Laboratory # 1

Goals

The goals of this lab are:

1. To give you experience with a 3D diffusion code,
2. To cement the differences between the linear system and the mesh. The purpose here will become more clear when we start working in 2D, which is simpler but can be confusing.
3. To expose you to scientific visualization,

Team Work: Math Primer Problems

1. Find the solution to the following system of equations.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 6 \\ x_2 - x_3 &= -3 \\ -x_1 - 3x_2 &= 12 \end{aligned}$$

2. Find the solution to the following system of equations.

$$\begin{aligned} x_1 - 2x_3 &= -1 \\ -2x_1 + x_2 + 6x_3 &= 7 \\ 3x_1 - 2x_2 - 5x_3 &= -3 \end{aligned}$$

3. Write the system in Problem #1 in matrix-vector notation, i.e., like this:

$$\begin{bmatrix} 1 & 0 & -2 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

4. Write the system in Problem #2 in matrix-vector notation.
5. Write out the result of the following matrix-vector product:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

6. Write out the result of the following matrix-vector product:

$$\begin{bmatrix} 3 & -2 & 2 \\ 1 & 4 & -2 \\ 2 & -5 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$$

During Lab: Team Development

Part 1: Code Development

You will be given a partially completed version of `fd_3d`, a three-dimensional (3D) steady-state diffusion solver for hexahedral domains. You are to complete the parts of the code as indicated in the source code provided. Specifically, you will complete the formation of the matrix for the internal cells as well as the boundary conditions.

1. Obtain `fd_3d` from the hpsc-2024 GitHub repository.
2. Complete the boundary condition application in `fd.cpp`.
3. Complete the formation of the matrix rows for internal cells in `fd.cpp`.

Part 2: Running the Code and Plotting Results

1. Run the case contained in the `fd_3d/src/run` script.
2. Open the resulting `.vtk` file (`Temp_0.vtk`) in Paraview. Produce plots of the Temperature variable on two cross-sections of the cube, where the cross-sections meet these specifications:
   - Both cross-sections should pass through the center point of the cube.
   - One of them should have a normal that points in the $x-$ direction.
   - The other should have a normal that points in the $y-$ direction.

   Your result should look like that in Figure 1 (Left).
3. While still in Paraview, produce a line-plot. Choose the `Temp_0.vtk` object, and select "Filters-Alphabetical-Plot Over Line" from the Main Menu. Specify the plot line to range from $(0.5, 0.5, 0)$ to $(0.5, 0.5, 1)$. Your result should look like that in Figure 1 (Right).

After Lab: Team Lab Report

For this lab, you are to submit a joint lab report, i.e., identical reports. In other words, you both submit the same PDF file to Canvas. You must both do all of the Math Primer problems, and you are to check your answers with each other. Then, submit one solution between the two of you. Include in your report the following:

1. In the main body, describe the problem being solved, including the shape of the object in which heat conduction is being simulated, along with a clear description of the boundary conditions, and the number of cells used in the $x-$, $y-$, and $z-$ directions. Also include the Paraview screenshot. **Note:** Do not only include the plots themeselves, since you could simply copy them from these instructions. Rather, include a screenshot of the entire Paraview window, e.g. showing the menus, etc., with the two plots displayed.
2. A self-evaluation section. This is the only place in the report where your tone can be informal. All other text in the report must be concise and formal.
3. The completed `fd_3d.cpp` code in Appendix A.
4. The Math Primer problem solutions in Appendix B.

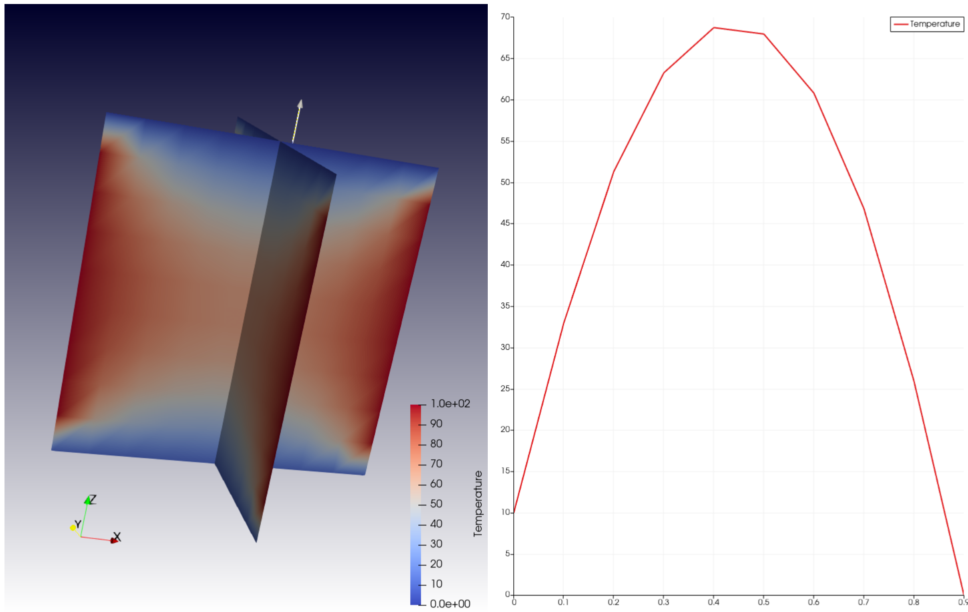

Figure 1: **Left:** Cross-section results expected in Paraview. **Right:** Line-plot results expected in Paraview.

Resources

- `paraview.org`
- `hpscLectureNotes_2024-09-06.pdf` on Canvas

Grading Rubric

Component	Expectations	Weight
Post-Lab Report		
Main Body	Simulation description and results discussion	25%
Main Body	Screenshot	20%
Self-evaluation	What worked what did not work	10%
Appendix A	Code listing with in-line comments	20%
Appendix B	Math Primer solutions	25%