

Assignment

11.9.1 - 9

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QUESTION

Find a_9 in the sequence $a_n = (-1)^{n-1} n^3$

SOLUTION

Symbol	Value	Description
$x(0)$	1	First term of the sequence
$x(n)$	$(-1)^n (n+1)^3 u(n)$	$(n+1)^{th}$ term of the sequence

TABLE 0
TABLE OF PARAMETERS

To obtain 9th term of the sequence put $n=8$ in $x(n)$

$$x(8) = 729 \quad (1)$$

From up-scaling property,

$$x(n) \xleftrightarrow{Z} X(z) \quad (2)$$

$$x(kn) \xleftrightarrow{Z} X(z^k) \quad (3)$$

From table,

$$x(2n) = (2n+1)^3 u(2n) \quad (4)$$

Using Z transform,

$$x(n) \xleftrightarrow{Z} X(z) \quad (5)$$

$$x(2n) \xleftrightarrow{Z} X(z^2) \quad (6)$$

$$u(2n) \xleftrightarrow{Z} \frac{1}{(1-z^{-2})}, |z| > 1 \quad (7)$$

$$2n u(2n) \xleftrightarrow{Z} \frac{z^{-2}}{(1-z^{-2})^2}, |z| > 1 \quad (8)$$

$$(2n)^2 u(2n) \xleftrightarrow{Z} \frac{z^{-2}(z^{-2}+1)}{(1-z^{-2})^3}, |z| > 1 \quad (9)$$

$$(2n)^3 u(2n) \xleftrightarrow{Z} \frac{z^{-2}(1+4z^{-2}+z^{-4})}{(1-z^{-2})^4}, |z| > 1 \quad (10)$$

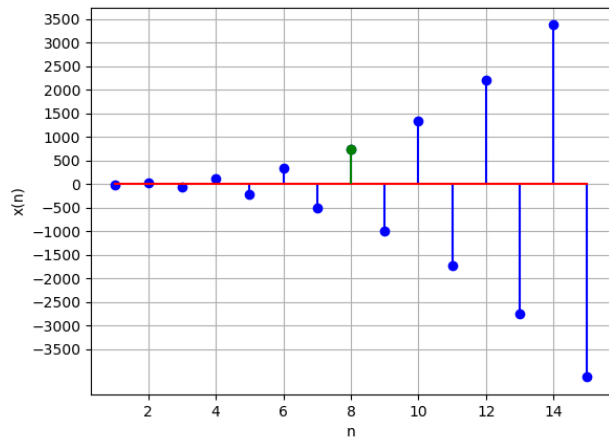
$$X(z^2) = \sum_{n=-\infty}^{n=\infty} (2n+1)^3 u(2n) z^{-2n} \quad (11)$$

$$= \sum_{n=-\infty}^{n=\infty} \left((2n)^3 + 3(2n)^2 + 3(2n) + 1 \right) u(2n) z^{-2n} \quad \{z : |z| > 1\} \quad (12)$$

$$= \frac{4z^6 + z^4 - 3z^2}{(z^2 - 1)^4}, |z| > 1 \quad (13)$$

Replace z^2 by z in (14)

$$X(z) = \frac{4z^3 + z^2 - 3z}{(z - 1)^4}, |z| > 1 \quad (14)$$



Graph of $x(n)$