Assignment

12.7 - 8

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Question

A charged 30 μF capacitor is connected to a 27 mH inductor. Suppose the initial charge on the capacitor is 6mC.What is the total energy stored in the circuit initially? What is the total energy at later time?

Solution

Given,
Initial charge on capacitor is 6mC.

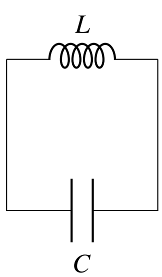

Fig. 0. Circuit diagram

Symbol	Description	Value
q(0^+)	Initial charge on capacitor	6 mC
q(t)	Charge on capacitor	-
L	Value of inductance	27 mH
C	Value of capacitance	30 μF
E	Total energy stored in circuit	-
E_L	Energy stored in inductor	-
E_C	Energy stored in capacitor	-
i(t)	current in the inductor	$\frac{dq}{dt}$
I(s)	Laplace transform of i(t)	-

TABLE 0
TABLE OF PARAMETERS

$$L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 0 \tag{1}$$

$$L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt = 0 \tag{2}$$

$$\mathcal{L}\{u(t)\} \leftrightarrow \frac{1}{s} \tag{3}$$

$$\mathcal{L}\left\{\frac{dq}{dt}\right\} \leftrightarrow sQ(s) \tag{4}$$

$$i(t) = \frac{dq}{dt} \tag{5}$$

From laplace transformations (3) and (4),

$$LsI(s) + \frac{1}{C}\frac{q(0^+)}{s} + \frac{1}{C}\frac{I(s)}{s} = 0 \tag{6}$$

$$I(s) = \frac{-q(0^+)}{LCs^2 + 1} \tag{7}$$

From initial value theorem ,

$$i(0^+) = \lim_{s\to\infty}[sI(s)] \tag{8}$$

$$i(0^+) = \lim_{s\to\infty}\left[s\frac{-q(0^+)}{LCs^2 + 1}\right] = 0 \tag{9}$$

From final value theorem ,

$$i(\infty) = \lim_{s\to 0}[sI(s)] \tag{10}$$

$$i(\infty) = \lim_{s\to 0}\left[s\frac{-q(0^+)}{LCs^2 + 1}\right] = 0 \tag{11}$$

$$i(0^+) = i(\infty) = 0 \tag{12}$$

Hence,

$$q(0^+) = q(\infty) = 6\ mC \tag{13}$$

$$E = E_L + E_C \tag{14}$$

from (9),

$$E_L = 0 \tag{15}$$

$$E_C = \frac{q^2}{2C} \tag{16}$$

$$E_C = 0.6\ J \tag{17}$$

$$E = 0.6\ J \tag{18}$$

Hence , the total energy stored in the circuit initially and at a later time is 0.6 J.