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Valuation & Risk Models

ASSIGNMENT 2

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Contents

Value-at-Risk Analysis of an Optimized Portfolio consisting of Human Capital Management Company Stocks

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Abstract

Human Capital Management (HCM) companies have been growing significantly in the last years due to high demand in the market. CDAY is the latest index in the market which will soon be followed by PYCR (Paycor HCM). The Value-at-Risk of fairly new stock indexes can be highly overestimated or underestimated from its historical values. We do not have enough data to make estimate the VaR. We study the historical VaR, parametric VaR and modified VaR of an optimized portfolio containing the 8 HCM companies. The optimized portfolio is then simulated for the next 30 days through 10,000 trials to compare the historical estimated VaR and simulated VaR. It is observed that the historical VaR is higher than the simulated VaR. This significant difference is explained by the lack of data. We observe that our 1-Day VaR for the portfolio for the multivariate normal distribution (MVT) and the GARCH (1,1) model is very close to the Gaussian parametric VaR that was estimated from the historical data. Estimating the 30th day VaR using the 1-Day VaR_{hist} and 1-Day VaR_{modi} was considerably higher simulated 30th day VaR for both MVN and $GARCH$ (1,1).

1. Introduction

Human Capital Management (HCM) is at the core of every company. Talent acquisition, workforce management and optimization, time and attendance, benefits administration, data intelligence, payroll, self-service, etc. are among the few tasks that an organization must handle to ensure day-to-day operation. The mentioned operations are not restricted to only one department; they are performed throughout a company which testifies the complexity of HCM nowadays. There exists many standalone software and/or hardware that can handle part of HCM, however, integration among multiple system is not always easy and at times unstable (specially on larger scales). The need for centralized networks and data centres are now an asset for companies to function efficiently. Thus, the demands and integration of HCM Software as a Service (SaaS) have increased substantially in the last decades whether for small, medium or larger organizations. With increased internet security, more and more companies have pivoted to cloud computing and storing most of their organizations' information virtually. As a result, the stock value of HCM SaaS companies has increased significantly after their initial public offering.

HCM SaaS have major advantages over traditional methods. They allow company operations to be conducted virtually which drastically reduce infrastructure cost. The integrated system makes organizations much more efficient as they only need to maintain one system. Data is shared among departments to ensure quality and compliant operations. Moreover, they allow ease of scalability as company undergoes rapid changes.

Most HCM products operate on a subscription basis, that is, after their implementation within a company there is a constant cash flow based for the services being provided by HCM SaaS. It is often based on the number of employees that is using the product. The business model ensures that there is a constant revenue. The greater the number of employees using the system; the greater the cash flow. The subscription fee is the major source of revenue for most HCM products. It ensures the financial stability of the company. Hence, generally the size of a company implementing a new HCM SaaS is a potential growth for them; driving the stock value. Additionally, such partnership greatly increases the visibility of the product across the market which is a determining factor now.

HCM companies such SAP, Oracle Corporation and ADP Management services are the among the oldest HCM public companies. The reasons for choosing a specific product to use depends on their different abilities and services they provide. For example, SAP delivers core HR and payroll data, time and attendance management, recruiting and onboarding; however, with the amount of services provided; the subscription fee tends to be relatively high and discouraging for small and medium enterprises.

We study the Value at Risk (VaR) of investing of an optimized portfolio consisting of 8 HCM companies (SAP, ORCL, WDAY, CDAY, CSOD, ADP, PCTY, PAYC) which includes a fairly new index CDAY – Ceridian HCM Holding Inc, which went public end of April, 2018. We focus our study from the day CDAY went live till today to understand its impact on the VaR of the portfolio. They have been performing relatively well on the stock market. We briefly review the performance of the 8 companies on the stock market and build a simple optimized portfolio consisting of all of them. We estimate their current Value-at-Risk and also simulate the portfolio value for the next 30 days using Monte Carlo simulation with Multivariance Normal Distribution and a GARCH process to predict their VaR in the future. Our goal is to see how bad the portfolio will be performing in a month to be able to advise whether this portfolio is potentially very risky in the next 30 days; taking into consideration that we are including a fairly new index, CDAY, from the optimization.

2. Market Analysis of HCM Companies

In order to understand the performance of HCM companies stock value, we study their stock prices in the last decade and beyond. We consider the index of 8 HCM companies; the data is retrieved for the companies from Yahoo Finance and displayed in Figure 1.

