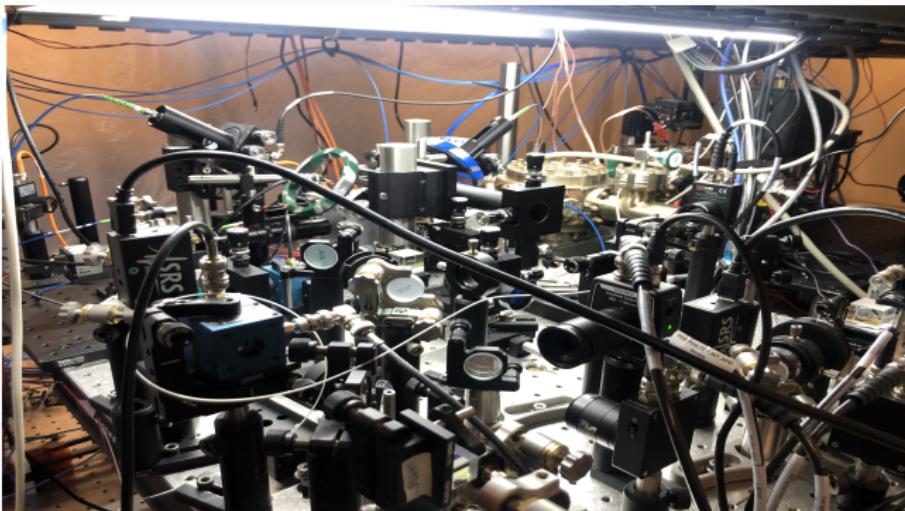


Optimizing Beam Splitters for Matter Waves

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Atom Interferometry

State of the art precision measurements

- Laser-cooled to millionths of a degree above absolute zero.
- Manipulate the state of the atoms using light pulses, steer the matter waves' paths and recombine the matter waves at the end.
- The interference signal manifests as a population difference between final momentum states.
- The energy and couplings along the atoms' path and interaction determine the phase shift.

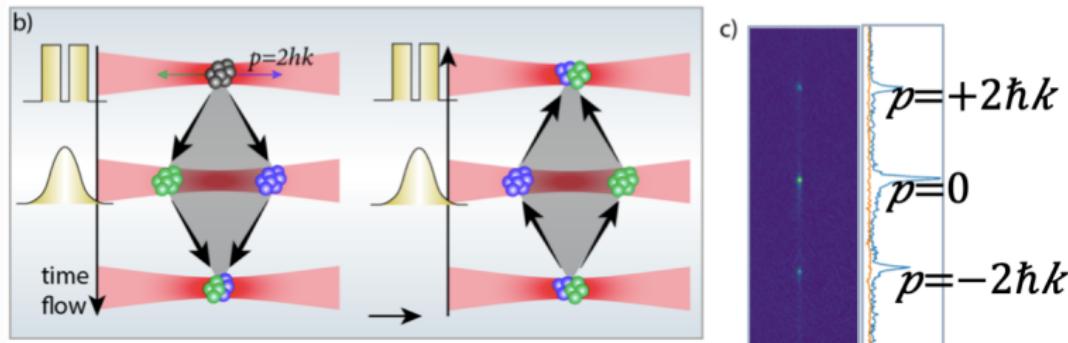
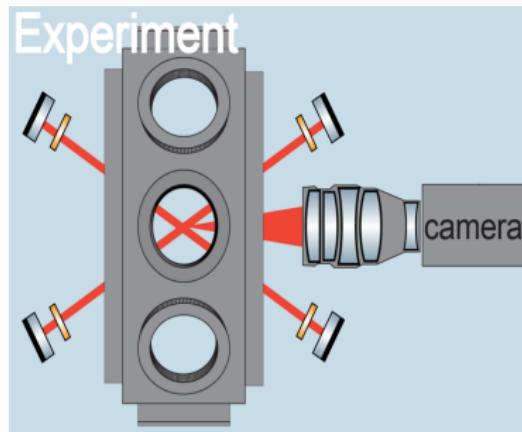


Figure: How do atom Interferometers work?

