Question: 1

Goals

1. Using the Finite Difference approach, discretize the 1-D wave equation with 2^{nd} order spatial accuracy and 1^{st} order time accuracy with the implicit time integration scheme.

Goal 1 Solution

Equation to be solved:

$$\frac{\partial u}{\partial t} + C\frac{\partial u}{\partial x} = 0$$ ………… Eq(1)

NOTE :

Suffix used below represent the following:

i represent x(space)

j represent t(time)

h = Δx

k = Δt

2^{nd} Order accurate spatial:

$$\frac{\partial u}{\partial x} = \frac{U_{i+1,j} - U_{i-1,j}}{2h} \text{ .... Eq(2)}$$

1^{st} Order accurate time derivative:

$$\frac{\partial u}{\partial t} = \frac{U_{i,j} - U_{i,j-1}}{k} \text{............Eq(3)}$$

Putting (ii) & (iii) in (i) we get

$$\frac{Ui,j - Ui,j-1}{k} + C\left(\frac{Ui+1,j - Ui-1,j}{2h}\right) = 0$$

$$-\left(\frac{Ui,j - Ui,j-1}{k}\right) = \frac{C}{2h}\left(Ui+1,j - Ui-1,j\right)$$

$$Ui,j-1 - Ui,j = \frac{Ck}{2h}\left(Ui+1,j - Ui-1,j\right)$$

Replace j to j + 1 and $\frac{Ck}{2h}$ = λ

$$Uij = Ui,j+1 + \lambda\, Ui+1,j+1 - \lambda\, Ui-1,j+1$$

$$\frac{Ck}{h} = 2 \quad \rightarrow \lambda = 1$$

$$Ui,j = Ui,j+1 + Ui+1,j+1 - Ui-1,j+1$$

RESULT:

$$\boxed{U_{i,j} = U_{i,j+1} + U_{i+1,j+1} - U_{i-1,j+1}}$$