Optimization in Energy and Power

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May 1, 2022

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Introduction

Applying optimization-based methods for the energy management of parallel hybrid vehicles to reach our desired optimal solution in less time.

Introduction

Few scenarios where convex optimization in Hybrid Electric Vehicle plays important role are:

- **Inter-conversion:** Based on the parameter, if the driver himself wants to change the power source what parameters would be optimum to do so.
- **Battery boundaries:** When battery State of Charge reaches boundary condition then choosing optimum solution.
- **Destination:** Depending on the distance travelled and journey logistics the optimum fuel and battery consumption.

Vehicle Model

- The Vehicle model conversion from non-linear vehicle model to approximated convex model.
- Expression of component model as convex optimization variables.
- Working with **3** decision variables i.e
  - Engine on/off state.
  - Torque spilt.
  - Gear split.
- Variable of our interest = Torque split.

Battery Model

Formation of a convex model based on the figure , observing the open Voltage depending on SOC.

$$SOC(k+1) = SOC(k) - \Delta t \cdot \frac{I_b(k)}{Q_0}$$

$$I_b(k) = \frac{V_{oc}\sqrt{V_{oc}^2 - 4R_i(P_m(k) + P_{aux})}}{2 \cdot R_i}$$

Battery Model

$$P_m(k) \leq V_{oc} \cdot I_b(k) R_i \cdot I_b^2(k) P_{aux}$$

In order to maintain the convexity the variable of interest and battery SOC are being constraint as follows:

$$I_{b,min} \leq I_b(k) \leq I_{b,max}$$

$$SOC_{min} \leq SOC(k) \leq SOC_{max}$$

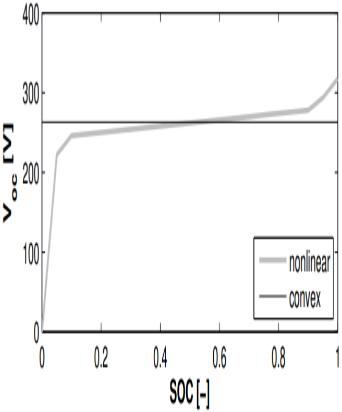

Figure: Open Voltage Model

Road Load Model

Let v(k) be speed of vehicle dependent on time k.

- Wheel Speed (ω_w):

$$\omega_w(k) = \frac{v(k)}{r_w}$$

- Wheel Torque (T_w):

$$T_w(k) = r_w \cdot \Big\{ \frac{1}{2}\rho_{air} c_d A v^2(k) + c_r m_v a_g cos(\alpha(k)) + m_v a_g sin(\alpha(k)) + (m_v + m_r(g(k)))a(k) \Big\}$$

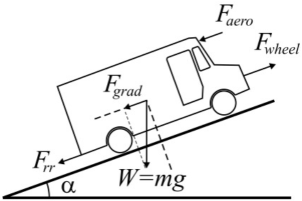

Figure: Road Load Model

Gearbox Model

Here i_g represents gearbox ratio.

- Gearbox speed (ω_g):

$$\omega_g(k) = \omega_w(k) \cdot i_g(k)$$

- Gearbox Torque (T_g):

$$T_g(k) = T_w(k) \cdot \frac{1}{i_g(k)} \cdot \left\{ \eta_{g,0} - \frac{\eta_{g,1}}{\omega_{g,1}} \cdot \omega_g(k) \right\}^{-sign(T_w(k))}$$

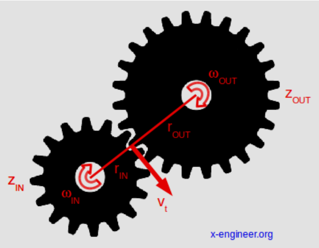

Figure: Roadload to Gearbox

Gearbox Model

Fuel consumption in Gear-shifting

- Equivalent fuel consumption per gearshift (m_g):

$$m_g(k) = \begin{cases} 0.01 & \text{if } g(k) \neq g(k-1) \\ 0 & \text{otherwise} \end{cases}$$

