Visualising Movement of Two Point Vortices

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January 27, 2017

1 AIM

To find the pathlines of the centres of two point vortices over a time period and visualising the same on plots

2 Theory

Let us assume vortices of strengths Γ_1, Γ_2 with their centres at A(x1,y1) and B(x2,y2) in the cartesian coordinate plane. Let the magnitude of the distance between the two points be r. We shall now compute the positions of these vortex centres after a small time dt. Iterating this process we can get the positions of these centres over any period of time. We then plot the graph from this data.

2.1 Computing positions after a time dt

We assumed the centres are at A(x1,y1) and B(x2,y2) then the vector K(x2-x1,y2-y1) is along the ray joining these two centres from A(x1,y1). At point B, the velocity it attains due the the vortex 1(at A) is given by

$$V_\theta = -\frac{\Gamma_1}{2\pi r} \tag{1}$$

Here the -ve sign denotes the rotation of the vortex to be clockwise. The vector direction of the movement at point B perpendicular to K in the clockwise sense is L(y2-y1, -(x2-x1)). Then from (1), L and dt the new position of B after a time dt can be given as

$$x_2^* = x_2 + [\frac{\Gamma_1(y_2 - y_1)}{2\pi r^2}]dt \tag{2}$$

$$y_2^* = y_2 - [\frac{\Gamma_1(x_2 - x_1)}{2\pi r^2}]dt \tag{3}$$

Similarly we can extend these equations to A:

$$x_1^* = x_1 + [\frac{\Gamma_2(y_1 - y_2)}{2\pi r^2}]dt \tag{4}$$

$$y_1^* = y_1 - [\frac{\Gamma_2(x_1 - x_2)}{2\pi r^2}]dt \tag{5}$$

Hence we now have the the new positions calculated. We add these to a list and then compute the new r, K, L by replacing A(x1,y1) and B(x2,y2) with A(x1*,y1*) and B(x2*,y2*). Iterating this process we obtain the pathlines of the centres over a period of time T= cdt where c is the no.of iterations.

3 Code Flow

- As discussed in the theory we have the variables c, t, G1, G2, x1, y1, x2, y2 to be input. As such we define these first in our code.
- We then define the lists Xl1, Xl2 ,Yl1 ,Yl2 with the initial points that will hold all the positions of our vortices and compute r.
- In a for loop with c iterations we compute the positions of the vortices using (2), (3), (4), (5) and append them into their respective lists and compute the new r over each iteration.
- We then plot Yl1 vs Xl1 and Yl2 vs Xl2 using the matplotlib library. If the plot is unsatisfactory we can always change c, t to get a better plot for readability.

4 Observations

From the plots we obtained we have the folowig observations

S.no.	Γ1	Γ2	Observed shape
1	3000	3000	Both trace the same circle over opposite ends of its diameters
2	-3000	3000	Two lines parallel to each other
3	5000	3000	Two concentric circles,vortex centres lie on opposite sides of the diameters drawn from inner circle towards the outer. Equal to the movement of twin star systems with unequal masses
4	-5000	3000	Two concentric circles, vortex centres lie on the same side of the diameters drawn towards the outer circle