Optimizing Beam Splitters for Matter Waves

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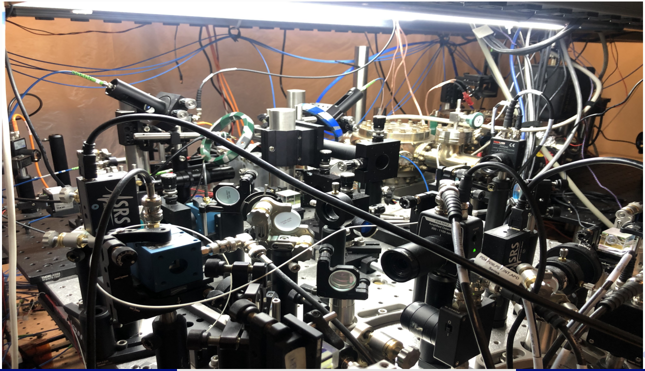


Atom Interferometry

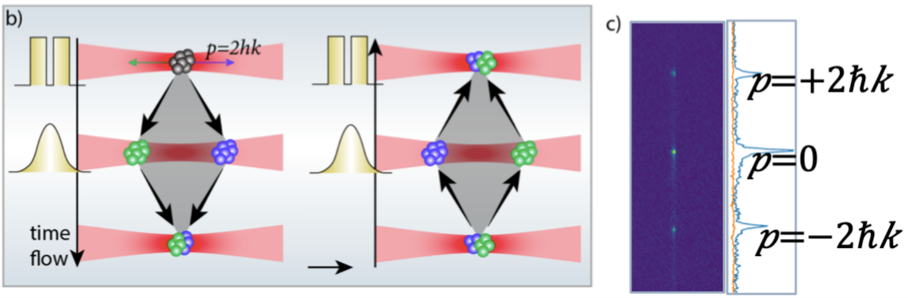
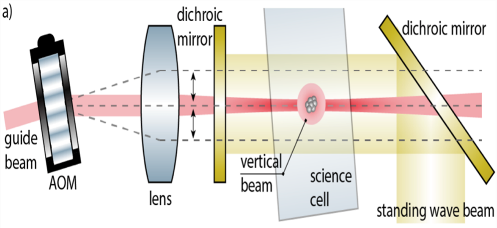
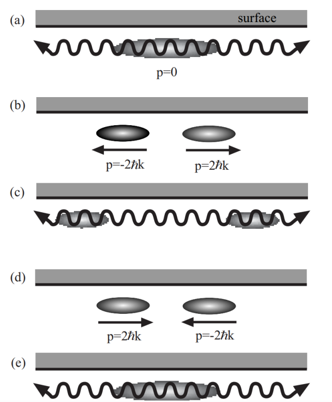


Figure: How do atom Interferometers work?

Overview

- Laser beams coherently split BEC into two equal and opposite momentum packets with $p = \pm 2n\hbar k$. [1]
- Higher momentum states are desired: sensitivity $\propto p$ wave-packets by beam splitting.

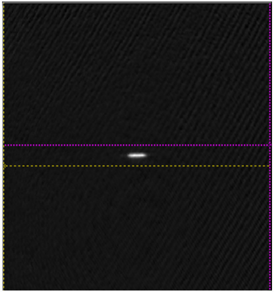




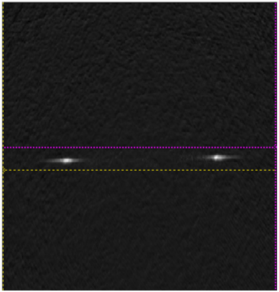
Wang et al. [1]

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

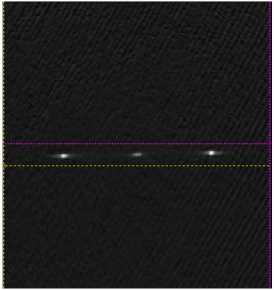
Experimental Setup to study Fidelity



(a) No Splitting
$t = 0\ ms$



(b) Good Splitting
$t = 20\ ms$



(c) Bad Splitting
$t = 20\ ms$

Figure: Absorption images of BECs held in an optical waveguide

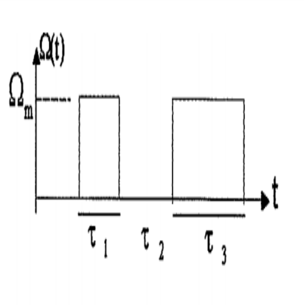
The Known and the Unknown

Desired Qualities

1. High fidelity for higher momentum states.
2. Low sensitivity to ΔI_{laser}

Current State

- Double squared pulses are used in the lab.
- Motivated by experimental discovery [1] and later justified by theoretical work by Wu et al.[2]
- Known: Optimal parameters for $2\hbar k$. [2]
- Low fidelity for higher momentum states.



