


Being Eve

Diffie-Hellman

Python Script



```
1  # --- Diffie-Hellman --- #
2
3  # Variable initialization
4  g, p = 7, 61
5  A, B = 30, 17
6
7  # Finding a and b using a brute-force method
8  # We check p values since the order of g in Z/pZ is p
9  for i in range(p):
10     num = (g ** i) % p
11     if num == A:
12         a = i
13     if num == B:
14         b = i
15
16  K_a = (B ** a) % p
17  K_b = (A ** b) % p
18
19  # Results
20  print("a:", a) # 41
21  print("b:", b) # 23
22  print("Alice's shared secret:", K_a) # 6
23  print("Bob's shared secret:", K_b) # 6
```

I used the Python script above to calculate the values of a and b , which I then used to calculate K_a and K_b , which are the shared keys from Alice's and Bob's perspectives, respectively.

As we can see, we can use a simple brute-force method to find a and b . Some algebraic reasoning lets us conclude that we only need to check at most p values, so we check p consecutive values of i to see if i was the random a/b used by Alice/Bob to generate A/B .

The Python script is my work for this problem. If the integers involved were much larger (specifically p , as the other numbers— g , A , B —can be represented with a congruent number (modulo p) that is at most p), the `for` loop in the Python script above would take significantly longer to run. If p is enormous, it might simply take too long to do the calculations shown above.

Side note: calculating quantities of the form $(x ** y) \% z$ wouldn't significantly slow this down. Due to some interesting properties of modular arithmetic, we can keep this computation relatively fast, even if y is large.

RSA (older version)

Python Script

```

1  # --- RSA --- #
2
3  e_Bob, n_Bob = (13, 5561)
4
5  encrypted = [1516, 3860, 2891, 570, 3483, 4022, 3437, 299,
6               570, 843, 3433, 5450, 653, 570, 3860, 482,
7               3860, 4851, 570, 2187, 4022, 3075, 653, 3860,
8               570, 3433, 1511, 2442, 4851, 570, 2187, 3860,
9               570, 3433, 1511, 4022, 3411, 5139, 1511, 3433,
10              4180, 570, 4169, 4022, 3411, 3075, 570, 3000,
11              2442, 2458, 4759, 570, 2863, 2458, 3455, 1106,
12              3860, 299, 570, 1511, 3433, 3433, 3000, 653,
13              3269, 4951, 4951, 2187, 2187, 2187, 299, 653,
14              1106, 1511, 4851, 3860, 3455, 3860, 3075, 299,
15              1106, 4022, 3194, 4951, 3437, 2458, 4022, 5139,
16              4951, 2442, 3075, 1106, 1511, 3455, 482, 3860,
17              653, 4951, 2875, 3668, 2875, 2875, 4951, 3668,
18              4063, 4951, 2442, 3455, 3075, 3433, 2442, 5139,
19              653, 5077, 2442, 3075, 3860, 5077, 3411, 653,
20              3860, 1165, 5077, 2713, 4022, 3075, 5077, 653,
21              3433, 2442, 2458, 3409, 3455, 4851, 5139, 5077,
22              2713, 2442, 3075, 5077, 3194, 4022, 3075, 3860,
23              5077, 3433, 1511, 2442, 4851, 5077, 3000, 3075,
24              3860, 482, 3455, 4022, 3411, 653, 2458, 2891,
25              5077, 3075, 3860, 3000, 4022, 3075, 3433, 3860,
26              1165, 299, 1511, 3433, 3194, 2458]
27
28  # https://factorization.info/prime-factors/0/prime-factors-of-5561.html
29  p, q = 67, 83
30
31  # Find d_Bob such that e_Bob * d_Bob = 1 (mod (p - 1)(q - 1))
32  mod = (p - 1) * (q - 1)
33
34  for i in range(mod):
35      if (e_Bob * i) % mod == 1:
36          d_Bob = i
37          break
38
39  print("d_Bob:", d_Bob) # 1249
40
41  # Decrypt the encrypted message
42  decrypted = []
43  for e in encrypted:
44      d = (e ** d_Bob) % n_Bob
45      decrypted.append(d)
46
47  # Decode the decrypted message
48  decoded = []
49  for d in decrypted:
50      decoded.append(chr(d))
51
52  # Print message
53  decoded_msg = "".join(decoded)
54  print(decoded_msg)
55
56  # Hey Bob. It's even worse than we thought! Your pal, Alice.
57  # https://www.schneier.com/blog/archives/2022/04/airtags-are-used-for-stalking-far-more-than-previously-reported.html

```

The encrypted message sent from Alice to Bob was the following:

- Hey Bob. It's even worse than we thought! Your pal, Alice.
<https://www.schneier.com/blog/archives/2022/04/airtags-are-used-for-stalking-far-more-than-previously-reported.html>

The Python script above demonstrates how I found this.