Magneto-Optical Trap

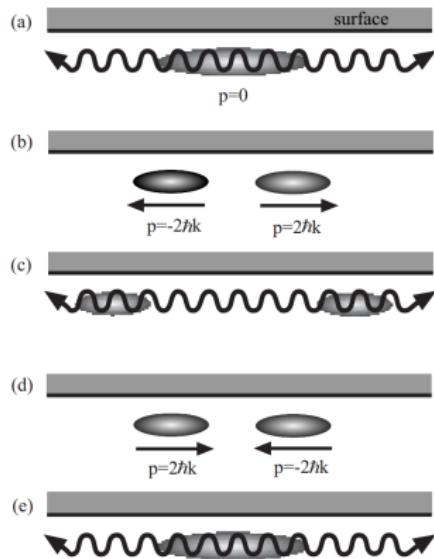
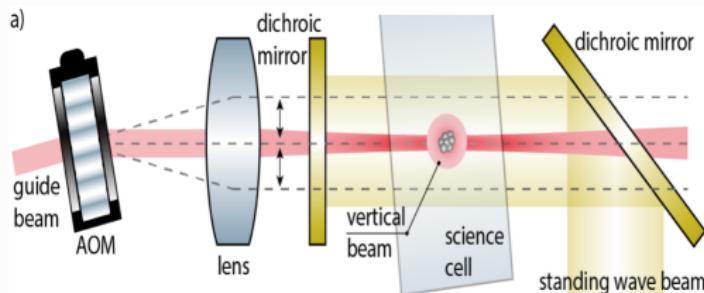
How are atoms cooled to such low temperatures?



- 3 pairs of counter-propagating beams close to resonance.
- $\sim 1\text{ms}$ and $5\text{e}4$ to slow atom from 300m/s (room temp) to 1m/s (10mK).
- Magnetic field to distinguish center of the trap.
- Doppler cooling, down to $\sim 140\mu\text{K}$.
- Sub-Doppler cooling, down to $\sim 15\mu\text{K}$.

Overview

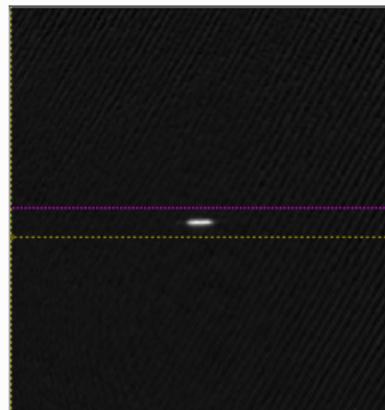
- Laser beams coherently split BEC into two equal and opposite momentum packets with $p = \pm 2n\hbar k$.¹
- Higher momentum states are desired: sensitivity $\propto p$ wave-packets by beam splitting.



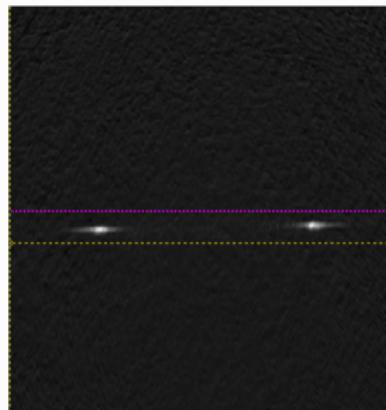
Wang et al. [1]