We want

- Arbitrary optimal shape for a given momentum state.

1: Wang, Ying-Ju, et al. "Atom Michelson interferometer on a chip using a Bose-Einstein condensate." Physical review letters 94.9 (2005): 090405.

2: Wu, Saijun, et al. "Splitting matter waves using an optimized standing-wave light-pulse sequence." Physical Review A 71.4 (2005): 043602.

Our Approach

Treat it as a combination of plane waves

- Expand the wave function in plane waves.

$$\Psi(x,t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_m C_{m,2n}(t)\, e^{i(2nk_0+k_m)x}$$

- Schrödinger Equation becomes a coupled system of linear equation. [2]

$$i\dot{C}_{2n} = \frac{\hbar}{2m} C_{2n}\,(2nk_0+k)^2 + \frac{\Omega}{2}\,(C_{2n-2}+C_{2n+2})$$

- Efficient method of solving for constant intensity:

$$C(t) = e^{-i\omega_r t A} C^{+}(0)$$

$$C = [C_0, C_2, C_4, C_{2m}, \cdots, C_{2n}]^T$$

C_{2n}, C_0: Amplitudes to be in a state

$\Omega(t)$: Strength of Coupling

$A(\Omega, \omega_r)$: Symmetric matrix

ω_r: Recoil frequency

$$\boxed{C_{2m} \uparrow \Longrightarrow \text{Fidelity}_{2m\hbar k} \uparrow}$$

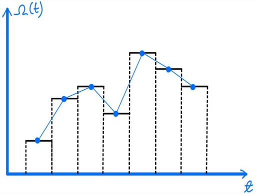
- For non square shapes treat as piece wise constant function.

Method

How do you find the best pulse?

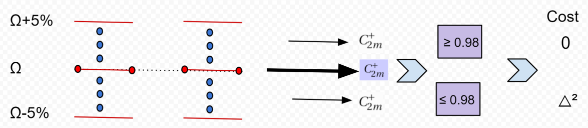
Stage 1: Hunting for highest fidelity

- Tried double squares of different intensities, triangle, and sinc squared pulses.
- Optimizing for intensity and time in each slice with different global and local minimizers.
- For arbitrary shapes, we optimize for time and intensity in each slice.

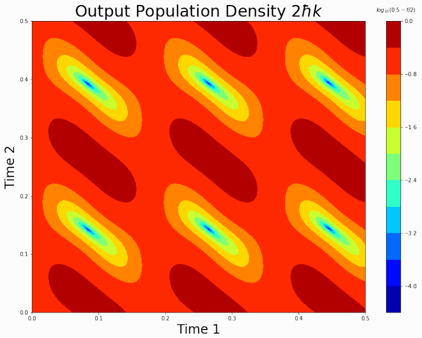


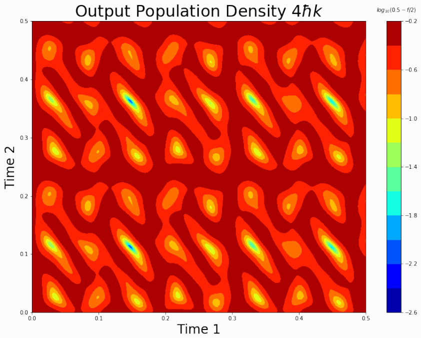
Stage 2: Fidelity is not everything

- Laser intensities drift with time.
- New function now has sensitivity accounted for, and we optimize on both fidelity and sensitivity at the same time.



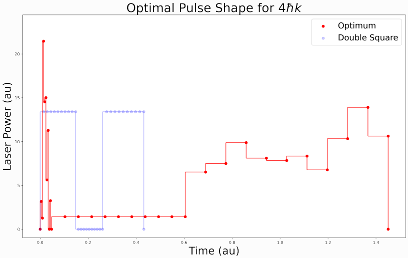
Visualizing the landscape

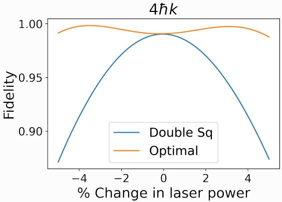


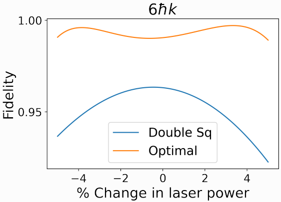


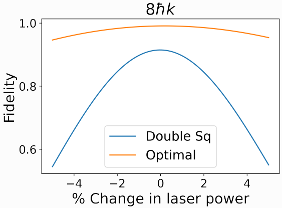
Results

Is the new pulse actually better than double square?