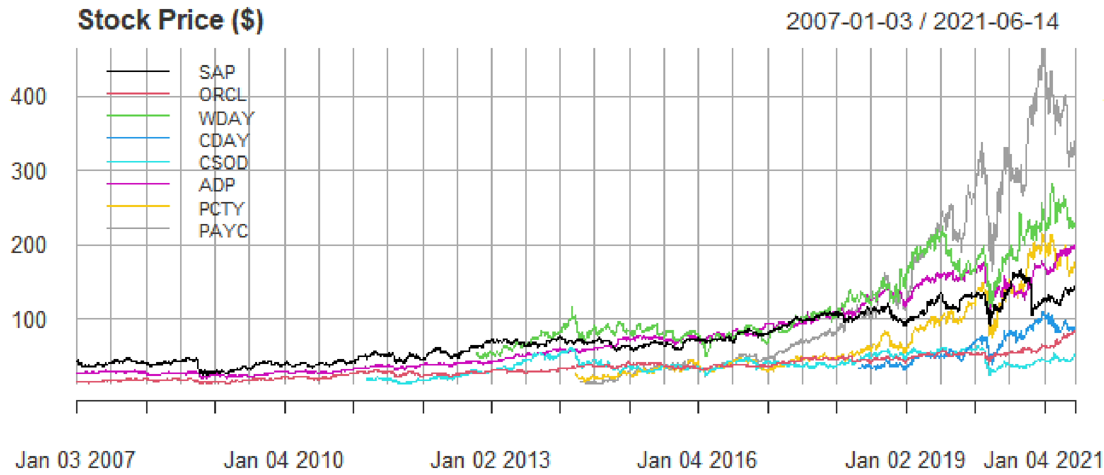


Figure 1 : Stock Price of 8 HCM Company [January 2007 – June 2021]

From Figure 1, we can observe that in 2019 there is major shift in growth of HCM companies followed by a major fall in February 2020. While HCM companies have been slowly growing in the past decade; the outbreak of COVID sparked a major interest for companies to start adapting to cloud computing; faster than expected. Leading to an increased demand of HCM product globally. However, following the outbreak of the virus in Wuhan in January 2020, the WHO declared that the corona virus was a public health emergency of international concern, which lead the stock market to crash suddenly explaining the major downfall of in February 2020. Following which, the value of the HCM stock prices increased substantially compared to previous years due to the demand increase for all companies to digitize their system to cope with the rapid changes.

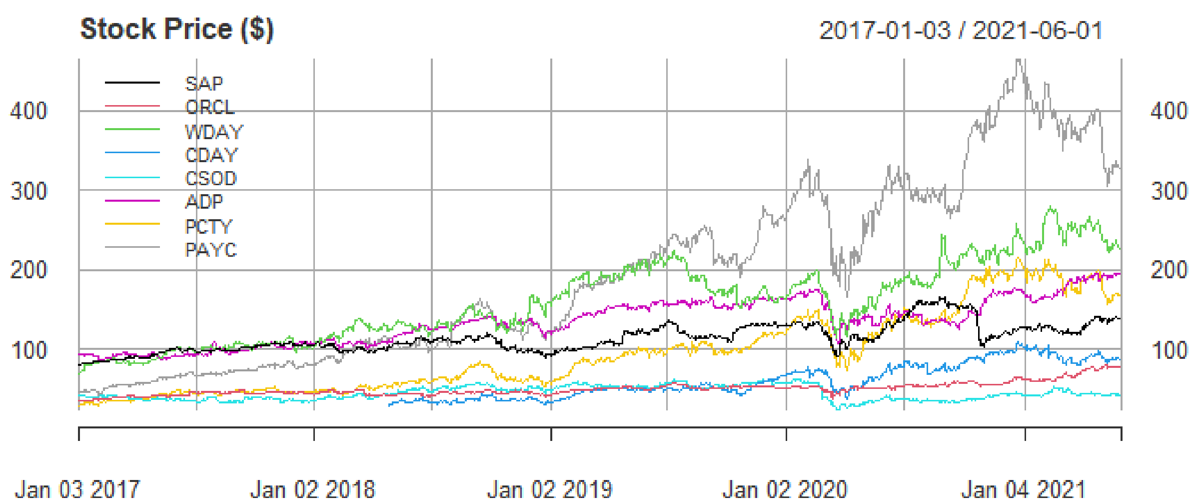


Figure 2 : Stock Prices of 8 HCM Companies [January 2017 - June 2021]

From Figure 2, we can observe that following the crash, the stock gained back their momentum fairly quickly. The observed trend indicates the stocks have potential in the market.

3. Simple Portfolio Optimization of HCM Companies

In order to get a realistic Value-at-Risk, we build a simple optimized portfolio and study the latter. We only consider the data as from April 2018 (start date of CDAY) till today to get a complete set with no null value from any company. The data set consists of 790 observations of the 8 companies in parallel. As observed in Figure 3, it is expected that there is some level of correlation between the stock value. PCTY and PAYC is relatively high and may not be adding value to the portfolio if both of them are present.

