

# INTRODUCTION TO LINEAR REGRESSION

# Linear Regression

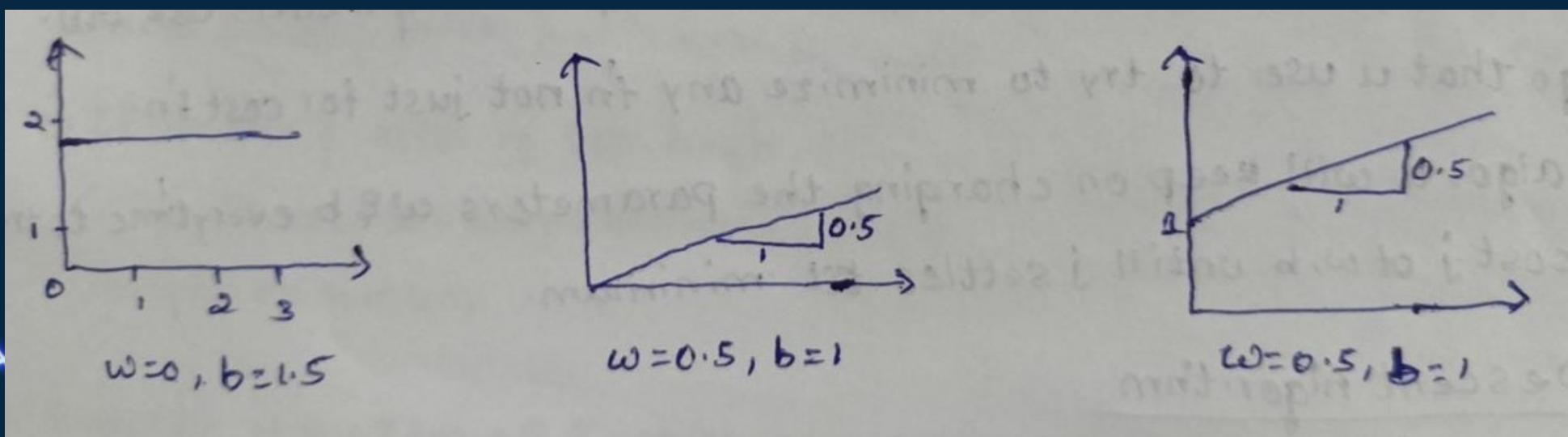
- Linear Regression is a supervised machine learning algorithm and performs a regression task
- In Regression there are infinitely many possible numbers that model could output.
- Linear regression is used in many different fields.
- **For example**, in finance, linear regression might be used to understand the relationship between a company's stock price and its earnings, or to predict the future value of a currency based on its past performance.
- Linear Regression includes both input features and output targets. To train the model, we feed both input features and output targets to our learning Algorithm.
- Here we fit a straight line to our data.

# Cost Function

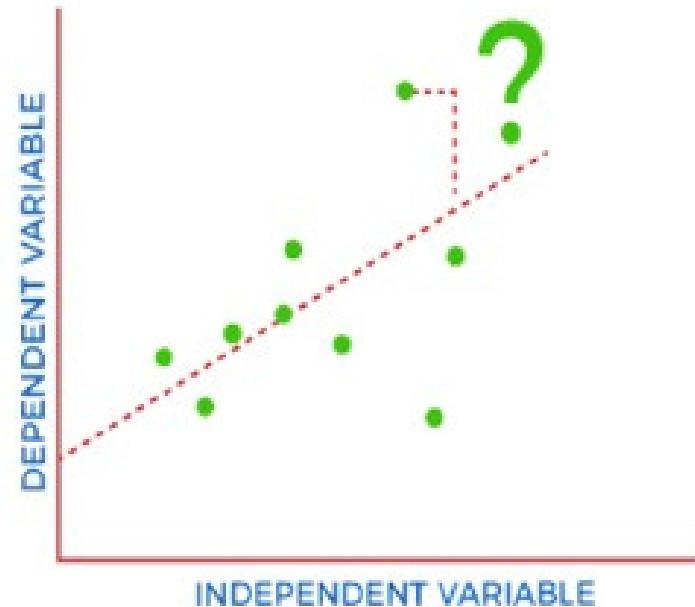
- In order to implement Linear Regression we need to define Cost Function which tells us how well the model is doing.

