Discrete Model Prodective Control on a Bicycle Dynamic Model

Apurva Sontakke,Nalin Bendapudi, Varun Shetty
{apurvaps, bnalin, vashetty}@umich.edu

***Abstract*—This report chronicles our the work done in EECS561-Digital Control Systems Project. The objective was to do trajectory optimization-based control of a car so that it smoothly traverses a given track. We have assumed that the dynamics of the car work according to the bicycle model. A proportional controller was used to obtain the initial trajectory of the car. The dynamic model was linearized around this trajectory, and a model predictive controller to find an optimized trajectory.**

***Index Terms*—Model predictive control, linearization, quadprog, bicycle model.**

I. Introduction

In this section, we'll talk about the dynamic model we used, our problem statement, and the motivation behind using MPC. Track taken from ROB 535 controls final project

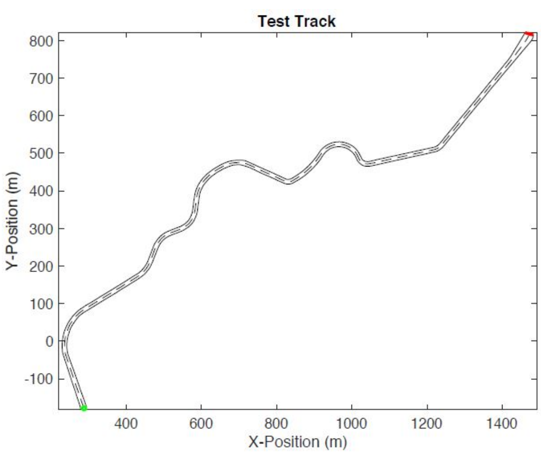

Fig. 1. Original Track

A. Bicycle Model

The non-linear bicycle model is given as follows:

$$\begin{bmatrix} \dot{X} \\ \dot{u} \\ \dot{Y} \\ \dot{v} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} ucos\psi - vsin\psi \\ \frac{1}{m}(-fmg + N_w F_x - F_{yf} sin(\delta_f)) + vr \\ usin\psi + vcos\psi \\ \frac{1}{m}(F_{yf} cos(\delta_f) + F_{yr}) - ur \\ r \\ \frac{1}{I_z}(aF_{yf} cos(\delta_f) - bF_{yr}) \end{bmatrix} \tag{1}$$

The lateral forces F_{yf} and F_{yr} are described using Pacejka "Magic Formula"

$$F_{zf} = \frac{b}{a+b} mg \tag{2}$$

$$F_{yf} = F_{zf} D_y sin(C_y \tan^{-1}(B_y \phi_{yf})) + S_{vy} \tag{3}$$

$$F_{zr} = \frac{a}{a+b} mg \tag{4}$$

$$F_{yr} = F_{zr} D_y sin(C_y \tan^{-1}(B_y \phi_{yr})) + S_v y \tag{5}$$

where

$$\phi_{yf} = (1 - E_y)(\alpha_f + S) \tag{6}$$

$$\phi_{yr} = (1 - E_y)(\alpha_r + S_{hy}) + \frac{E_y}{B_y} \tan -1(B_y(\alpha_r + S_{hy})) \tag{7}$$

where α_f and α_r are the front and rear lateral slip angles which are given in degrees in the previous formulas. The front and rear lateral slip angles which is described in radians is given by:

$$\alpha_f = \delta_f - \tan^{-1}(\frac{v + ar}{u}) \tag{8}$$

$$\alpha_r = -\tan^{-1}(\frac{v - br}{u}) \tag{9}$$

Additionally, combined longitudinal and lateral loading of tires will be limited to F_x^* and F_{yr}^* in the following manner:

$$F_{total} = \sqrt{(N_w F_x)^2 + (F_{yr})^2} \tag{10}$$

$$F_{max} = 0.7mg \tag{11}$$

If $F_{total} > F_{max}$:

$$F^{x} = \frac{F_{max}}{F_{total}} F_x \tag{12}$$

$$F^{yr} = \frac{F_{max}}{F_{total}} F_{yr} \tag{13}$$

The inputs into this model are δ_f , which is the front wheel steering angle; and F_x, which is the traction force generated at each tire by the vehicle's motor. The vehicle begins from the following initial condition:

$$\begin{bmatrix} x \\ u \\ y \\ v \\ \psi \\ r \end{bmatrix} = \begin{bmatrix} 287[m] \\ 5[m/s] \\ -176[m] \\ 0[m/s] \\ 2[rad] \\ 0[rad/s] \end{bmatrix} \tag{14}$$

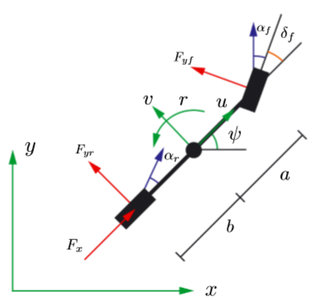

Fig. 2. An illustration of the bicycle model used to define the vehicle's dynamics.

TABLE I
DYNAMIC MODEL CONSTANTS

Vehicle Parameter	Value
δ	[-0.5,0.5]
F_x	[-5000,5000]
m	1400
N_w	2
f	0.01
I_x	2667
a	1.35
b	1.45
B_y	0.27
C_y	1.2
D_y	0.7
E_y	-1.6
S_{hy}	0
S_{vy}	0
g	9.806

II. Methodology

MPC (Model Predictive Control) is a time control strategy in which the future steps i.e the sequence of the control steps is determined my minimizing the cost function (objective function) at each time step over a horizon dependent upon the equations and constraints of the model.In short Model Predictive Control is used to predict the future behaviour of the system. One of the most important advantage of using MPC is that it can handel multiple input and multiple output systems and interactions between them.