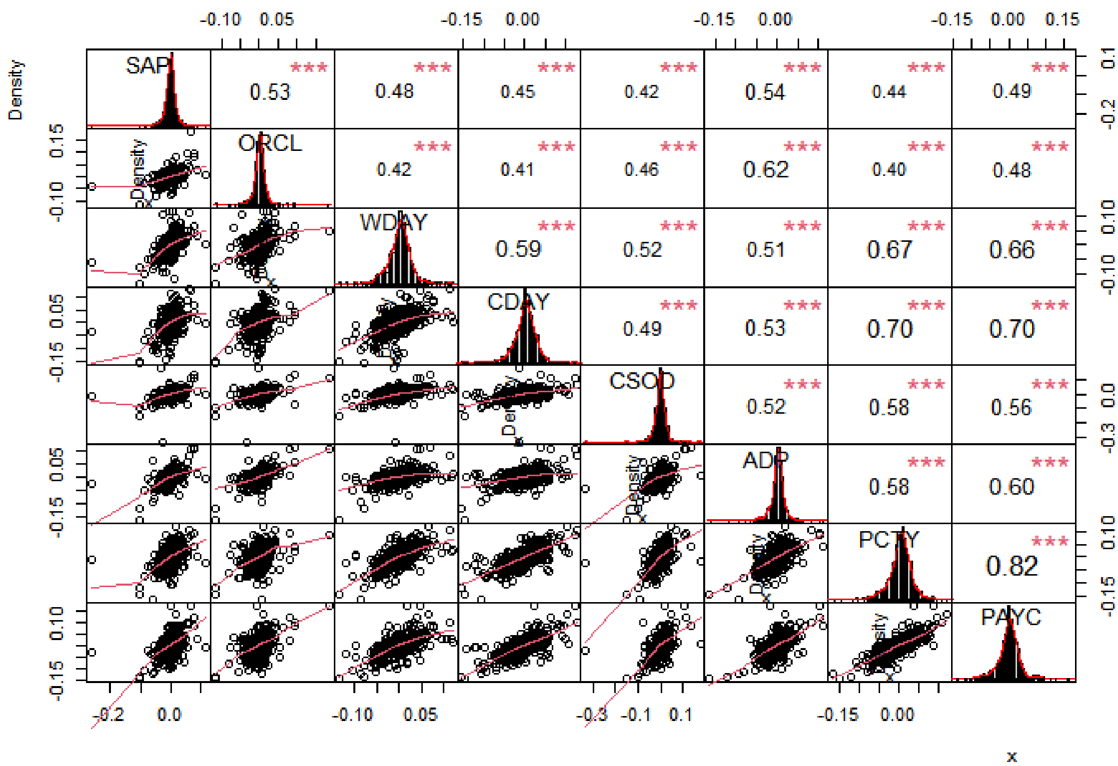


Figure 3 : Correlation between stock prices of HCM companies

We compute the log returns of the 8 index and optimized the latter. Our constraint is as described below in the R code.

```
##CONSTRAINTS
### No Shorts is allowed (#100% of Initial value is invested)
portf <- add.constraint(portf,type="weight_sum", min_sum = 1, max_sum = 1)
### 0% can be invested in a stock; not more than 50% should be invested in a
single stock
portf <- add.constraint(portf, type='box',min=0,max = 0.5)
### Maximize the Return
portf <- add.objective(portf, type="return", name="mean")
### Minimize the Variance
portf <- add.objective(portf, type="risk", name="StdDev")
### Optimizing Portfolio using ROI (simple optimization)
optPort <- optimize.portfolio(Port.returns, portf, optimize_method = "ROI")
```

The optimal percentage to be invested from the constraint above is as follows.

Index	SAP	ORCL	WDAY	CDAY	CSOD	ADP	PCTY	PAYC
Percentage	0%	36%	0%	14%	0%	0%	50%	0%

Using the optimal weight calculated; we estimate the historical portfolio log returns by applying the weight on the stock log return. A histogram of the results in displayed in Figure 4.

