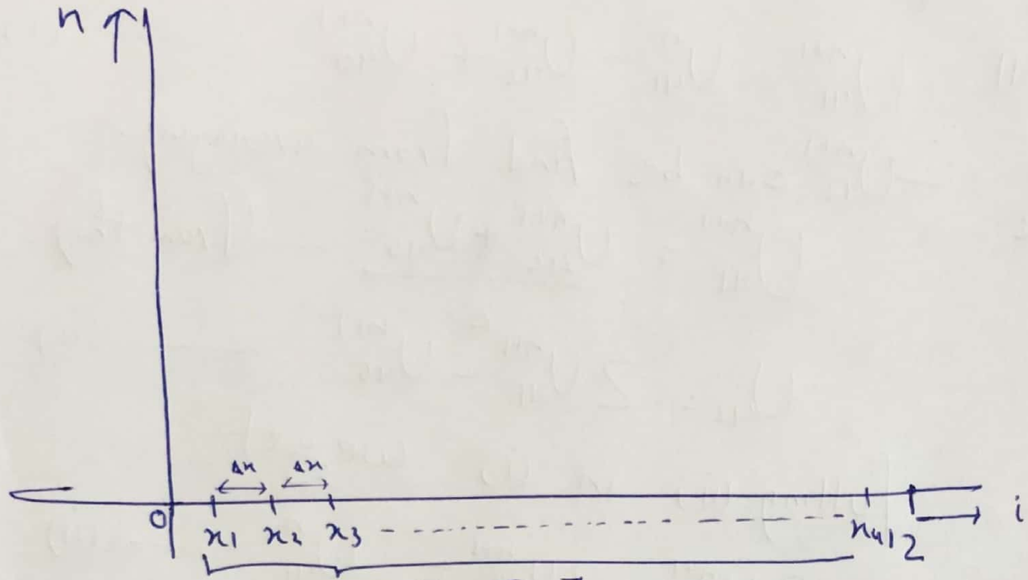
Q-2

$$U_i^{n+1} = U_i^n - \left(U_{i+1}^{n+1} - U_{i-1}^{n+1}\right) \quad \text{— from Goal 1}$$



41 pt b/w [0,2]

Assuming them to be equally sep (Δx), we get

$$\Delta x = \frac{2}{42}$$

$x_0 = 0$

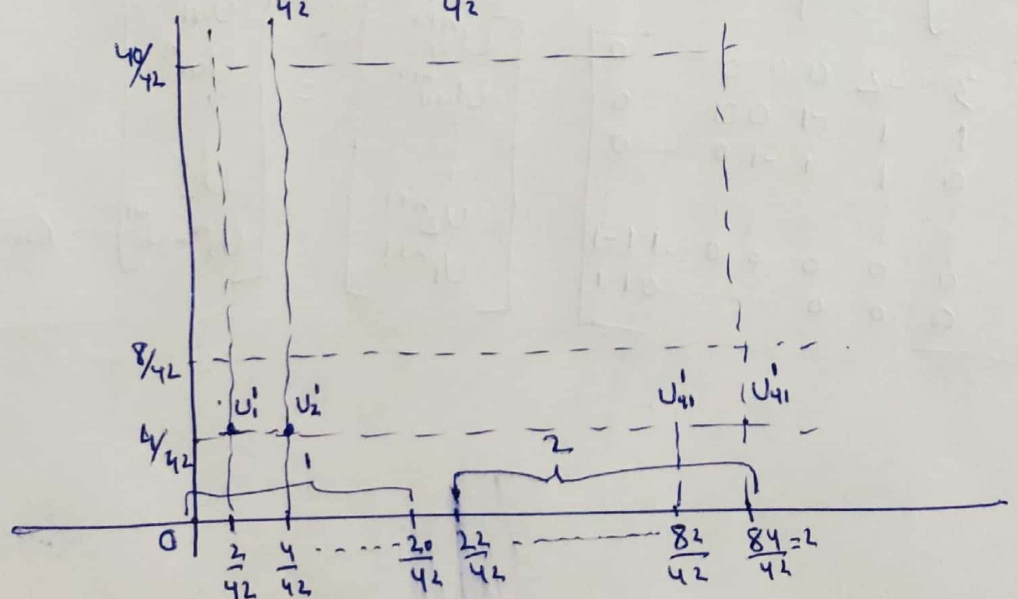
So, $x_1 = \frac{2}{42}$ $\quad x_2 = \frac{4}{42}$ $\quad$ ------- $\quad x_{41} = \frac{2\times 41}{42}$, $\quad x_{42} = \frac{2\times 42}{42} = 2$

As $\boxed{\frac{C\Delta t}{2\Delta x} = 2}$ — given & $C = 1$

$$\Delta t = 2\Delta x$$

$$\Delta t = \frac{4}{42}$$

10 time $\quad n_1 = \frac{4}{42}$ $\quad n_2 = \frac{8}{42}$ ------- $\quad n_{10} = \frac{4\times 10}{42}$



Equation to Solve:

$$U_i^{n+1} = U_i^n - \left(U_{i+1}^{n+1} - U_{i-1}^{n+1}\right) \text{ — (from Goal 1)}$$

for General n & $i = 1, 2, 3 \ldots 41$

for $i = 41$ $\quad U_{41}^{n+1} = U_{41}^n - U_{42}^{n+1} + U_{40}^{n+1}$ ——— (i)

Ghost value $\quad U_{42}^{n+1}$ can be find from averaging.

$$U_{41}^{n+1} = \frac{U_{42}^{n+1} + U_{40}^{n+1}}{2} \text{ — (from this)}$$

$$U_{42} = 2U_{41}^{n+1} - U_{40}^{n+1} \text{ ——— (ii)}$$

Putting (ii) in (i) we get.

$$3U_{41}^{n+1} - 2U_{40}^{n+1} = U_{41}^n \text{ ——— (iii)}$$

for $i = 2$ to 40 Equation is

$$U_i^{n+1} + U_{i+1}^{n+1} - U_{i-1}^{n+1} = U_i^n \text{ ——— (iv)}$$

$i = 1$

$$U_1^{n+1} + U_2^{n+1} - U_0^{n+1} = U_1^n$$

Known can be shift to LHS

$$U_1^{n+1} + U_2^{n+1} = U_1^n + U_0^{n+1} \text{ ——— (v)}$$

Combining eq (iii), (iv) & (v) in matrix.

$$\begin{bmatrix} 3 & -2 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 1 & -1 \\ 0 & 0 & 0 & \cdots & & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} U_{41}^{n+1} \\ U_{40}^{n+1} \\ \vdots \\ U_2^{n+1} \\ U_1^{n+1} \end{bmatrix} = \begin{bmatrix} U_{41}^n \\ U_{40}^n \\ \vdots \\ U_2^n \\ U_1^n + U_0^{n+1} \end{bmatrix} \text{ — Know}$$