Assignment

11.9.1 - 9

EE23BTECH11220 - R.V.S.S Varun

Question

Find a_9 in the sequence $a_n = (-1)^{n-1} n^3$

Solution

Symbol	Value	Description
x(0)	1	First term of the sequence
x(n)	$(-1)^n (n+1)^3 u(n)$	$(n+1)^{th}$ term of the sequence

TABLE 0
Table of parameters

To obtain 9^{th} term of the sequence put n=8 in x(n)

$$x(8) = 729 \tag{1}$$

From up-scaling property,

$$x(n) \overset{Z}{\leftrightarrow} X(z) \tag{2}$$

$$x(kn) \overset{Z}{\leftrightarrow} X\left(z^k\right) \tag{3}$$

From table,

$$x(2n) = (2n+1)^3 u(2n) \tag{4}$$

Using Z transform,

$$x(n) \overset{Z}{\leftrightarrow} X(z) \tag{5}$$

$$x(2n) \overset{Z}{\leftrightarrow} X\left(z^2\right) \tag{6}$$

$$u(2n) \overset{Z}{\leftrightarrow} \frac{1}{(1-z^{-2})}, |z| > 1 \tag{7}$$

$$2n\ u(2n) \overset{Z}{\leftrightarrow} \frac{z^{-2}}{(1-z^{-2})^2}, |z| > 1 \tag{8}$$

$$(2n)^2 u(2n) \overset{Z}{\leftrightarrow} \frac{z^{-2}\left(z^{-2}+1\right)}{(1-z^{-2})^3}, |z| > 1 \tag{9}$$

$$(2n)^3 u(2n) \overset{Z}{\leftrightarrow} \frac{z^{-2}\left(1+4z^{-2}+z^{-4}\right)}{(1-z^{-2})^4}, |z| > 1 \tag{10}$$

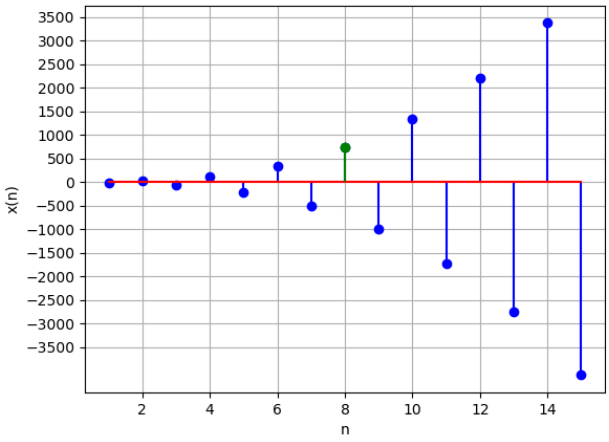
$$X\left(z^2\right) = \sum_{n=-\infty}^{n=\infty} (2n+1)^3 u(2n) z^{-2n} \tag{11}$$

$$= \sum_{n=-\infty}^{n=\infty} \left((2n)^3 + 3(2n)^2 + 3(2n) + 1\right) u(2n) z^{-2n} \tag{12}$$

$$= \frac{4z^6 + z^4 - 3z^2}{(z^2-1)^4}, |z| > 1 \tag{13}$$

Replace z^2 by z in (14)

$$X(z) = \frac{4z^3 + z^2 - 3z}{(z-1)^4}, |z| > 1 \tag{14}$$



Graph of x(n)