For Example, the model we use to fit in the training set is  $f(x) = w \cdot x + b$  where  $w, b$  are parameters.

Depending on the values of  $w, b$  we get different  $f(x)$  which generates a different line.



## COST FUNCTION IN MACHINE LEARNING



**How do you find the values of  $w$  and  $b$  so that we get different  $f(x)$  which generates a line on the graph?**

model:

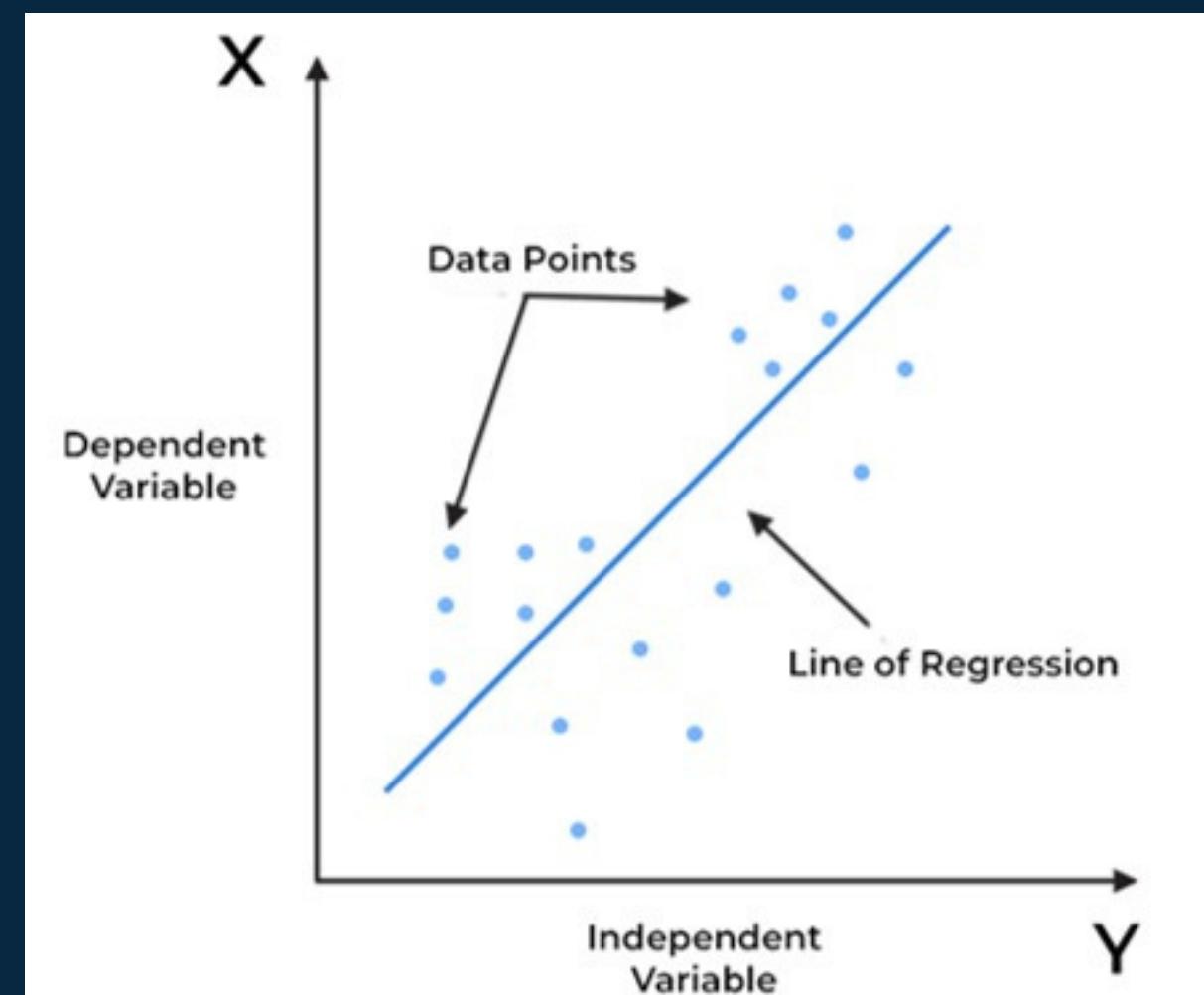
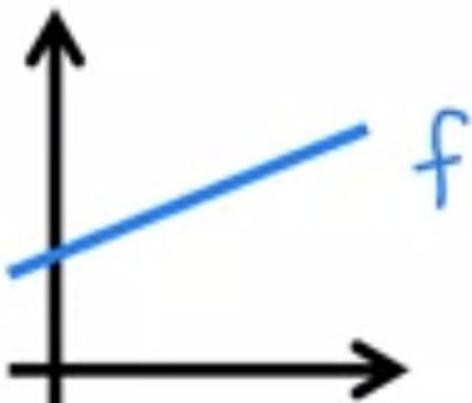
$$\underline{f_{w,b}(x) = wx + b}$$

parameters:

$$\underline{w, b}$$

cost function:

