Optimizing Beam Splitters for Matter Waves

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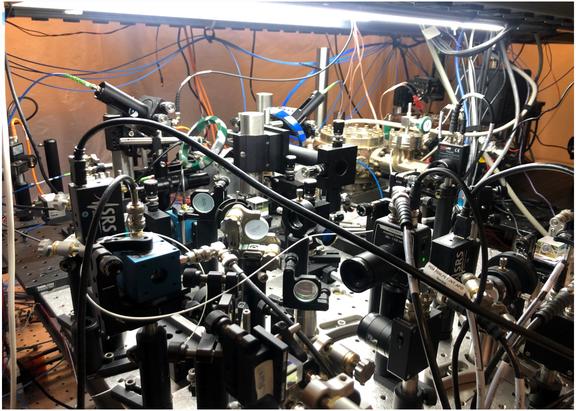


Atom Interferometry

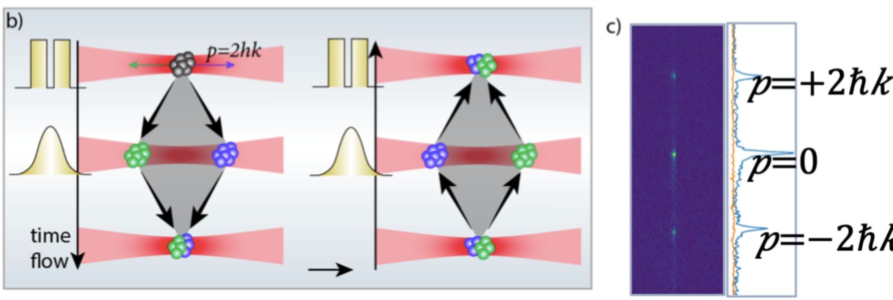
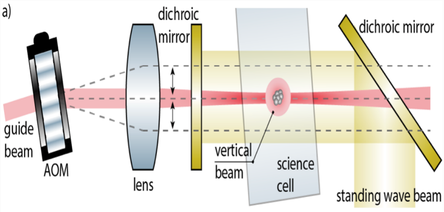
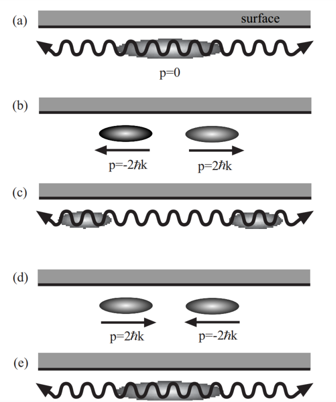


Figure 1: How do atom Interferometers work?

Overview

- Laser beams coherently split BEC into two equal and opposite momentum packets with $p = \pm 2n\hbar k$. [1]
- Higher momentum states are desired: sensitivity $\propto$ p wave-packets by beam splitting.

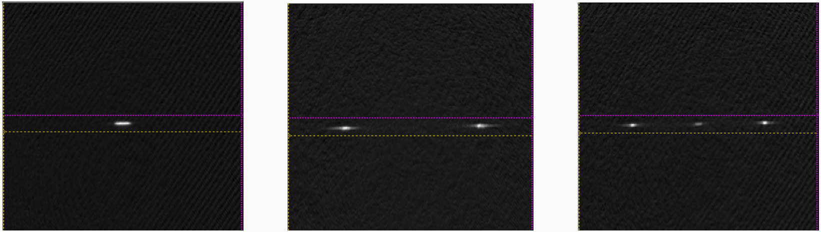




Wang et al. [1]

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

Experimental Setup to study Fidelity



(a) No Splitting
$t = 0\ ms$

(b) Good Splitting
$t = 20\ ms$

(c) Bad Splitting
$t = 20\ ms$

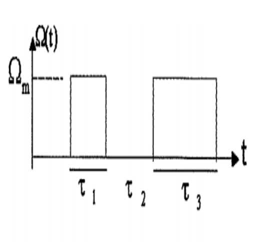
Figure 2: Absorption images of BECs held in an optical waveguide

Desired Qualities

1. High fidelity for higher momentum states.
2. Low sensitivity to ΔI_{laser}

Current State

- Double squared pulses are used in the lab.
- Motivated by experimental discovery [1] and later justified by theoretical work by Wu et al.[2]
- Known: Optimal parameters for $2\hbar k$. [2]
- Low fidelity for higher momentum states.



We want

- Arbitrary optimal shape for a given momentum state.

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

2: Wu, Saijun, et al. "Splitting matter waves using an optimized standing-wave light-pulse sequence." Physical Review A 71.4 (2005): 043602.