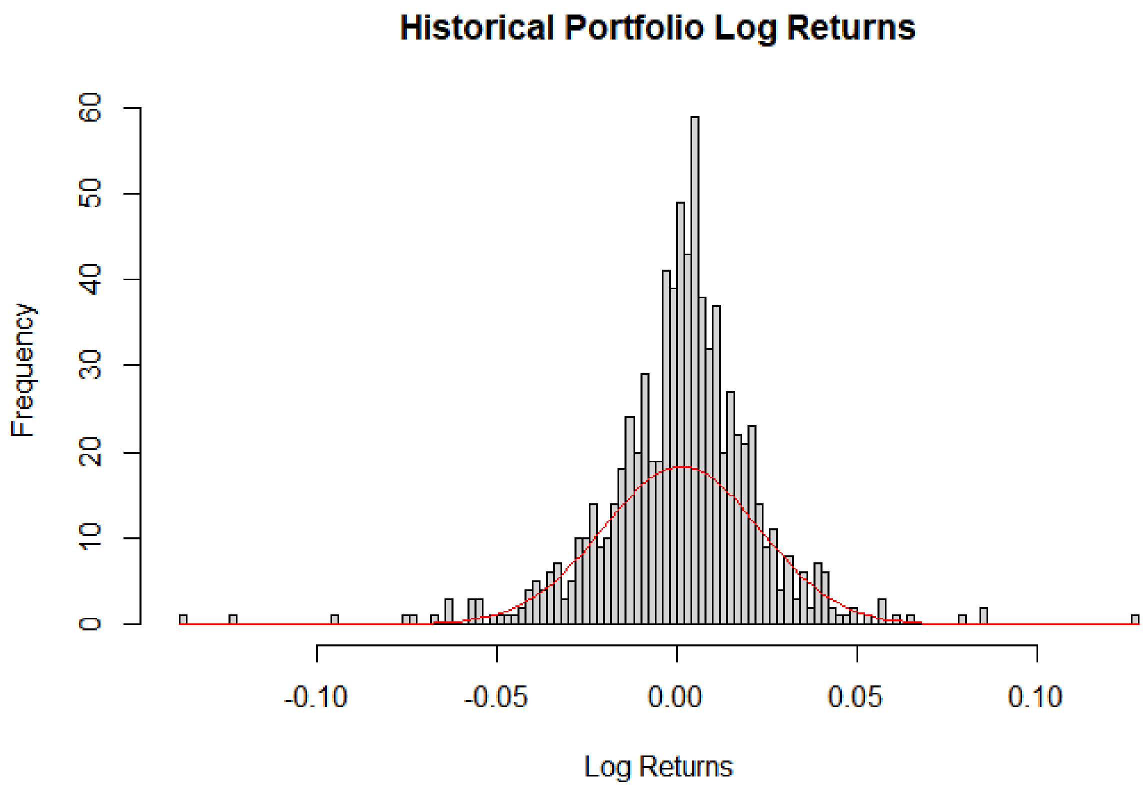


Figure 4 :Histogram of Historical Portfolio Log Returns calculated using the optimal weights

We can observe from Figure 4 that the historical log returns have an approximate normal distribution with mean $\mu = 0.00123$ and $\sigma = 0.0217$ which we can be used to derive the Gaussian Value-at-Risk from the historical data.

4. Value-at-Risk and Expected Shortfall

The Value-at-Risk and Expected Shortfall are statistic measures that describes the exposure to loss one may face when investing in a financial instrument. It is way of estimating our potential loss on an investment in the worst-case scenario with a certain degree of confidence.

4.1. Historical Method

The VaR at a confidence level of p% is calculated using the historical data is the p-quantile of the negative returns. The data is first sorted in descending order then the p% position is looked up from the sorted array. This method can be very effective if we have large number of data. However, it can be overestimating the VaR of we have small dataset. The VaR_{hist} is given by:

$$VaR_{hist} = quantile(R, p)$$

where R is the return value.

4.2. Parametric Method

The Parametric VaR is calculated by fitting an approximate model to the data that we have. It is common to fit a Gaussian Normal distribution and derive the VaR from the estimated parameters. This method is especially useful when we do not have enough data to measure the VaR accurately from historical data. The VaR_{gaus} is given by:

$$\sigma = \sqrt{\sigma^2}$$
$$VaR_{gaus} = \mu - \sigma Z$$

where R is the return value and $Z = z^{-1}(p)$ is the p-quantile of the standard normal distribution, σ^2 is the variance of the return values and μ is the mean.

A similar method can be used with different distribution to estimate the VaR. These maybe useful to catch fat tail phenomenon which the standard normal distribution does not take into consideration.

4.3. Modified VaR

The modified value at risk calculation is an adjustment of the standard deviation to account for skew and kurtosis in the return distribution. The VaR_{modi} is given by:

$$Z = \left(z_p + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2\right)$$
$$VaR_{modi} = \mu - Z\sigma$$

where μ and σ are the mean and standard deviation, S is the skew, K is the kurtosis, $z_p = z^{-1}(p)$ is the quantile of the distribution.

4.4. Portfolio VaR

The portfolio VaR is given by applying the weights related to the assets in the portfolio to the covariance matrix. The general formula for the variance of a portfolio is given by:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

$$\mu_p = \sum_{i=n}^{n} w_i \, \mu_i(r_i)$$

where w_i is the weight, $Cov(r_i, r_j)$ is the covariance between asset r_i and r_j and μ_i is the mean return of asset r_i.

Using the historical value and derived parameters; the VaR can be estimated for the portfolio using the historical, parametric or modified VaR method.

4.5. Conditional Value at Risk CVaR (Expected Shortfall)

The Expected Shortfall is the expected return on a financial instrument in the worst percentile of the return value. It is an alternative to the value at risk. The CVaR can be more intuitive in scenarios where fat tails occur on the left of the distribution. The CVaR is derived from the VaR value itself; it is the expected VaR in the worst percentile. It is given by the generalised summation:

$$CVaR_t^{\alpha} = \frac{1}{\alpha} \sum_{i=1}^{\alpha} VaR_t^{\left[1+\alpha\left(\frac{i}{\alpha+1}-1\right)\right]}$$

where VaR_t^{γ} represents the Value-at-Risk at t.