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (f_{w,b}(x^{(i)}) - y^{(i)})^2$$



# GOAL

Minimize cost Function (i.e.)  $J(w,b)$

# Gradient Descent

- Gradient Descent is an algorithm that we use to try to minimize any function not just for cost function.
- So by using Gradient Descent algorithm we keep on changing the parameters  $w, b$  everytime to try to reduce the cost  $J(w, b)$  until  $J$  settles at minimum.

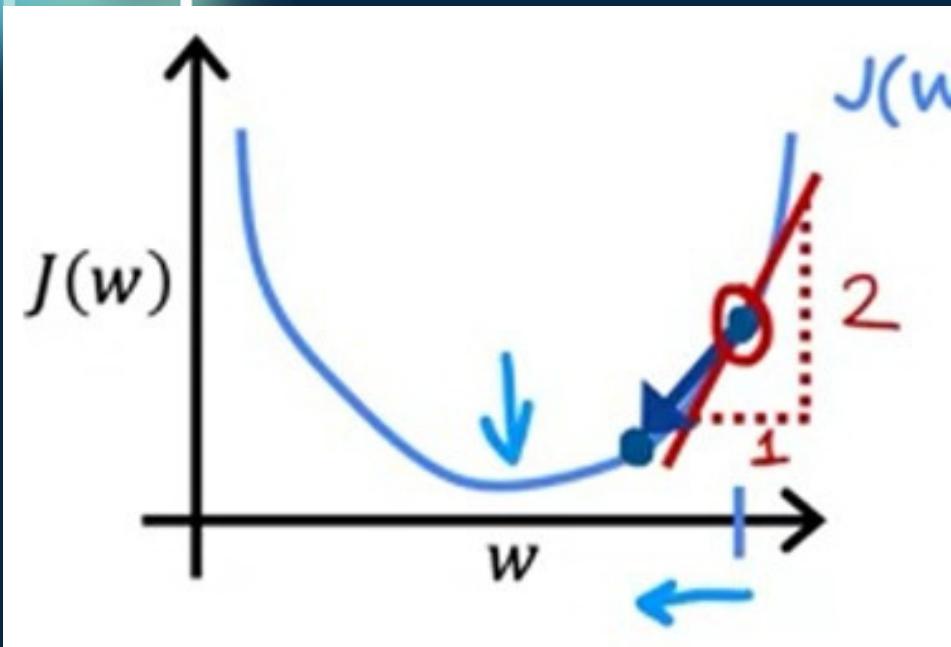
# Gradient Descent

- Here alpha is learning rate which is small +ve number between 0 and 1
- If alpha is large a very aggressive gradient descent (i.e.) you are trying to take huge steps downhill.
- If alpha is small you are taking small steps downhill.