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

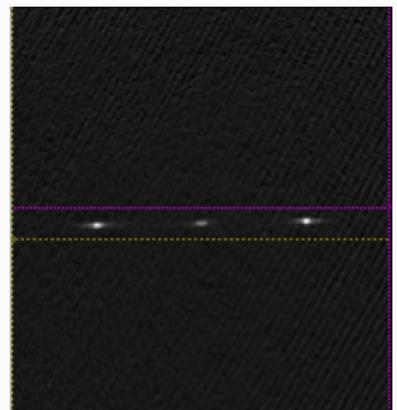
Experimental Setup to study Fidelity



(a) No Splitting
 $t = 0 \text{ ms}$



(b) Good Splitting
 $t = 20 \text{ ms}$



(c) Bad Splitting
 $t = 20 \text{ ms}$

Figure: Absorption images of BECs held in an optical waveguide

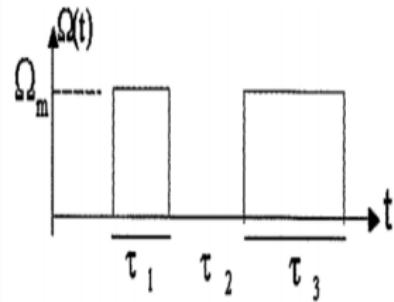
The Known and the Unknown

Desired Qualities

- ① High fidelity for higher momentum states.
- ② Low sensitivity to ΔI_{laser}

Current State

- Double squared pulses are used in the lab.
- Motivated by experimental discovery¹ and later justified by theoretical work by Wu et al.²
- Known: Optimal parameters for $2\hbar k$.²
- Low fidelity for higher momentum states.



We want

- Arbitrary optimal shape for a given momentum state.

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

2: Wu, Sajun, et al. "Splitting matter waves using an optimized standing-wave light-pulse sequence." Physical Review A 71.4 (2005): 043602.

Our Approach

Treat it as a combination of plane waves

- Expand the wave function in plane waves.

$$\Psi(x, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_m C_{m,2n}(t) e^{i(2nk_0+k_m)x}$$

- Schrödinger Equation becomes a coupled system of linear equation

$$i\dot{C}_{2n} = \frac{\hbar}{2m} C_{2n} (2nk_0 + k)^2 + \frac{\Omega}{2} (C_{2n-2} + C_{2n+2})$$

- Efficient method of solving for constant intensity:

$$C(t) = e^{-i\omega_r t A} C^+(0)$$

$$C = [C_0, C_2, C_4, C_{2m}, \dots, C_{2n}]^T$$

C_{2n}, C_0 : Probability Amplitudes

$A(\Omega, \omega_r)$: Symmetric matrix

$\Omega(t)$: Strength of Coupling

ω_r : Recoil frequency

$$C_{2m} \uparrow \implies \text{Fidelity}_{2m\hbar k} \uparrow$$

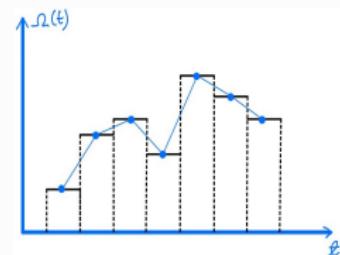
- For non-square shapes, treat as piece-wise constant functions

Method

How do you find the best pulse?

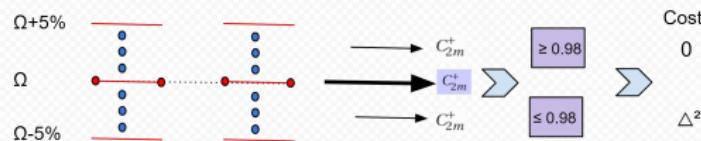
Stage 1: Hunting for highest fidelity

- Tried different intensity double sq. , triangle, and sinc squared pulses.
- Optimizing for intensity & time in each slice with different global & local minimizers.
- For arbitrary shapes, we optimize for time and intensity in each slice.