5. VaR and Expected Shortfall of the HCM optimized portfolio

We make use of the inbuild functions $VaR(X)$ and $CVaR(X)$ from the Performance Analytics package in R and summarise our calculation in table 2. Both the VaR and $CVaR$ are calculated for our portfolio log returns calculated in Section 3. We consider an initial investment of $10,000 in the portfolio of estimate the VaR and $CVaR$ at 99% confidence interval which is commonly used by the Basel Committee. The 30 day VaR^{30} is estimated using the 1-day $VaR \times \sqrt{30}$

5.1. Value at Risk – VaR of the Optimized Portfolio

1-Day				**Index**					
Method	SAP	ORCL	WDAY	CDAY	CSOD	ADP	PCTY	PAYC	Portfolio
VaR_{hist}	-538.22	-492.49	-718.10	-988.38	-690.99	-510.25	-782.08	-871.60	-612.51
VaR_{gaus}	-472.22	-410.2	-600.37	-668.03	-604.37	-427.70	-651.65	-688.81	-480.11
VaR_{modi}	-1859.24	-1078.34	-826.46	-1033.15	-2377.77	-1071.79	-953.62	-1006.30	-817.91
30-Days									
VaR_{hist}^{30}	-2947.95	-2697.48	-3933.20	-5413.58	-3784.71	-2794.75	-4283.63	-4773.95	-3354.86
VaR_{gaus}^{30}	-2586.46	-2246.76	-3288.36	-3658.95	-3310.27	-2342.61	-3569.23	-3772.77	-2629.67
VaR_{modi}^{30}	-10183.48	-5906.31	-4526.71	-5658.80	-13023.58	-5870.44	-5223.19	-5511.73	-4479.88

Table 1 : 1-Day VaR and 30-Day VaR from historical data

5.2. Conditional Value at Risk – CVaR of the Optimized Portfolio

1-Day					Index				
Method	SAP	ORCL	WDAY	CDAY	CSOD	ADP	PCTY	PAYC	Portfolio
$CVaR_{hist}$	-975.94	-748.01	-949.27	-1181.80	-1171.67	-833.10	-1082.92	-1078.25	-835.32
$CVaR_{gaus}$	-539.72	-469.74	-685.81	-763.33	-689.50	-489.49	-745.00	-786.99	-549.79
$CVaR_{modi}$	-1859.21	-1078.34	-826.46	-1033.15	-2377.77	-1071.79	-953.62	-1006.30	-817.91
30-Days					Index				
$CVaR_{hist}$	-5345.44	-4097.02	-5199.37	-6472.99	-6417.50	-4563.08	-5931.40	-5905.82	-4575.24
$CVaR_{gaus}$	-2956.17	-2572.87	-3756.34	-4180.93	-3776.55	-2681.05	-4080.53	-4310.52	-3011.32
$CVaR_{modi}$	-10183.31	-5906.31	-4526.71	-5658.80	-13023.58	-5870.44	-5223.19	-5511.73	-4479.88

Table 2 : 1-Day CVaR and 30-Day CVaR from historical data

6. Portfolio Simulation for next 30 Days

We have calculated the Value-at-Risk using our historical data which consisted of only 790 observations which may not be very accurate. This tends to create large gaps in our probability distribution when estimating the VaR historically or using a parametric method. To solve this problem; we result to simulation of the portfolio values in the next 30 days and estimate the VaR from our simulated data. We choose to simulate the path of the portfolio 10000 times with an initial investment of $\$10000$ to get a fairly clustered set of data to perform further VaR estimates.

6.1. Monte Carlo Simulation – Multivariate Normal Distribution

We first assume that the daily log returns, R_t, of the portfolio are distributed by a multivariate normal distribution.

$$R_t \sim MVN(\mu, \Sigma)$$

where μ is the mean return vector of R_t and Σ is the covariance matrix of the portfolio.

The Cholesky decomposition of Σ is calculated to obtain the lower triangular decomposition of the matrix. The decomposition is used in the simulation process to generate simulation quicker than using the covariance matrix method.

$$LL^T = \Sigma$$

Using the lower triangular matrix L and a sampled vector of Z_t from a normal distribution; we simulate a log return value of the portfolio.

$$R_t = \mu + LZ_t$$

where $Z_t \sim N(0,1)$.

```
### Setting up MC for 10000 Simulation for 30 Trading Days
mc_sims = 10000
days = 30
Mean.Assets = matrix( MEAN.PortAssets , length(MEAN.PortAssets) , 30 )
Varianceby2 = matrix(0.5*diag(COVM.Port) , length(0.5*diag(COVM.Port)) , 30 )
simulation = data.frame(Days = 0:30)
InitialPortfolio = 10000
seed = 1
for (i in 1:mc_sims){
         DailyLogReturns = ColTCOVM.port %*% matrix(rnorm(8*30, mean=0,
sd=1),8,30) + Mean.Assets - Varianceby2
         sim = t(OptimalWeights) %*% DailyLogReturns
         sim2  = rbind(rep(0,1), matrix(cumsum(sim)))
         simulation[i+1] = InitialPortfolio*exp(sim2)
}
```