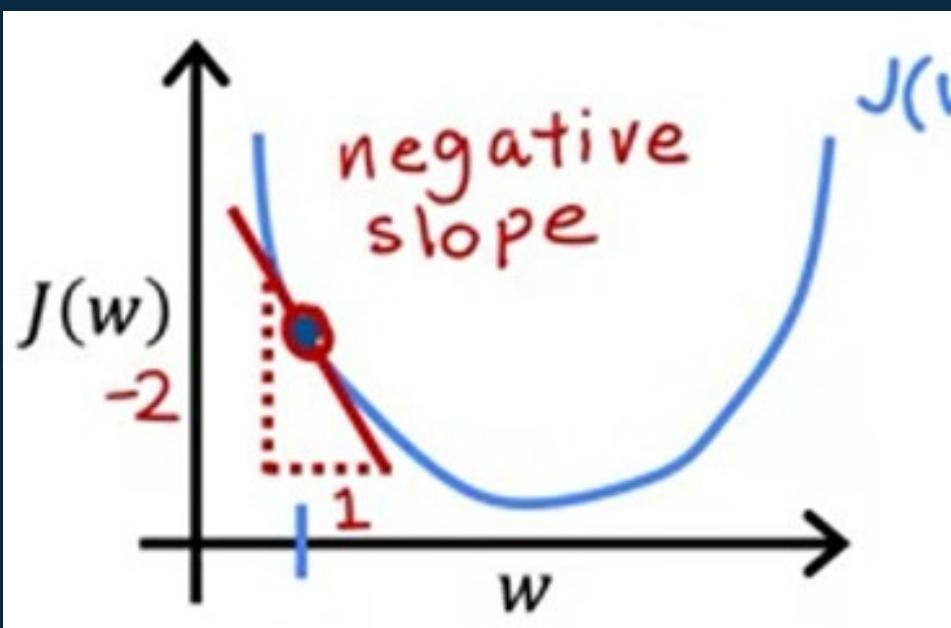
$$w \leftarrow w - \alpha \frac{\partial}{\partial w} J(w, b)$$
$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

- For Gradient Descent Algorithm we are going to repeat the above 2 steps repeatedly until the algorithm converges.

- Here we need to simultaneously update w,b.



- Here let's initialize Gradient Descent with some starting value of  $w$ .
  - Here Gradient Descent will update  $w$  to
- $w - \alpha \frac{\partial}{\partial w} (J(w, b))$
- Here alpha and slope both are +ve.
  - So we move to left side by decreasing value of  $w$ .



- Here let's initialize Gradient Descent with some starting value of  $w$ .
  - Here Gradient Descent will update  $w$  to
- $w - \alpha \frac{\partial}{\partial w} (J(w, b))$
- Here alpha is +ve and slope is -ve.
  - This step of Gradient Descent causes  $w$  to increase which means we are moving towards right of graph.

# Evaluation metrics

- The sum of squares due to error (SSE)

$$Err(i) = Y_i - f(x(i))$$

$$\text{SSE}(\text{sum of squared errors}) = \sum_{i=1}^n Err(i)^2$$

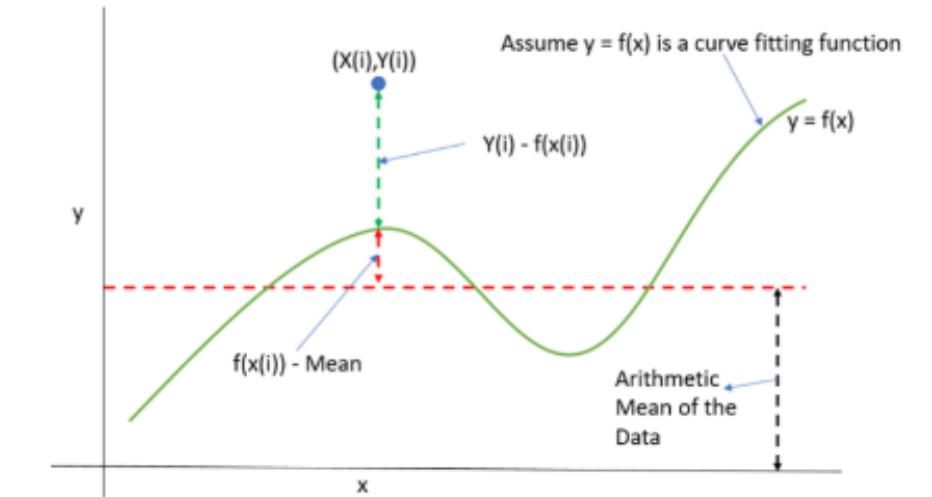
- R-square

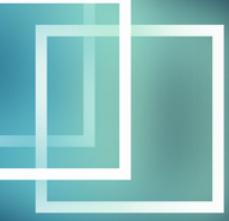
$$R^2 = \frac{SSR}{SST}$$

$$SSR = \sum_{i=1}^n (f(x(i)) - \text{Mean})^2$$

$$SSE = \sum_{i=1}^n (Y_i - f(x(i)))^2$$

$$SST = SSR + SSE$$





# Evaluation metrics

- Adjusted R-square

$$R_{adj}^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - k - 1}$$

- Root mean squared error (RMSE)

$$RMSE = \sqrt{\left(\frac{SSE}{n}\right)}$$