

GATE 2021 AG -Q.2

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QUESTION

If x is an integer with $x > 1$, the solution of

$$\lim_{x \rightarrow \infty} \left(\frac{1}{x^2} + \frac{2}{x^2} + \frac{3}{x^2} + \cdots + \frac{x-1}{x^2} + \frac{1}{x} \right)$$

- (A) Zero
(B) 0.5
(C) 1.0
(D) ∞

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We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$R = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{z^{x+1}}{x^2 (z-1)^3} \right) \quad (12)$$

$$R = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \frac{z^{x+1}}{x^2} \quad (13)$$

$$R = \frac{1}{2} \frac{(x+1)(x)}{x^2} \quad (14)$$

SOLUTION

$$\lim_{x \rightarrow \infty} q(x-1) = \frac{1}{2} \quad (15)$$

Hence , option (B) is correct.

Parameter	Description	Value
$p(n)$	$(n+1)^{th}$ term of the sequence	$\frac{n+1}{x^2}$
$q(n)$	sum of $(n+1)$ terms of the sequence	-

TABLE 0

TABLE OF PARAMETERS

From table ,

$$P(z) = \frac{1}{x^2} \sum_{n=-\infty}^{n=\infty} (n+1) u(n) z^{-n} \quad (1)$$

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1 \quad (2)$$

$$u(n) \xleftrightarrow{Z} \frac{1}{(1-z^{-1})}, |z| > 1 \quad (3)$$

$$P(z) = \frac{1}{x^2} \frac{1}{(1-z^{-1})^2}, |z| > 1 \quad (4)$$

$$q(n) = p(n) * u(n) \quad (5)$$

$$\Rightarrow Q(z) = P(z) U(z) \quad (6)$$

$$Q(z) = \left(\frac{1}{x^2} \frac{1}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right) \quad (7)$$

$$= \frac{1}{x^2} \frac{1}{(1-z^{-1})^3}, |z| > 1 \quad (8)$$

Using Contour Integration to find the inverse Z-transform,

$$q(x-1) = \frac{1}{2\pi j} \oint_C Q(z) z^{x-2} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{x-2}}{x^2 (1-z^{-1})^3} dz \quad (10)$$