tossed simultaneously. Find the probability of getting (i) atleast 7 heads.
   (ii) atmost 7 heads.

2. A manufacturer of cotter pins known that 5% of his product is defective pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet the guaranteed quality?

3. Three cards are drawn at random successively with replacement from a well shuffled pack of 52 cards. Getting a card of diamonds is termed as success. Obtain the probability distribution of the number of success.

4. If X is a Poisson variate such that P(X=2)=9P(X=4)+90P(X=6) find P(X>=2).

5. If the probability of success is 1/100, how many trails are necessary in order that probability of at least on success is greater than 1/2?

6. The Joint probability density of continuous random variable is given by

$f(x,y) = \begin{cases} 8xy, 0 \le x \le y \le 1 \\ 0, \quad otherwise \end{cases}$, find COV(X, Y).

7. if X,Y are random variables variances $\sigma_x^2 = 2, \sigma_y^2 = 4$ & covariance $\sigma_{xy} = -2$ then find the variance of random variable $Z = 3x - 4y + 8$.

8. Use Chebyshev's Theorem , a Random variable X has mean $\mu = 8$ & $variance\ \sigma^2 = 9$ for a probability distribution ,

Then find (i) p(-4<x<20) (ii) $P(|X - 8|) \ge 6)$

9. 6 dice are thrown simultaneously 720 times, getting 3 or 4 is considered as successes. Find the number of times at least 3 dice to show 3 or 5.

10.Find mean & Variance of Geometric Distribution with $P(x) = \frac{1}{2^x}$, x=1,2,3,………

11.Derive Mean & Variance of Geometric Distribution

12. Derive Mean & Variance of Binomial Distribution

13.Derive mean & Variance of Poission Distribution

Unit-3

1. In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution.
2. If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kgs, how many students have masses (i) Greater than 72 kgs (ii) Less than or equal to 64 kgs (iii) Between 65 and 71 kgs inclusive
3. In a sample of 1000 cases, the mean of a certain test is 14 and the standard deviation is 2.5. Assuming the distribution to be normal, find (i) How many students score between 12 and 15? (ii) How many score above 18? (iii) How many score below 18?
4. Define Exponential, Gamma and Uniform distribution with their mean and variance ?
5. A population consists of five numbers 2,3,6,8 and 11.Consider all possible samples of size

two which can be drawn without replacement from this population. Find a)The mean of the population. b)The standard deviation of the population. c)The mean of the sampling distribution of means. d)The standard deviation of the sampling distribution of means.

6. A normal population has a mean of 0.1 and standard deviation of 2.1.Find the probability that mean of a sample size 900 will be negative.
7. What happens to the Standard error if the sample size is 1) increased from 50 to 100, 2) decreased from 200 to 50?
8. Explain characteristics of t-distribution, F-distribution.
9. A random sample of size 100 is taken from an infinite population having the mean 76 and the variance 256. What is the probability that $\bar{x}$ will be between 75 and 78.
10. The marks obtained in mathematics by 1000 students is normally distributed, with mean 78% and standard deviation 11%. Determine
    (i) How many students got marks above 90%
    (ii) What was the highest mark obtained by the lowest 10% of the students?
    (iii)Within what limits did the middle of 90% of the students lie.

Unit-4 :

1. A sample of 400 items is taken from population whose S.D. IS 10 .the mean of sample is 40. Test whether the sample has come from a population with mean 38.Also calculate 95% confidence interval for the population.
2. The research investigator is interested in studying whether there is a significant difference in the salaries of MBA grades in two metropolitan cities. A random sample of size 100 from Mumbai yields on average income of Rs. 20,150. Another random sample of 60 from Chennai results in an average income of Rs. 20,250. If the variances of both the populations are given as $\sigma^2{}_1$ = Rs. 40,000 and $\sigma^2{}_2$ = Rs. 32,000 respectively.
3. Find 95% confidence interval for mean of normal distribution with variance 0.25 using sample of n=100 values with mean 212.3.
4. In two large populations, there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations.
5. A Coin was tossed 400 and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.
6. A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance.
7. In a random sample of 160 workers exposed to a certain amount of radiation 24 experienced some ill effects. Construct 99% confidence interval for corresponding true percentage.
8. Define following (i) Null Hypothesis (ii) Alternative hypothesis (iii) Type-I & Type-II Errors (iv) Unbiased estimator (v) Good Estimator (vi) Most Efficient Estimator.
9. A random sample of size 100 has a standard deviation 5. What is maximum error with 95% confidence.
10. What is the size of the smallest sample required to estimate unknown proportion to within a maximum error of 0.06 with atleast 95% confidence.
11. Two horses A & B were tested according to the time(sec's) to run a particular track with following results.

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether the two horses have same running capacity?

Unit-5 :

1. Define (i) Stochastic matrix (ii) Regular matrix (iii) Absorbing state (iv) Irreducible state (v) Ergodic state.
2. if the transition probability matrix is $\begin{pmatrix} 0 & 0.2 & x \\ x & 0.1 & y \\ 0.1 & 0.2 & z \end{pmatrix}$ then find the values of x,y and z.
3. Three boys A,B,C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. S.T. the process is Markovian. Find the transition matrix and classify the states.
4. The TPM of Markov chain $\{X_n\}$, with n=1,2,3...... having 3 states 1,2,3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7 \quad 0.2 \quad 0.1)$.
   Find (i) $P(X_2 = 3)$, and (ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.
5. House wife buys three kinds of cereals A,B,C. She never buy the same in successive weeks. If she buys cereal A ,next week she buys cereal B. However she buys B or C next she is three times as likely to buy A as other cereal, How often she buys each of the three cereals.
6. (a) Define (i) Markov chain (ii) Absorbing markov chain
   (b) Find the equilibrium vector of $\begin{bmatrix} 0.25 & 0.75 \\ 0.5 & 0.5 \end{bmatrix}$
7. If the transition probability matrix of a markov chain is $\begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$ Find
   (a) $P[X_3=1/X_1=0]$ (b) $P[X_3=1/X_0=0]$ (c) $P[X_3=2/X_0=1]$
8. If the transition probability matrix is $\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$ and the initial probabilities are $\left(\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}\right)$ then find: (a) the probabilities after three periods (b) Equilibrium vector.
9. The transition probability matrix of a markov chain is given by $\begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$. Is this matrix irreducible?
10. If the transition probability matrix of shares of three brands A, B, C is $\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.8 & 0.1 & 0.1 \\ 0.35 & 0.25 & 0.4 \end{bmatrix}$ and the initial market shares are 50%, 25% and 25% . Find :
    (i) The Market Shares in second and third periods.
    (ii) The limiting probabilities.