GATE 2021 AG -Q.2

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Question

If x is an integer with x>1, the solution of

$$\lim_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+\frac{3}{x^2}+\cdots+\frac{x-1}{x^2}+\frac{1}{x}\right)$$

a) Zero
b) 0.5
c) 1.0
d) ∞ $(GATE\ 2021\ AG\ Q.2)$

Solution

Parameter	Description	Value
p(n)	$(n+1)^{th}$ term of the sequence	$\frac{n+1}{x^2}$
q(n)	sum of $(n+1)$ terms of the sequence	-

TABLE 4
Table of parameters

From table ,

$$P(z)=\frac{1}{x^2}\sum_{n=-\infty}^{n=\infty}(n+1)\,u(n)\,z^{-n} \tag{1}$$

$$n^2u(n)\overset{\mathcal{Z}}{\longleftrightarrow}\frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3},|z|>1 \tag{2}$$

$$nu(n)\overset{\mathcal{Z}}{\longleftrightarrow}\frac{z^{-1}}{(1-z^{-1})^2},|z|>1 \tag{3}$$

$$u(n)\overset{\mathcal{Z}}{\longleftrightarrow}\frac{1}{(1-z^-1)},|z|>1 \tag{4}$$

From (1)

$$P(z)=\frac{1}{x^2}\frac{1}{(1-z^{-1})^2},|z|>1 \tag{5}$$

$$q(n)=p(n)*u(n) \tag{6}$$

$$\implies Q(z)=P(z)\,U(z) \tag{7}$$

$$=\left(\frac{1}{x^2}\frac{1}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{8}$$

$$=\frac{1}{x^2}\frac{1}{(1-z^{-1})^3},\quad |z|>1 \tag{9}$$

Using Z-transform pairs (2) ,(3) ,(4) to find the inverse Z-transform,

$$\frac{1}{(1-z^{-1})^3}=\frac{1}{(1-z^{-1})}+\frac{3z^{-1}}{2(1-z^{-1})^2}+\frac{z^{-1}\left(z^{-1}+1\right)}{2(1-z^{-1})^3} \tag{10}$$

From equations (9) ,(10)

$$q(n)=\frac{1}{x^2}\left(\frac{n^2}{2}+\frac{3n}{2}+1\right) \tag{11}$$

$$=\frac{1}{x^2}\frac{(n+1)(n+2)}{2} \tag{12}$$

put $n=x-1$ to obtain the sum of first x terms

$$q(x-1)=\frac{1}{x^2}\frac{(x)(x+1)}{2} \tag{13}$$

$$\lim_{x\to\infty}q(x-1)=\frac{1}{2} \tag{14}$$

Hence , option (B) is correct.