Assignment - 2

Varunaditya Singhal

***Abstract*—This document contains the solution to Exercise 2.42 (b) of Oppenheim.**

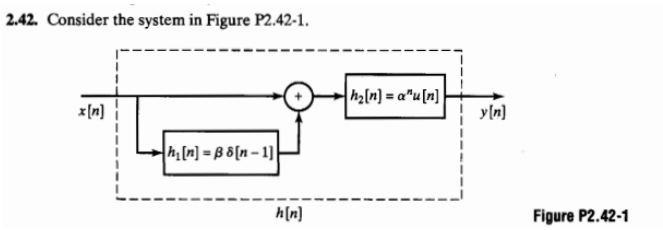

Fig. 1: Plot of the Figure

Problem 1. In the above diagram, find the frequency response of the system.
Solution: Taking the Fourier transform of h[n] from part (a), we get

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \tag{1}$$

$$= \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-j\omega n} + \beta \sum_{n=-\infty}^{\infty} \alpha^{n-1} u[n-1]e^{-j\omega n} \tag{2}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} + \beta \sum_{t=0}^{\infty} \alpha^{t-1} e^{-j\omega t} \tag{3}$$

where we have used l=(n-1) in the second summation. Now, we can write the above equation as -

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}} + \frac{\beta e^{-j\omega}}{1-\alpha e^{-j\omega}} \tag{4}$$

$$= \frac{1+\beta e^{-j\omega}}{1-\alpha e^{-j\omega}} \tag{5}$$

Kindly note that the Fourier tranform of α^nu[n] is well known, and the second term of h[n] (see part (a)) is just a scaled and shifted version of α^nu[n]. So, we could have used the properties of the Fourier tranform to reduce the algebra.