Assignment

11.9.1 - 9

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Question

Find a_9 in the sequence $a_n = (-1)^{n-1} n^3$

Solution

Symbol	Value	Description
x(0)	1	First term of the sequence
x(n)	$(-1)^n (n+1)^3 u(n)$	$(n+1)^{th}$ term of the sequence

TABLE 0
Table of parameters

To obtain 9^{th} term of the sequence put n=8 in x(n)

$$x(8) = 729 \tag{1}$$

Using Z transform,

$$X(z) = \sum_{n=-\infty}^{\infty} (-1)^n (n+1)^3 u(n) z^{-n} \tag{2}$$

$$= \sum_{n=-\infty}^{\infty} (n+1)^3 u(n) (-z)^{-n} \tag{3}$$

$$= \sum_{n=-\infty}^{\infty} \left(n^3 + 3n^2 + 3n + 1\right) u(n) (-z)^{-n} \tag{4}$$

Replace z by $-z$ in known z-transforms,

$$u(n) \overset{Z}{\leftrightarrow} \frac{1}{1+z^{-1}}, |z| > 1 \tag{5}$$

$$nu(n) \overset{Z}{\leftrightarrow} \frac{-z^{-1}}{(1+z^{-1})^2}, |z| > 1 \tag{6}$$

$$n^2 u(n) \overset{Z}{\leftrightarrow} \frac{z^{-1}\left(z^{-1}-1\right)}{(1+z^{-1})^3}, |z| > 1 \tag{7}$$

$$n^3 u(n) \overset{Z}{\leftrightarrow} \frac{-z^{-1}\left(1-4z^{-1}+z^{-2}\right)}{(1+z^{-1})^4}, |z| > 1 \tag{8}$$

$$X(z) = \frac{z^{-2}-z^{-1}+1}{(1+z^{-1})^4}, |z| > 1 \tag{9}$$