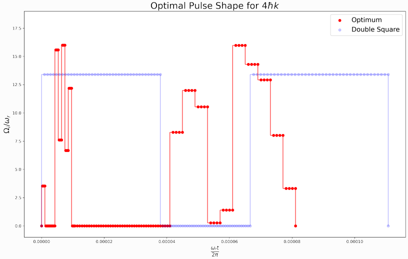


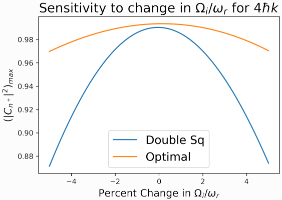


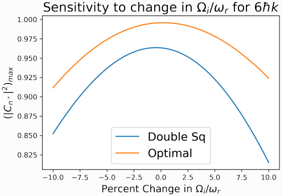


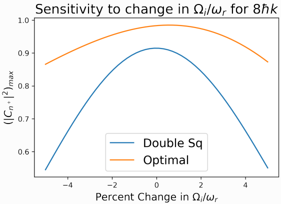
Results

Don't forget the Fourier relationship $\Delta E \Delta t \geq \frac{\hbar}{2}$





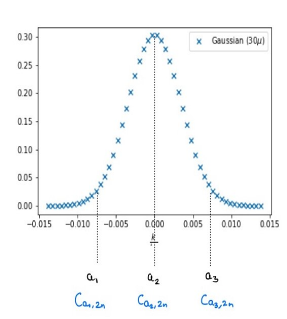




Results

Does it hold the test of momentum superposition?

- So far, our work assumed no momentum spread (δ function).
- In the lab, we have a confined BEC that has some momentum spread which can be approximated by a gaussian.
- Does this superposition affect the results?
  - Our BEC is of full width half max around 40 μ meters.
  - Given this spread in position space, the fidelity of the optimal pulse virtually unaffected.



Experimental Verification

$4\hbar k$ Pulses

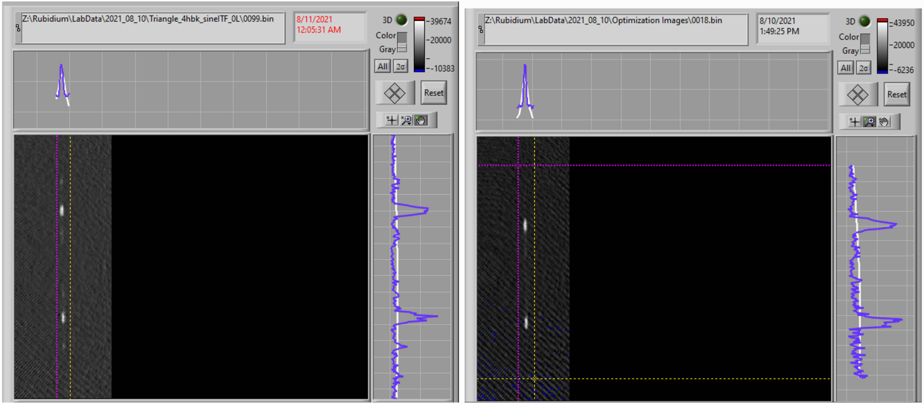


Figure: Characeterising the performance of $4\hbar k$ pulses
(Left: Double Square Pulse) (Right: Optimized Pulse)

Experimental Verification

$8\hbar k$ Pulses

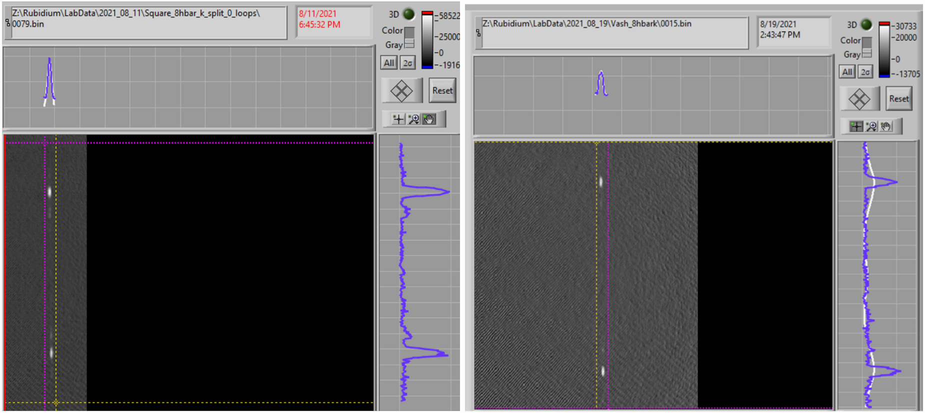


Figure: Characeterising the performance of $8\hbar k$ pulses
(Left: Double Square Pulse) (Right: Optimized Pulse)

Summary & Outlook

Summary

- We have found pulse shapes that do better than the current state of art method, i.e., double square.
- The plane wave approximation holds well given the dimensions of our BEC.