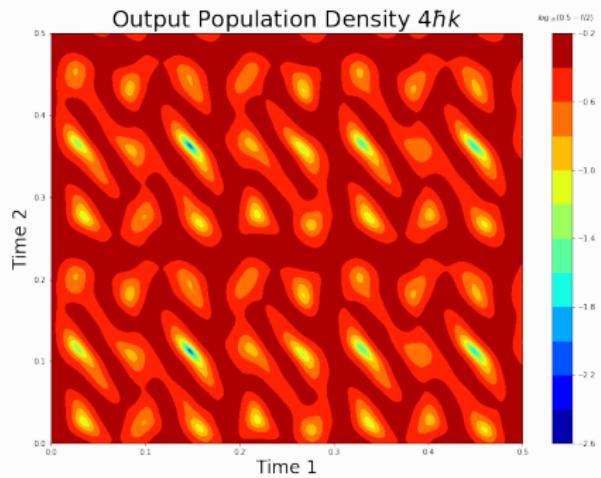
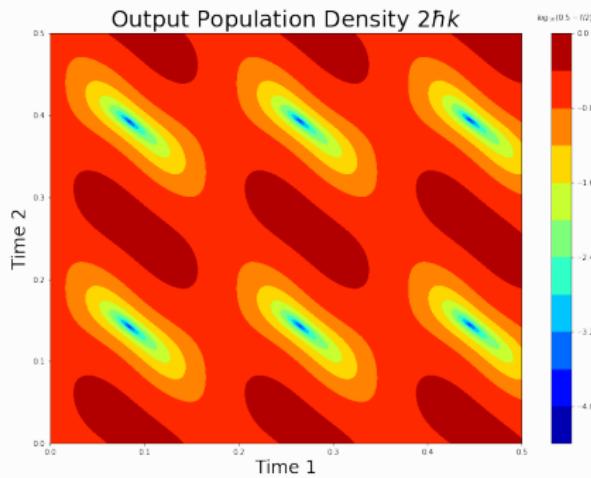


Stage 2: Fidelity is not everything

- Laser intensities drift with time.
- New function now accounts for sensitivity.
- Optimize concurrently on both fidelity and sensitivity .

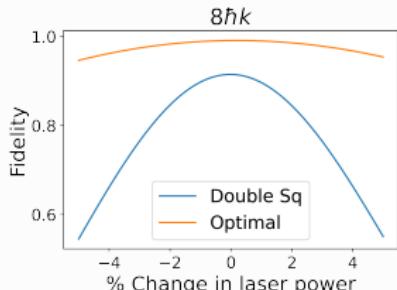
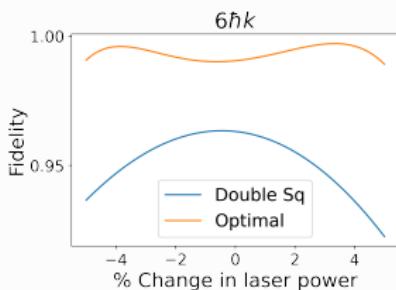
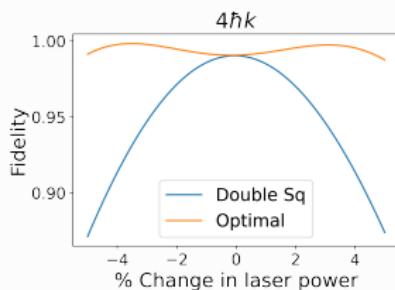
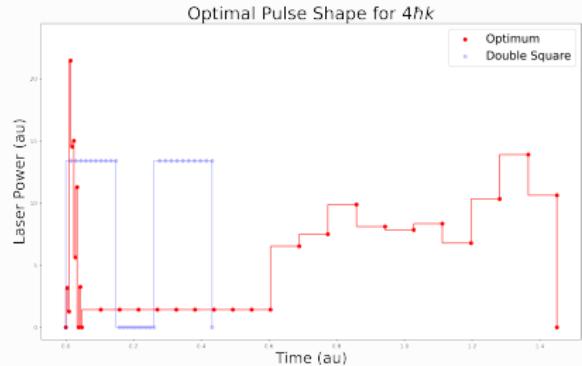