- First, I used an online resource to find p and q , the prime factors of n_{Bob} that were used to generate Bob's secret key and public key. This alone could be very difficult for large values of n_{Bob} , as factoring can be fairly computationally intensive.
- Then, I used a brute-force algorithm to find d_{Bob} . I knew that $d_{Bob} * e_{Bob}$ had to equal 1 (mod $(p - 1)(q - 1)$), so I checked all values of d_{Bob} from 0 to $(p - 1)(q - 1) - 1$. This step could also be extremely time-intensive for large values of p and q .
- Once I found d_{Bob} (1249), I had Bob's secret key. I used this to decrypt the encrypted buffer character-by-character.
- I noticed that every value in the decrypted buffer was less than 128, so I guessed that this was encoded using ASCII. To decode this, I simply used Python's *chr* function.
- Then, I printed the decoded buffer to get the original message.

To summarize, the most time-intensive steps if we had larger integers would be factoring n_{Bob} into primes p, q and using a brute-force algorithm to find d_{Bob} .

Even if Bob's keys involved larger integers, this encryption would still be insecure. By encrypting the message character-by-character, Alice has basically created a complicated-looking substitution cipher. Analyzing character frequencies could allow Eve to crack the message.

For example, this sequence could be cracked somewhat easily, as it "looks" like the beginning of a URL, even when we can only see character frequencies. (Repeated characters underlined.)

- 1511, 3433, 3433, 3000, 653, 3269, 4951, 4951, 2187, 2187, 2187, 299
- Actual text: <https://www>.

RSA (newer version)

Not doing a full explanation for this one since it is optional. (Encrypted message at the bottom.)

```

1  # --- RSA (Newer Version) --- #
2
3  e_Bob, n_Bob = (17, 170171)
4
5  encrypted = [65426, 79042, 53889, 42039, 49636, 66493, 41225, 58964,
6               126715, 67136, 146654, 30668, 159166, 75253, 123703, 138090,
7               118085, 120912, 117757, 145306, 10450, 135932, 152073, 141695,
8               42039, 137851, 44057, 16497, 100682, 12397, 92727, 127363,
9               146760, 5303, 98195, 26070, 110936, 115638, 105827, 152109,
10              79912, 74036, 26139, 64501, 71977, 128923, 106333, 126715,
11              111017, 165562, 157545, 149327, 60143, 117253, 21997, 135322,
12              19408, 36348, 103851, 139973, 35671, 93761, 11423, 41336,
13              36348, 41336, 156366, 140818, 156366, 93166, 128570, 19681,
14              26139, 39292, 114290, 19681, 149668, 70117, 163780, 73933,
15              154421, 156366, 126548, 87726, 41418, 87726, 3486, 151413,
16              26421, 99611, 157545, 101582, 100345, 60758, 92790, 13012,
17              100704, 107995]
18
19  # https://www.calculatorsoup.com/calculators/math/prime-factors.php
20  p, q = 379, 449
21
22  # Find d_Bob such that e_Bob * d_Bob = 1 (mod (p - 1)(q - 1))
23  mod = (p - 1) * (q - 1)
24
25  for i in range(mod):
26      if (e_Bob * i) % mod == 1:
27          d_Bob = i
28          break
29
30  print("d_Bob:", d_Bob) # 1249
31
32  # Decrypt the encrypted message
33  decrypted = []
34  for e in encrypted:
35      d = (e ** d_Bob) % n_Bob
36      decrypted.append(d)
37
38  two_byte_decrypted = []
39  for d in decrypted:
40      two_byte_decrypted.append(d >> 8)
41      two_byte_decrypted.append(d % (2 ** 8))
42
43  # Decode the decrypted message
44  decoded = []
45  for d in two_byte_decrypted:
46      decoded.append(chr(d))
47
48  # Print message
49  decoded_msg = "".join(decoded)
50  print(decoded_msg)
51
52  # Hi Bob. I'm walking from now on. Your pal, Alice.
53  # https://foundation.mozilla.org/en/privacynotincluded/articles
54  # /its-official-cars-are-the-worst-product-category-we-have-ever-reviewed-for-privacy/

```