Visualising movement of a particle in a field of two point vortices and plotting stream plots

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1 AIM

- To find the pathline of a particle at any position in the field of two point vortices over a time period and visualising the same on a plot.
- Plotting the streamlines (read stream plot) of the two vortices at any instant of time.

2 Theory

In report-1 we have found the pathlines of two point vortices, we shall use the positional data here to compute the position data of the particle and the streamlines for the stream plots.

2.1 Computing position of the particle after a time dt

We assume the vortex centres are at A(x1,y1) and B(x2,y2) and the particle at P(x3,y3) then the vector K1(x3-x1,y3-y1) is along the ray joining the the particle and the vortex from A(x1,y1). The vector K2(x3-x2,y3-y2) is along the ray joining the the particle and the vortex from B(x2,y2). The distance between the particle to the corresponding vortex can be found by finding the absolute value of the vectors K1, K2, we name these as r1 and r2 respectively.

At point P, the velocity it attains due the vortex 1(at A) is given by

$$V_\theta = -\frac{\Gamma_1}{2\pi r_1} \tag{1}$$

At point P, the velocity it attains due the vortex 2(at B) is given by

$$V_\theta = -\frac{\Gamma_2}{2\pi r_2} \tag{2}$$

Here the -ve sign denotes the rotation of the vortex to be clockwise. We then rotate the vectors K1 and K2 by 90 in the clockwise sense to obtain the vector directions of their corresponding velocities. We then add the resultant dispalce ments by both the vortices on this particle and compute the new positions.

$$x_3^* = x_3 + [\frac{\Gamma_1(y_3 - y_1)}{2\pi r_1^2}]dt + [\frac{\Gamma_2(y_3 - y_2)}{2\pi r_2^2}]dt \tag{3}$$

$$y_3^* = y_3 - [\frac{\Gamma_1(x_3 - x_1)}{2\pi r_1^2}]dt + [\frac{\Gamma_2(x_3 - x_2)}{2\pi r_2^2}]dt \quad (4)$$

Hence we now have the the new positions calculated. We add these to a list and then compute the new r1,r2, K1,K2 by replacing P(x3,y3) with P(x3*,y3*). Iterating this process we obtain the pathline of the particle over a period of time T.

3 Code Flow

- Define T, t, the period of time and dt respectively.
- Define x, p, the list containing the initial positions of the votex centres and the particle in complex form. Define G, list of the corresponding vortex strengths
- Define Xl, List of lists containing positional data of the vortex centres and the particle we also add the intial data when defining this list
- Define meshgrid N vs M
- In the for loop we perform the following actions in sequence:
  - calculate the vectors from one vortex to another [l]
  - calculate vectors from the vortex centres to the particle [pl0,pl1]
  - find absolute value of these vectors (read as distance between them) [rp,r]
  - find the new position of vortex centres A,B (we calculate the velocity vector by turning the vector l by 90 using complex math i.e. multiplying by -i)[x]
  - find the new position vector of P in a similar manner[p]
  - find the distance between each point in the meshgrid to the vortex centres[Rs]
  - find the velocity vector magnitude in the x,y directions for each point in the meshgrid[Ux, Uy]
  - append the positional data of the vortex centres and particle in to their corresponding lists.[Xl]
  - plot the stream plot from Ux,Uy data.
- We finally code the placeholders for plotting the streamplot at the end of the time T, pathlines of the vortex centres and the pathline for the particle under consideration

4 Observations

From the plots we obtained we have the folowing observations

- The particle path is very difficult to predict at first glance, plotting it gives us a clear picture of how the particle might move

- The stream plot data is computation heavy and hence should be plotted cautiously varying the parameters for the ease of computation
- The stream plots change as expected and when seen at a very low resolution such that the mesh grid is much larger that the distance between the vorte centres we obtain streamlines almost circular. Thereby we could theroetically club vortices that are close into a single vortex for computational ease when dealing with large number of vortices.