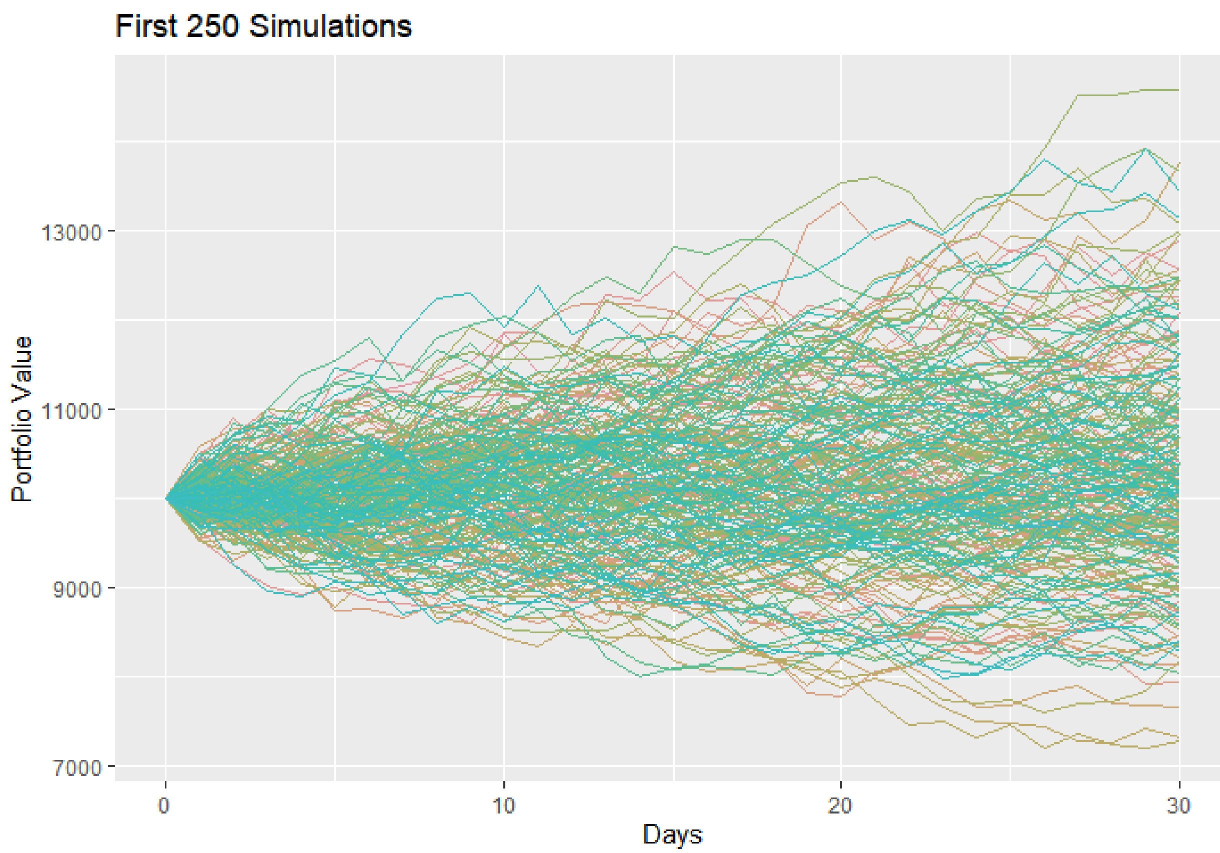


Figure 5 : Monte Carlo Simulation using MVN of Optimized Portfolio

6.2. Monte Carlo Simulation – GARCH model

In order to simulate the GARCH model; we test the log return data of the portfolio for ARCH effect and identify the best parameter for the GARCH model to be used for the portfolio.