Our Approach

- Expand the wave function in plane waves.

$$\Psi(x,t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_m C_{m,2n}(t)\, e^{i(2nk_0+k_m)x}$$

- Schrödinger Equation becomes a coupled system of linear equation

$$i\dot{C}_{2n} = \frac{\hbar}{2m} C_{2n}\,(2nk_0+k)^2 + \frac{\Omega}{2}\,(C_{2n-2}+C_{2n+2})$$

- Efficient method of solving for constant intensity:

$$C(t) = e^{-i\omega_r t A} C^{+}(0)$$

$$C = [C_0, C_2, C_4, \boxed{C_{2m}}, \cdots, C_{2n}]^T$$

C_{2n}, C_0: Probability Amplitudes

$A(\Omega, \omega_r)$: Symmetric matrix

$\Omega(t)$: Strength of Coupling

ω_r: Recoil frequency

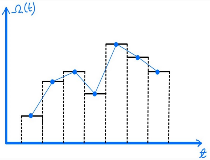
$$\boxed{C_{2m}\uparrow \implies \text{Fidelity}_{2m\hbar k}\uparrow}$$

- For non square shapes treat as piece wise constant function.

Method

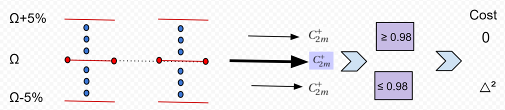
Stage 1: Hunting for highest fidelity

- Tried different intensity double sq. , triangle, and sinc squared pulses.
- Optimizing for intensity & time in each slice with different global & local minimizers.
- For arbitrary shapes, we optimize for time and intensity in each slice.

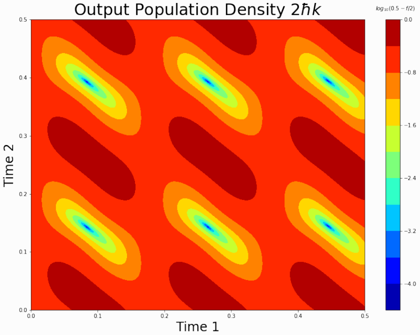


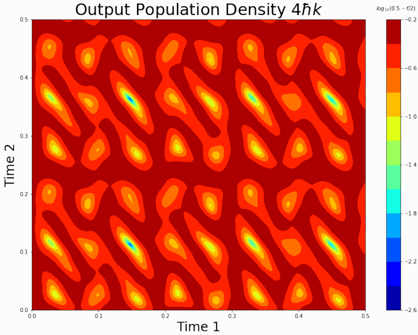
Stage 2: Fidelity is not everything

- Laser intensities drift with time.
- New function now accounts for sensitivity.
- Optimize concurrently on both fidelity and sensitivity .

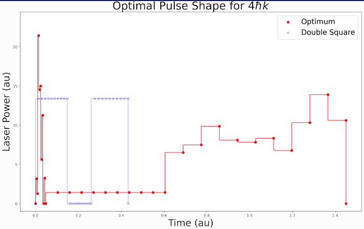


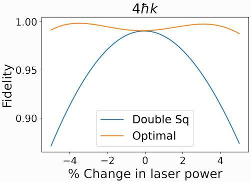
Visualizing the landscape

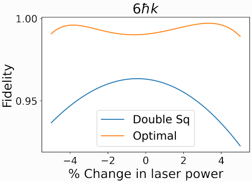


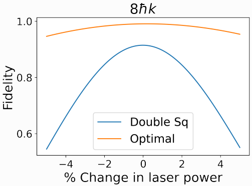


Results

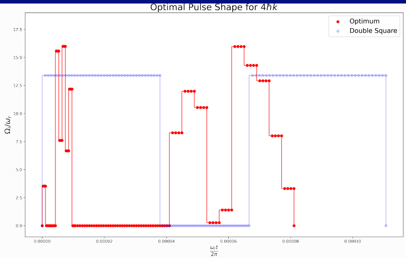

Optimal Pulse Shape for $4\hbar k$
Optimum
Double Square
Laser Power (au)
Time (au)

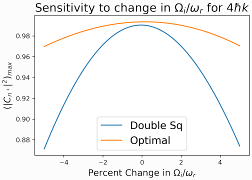

$4\hbar k$
1.00
0.95
0.90
Fidelity
Double Sq
Optimal
−4
−2
0
2
4
% Change in laser power

