Chalmers University of Technology

SSY190: Homework 1

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1 Introduction

Here, a PID controller is to be implemented on the plant. The transfer function of the plant is given by equation (1.1) and the controller by equation (1.2) as follows:

$$G(s) = \frac{K}{1 + sT} \tag{1.1}$$

$$F(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + T_f s} \tag{1.2}$$

Where, K and T are the plant parameters and the K_p, K_i, K_d, T_f are the controller parameters.
1.1 is the block diagram plant with controller.

2 Discretization

Discretization is to be performed in order to process the real time data on the processor. Both plant and controller are discretized using the sampling time h.

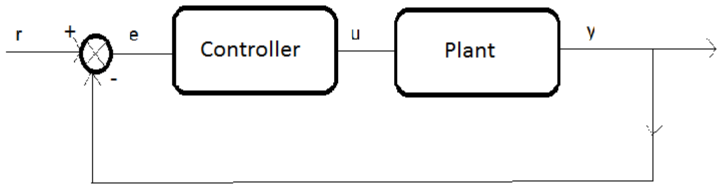

Figure 1.1: Block Diagram of the plant

2.1 Zero order hold

The plant is discretized by using the zero order hold. Zero order hold for following continuous time transfer function is given as below.

$$K\frac{a}{a+S} \rightarrow K\frac{(1-e^{-ah})z^{-1}}{1-e^{-ah}z^{-1}} \tag{2.1}$$

Using equation (2.1) with some algebraic manipulation, we obtain the following discretized transfer function for the plant.

$$\frac{y}{x} = G(z) = K\frac{(1-e^{-h/T})z^{-1}}{1-e^{-h/T}z^{-1}} \tag{2.2}$$

Using discrete time transfer function in (2.2), the following difference equation is obtained showing relation between the input and output of the plant.

$$y(n) = e^{-h/T}y(n-1) + K(1-e^{-h/T})x(n-1) \tag{2.3}$$

2.2 Tustin/bilinear transformation

The controller is discretized using Tustin bilinear transformation. In this transformation, we replace the 's' in the continuous transfer function by following function in 'z'.

$$s = \frac{2}{T_s}\frac{z-1}{z+1} \tag{2.4}$$

We obtain F(z) using (1.2) and (2.4). Difference equation for the plant and the controller is obtained using $F(z) = \frac{u}{e}$ as shown in 1.1. Simplified difference equation is given below.

$$\begin{aligned} u(n) = -(\alpha-1)u(n-1) + \alpha u(n-2) + (a_0 + T_sK_i + \beta(1+\alpha^{-1}))e(n) \\ +(a_0(\alpha-1) + T_sK_i\alpha - \beta(1+\alpha^{-1}))e(n-1) \\ -\alpha a_0 e(n-2) \end{aligned} \tag{2.5}$$

Where, $\alpha = \frac{T_s-2T_f}{T_s+2T_f}$, $\beta = \frac{2K_d}{T_s+2T_f}$, $a_0 = K_p - \frac{T_sK_i}{2} - \frac{\beta}{\alpha}$ are simplified constants.
e is the feedback error and u is the control input.
Once we obtain the difference equations, we can implement the system by defining initial conditions.

K_p	9.0
K_d	6.0
K_i	5.0
T_f	2.0
h	0.1
T	3.0
K	4.0

Table 3.1: System and controller parameter values

3 Implementation in C

3.1 System parameters and initialization

Since there was no specification, tuning was performed to achieve reference point. These parameters along with the assumed system parameters are given below: All parameters at $t < 0$ are set to 0 (refer to code). Also, initial output was set to 0. After running the simulation for the above values a significant reference tracking was achieved.