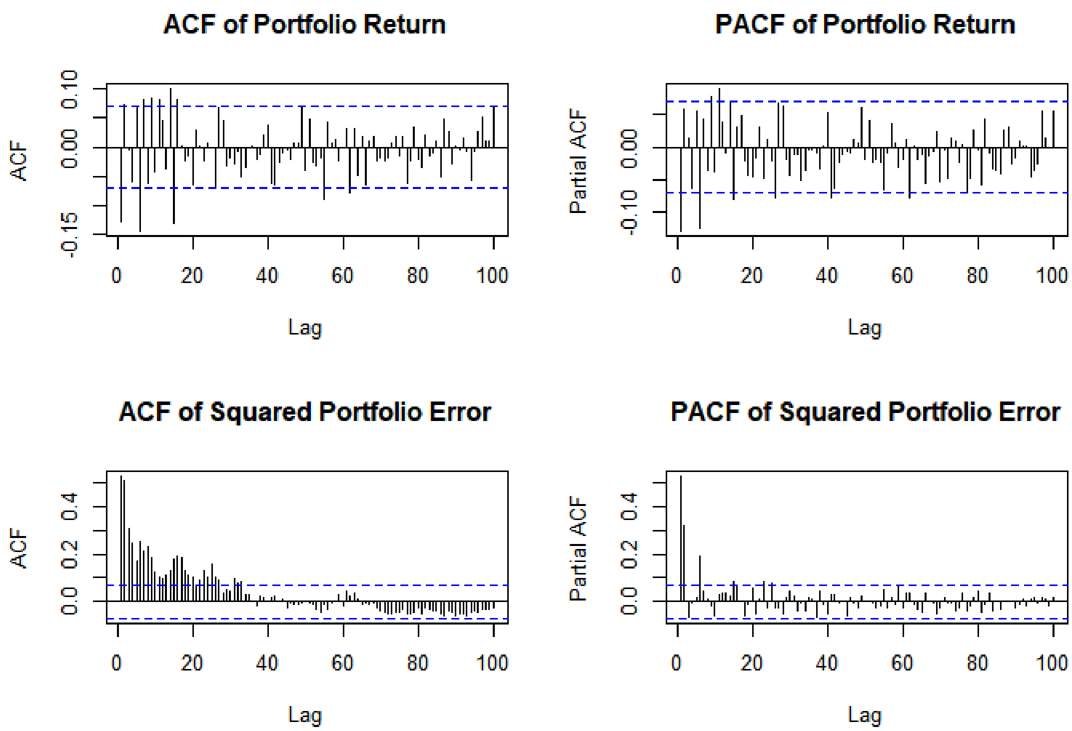


Figure 6 : ACF and PACF plot of historical Portfolio Return using optimal weights

From Figure 6; we can observe that the AR and MA process is insignificant but there is presence or ARCH effect in our portfolio data. We use the simple GARCH (1,1) model and test for the significance of the parameters using the *rugarch* package in R.

```
#Fitting the GARCH(1,1) Model using rugarch package

##Model Specification
x = ugarchspec(variance.model = list(garchOrder=c(1,1)),mean.model =
list(armaOrder=c(0,0)))
##Fitting Model Specification
x_fit = ugarchfit(x,data=Port.timeseries)
```

The optimal parameters of our fitted model are given by:

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001777    0.000570   3.1165 0.001830
omega   0.000030    0.000008   3.5868 0.000335
alpha1  0.203478    0.042725   4.7625 0.000002
beta1   0.727717    0.048897  14.8828 0.000000
```

From the fitted results, the p-value is relatively low; which imply that the parameters are statistically significant and that there is not enough evidence to reject them.

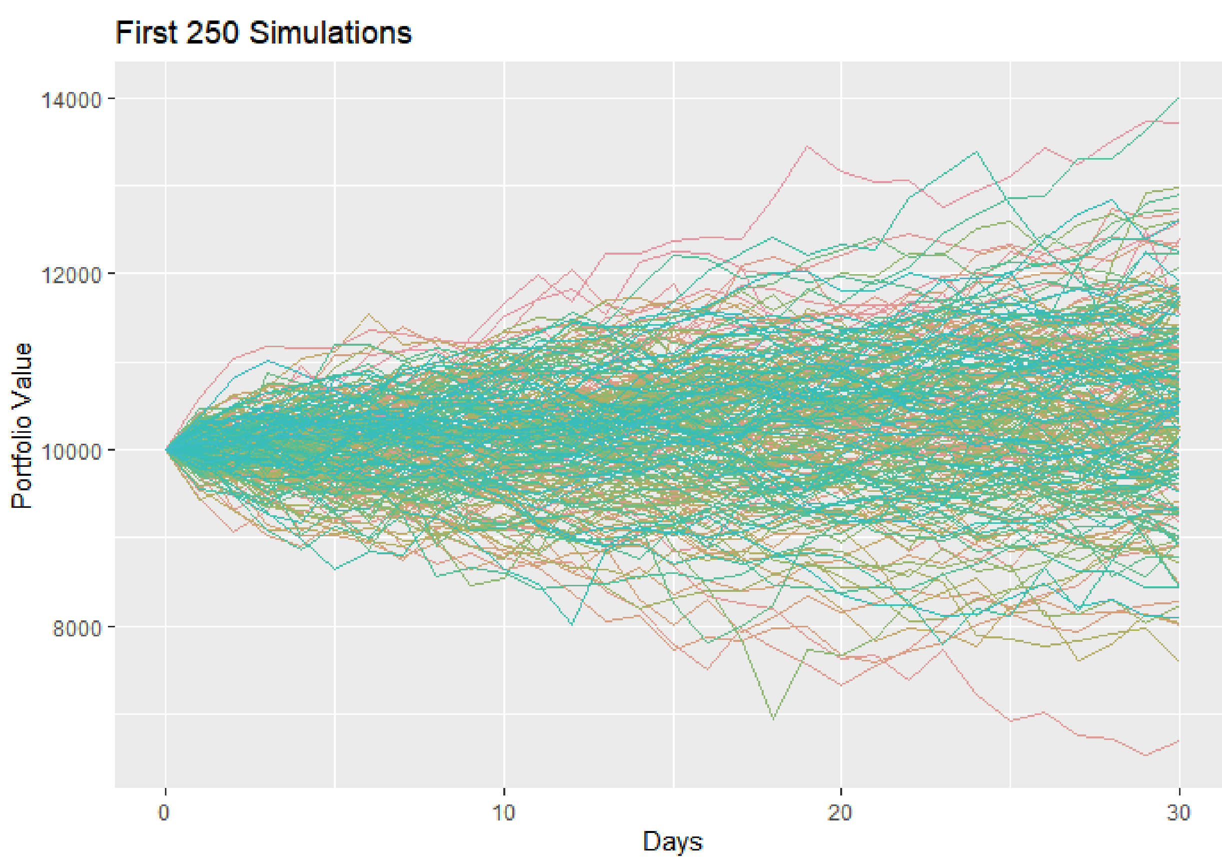
Fitted GARCH (1,1) Model is given by:

$$R_t = \mu + e_t = 0.001777 + e_t$$
$$\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 = 0.000030 + 0.203478 e_{t-1}^2 + 0.727717 \sigma_{t-1}^2$$