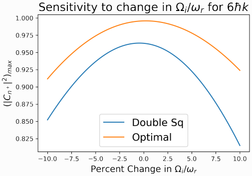

$6\hbar k$
1.00
0.95
Fidelity
Double Sq
Optimal
−4
−2
0
2
4
% Change in laser power

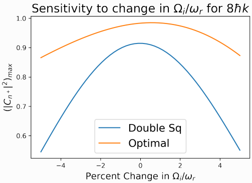

$8\hbar k$
1.0
0.8
0.6
Fidelity
Double Sq
Optimal
−4
−2
0
2
4
% Change in laser power

Results


Optimal Pulse Shape for $4\hbar k$
Optimum
Double Square
Ω_i/ω_r
$\frac{\omega_r t}{2\pi}$

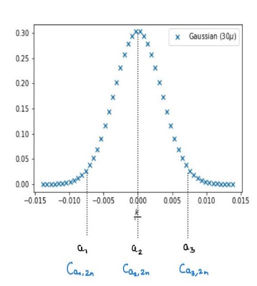

Sensitivity to change in Ω_i/ω_r for $4\hbar k$
$(|C_{n^+}|^2)_{max}$
Double Sq
Optimal
Percent Change in Ω_i/ω_r


Sensitivity to change in Ω_i/ω_r for $6\hbar k$
$(|C_{n^+}|^2)_{max}$
Double Sq
Optimal
Percent Change in Ω_i/ω_r


Sensitivity to change in Ω_i/ω_r for $8\hbar k$
$(|C_{n^+}|^2)_{max}$
Double Sq
Optimal
Percent Change in Ω_i/ω_r

Results

- So far, our work assumed no momentum spread (δ function).
- In the lab, we have a confined BEC that has some momentum spread which can be approximated by a gaussian.
- Does this superposition affect the results?
  - ☐ Our BEC is of full width half max around 40 μ meters.
  - ☐ Given this spread in position space, the fidelity of the optimal pulse virtually unaffected.



Experimental Verification

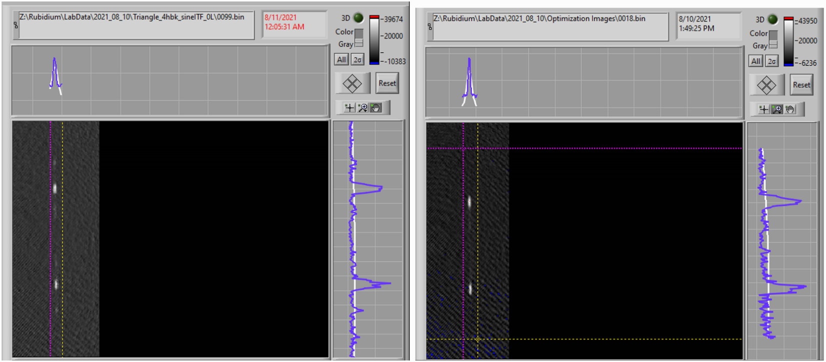


Figure 3: Characeterising the performance of $4\hbar k$ pulses

(Left: Double Square Pulse) (Right: Optimized Pulse)

Experimental Verification

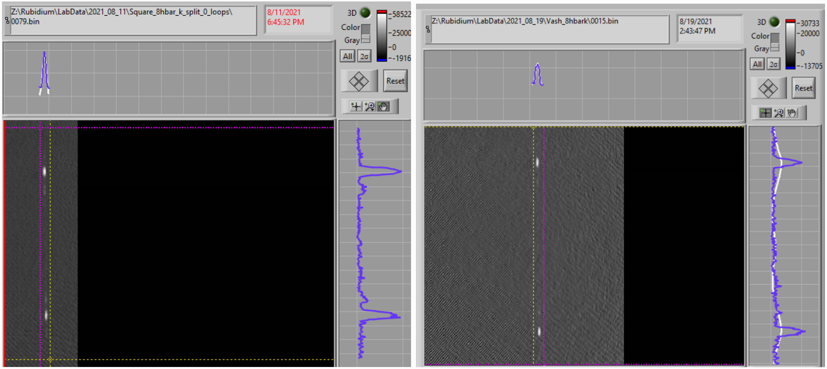


Figure 4: Characeterising the performance of $8\hbar k$ pulses

(Left: Double Square Pulse) (Right: Optimized Pulse)

Summary & Outlook

Summary

- We have found pulse shapes that do better than the current state of art method, i.e., double square.
- The plane wave approximation holds well given the dimensions of our BEC.

Outlook

- Experimental Verification of the results.
- 2D optimization that takes into account sensitivity to both intensity and momentum.

