On Properties of Mechanical Resonance

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1 Introduction

This experiment investigates properties of mechanical resonance of a mass attached to a clamped hacksaw blade. It begins with finding the natural frequency of our system, then introducing Eddy damping [1] to see any change in oscillation frequency under different damping conditions. We finish by adding a mechanical wave driver [2] and analyzing oscillation amplitudes and frequency phase distinctions at multiple driving frequencies.

2 Abstract

After inputting a mechanical wave driver and taking multiple data points at different driving frequencies, we concluded that the model of a simple harmonic oscillator sufficiently described the resonant behaviour observed. The amplitude was largest when the driving and natural frequencies were out of phase by around $\frac{\pi}{2}$ radians.

3 Theory and Procedure

We consider a damped oscillator being described by $\ddot{x}+2\gamma\dot{x}+\omega_o^2 x=0$. Where γ and ω_o are the damping coefficient and natural frequency respectively. Therefore, its forced oscillator counterpart will be characterized by

$$\ddot{x}+2\gamma\dot{x}+\omega_o^2 x=\frac{F_o}{m}e^{i\omega t} \tag{1}$$

Where F_o and ω are the driving force and driving frequency respectively. Giving us a solution of $x(t) = x_o(\omega)e^{i\omega t}$ and knowing $a(t)=\ddot{x}(t)$, gives us a final solution

$$|a_o(\omega)| = \frac{\omega^2(F_o/m)}{\sqrt{(\omega_o^2-\omega^2)^2+4\gamma^2\omega^2}} \tag{2}$$

Where the phase of this acceleration is given by

$$tan(\phi) = \frac{2\gamma\omega}{\omega^2-\omega_o^2} \tag{3}$$

Equations 2 and 3 are what we analyzed in our experiment.

To examine properties of mechanical resonance, we attached an accelerometer [3] to our mass. A magnet on the accelerometer and an external copper magnet plate allowed us to incorporate Eddy damping as a way to control damping. We used a data acquisition device (DAQ) [4] to oscillate our mass, and an oscilloscope [5] to control the natural oscillation of the mass. We then introduced the mechanical wave driver, which was set perpendicular to the hacksaw blade and connected it to a function generator [6] to exert fine control over the driving frequency. The system was then clamped to fix their relative positions throughout the experiment. The mass/accelerometer system was then subjected to many driving frequencies near and far from a calculated natural frequency of 13.16 ± 0.09 Hz under three levels of damping.

4 Results

Fig.1a shows the phase shift as a function of driving frequency, in which the phase shift is relative to the mass-blade systems natural frequency. We acquired at least five data points for each damping condition, and fit it using equation 3 (note that γ is our damping coefficient). The error bars propagate from the time step error when calculating phase shift with LabVIEW [7] data. Our sample rate and number of samples limited us to a 5×10^{-4}s difference between each data point. Fig.1b shows the acceleration amplitude of our system as a function of driving frequency which we fit with a slight variation of equation 2, where we fit $\frac{|a_o(\omega)|}{F_o/m}$. The error for Fig.1b (although negligible when plotting) propagates from the standard deviation we acquire when converting amplitudes. We calibrated our system by scaling our voltage data by a factor of $\frac{g}{\Delta V}$ to measure acceleration amplitude, which is a more appropriate measurement for our system.