Visualizing the landscape



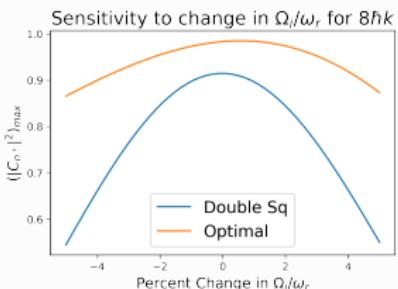
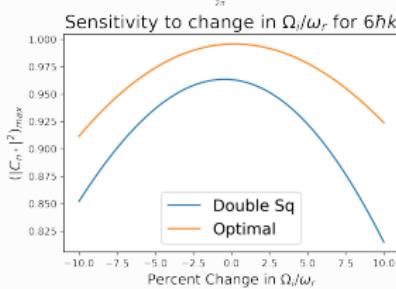
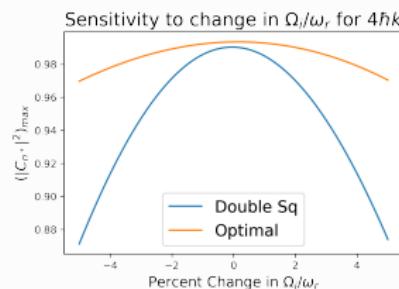
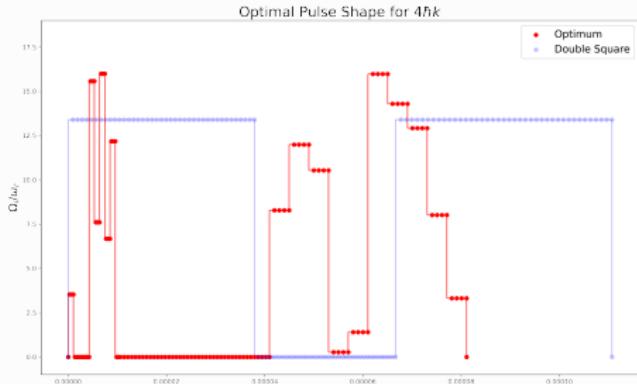
Results

Is the new pulse actually better than double square?



Results

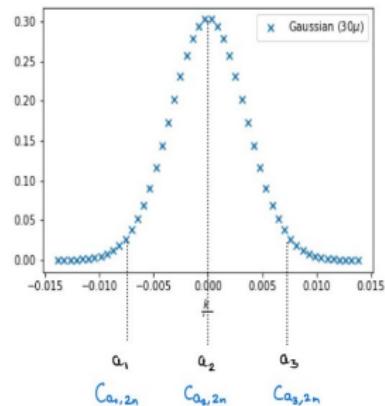
Don't forget the Fourier relationship $\Delta E \Delta t \geq \frac{\hbar}{2}$



Results

Does it hold the test of momentum superposition?

- So far, our work assumed no momentum spread (δ function).
- In the lab, we have a confined BEC that has some momentum spread which can be approximated by a Gaussian.
- Does this superposition affect the results?
 - Our BEC is of full width half max around $40\ \mu$ meters.
 - Given this spread in position space, the fidelity of the optimal pulse virtually unaffected.