```
#Setting up Monte Carlo Simulation for Fitted GARCH(1,1) for 30 Trading Days
xfinal = x
setfixed(xfinal)<-as.list(coef(x_fit))

sim = ugarchpath(spec = xfinal,m.sim=10000,n.sim=1*30,rseed=1)

p = data.frame(Days = 0:30)
p = cbind(p,rbind(T=10000,InitialPortfolio*apply(fitted(sim),2,'cumsum')+
InitialPortfolio))
```



6.1.1 Results and Interpretation of Monte Carlo Simulations

The simulations are run for 10000 trials. The data is then sorted in a descending order to identify the VaR and $CVaR$ after the first day and the 30th day. We consider the VaR and $CVaR$ at 99% confidence interval.

Method	Portfolio VaR – 1 Day	Portfolio VaR – 30 Days
VaR_{MVN}	-483.49	-2214.59
$VaR_{GARCH(1,1)}$	-417.80	-2232.61

Table 3 : 1-Day VaR and 30-Day VaR from historical data

Method	Portfolio CVaR – 1 Day	Portfolio CVaR – 30 Days
$CVaR_{MVN}$	-559.25	-2521.34
$CVaR_{GARCH(1,1)}$	-482.57	-2993.38

Table 4 : 1-Day CVaR and 30-Day CVaR from simulated data

7. Discussion and Conclusion

Method	Portfolio VaR – 1 Day	Portfolio VaR – 30 Days	Portfolio CVaR – 1 Day	Portfolio CVaR – 30 Days
VaR_{hist}	-612.51	-3354.86	-835.32	-4575.24
VaR_{gaus}	-480.11	-2629.67	-549.79	-3011.32
VaR_{modi}	-817.91	-4479.88	-817.91	-4479.88
VaR_{MVN}	-483.49	-2214.59	-559.25	-2521.34
$VaR_{GARCH(1,1)}$	-417.80	-2232.61	-482.57	-2993.38

Table 5 : Summary of VaR for optimized portfolio

The VaR calculated from the historical data for the 1-day VaR was relatively high for the $VaR_{hist}(-612.51)$ and $VaR_{modi}(-817.91)$ compared to the $VaR_{gaus}(-480.11)$ as shown in Table 1. Consequently the 30-day VaR estimate was significantly high as well. Due to the lack of data; we can't make conclusive inference from the latter estimation. It is also observed that historically the portfolio VaR is relatively lower compared to the other individual index. With only 790 observations, it is likely that the probability distribution is not properly spread out, which lead to a high VaR. Using the available information; the simulated data is likely to provide us with better estimate of the VaR. From the simulation the 1-day VaR for the MVN and $GARCH(1,1)$ model were relatively low and close to the 1-day VaR_{gauss}. However, from simulation, it was observed that the 30-day VaR was being highly valued for by all 30-day VaR calculated using the 1-day VaR. The $CVaR$ for both MVN and $GARCH(1,1)$ were relatively lower compared to the other method due to the number of simulations that was run. The simulation result for both the MVN and $GARCH(1,1)$ are relatively close and are derived from historical data. The decision for someone to invest in such a portfolio depends on its risk averseness. The portfolio consists of a new index which we do not have much information. It would not be recommended that someone being risk adverse to invest in such a portfolio specially knowing that the historical VaR is quite large even though the simulation is inferring otherwise.