Discrete Model Prodective Control on a Bicycle Dynamic Model

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***Abstract*—This report chronicles our the work done in EECS561-Digital Control Systems Project. The objective was to do trajectory optimization-based control of a car so that it smoothly traverses a given track. We have assumed that the dynamics of the car work according to the bicycle model. A proportional controller was used to obtain the initial trajectory of the car. The dynamic model was linearized around this trajectory, and a model predictive controller to find an optimized trajectory.**

***Index Terms*—Model predictive control, linearization, quadprog, bicycle model.**

I. Introduction

In this section, we'll talk about the dynamic model we used, the motivation behind doing MPC, and the results we expect to achieve through our approach.

A. Bicycle Model

The non-linear bicycle model is given as follows:

$$\begin{bmatrix} \dot{X} \\ \dot{u} \\ \dot{Y} \\ \dot{v} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} ucos\psi - vsin\psi \\ \frac{1}{m}(-fmg + N_w F_x - F_{yf} sin(\delta_f)) + vr \\ usin\psi + vcos\psi \\ \frac{1}{m}(F_{yf} cos(\delta_f) + F_{yr}) - ur \\ r \\ \frac{1}{I_z}(aF_{yf} cos(\delta_f) - bF_{yr}) \end{bmatrix} \tag{1}$$

The lateral forces F_{yf} and F_{yr} are described using Pacejka "Magic Formula"

$$F_{zf} = \frac{b}{a+b} mg \tag{2}$$

$$F_{yf} = F_{zf} D_y sin(C_y \tan^{-1}(B_y \phi_{yf})) + S_{vy} \tag{3}$$

$$F_{zr} = \frac{a}{a+b} mg \tag{4}$$

$$F_{yr} = F_{zr} D_y sin(C_y \tan^{-1}(B_y \phi_{yr})) + S_v y \tag{5}$$

where

$$\phi_{yf} = (1 - E_y)(\alpha_f + S) \tag{6}$$

$$\phi_{yr} = (1 - E_y)(\alpha_r + S_{hy}) + \frac{E_y}{B_y} \tan -1(B_y(\alpha_r + S_{hy})) \tag{7}$$

where α_f and α_r are the front and rear lateral slip angles which are given in degrees in the previous formulas. The front and rear lateral slip angles which is described in radians is given by:

$$\alpha_f = \delta_f - \tan^{-1}(\frac{v + ar}{u}) \tag{8}$$

$$\alpha_r = - \tan^{-1}(\frac{v - br}{u}) \tag{9}$$

Additionally, combined longitudinal and lateral loading of tires will be limited to F_x^* and F_{yr}^* in the following manner:

$$F_{total} = \sqrt{(N_w F_x)^2 + (F_{yr})^2} \tag{10}$$

$$F_{max} = 0.7mg \tag{11}$$

If $F_{total} > F_{max}$:

$$F^x = \frac{F_{max}}{F_{total}} F_x \tag{12}$$

$$F^{y^r} = \frac{F_{max}}{F_{total}} F_{yr} \tag{13}$$

The inputs into this model are δ_f , which is the front wheel steering angle; and F_x, which is the traction force generated at each tire by the vehicle's motor. The vehicle begins from the following initial condition:

II. Methodology

III. Results and Discussion

IV. Conclusion

V. Appendix

The code for this project is uploaded on GitHub and can accessed through this link.

TABLE I
DYNAMIC MODEL CONSTANTS

Vehicle Parameter	Value
δ	[-0.5,0.5]
F_x	[-5000,5000]
m	1400
N_w	2
f	0.01
I_x	2667
a	1.35
b	1.45
B_y	0.27
C_y	1.2
D_y	0.7
E_y	-1.6
S_{hy}	0
S_{vy}	0
g	9.806