Experimental Verification

$4\hbar k$ Pulses

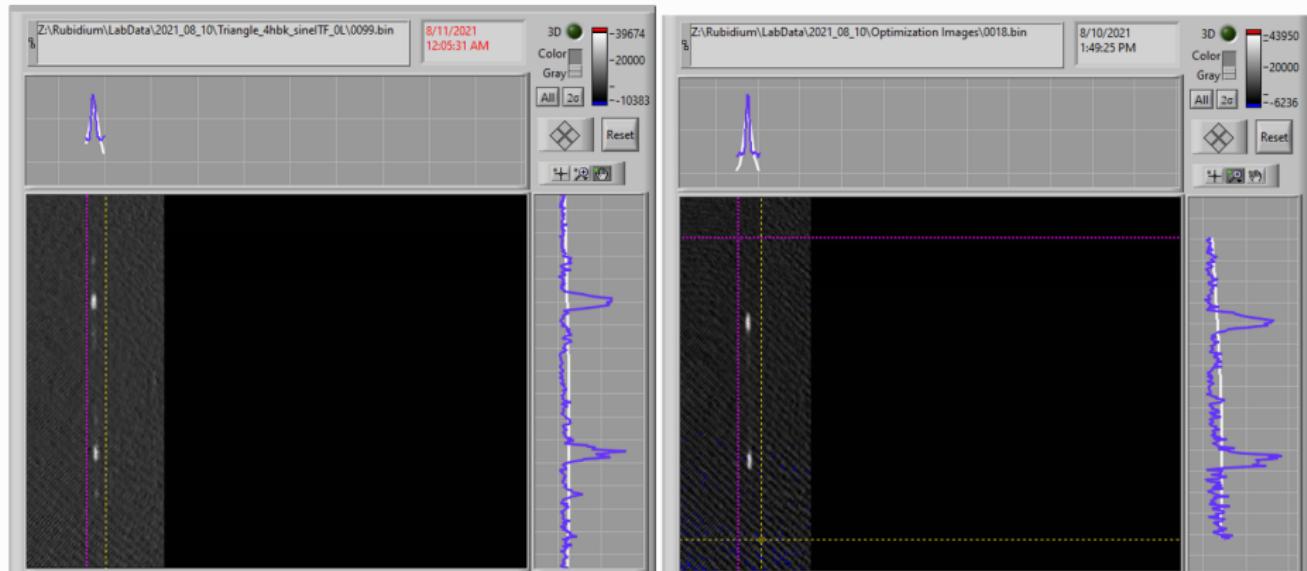


Figure: Characterising the performance of $4\hbar k$ pulses
(Left: Double Square Pulse) (Right: Optimized Pulse)

Experimental Verification

$8\hbar k$ Pulses

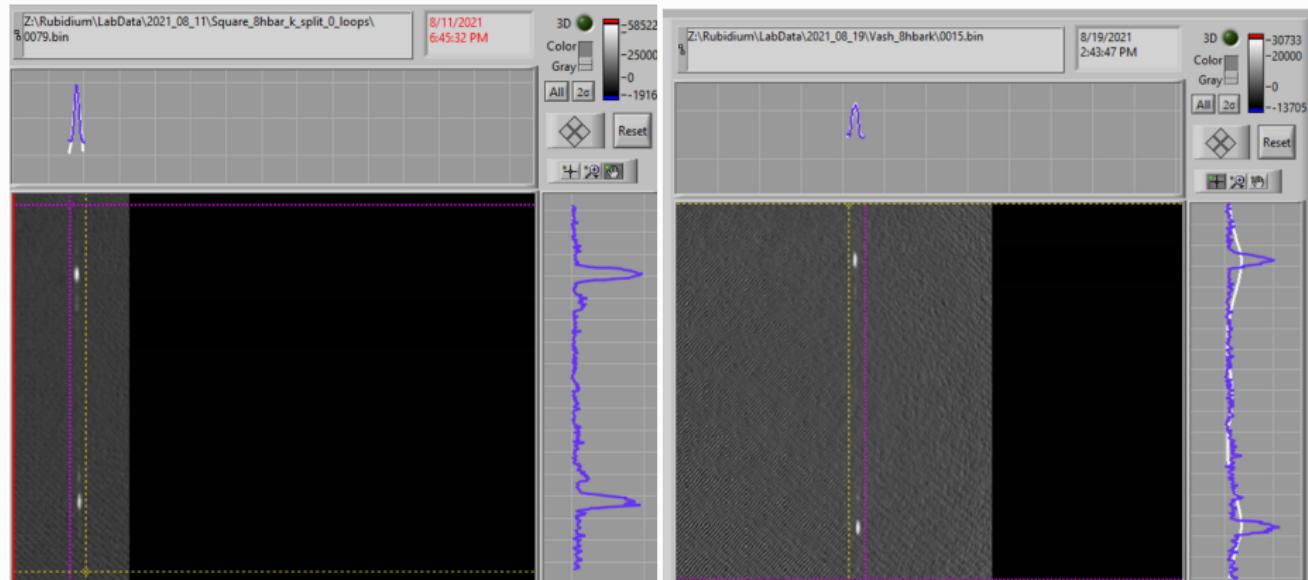


Figure: Characterising the performance of $8\hbar k$ pulses
(Left: Double Square Pulse) (Right: Optimized Pulse)

Summary & Outlook

Summary

- We have found pulse shapes that do better than the current state of art method, i.e., double square.
- The plane wave approximation holds well given the dimensions of our BEC.