We use the linear MPC because of its ability to find a global and optimal minimum by placing constraints in the inputs and states. We use the linear MPC algorithm where we apply successive linearization to develop a time-variant linear system.

We obtain a linear model of our dynamics by calculating an error model about the reference trajectory. In order to generate the reference trajectory we used a Proportional controller by calculating the lateral error between the center-line of the track and the current position of our vehicle. This error is used as the feedback reference to then generate the steering command.

A. Trajectory Tracking using MPC

While obtaining the nominal trajectory we set a constant lateral velocity and this leads to a reduced order model. This reduced order model is:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{v} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} u cos\psi - v sin\psi \\ u sin\psi + v cos\psi \\ \frac{1}{m}(F_{yf}cos(\delta_f) + F_{yr}) - ur \\ r \\ \frac{1}{I_z}(aF_{yf}cos(\delta_f) - bF_{yr}) \end{bmatrix} \tag{15}$$

We apply Euler integration to generate a discrete time system by approximating the dynamics as follows:

$$x(k+1) = x(k) + dt \times (A(k)x(k) + B(k)u(k)) \tag{16}$$

where $x(k) \in \mathbb{R}^n, A(k) \in {}^{n\times n}, B(k) \in {}^{n\times m}$. We discretized the system at a sampling time of 0.01s and used a prediction horizon of 10 steps. For our model, A(k) and B(k) are as follows:

$$A(k) = \begin{bmatrix} 0, 0, -u_0 sin(\psi_{ref}) - v_{ref}cos(\psi_{ref}), -sin(\psi_{ref}), 0 \\ 0, 0, 0, 0, 1 \\ 0, 0, 0, \frac{-(Ca_r + Ca_f)}{mu_0}, \frac{(Ca_r b - Ca_f a - mu_0^2)}{mu_0} \\ 0, 0, 0, \frac{(Ca_r b - aCa_f)}{I_z u_0}, \frac{-((b^2 Ca_r) + (a^2 Ca_f))}{I_z u_0} \end{bmatrix} \tag{17}$$

$$B(k) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{Ca_f}{m} \\ \frac{aCa_f}{I_z} \end{bmatrix} \tag{18}$$

To find the optimized solution we use quadratic programming that is specified by:

$$min\frac{1}{2}x^T Hx + f^T x \tag{19}$$

such that $Aeq * x = beq$ and $lb \leqslant x \leqslant ub$. H and Aeq are matrices and beq, lb abd ub are vectors.

The initial conditions of the dynamics are satisfied in the first three rows of Aeq and beq. All subsequent rows of beq are zero. Each subsequent row of Aeq represents the Euler integration steps in order from time step k to time step $k + 10$. Aeq encodes the linear equalities and is a $M \times N$ matrix where M are the number of equalities i.e. in our case $55(numberofstates \times numberofstatedecisionvariables)$; N is the number of total decision variables. beq is a vector with M rows. An example of Aeq for the first time step is:

$$\begin{bmatrix} Aeq(6:15, 6:15) \\ Aeq(6:15, 6:10) \\ Aeq(6:15, 56) \end{bmatrix} = \begin{bmatrix} -eye(5) \\ A(2) \\ B(2) \end{bmatrix} \tag{20}$$

The remaining elements of Aeq can be filled in a similar fashion. H is the quadratic objective term. We construct it using a quadratic penalty of $Q = [1, 1, 0.5, 1, 1]$ and $R = 0.1$.

We compute the states using ode45 and using the inputs generated from MPC.

III. Results and Discussion

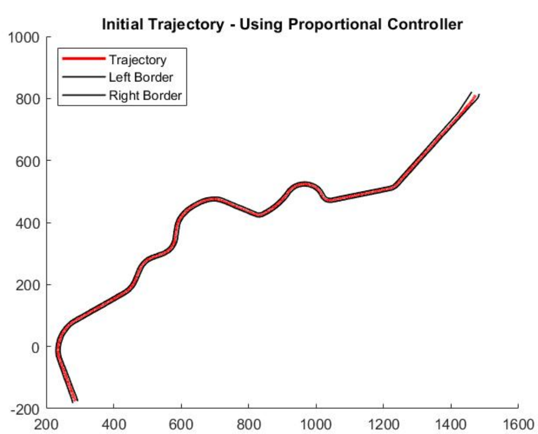

Fig. 3. Initial trajectory generated

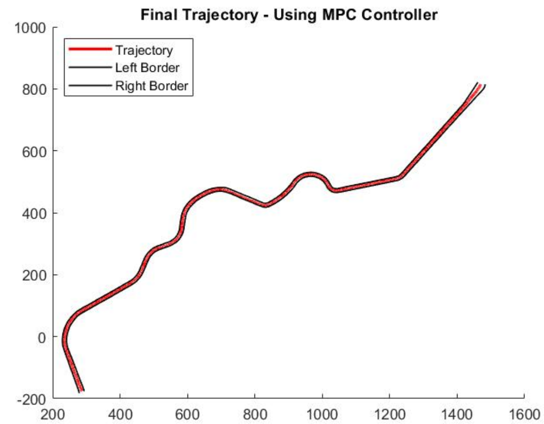

Fig. 4. Final trajectory generated

IV. Conclusion

V. Appendix

The code for this project is uploaded on GitHub and can accessed through this link.