- The gear g is being controlled by gear controlled variable (u_g):

$$g(k) = g(k-1) + u_g(k), \quad u_g \in \{-1, 0, 1\}$$

Torque Split Model

- The torque split between the electric motor and the internal combustion engine is determined by the motor torque (T_m).
- $$T_e(k) = T_g(k) - T_m(k)$$

Electric Motor Model

- Motor Power (P_m):

$$P_m \geq b_0(\omega_m) + b_1(\omega_m)T_m + b_2(\omega_m)T_m^2$$

- Convexity of the equation
- Limitation on motor speed:

$$0 \leq \omega_m(k) \leq \omega_{m,max}$$

- Limitation on motor torque:

$$T_{m,min}(\omega_m(k)) \leq T_m(k) \leq T_{m,max}(\omega_m(k))$$

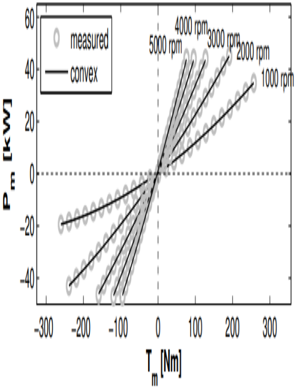

Figure: Convex Motor Model

Engine Model

- Mass of fuel consumed ($\dot{\mathbf{m}}_f$)
-

$$\dot{\mathbf{m}}_f(k) \geq u_e(k) \cdot \Big\{ a_0(k) + a_1(k) \cdot \mathbf{T}_e k + a_2(k) \cdot \mathbf{T}_e^2(k) \Big\}$$

- Convexity of the equation
- Limitation on engine speed:

$$\omega_{e,min} \leq \omega_e(k) \leq \omega_{e,max}$$

- Limitation on engine torque:

$$0 \leq T_e(k) \leq T_e(\omega_e(k))$$

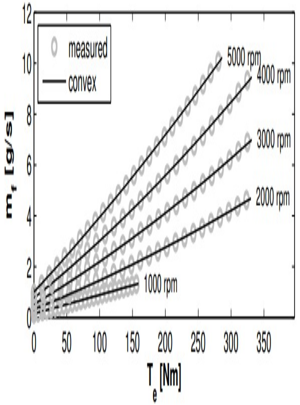

Figure: Convex Engine Model

Engine Model

Fuel consumption in change in state of engine.

- The engine on/off decision is determined via the variable $u_e \in \{0, 1\}$ such that the engine state (e) is given by

$$e(k) = u_e(k)$$

- Equivalent fuel consumption in changing state of engine (m_e):

$$m_e(k) = \begin{cases} 0.3g & \text{if } e(k) = 1 \text{ and } e(k-1) = 0 \\ 0 & \text{otherwise} \end{cases}$$

Problem Formulation

- The problem here for us is to minimize the overall fuel consumption, whenever the engine's running.
- We are dealing with 3 state variables here; Which are for,
  - Engine On/off
  - Gearshift
  - State of Charge
- We deal with Engine start and Gearshift through Dynamic Programming.
- For SOC optimization we use *Convex Optimization*.

Note: We calculate the gearshift and engine On/Off variables in advance to solve the problem.

Problem Formulation

Objective

Minimize:

$$\sum_{k=1}^{N} \dot{\mathbf{m}}_f(k) \cdot \Delta t$$

- The following objective depicts the fuel consumption which is being minimized by using the Electric power upon the intervals we are running the engine.
- Most of the constraints are derived on the basis of equations we had in Vehicle Model Description.
- Another constraint is the charge-sustenance condition which is-

$$SOC(N+1) = SOC(1) = SOC_0$$

where SOC_0 is the initial SOC.

Iterative Model

We begin by defining s as our equivalence factor between fuel consumption and electrical energy. We find this equivalence factor by solving the aforementioned Convex Problem.

