Instructions

1. All answers are to be written on one side of a single A4 sheet (could be a single page from your notebook also). We will refer to this as the **answer sheet**.
2. All Qs are either MCQs or fill-in-the-blanks.
3. Each Q carries 2 marks.
4. For MCQ Qs, write down the Q number followed by the option (A,B,C,D)
5. For fill-in-the-blanks Qs just write down the Q number followed by the answer (the answer could be a scalar, a vector or a matrix)
6. On the **answer sheet**, we only expect you to write the answer and not provide any explanations
7. You can write the explanations separately in rough sheets
8. By 9:50 am, you will take a photo of your **answer sheet** and upload them on gradescope.
9. By 10:00 am, you need to upload the **rough sheets**.

Questions

1. Write down a 5×5 singular matrix whose null space is orthogonal to its column space. (Or explain why such a matrix can never exist.)
2. Consider a matrix $A \in \mathbb{R}^{n\times 2}$ such that $\frac{n}{2}$ is an even number. The first $\frac{n}{2}$ entries of the first column of A contains alternating 1s and -1s and the last $\frac{n}{2}$ entries of the first column of A contains alternating -1s and 1s. The second column of A contains the numbers 1 to n. For example, if $n = 8$ then
$$A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 1 & 3 \\ -1 & 4 \\ -1 & 5 \\ 1 & 6 \\ -1 & 7 \\ 1 & 8 \end{bmatrix}$$
Now consider the vector $\mathbf{b} \in \mathbb{R}^n$ which contains all 1s, i.e., $b^T = [111....1]$. What will be the projection of $\mathbf{b}$ onto the column space of A? (Please note that we expect a solution for the generic case and not the specific case on $n = 8$.)

3. Find the eigenvalues of the following matrix.

$$\begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & \frac{1}{3} \\ \sqrt{3} & 1 & \frac{1}{\sqrt{3}} \\ 3 & \sqrt{3} & 1 \end{bmatrix}$$

4. Consider 4 independent vectors $a_1, a_2, a_3, a_4 \in \mathbb{R}^n$. These 4 vectors form the basis of a 4 dimensional subspace in $\mathbb{R}^n$. If one of these vectors is the standard basis vector (i.e., its i-th element is 1 and all other elements are 0, $0 < i < n$) then will using the Gram-Schmidth process to find an orthonormal basis always result in the 4 standard basis vectors. Explain your answer.

5. Consider the following matrix

$$A = \begin{bmatrix} 0.17 & 0 & 0 & 0.83 & 0 \\ 0.30 & 0.20 & 0.10 & 0.40 & 0 \\ 0.25 & 0.20 & 0.10 & 0.15 & 0.30 \\ 0.40 & 0 & 0 & 0.60 & 0 \\ 0.18 & 0.20 & 0.30 & 0.12 & 0.20 \end{bmatrix}$$

What number should you subtract from the diagonal so that the determinant of the resulting matrix would be 0? (There may be multiple answers. You can write any one.)

6. Suppose $A = BCB^{-1}$ where $A, B, C \in \mathbb{R}^n$ and none of them is a diagonal matrix. If $\mathbf{x} = [1, 1, 1]$ is an eigenvector of C and $B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ then write down at least one eigenvector of A.

7. Consider a square symmetric matrix S such that $x^T S x \geq 0 \; \forall x \in \mathbb{R}^n$. Which of the following cannot be an eigenvalue of any such matrix S: $\{-3, -1, 0, 1, 3\}$. Explain your answer.

8. Consider the following matrix:

$$A = \begin{bmatrix} 0.64 & 0.36 \\ 0.31 & 0.69 \end{bmatrix}$$

Write down the eigenvalues and eigenvectors of A^∞.

9. Write down a 3×3 matrix whose eigenvalues are 0, 1, 1. The matrix should not be a diagonal matrix.

10. A class contains 12 students named A, B, C, D, E, F, G, H, I, J, K, L. In how many ways can you arrange these students such that H and I are not adjacent to each other in the queue.