Instructions

- All answers are to be written on one side of a single A4 sheet (could be a single page from your notebook also). We will refer to this as the **answer sheet**.
- All Qs are either MCQs or fill-in-the-blanks.
- Each Q carries 2 marks.
- For MCQ Qs, write down the Q number followed by the option (A,B,C,D)
- For fill-in-the-blanks Qs just write down the Q number followed by the answer (the answer could be a scalar, a vector or a matrix)
- On the **answer sheet**, we only expect you to write the answer and not provide any explanations
- You can write the explanations separately in rough sheets
- By 9:50 am, you will take a photo of your **answer sheet** and upload them on gradescope.
- By 10:00 am, you need to upload the **rough sheets**.

1. Suppose $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$. From the triangular law of inequality we know that $||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$. Give an example of $\mathbf{u}, \mathbf{v}$ such that $\mathbf{u} \neq \mathbf{0}$, $\mathbf{v} \neq \mathbf{0}$ and $||\mathbf{u} + \mathbf{v}|| = ||\mathbf{u} - \mathbf{v}||$

2. Write down all values of c for which the columns of this matrix will be dependent. $\begin{bmatrix} c & c & c \\ 8 & 1 & 3 \\ 7 & 2 & 5 \end{bmatrix}$

3. Consider the following matrix $A = \begin{bmatrix} a & b & c \\ 1 & f & e \\ 0 & i & i \end{bmatrix}$ If $a, f \neq 0$ then A would have dependent columns if

   A. ae - 2b = ac - 2e

   B. ae + b = ac + e

   C. ae - b = ac - e

   D. ae + 2b = ac + 2e

4. Write down some value of d and t for which this system of equations will have 0 solutions.

$$\begin{aligned} 2x + 5y + z &= 0 \\ 4x + dy + z &= 3 \\ y - z &= t \end{aligned}$$

5. Write down two matrices A and B such that $A, B \in \mathbb{R}^{2\times 2}$ and $A \neq B$, $A \neq I$, $B \neq I$ and $AB = B, BA = B$.

6. If A in an invertible $n \times n$ matrix such that $n = 2^k$ and P is a permutation matrix then PA is

    A. invertible only if it exchanges more than one pair of rows in A

    B. invertible only if it exchanges exactly one pair of rows in A

    C. always invertible

7. Write down a 4×4 matrix A whose column space is the same as its nullspace or explain why this is not possible.

8. Consider the following matrix

$$A = \begin{bmatrix} 5 & 7 & 2 & -1 & 3 \\ 7 & 2 & -5 & -8 & 5 \\ 2 & -5 & -8 & -5 & 6 \\ -1 & -8 & -5 & 6 & 7 \\ 3 & 5 & 6 & 7 & -1 \end{bmatrix}$$

I performed LDU factorisation on the above matrix and got

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{7}{5} & 1 & 0 & 0 & 0 \\ 0.4 & 1 & 1 & 0 & 0 \\ -\frac{1}{5} & \frac{11}{13} & -2 & 1 & 0 \\ 0.6 & -0.102 & -4 & 0.97 & 1 \end{bmatrix}$$

Write down U.

9. Write down a 4×4 matrix A such that the matrix U obtained after the LDU factorisation of A is a symmetric matrix.

10. Which of the following statements are true? (select all statements that are true)

    A. Any rank-1 matrix $A(m \times n)$ can **always** be written as $\mathbf{u}\mathbf{v}^\top$ where $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{v} \in \mathbb{R}^n$.

    B. If A is a $m \times p$ matrix and B is a $p \times n$ matrix then $rank(A) \leq p$ (**always**) and $rank(B) \leq p$ (**always**) but the rank of AB can be greater than p.

    C. If A and B are two rank-1 matrices then the rank of their product AB can **never** be greater than 1.

    D. If A and B are two rank-1 matrices then the rank of their sum $A + B$ can **never** be greater than 1.