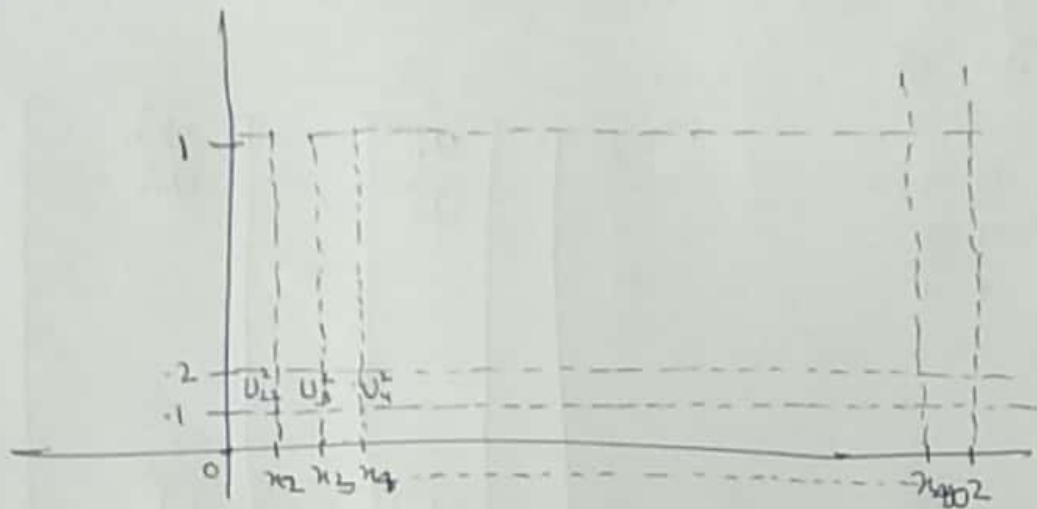


Q2

$$U_i^{n+1} = U_i^n - (U_{i+1}^{n+1} - U_{i-1}^{n+1}) \quad \text{--- (from Goal 1)}$$



$$\Delta x = \frac{2}{40} = \frac{1}{20} = 0.05$$

$$\frac{C \Delta t}{\Delta x} = 2 \quad C = 1$$

$$\Delta t = 2 \Delta x = 2 \times 0.05 = 0.1$$

$$x_1 = 0 \quad x_2 = 0.05 \quad x_3 = 0.1 \quad \dots \quad x_{41} = 0.5 \quad \dots \quad x_{40} = 1.95 \quad x_{41} = 2$$

Eq. to Solve :- $U_i^{n+1} = U_i^n - (U_{i+1}^{n+1} - U_{i-1}^{n+1})$

for General n & $i = 1, 2, 3, \dots, 41$

$$\text{for } i=41 \quad U_{41}^{n+1} = U_{41}^n - U_{41}^{n+1} + U_{40}^{n+1} \quad \text{--- (i)}$$

Ghost Value $-U_{42}^{n+1}$ can be find from averaging

$$U_{41}^{n+1} = \frac{U_{42}^{n+1} + U_{40}^{n+1}}{2}$$

$$U_{42} = 2U_{41}^{n+1} - U_{40}^{n+1} \quad \text{--- (ii)}$$

Putting (ii) in (i) we get

$$3U_{41}^{n+1} - 2U_{40}^{n+1} = U_{41}^n \quad \text{--- (iii)}$$

for $i=2$ to 40 Eq. is

$$U_i^{n+1} + U_{i+1}^{n+1} - U_{i-1}^{n+1} = U_i^n \quad \text{--- (iv)}$$

for $i=1 \quad C=1$

Using interpolation we get

$$U_{i+1}^{n+1} = U_i^{n+1}$$

$$\boxed{U_1^1 = U_0^0} \quad \text{--- (v)}$$

Combining eq (iii), (iv), & in matrix form we can rewrite it as.

$$\begin{bmatrix}
 3 & -2 & 0 & 0 & \cdots & 0 \\
 1 & 1 & -1 & 0 & \cdots & 0 \\
 0 & 1 & 1 & -1 & \cdots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & 0 & 0 & \cdots & +1 & 1 & -1 \\
 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 U_{41}^{n+1} \\
 U_{40}^{n+1} \\
 \vdots \\
 U_1^{n+1}
 \end{bmatrix}
 =
 \begin{bmatrix}
 U_{41}^n \\
 U_{40}^n \\
 \vdots \\
 U_0^n
 \end{bmatrix}$$