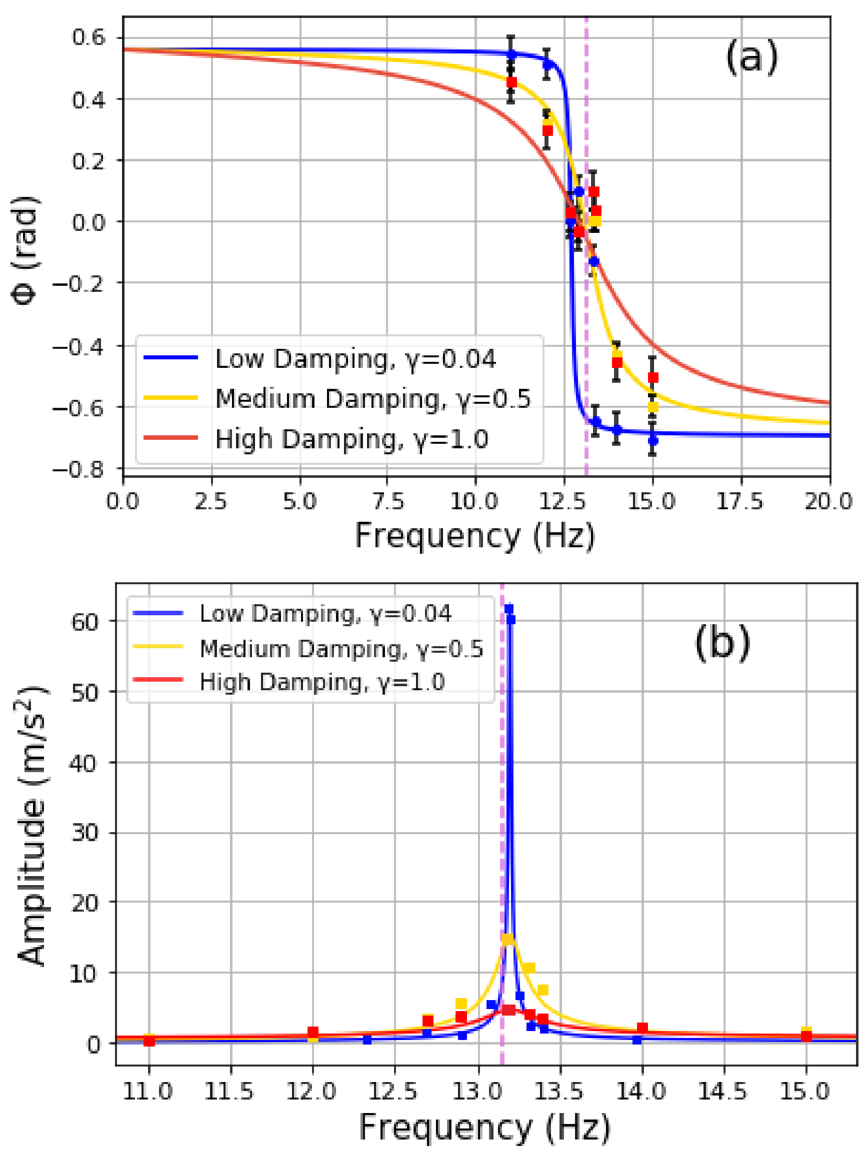

Figure 1: (a) Phase shift of the driven oscillation as a function of frequency under different damping conditions γ. Errors were taken from the root of the variance. (b) Is the acceleration amplitude of the system as a function of frequency. The error bars are too small to see, hence is excluded. The dotted vertical line represents an experimentally calculated natural frequency of 13.16 ± 0.09 Hz

5 Discussion

We found that our data properly models resonant behaviour. Near our natural frequency where Fig.1a has $\delta\phi = 1.2 \pm 0.3\ rad$, we see a large increase in the systems acceleration amplitude with a peak of $61.3\ m/s^2$, concluding that the applied force is in harmonic proportion to the natural frequency of the system. These findings are satisfying given equations 2 and 3 predict this. As ω approaches ω_o, our $\omega_o^2 - \omega^2$ goes to zero, hence ϕ goes to $\frac{\pi}{2}\ rad$. We then reach harmonic proportion as mentioned above, which increases $|a_o(\omega)|$ significantly. It is also important to recognize the invariant resonant frequency under different Eddy damping conditions, as was introduced at the start. Note that there is a slight discrepancy from our peak amplitude and phase shifts with our natural frequency which will be discussed in our sources of error.

We find discrepancy between our data and our modeled prediction. Our data points in 1a are shifted to the right in comparison to our theoretical models, and the horizontal errors are too small to be in agreement. This problem most likely arose from preliminary calculations of our natural frequency. There was dilemma between what we calculated to be our natural frequency, and what LabVIEW [7] was showing. Furthermore, no value of frequency seemed to have a $\frac{\pi}{2}$ phase shift according to our LabVIEW graphing software. For this issue we (after addressing to a supervisor) decided to proceed with our experimentally calculated frequency, as time was a constraint. This is what we reasoned to be the cause of offset of natural frequency that we see in figures 1a and 1b. A big case for discrepancy would also be the external vibrations from other lab stations, which would deviate our oscillation values.

6 Conclusion

We see that the model of a simple harmonic oscillator sufficiently describes the resonant behaviour observed. The amplitude was largest when the driving and natural frequencies were out of phase by around $\frac{\pi}{2}$ radians. This behaviour was consistent for different damping conditions, so we conclude that Eddy damping did not shift our natural frequency. Our predictions were satisfied by our data; although there was some discrepancy, the general trend matched our theoretical model. Of course our analysis of this has a limited range. This may not hold for larger amplitudes as the saw blade would break, so our range of frequencies we can investigate are limited. Furthermore, this whole experiment was done under a small angle approximation. At larger amplitudes, this model would break down and the acceleration of the blade would no longer be proportional to the angular displacement.

References

[1] Phys 231 Labscripts 4-5. See https://canvas.sfu.ca/courses/48548/files/folder/labscripts.

[2] Pasco Mechanical Wave Driver SF-9324. See https://www.pasco.com/index.cfm.

[3] SparkFun Triple Axis Accelerometer Breakout - ADXL335. see https://www.sparkfun.com/

[4] Data Acquisition Device P231. See https://www.dataq.com/data-acquisition/

[5] Oscilloscope TDS1002B. See https://www.tek.com/

[6] Xantrex XT60-1 power supply, with programmable analog interface. See www.xantrex.com.

[7] LabVIEW 2019-National Instruments. See www.ni.com.