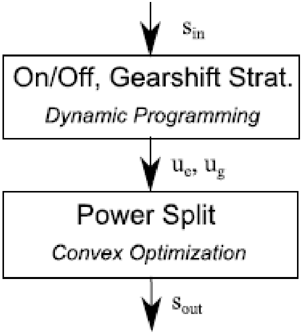

Figure: Sequential Model for Algorithm

Iterative Model

Optimality Condition

$$s_{out} \equiv s^* \text{ iff } s_{in} \equiv s_{out}$$

Keeping in mind the derived *condition for optimality* we devise a iterative algorithm to find optimal values, starting off with a s_{in} we begin our algorithm and iterate in the following way as shown in the upcoming slide.

Iterative Model

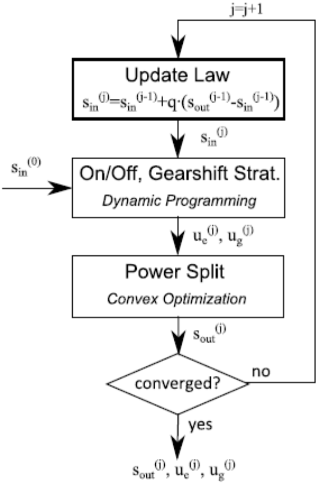

Figure: Iterations on Sequential Model

Iterative Model

Steps in the Iterative model:

- **Step 1: Initialization of the model**
  - Includes initializing of the counter and equivalence factor.
- **Step 2: Use of DP to obtain $u_g^{(j)}, u_e^{(j)}$**
  - Values of $u_g^{(j)}$ and $u_e^{(j)}$ are obtained which are optimal for $s_{in}^{(j)}$.
- **Step 3: Use of convex optimization to obtain $s_{out}^{(j)}$**
  - For the acquired values of $u_g^{(j)}$ and $u_e^{(j)}$, we calculate the s_{out}.

Iterative Model

- **Step 4: Check for Convergence**
  - We calculate the root mean square error of the solution that we obtain for checking the tolerance of the solution using the following equation-

$$E = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(s_{in}(k)^{(j)} - s_{out}(k)^{(j)})^2}$$

  - If tolerance is higher than the tolerance level, increase j and update law as in step-5.

Iterative model

- **Step 5: Update law**
  - If Tolerance level is higher than the Maximum tolerance level, perform damping. We do the following update-

    $$s_{in}^{(j)} = s_{in}^{(j-1)} + q * (s_{out}^{(j-1)} - s_{in}^{(j-1)})$$

    Here q is initialized at 0.2 and whenever the error is high we set $q \rightarrow 0.7.q$
    Using this law s_{out} converges to the optimal solution.

Result

- The minimum and maximum values of the SOC are assumed to be 20% and 80%, respectively.
- The given table summarizes the fuel consumption, the number of engine starts, the number of gearshifts, and the computation time for the four cases considered-
- Summarizing the findings described in this section, the DP-C method yields more accurate results in much less time than a pure DP implementation.

Results

Cycle	Performance Index	DP	DP-C
NEDC	FC[L/100 km]	4.405	4.399
	ICE starts [#]	7	7
	Gearshifts [#]	86	86
	CPU time [s]	791	6
FTP	FC [L/100 km]	4.288	4.279
	ICE starts [#]	28	28
	Gearshifts [#]	180	182
	CPU time [s]	1287	14
CADC	FC [L/100 km]	5.635	5.627
	ICE starts [#]	78	75
	Gearshifts [#]	316	310
	CPU time [s]	2363	105
CADC (bounded)	FC [L/100 km]	5.657	5.643
	ICE starts [#]	90	85
	Gearshifts [#]	322	310
	CPU time [s]	527	122

Figure: Comparison

Conclusion

- This paper presents a method to calculate the globally optimal energy management strategy for a parallel HEV on a given driving cycle taking into account penalties to avoid frequent engine start and/or gearshift events.
- The proposed method can be extended to other vehicle topologies and different formulations of the energy management problem

THANK YOU

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