Outlook

- Experimental Verification of the results.
- 2D optimization that takes into account sensitivity to both intensity and momentum.

References I



Ying-Ju Wang et al. “Atom Michelson Interferometer on a Chip Using a Bose-Einstein Condensate”. In: *Physical Review Letters* 94.9 (Mar. 2005). ISSN: 1079-7114. DOI: [10.1103/physrevlett.94.090405](https://doi.org/10.1103/physrevlett.94.090405). URL: <http://dx.doi.org/10.1103/PhysRevLett.94.090405>.

Bonus Slides

```
"""Input: List of fidelity values
Returns: Cost"""
def cost_function(list_functions):
    cost = 0
    for i in list_functions:
        delta = 0.98-i
        if delta <0:
            cost +== 0
        else:
            cost +== delta**2
    return cost
```

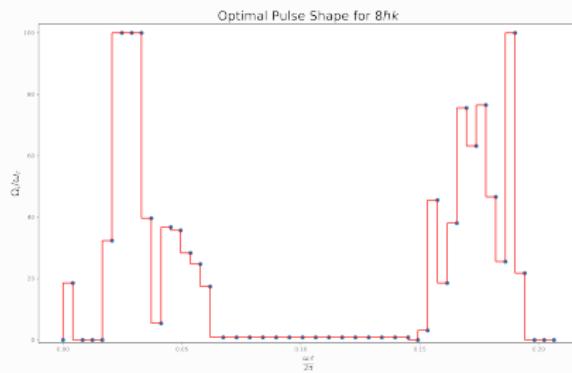
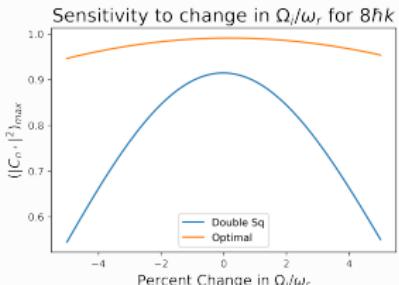
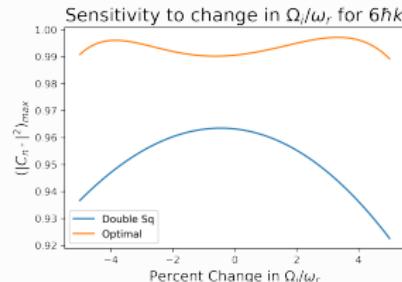
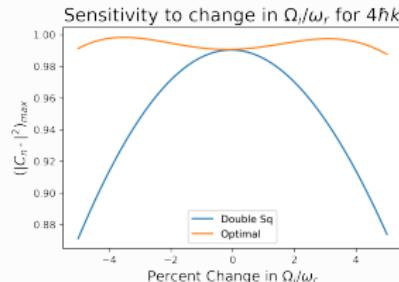
```
"""Input: Pulse Parameters, which column to maximize
Returns: Cost when I changes by different percentages"""
def real_function(params, number):
    list_arg_sen = params

    # make a list of values of the function for different changes
    percent_list = np.linspace(-5,5,50)
    function_values = []
    function_values.append(square_opt_fixed(params, number))
    for i in percent_list:
        params_changed = changing_intensity(params, i)
        function_values.append(square_opt_fixed(params_changed, number))

    optimum_cost = cost_function(function_values)
    return optimum_cost
```

Results

Optimizing the laser shape for higher momentum



Results

Does it hold the test of momentum superposition?

All works so far assume a BEC with 0 initial momentum. In the lab, we have a confined BEC that has some momentum spread which is given by a gaussian.

- Take in fwhm in position space to make a gaussian for k/k_0 (k_0 initial wavenumber)
- Sample points from -3σ to 3σ , call these a'_i 's
- Enforce the normalization condition st $\sum_{i=1}^n |a_i|^2 = 1$.
- Then use these coefficients to get the amplitudes in the superposition states

$$P(C_n) = \sum_{i=1}^n |a_i|^2 |C_{k_i, n}|^2$$

