

Assignment

12.7 - 8

EE23BTECH11220 - R.V.S.S Varun

QUESTION

A charged $30 \mu\text{F}$ capacitor is connected to a 27 mH inductor. Suppose the initial charge on the capacitor is 6 mC . What is the total energy stored in the circuit initially? What is the total energy at later time?

SOLUTION

Given,

Initial charge on capacitor is 6 mC .

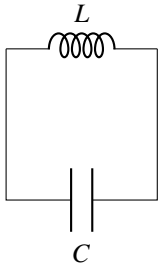


Fig. 0. Circuit diagram

Symbol	Description	Value
$q(0^+)$	Initial charge on capacitor	6 mC
q	Charge on capacitor	-
L	Value of inductance	27 mH
C	Value of capacitance	$30 \mu\text{F}$
E	Total energy stored in circuit	-
E_L	Energy stored in inductor	-
E_C	Energy stored in capacitor	-
i	current in the inductor	-
$I(s)$	Laplace transform of $i(t)$	-

TABLE 0
TABLE OF PARAMETERS

$$L \frac{di(t)}{dt} + \frac{1}{C} \int_{-\infty}^t i(t) dt = 0 \quad (1)$$

$$L \frac{di(t)}{dt} + \frac{1}{C} \int_{-\infty}^0 i(t) dt + \frac{1}{C} \int_0^t i(t) dt = 0 \quad (2)$$

$$\mathcal{L}\{u(t)\} \leftrightarrow \frac{1}{s} \quad (3)$$

$$\mathcal{L}\left\{\frac{dq}{dt}\right\} \leftrightarrow sQ(s) \quad (4)$$

$$i(t) = \frac{dq}{dt} \quad (5)$$

Laplace transform with variable s ,

$$LsI(s) + \frac{1}{C} \frac{q(0^+)}{s} + \frac{1}{C} \frac{I(s)}{s} = 0 \quad (6)$$

$$I(s) = \frac{-q(0^+)}{LCs^2 + 1} \quad (7)$$

To find initial value of current ,

$$i(0^+) = \lim_{s \rightarrow \infty} [sI(s)] \quad (8)$$

$$i(0^+) = \lim_{s \rightarrow \infty} \left[s \frac{-q(0^+)}{LCs^2 + 1} \right] = 0 \quad (9)$$

To find final value of current ,

$$i(\infty) = \lim_{s \rightarrow 0} [sI(s)] \quad (10)$$

$$i(\infty) = \lim_{s \rightarrow 0} \left[s \frac{-q(0^+)}{LCs^2 + 1} \right] = 0 \quad (11)$$

$$i(0^+) = i(\infty) = 0 \quad (12)$$

Hence,

$$q(0^+) = q(\infty) = 6 \text{ mC} \quad (13)$$

$$E = E_L + E_C \quad (14)$$

from (9),

$$E_L = 0 \quad (15)$$

$$E_C = \frac{q^2}{2C} \quad (16)$$

$$E_C = 0.6 \text{ J} \quad (17)$$

$$E = 0.6 \text{ J} \quad (18)$$

Hence , the total energy stored in the circuit initially and at a later time is 0.6 J .