References

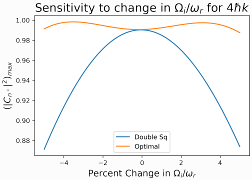
Ying-Ju Wang et al. "Atom Michelson Interferometer on a Chip Using a Bose-Einstein Condensate". In: *Physical Review Letters* 94.9 (Mar. 2005). ISSN: 1079-7114. DOI: *10.1103/physrevlett.94.090405*. URL: *http://dx.doi.org/10.1103/PhysRevLett.94.090405*.

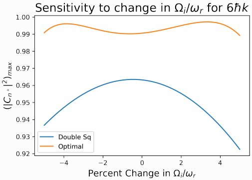
Bonus Slides

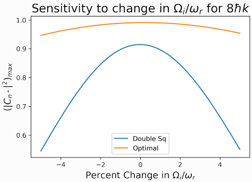
```python
"""Input: List of fidelity values
Returns: Cost"""
def cost_function(list_functions):
    cost = 0
    for i in list_functions:
        delta = 0.98-i
        if delta <0:
            cost +=0
        else:
            cost += delta**2
    return cost
```

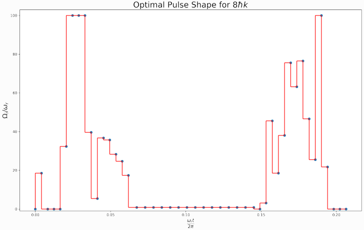
```python
"""Input: Pulse Parameters, which coloumn to maximize
Returns: Cost when I changes by different percentages"""
def real_function(params, number):
    list_arg_sen = params

    # make a list of values of the function for different changes
    percent_list = np.linspace(-5,5,50)
    function_values = []
    function_values.append(square_opt_fixed(params, number))
    for i in percent_list:
        params_changed = changing_intensity(params, i)
        function_values.append(square_opt_fixed(params_changed, number

    optimum_cost = cost_function(function_values)
    return optimum_cost
```

Results

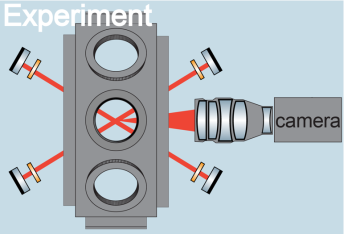

Sensitivity to change in Ω_i/ω_r for $4\hbar k$
$(|C_{n^+}|^2)_{max}$
Percent Change in Ω_i/ω_r
Double Sq
Optimal

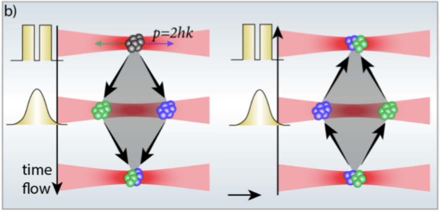

Sensitivity to change in Ω_i/ω_r for $6\hbar k$
$(|C_{n^+}|^2)_{max}$
Percent Change in Ω_i/ω_r
Double Sq
Optimal


Sensitivity to change in Ω_i/ω_r for $8\hbar k$
$(|C_{n^+}|^2)_{max}$
Percent Change in Ω_i/ω_r
Double Sq
Optimal


Optimal Pulse Shape for $8\hbar k$
Ω_i/ω_r
$\frac{\omega_r t}{2\pi}$

Magneto-Optical Trap



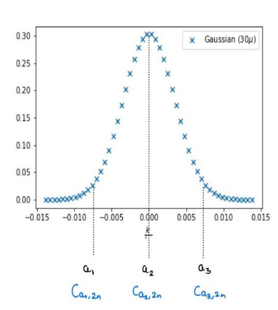


- 3 pairs of counter-propagating beams close to resonance.
- $\sim$ 1ms and 5e4 to slow atom from 300m/s (room temp) to 1m/s (10mK).
- Magnetic field to distinguish center of the trap.
- Doppler cooling, down to $\sim 140\mu K$.
- Sub-Doppler cooling, down to $\sim 15\mu K$.

Results

All works so far assume a BEC with 0 initial momentum. In the lab, we have a confined BEC that has some momentum spread which is given by a gaussian.

- Take in fwhm in position space to make a gaussian for k/k_0 (k_0 initial wavenumber
- Sample points from -3σ to 3σ, call these $a_i's$
- Enforce the normalization condition st $\sum_{i=1}^{n} |a_i|^2 = 1$.
- Then use these coefficients to get the amplitudes in the superposition states



$$P(C_n) = \sum_{i=1}^{n} |a_i|^2 |C_{k_i,n}|^2$$