EP219: Data Analysis and Interpretation Tutorial Sheet 1, Fall 2018

Problem 1 Sketch the following probability distribution functions and find the mean, variance/standard deviation of each:

a) Gaussian distribution

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-(x-\mu)^2/2\sigma^2\right), \; x \,\epsilon\, \mathbb{R}$$

b) Poisson distribution

$$P(k|\lambda) = \frac{\lambda^k}{k!} \exp(-\lambda), \; k \,\epsilon\, \mathbb{Z}, \, k \geq 0$$

c) Exponential decay

$$f(x|\lambda) = \lambda \exp(-\lambda x), \; x \geq 0$$

d) Uniform distribution

$$f(x) = \frac{1}{2\alpha}, \text{ for } \mu - \alpha \leq x \leq \mu + \alpha$$

e) Dice rolls

$$P(x) = 1/6, \text{for } x \,\epsilon\, \{1, ..., 6\}$$

Problem 2 Find the mean value of the volume of the sphere with a gaussian (normally) distributed radius with parameters $(\mu = r_0, \sigma^2)$. Is this the same as the volume calculated using the mean radius?

Problem 3 Consider a particle in a gas at temperature T moving in a 1-d box. Its velocity distribution is given by a normal distribution with zero mean and variance $\sigma^2 = kT/m$, where k is the Boltzmann constant and m is the mass of the particle. Find the distribution function $f(E)$ and expected value $\langle E \rangle$ of the energy of this particle.

Problem 4 Monte Carlo method to estimate the value of π. We can use a computer to generate random numbers to estimate the value of π. We consider a square of side 2 cm and a circle of radius 1 cm with the same center. Now we generate N random points (x, y) inside the square by drawing x and y from a uniform distribution. If the points are within 1 cm of the origin (i.e. $x^2 + y^2 \leq 1$) , we can identify those points as lying within the circle and we call these *good points.* After generating a large enough number of random points, we can estimate the ratio of the area of the circle to the area of the square by taking the ratio of the number of good points (N_g) to the total number of points (N). Hence, we can numerically estimate the value of π.

a) We can treat the ratio of number of good points to the total number of points as a new random variable u, where $u = N_g/N$. What is the mean value of u?

If we generate an infinite number of points we would find the true value of π. However, we have finite computing time and we would like to know how many random points we need to generate to measure the value of π to a given accuracy.

b) Find the error in our inferred value of u by calculating the standard deviation of u, denoted as σ_u.

c) In order to get the value of π to an accuracy of 1%, how many points N do we need to generate?

Problem 5 After taking this class you are super confident of your ability to play the stock market. Let us suppose you decide to invest Rs. 500 in stocks. Let us assume for simplicity that each company you invest in yields a 100% profit on your investment with probability 0.7. However, with probability 0.3 you will lose your full investment in that company. We can also assume that each company's performance is independent of the others. Let us now consider two investment strategies.

- Strategy 1: You invest all of your Rs 500 in one company (BlueChip Co.). What is your expected profit? What is the expected fluctuation (standard deviation) in the profit?
- Strategy 2: You invest Re 1 in each of 500 different companies. What is your expected profit in this case? What is the expected fluctuation in the profit?

Define the risk factor as the expected fluctuation in profit divided by expected profit. A high risk factor indicates large volatility with large possible swings in your profit margins. Based on your answers, is there any difference in these investment strategies? Does either result in a higher expected profit? Does either result in higher risk?

Problem 6 Consider a bullet shot from a hill of height 120 m above sea-level. The initial speed of the bullet is 100 m/s and the gun is at an angle of 30° from the horizontal (upwards). Show that it will take 12 seconds for the bullet to hit the ground (at sea-level). Your friend simultaneously starts a stopwatch ($t = 0$) when you fire the bullet and stops it at a random time $t < 12s$. The probability distribution function for t is assumed to be uniform. Let y denote the height of the bullet above sea level at the instant when the stopwatch is stopped. Find the probability density function for y and sketch it. (You may take g = 10 m s^{-2}.)