Outlook

- Experimental Verification of the results.
- 2D optimization that takes into account sensitivity to both intensity and momentum.

References I

Ying-Ju Wang et al. “Atom Michelson Interferometer on a Chip Using a Bose-Einstein Condensate”. In: *Physical Review Letters* 94.9 (Mar. 2005). ISSN: 1079-7114. DOI: 10.1103/physrevlett.94.090405. URL: http://dx.doi.org/10.1103/PhysRevLett.94.090405.

Saijun Wu et al. “Splitting matter waves using an optimized standing-wave light-pulse sequence”. In: *Phys. Rev. A* 71 (4 Apr. 2005), p. 043602. DOI: 10.1103/PhysRevA.71.043602. URL: https://link.aps.org/doi/10.1103/PhysRevA.71.043602.

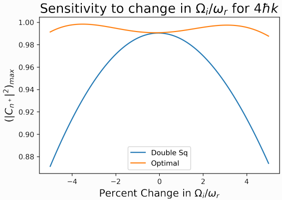
Bonus Slides

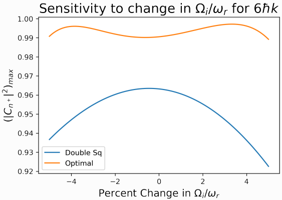
```python
"""Input: List of fidelity values
Returns: Cost"""
def cost_function(list_functions):
    cost = 0
    for i in list_functions:
        delta = 0.98-i
        if delta <0:
            cost +=0
        else:
            cost += delta**2
    return cost
```

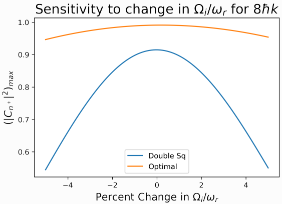
```python
"""Input: Pulse Parameters, which coloumn to maximize
Returns: Cost when I changes by different percentages"""
def real_function(params, number):
    list_arg_sen = params

    # make a list of values of the function for different changes
    percent_list = np.linspace(-5,5,50)
    function_values = []
    function_values.append(square_opt_fixed(params, number))
    for i in percent_list:
        params_changed = changing_intensity(params, i)
        function_values.append(square_opt_fixed(params_changed, number))

    optimum_cost = cost_function(function_values)
    return optimum_cost
```

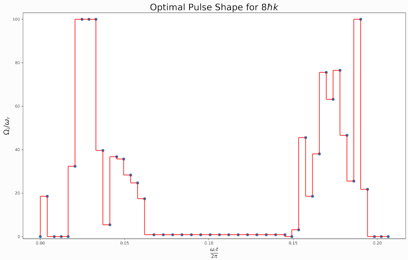
Results

Optimizing the laser shape for higher momentum

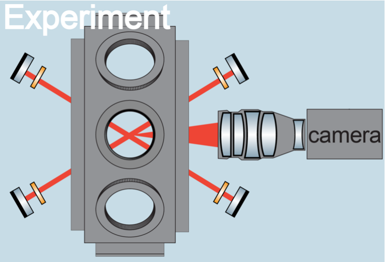


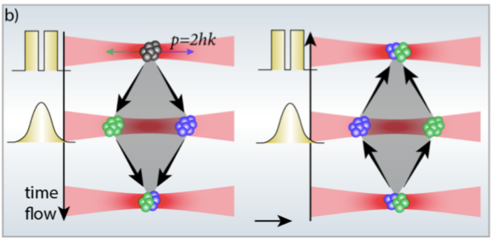






Magneto-Optical Trap





- 3 pairs of counter-propagating beams close to resonance.
- $\sim$ 1ms and 5e4 to slow atom from 300m/s (room temp) to 1m/s (10mK).
- Magnetic field to distinguish center of the trap.
- Doppler cooling, down to $\sim 140\mu K$.
- Sub-Doppler cooling, down to $\sim 15\mu K$.

Results

Does it hold the test of momentum superposition?

All works so far assume a BEC with 0 initial momentum. In the lab, we have a confined BEC that has some momentum spread which is given by a gaussian.

- Take in fwhm in position space to make a gaussian for k/k_0 (k_0 initial wavenumber
- Sample points from -3σ to 3σ, call these $a_i's$
- Enforce the normalization condition st $\sum_{i=1}^{n}|a_i|^2 = 1$.
- Then use these coefficients to get the amplitudes in the superposition states

$$P(C_n) = \sum_{i=1}^{n} |a_i|^2 |C_{k_i,n}|^2$$