(1.) $w = (X^T \cdot X)^{-1} \cdot X^T \cdot z$

Definimos então as matrizes X e z.

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 3 & 2 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{bmatrix} \qquad z = \begin{bmatrix} 1{,}25 \\ 7 \\ 2{,}7 \\ 3{,}2 \\ 5{,}5 \end{bmatrix}$$

Utilizando numpy para os cálculos, temos

$$W = \begin{bmatrix} 1{,}39394 \\ -1{,}02929 \\ 1{,}76717 \end{bmatrix}$$

(2.) $w = (X^T \cdot X + \lambda I)^{-1} \cdot X^T \cdot z$

Utilizando as mesmas matrizes X e z do exercício 1 e numpy para os cálculos, temos

$$W = \begin{bmatrix} 0{,}60772 \\ -0{,}55914 \\ 1{,}66116 \end{bmatrix}$$

(3.) $RMSE(\hat{z}, z) = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (z_i - \hat{z}_i)}$

D	OLS	Ridge
x_1	2,13182	1,70974
x_2	5,66616	5,03207
x_3	1,84040	2,25261
x_4	3,60758	3,91378
x_5	6,40404	6,13409
x_6	2,86970	2,81176
x_7	3,89899	3,37090
x_8	-1,98535	-0,52684

Com os dados do enunciado, obtemos z e, com os dados desta tabela, obtemos $\hat{z}$.

(Treino, OLS)

$z = (1{,}25 ; 7 ; 2{,}7 ; 3{,}2 ; 5{,}5)$ $\qquad$ $\hat{z} = (2{,}13182 ; 5{,}66616 ; 1{,}84040 ; 3{,}60758 ; 6{,}40404)$

$RMSE = 0{,}42790$

(Treino, Ridge)

$z = (1{,}25 ; 7 ; 2{,}7 ; 3{,}2 ; 5{,}5)$ $\qquad$ $\hat{z} = (1{,}70974 ; 5{,}03207 ; 2{,}25261 ; 3{,}91378 ; 6{,}13409)$

$RMSE = 0{,}51958$

(Teste, OLS)

$y = (0{,}7 \,;\, 1{,}1 \,;\, 2{,}2)$ $\qquad\qquad$ $\hat{y} = (2{,}86970 \,;\, 3{,}89899 \,;\, -1{,}98535)$

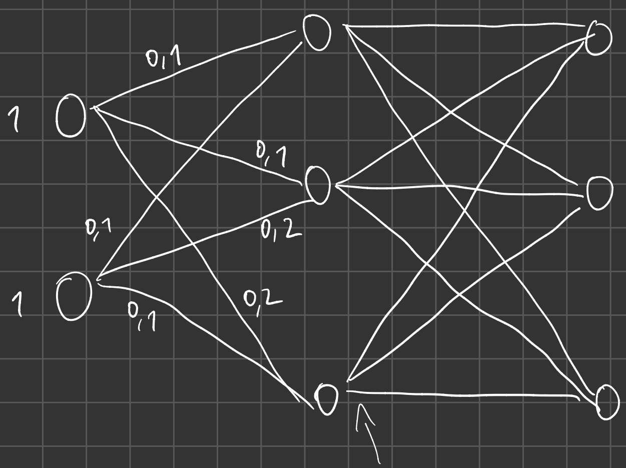
$RMSE = 5{,}00985$

(Teste, Ridge)

$y = (0{,}7 \,;\, 1{,}1 \,;\, 2{,}2)$ $\qquad\qquad$ $\hat{y} = (2{,}81176 \,;\, 3{,}37090 \,;\, -0{,}52684)$

$RMSE = 2{,}84203$

4



$$\begin{bmatrix} 0{,}1 & 0{,}1 \\ 0{,}1 & 0{,}2 \\ 0{,}2 & 0{,}1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0{,}1 \\ 0 \\ 0{,}1 \end{bmatrix} = \begin{bmatrix} 0{,}3 \\ 0{,}3 \\ 0{,}4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0{,}3 \\ 0{,}3 \\ 0{,}4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2{,}7 \\ 2{,}3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{e^{2,7}}{e^{2,7} + e^{2,3} + e^{2}} \\[2ex] \dfrac{e^{2,3}}{e^{2,7} + e^{2,3} + e^{2}} \\[2ex] \dfrac{e^{2,3}}{e^{2,7} + e^{2,3} + e^{2}} \end{bmatrix} = \begin{bmatrix} 0{,}46149 \\ 0{,}30934 \\ 0{,}22917 \end{bmatrix}$$

cross-entropy loss $= -\left(0 \cdot \log(0{,}46149) + 1 \cdot \log(0{,}30934) + 0 \cdot \log(0{,}22917)\right) =$
$= 1{,}17331$

$$w' = w - \eta \frac{\partial E(w)}{\partial w}$$

$$\text{output}(x) = \frac{e^{x}}{\sum_j e^{x_j}}$$

$$y = (0,1,0)$$

$$\hat{y} = \text{output}(x, w)$$

$$\hat{y}_k = \text{output}(x_k, w) =$$

$$\frac{\partial Loss}{\partial z_i} = p_i - y_i$$

$$\frac{\partial Loss}{z_1} = p_1 - y_1 = 0{,}46149 - 0 = 0{,}46149$$

$$\frac{\partial Loss}{\partial z_2} = p_2 - y_2 = 0{,}30934 - 1 = -0{,}69066$$

$$\frac{\partial Loss}{\partial z_3} = p_3 - y_3 = 0{,}22917 - 0 = 0{,}22917$$

$$\frac{\partial E(w)}{\partial w} = \begin{bmatrix} 0{,}46149 \\ -0{,}69066 \\ 0{,}22917 \end{bmatrix} \times \begin{bmatrix} 0{,}3 \\ 0{,}3 \\ 0{,}4 \end{bmatrix} = \Bigg[$$

$$w' = w - \eta \frac{\partial E(w)}{\partial w} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} - 0{,}1 \begin{bmatrix} 0{,}138447 & 0{,}138447 & 0{,}184596 \\ -0{,}207198 & -0{,}207198 & -0{,}276264 \\ 0{,}068751 & 0{,}068751 & 0{,}091668 \end{bmatrix} =$$

$$= \Bigg[$$