

## First Order Logic: Conversion to CNF

### 1. Eliminate biconditionals and implications:

- Eliminate  $\Leftrightarrow$ , replacing  $\alpha \Leftrightarrow \beta$  with  $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ .
- Eliminate  $\Rightarrow$ , replacing  $\alpha \Rightarrow \beta$  with  $\neg \alpha \vee \beta$ .

### 2. Move $\neg$ inwards:

- $\neg(\forall x p) \equiv \exists x \neg p$ ,
- $\neg(\exists x p) \equiv \forall x \neg p$ ,
- $\neg(\alpha \vee \beta) \equiv \neg \alpha \wedge \neg \beta$ ,
- $\neg(\alpha \wedge \beta) \equiv \neg \alpha \vee \neg \beta$ ,
- $\neg \neg \alpha \equiv \alpha$ .

### 3. Standardize variables apart by renaming them: each quantifier should use a different variable.

### 4. Skolemize: each existential variable is replaced by a *Skolem constant* or *Skolem function* of the enclosing universally quantified variables.

- For instance,  $\exists x \text{Rich}(x)$  becomes  $\text{Rich}(G1)$  where  $G1$  is a new Skolem constant.
- "Everyone has a heart"  $\forall x \text{Person}(x) \Rightarrow \exists y \text{Heart}(y) \wedge \text{Has}(x,y)$  becomes  $\forall x \text{Person}(x) \Rightarrow \text{Heart}(H(x)) \wedge \text{Has}(x,H(x))$ , where  $H$  is a new symbol (Skolem function).

### 5. Drop universal quantifiers

- For instance,  $\forall x \text{Person}(x)$  becomes  $\text{Person}(x)$ .

### 6. Distribute $\wedge$ over $\vee$ :

- $(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$ .

## Output

```
#Test 1  
main()
```

```
Enter FOL:  
∀x food(x) => likes(John, x)  
The CNF form of the given FOL is:  
¬ food(A) ∨ likes(John, A)
```

```
#Test 2  
main()
```

```
Enter FOL:  
∀x[∃z[loves(x,z)]]  
The CNF form of the given FOL is:  
[loves(x,B(x))]
```

```
#Test 3  
main()
```

```
Enter FOL:  
[american(x)^weapon(y)^sells(x,y,z)^hostile(z)] => criminal(x)  
The CNF form of the given FOL is:  
[¬american(x)∨¬weapon(y)∨¬sells(x,y,z)∨¬hostile(z)] ∨ criminal(x)
```