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October 10, 2024

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|  | - |  | (sgn(y)) @@2u (x; y) + @2u (x; y) = 0  x2 @y2 | | |  |  | (1) |
| D=D+[D«, D+ =f(x;y): | | | 0<x<Ü; | 0<y<+1g;D« =f(x;y): «y<x<y+Ü; «Ü=2<y<0g | | | |  |
| u(x;y) 2 C2(D+)\C2(D«)\C(D+ [D«) | | | |  |  |  |  |  |
|  |  |  | u(0;y)=0; u(Ü;y)=0; 0<y<+1; | | |  |  | (2) |
|  |  |  | u(x;«x)=f(x); 0×x×Ü=2; f(0)=0; | | |  |  | (3) |
|  |  |  |  | u(x;y)Ö0; y!+1 | | |  | (4) |
|  |  |  | k1 @@uy (x; +0) =@@uy (x; «0); | | | 0<x<Ü; | |  | (5) |
| k2(«1;+1);k=6 0.  D« D+ | |  | (2), (4) | |  |  |  |  |
|  | 1.  2. | k1 @@uy (x; 0 + 0) «@@ux (x; 0 + 0) = «f 0 Þ x2 ß ;  (1) - (5) .  jkj<1; k6=0,f(x)2C[0;Ü=2]\C2(0;Ü=2),f0(x)2L2(0;Ü=2).  X  u(x;y)= 1 Ane«ny sinnx;  n=1 | | | |  | (8)-(11) , | (6) |
|  | An |  | X1 nAn sin [nx + k] = p1 k+ k2 f 0 Þ x2 ß n=1 | | |  |  | (1) |
| 2  1. (1) - (5) .  ., u@1u(x;«y@)u; u2(x; y) (1)-(5). u(x; y) =  @y @x=0 x y . (5) | | | | u1(x;y)«u2(x;y) (1)-(5) k1 @@uy « @@ux jy=0+0 = 0: | | |  |  | f(x)Ô0. u(x;y)=F(x+y)«F(0).  (2) | |
|  | u(x;y)  - (6) | D+ (2),(4),(6).  f(x;y): 0<x<Ü; y=0g. | | | (2) | (4). |  |  |

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|  | , | D« | (1) |  | u(x; y) = F (x + y) + f ( x «2 y ) « F (0): | | | | | (3) |
|  | (7): | |  | @@uy (x; y) «@@ux (x; y)jy=0+0 = «f 0 Þ x2 ß ; | | | | 0<x<Ü: | |  |  |
|  | (5), | |  | k1 @@uy (x; 0 + 0) «@@ux (x; 0 + 0) = «f 0 Þ x2 ß ; | | | | 0<x<Ü: | |  |  |
|  |  | D+ |  |  | @@2u (x; y) + @2u (x; y) = 0 x2 @y2 | | | |  | (4) |
|  |  |  |  | u(0;y)=0; u(Ü;y)=0; 0<y<+1; k1 @@uy (x; 0 + 0) «@@ux (x; 0 + 0) = «f 0 Þ x2 ß ; | | | | |  | (5)  (6) |
|  |  |  |  |  | u(x;y)Ö0; y!+1 | | | |  | (7) |
|  | 2. | jkj < 1; | k=6 0,f(x)2C[0;Ü=2]\C2(0;Ü=2),f0(x)2L2(0;Ü=2).  X  u(x;y)= 1 Ane«ny sinnx;  n=1 | | | | | (8)-(11) |  |  |
|  | (10) | |  | ZÜ â k1 @@uy (x; y) «@@ux (x; y) + f 0 Þ x2 ßã2 dx ! 0; y ! 0 + 0;  0 | | | | |  |  |
|  | An |  |  |  | X1 nAn sin [nx + k] = p1 k+ k2 f 0 Þ x2 ß n=1 | | | | | (8) |
| . |  | fsin[nx + k]g1n=1 | | L2(0;Ü) | k 2 («1;1) | | [2]. |  |  |  |
|  |  |  |  | X  C1kf 0kL2(0;Ü) × n1=1 n2A2n × C2kf 0kL2(0;Ü); 0 < C1 < C2; | | | | |  |  |
|  | C1;C2 | P  f 0. n1=1 jAnj | | (12). (12) | | (8) | (9) | . (11) | P  n1=1 e«ny = 1«e«e«y y = ey1«1 . | (10). |  |

M (x) = k1 @@uy «@@ux + f 0 Þ x2 ß

M (x) = « k1 P1 nAâ ne«ny sin nx « P1 nAne«ny cos nx + f 0 Þ x2 ß =

n=1 nã

= « pnP1=1 nAne«ny k1 sin nxâ+ cos nx=1+ f 0 Þ x2 ß = ã Þ ß

1+k2 P 1 k x = « p k n1=1 nAne«ny p1 + k2 sin nx Þ+ pß1 + k2 cos nx + f 0 2 =

1+k2 P x

== «p1 +kk2 P1n1=1nAnA(n1e«ny sin [nx + k] + f 0 2 =

k n=1 n «e«ny)sin[nx+k]:

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| , | y!lim0+0 I(y) = 0. | ZÜ  I(y)= M(x)2dx×I1(y)+I2(y);  0 |
|  |  | I1(y) = 2p1k+ k2 ZÜ "Xm nAn sin [nx + k] «1 « e«ny¬#2 dx  0 n=1 |
|  |  | I2(y) = 2p1k+ k2 ZÜ "n=X+1 nAn sin [nx + k] «1 « e«ny¬#2 dx  0 m+1 |
| ",  , m , .. | . | X X  I2(y) × C3 n=1m+1 n2A2n(1 « e«ny)2 × C3 n=1m+1 n2A2n < 2" :  X  I1(y) × C4 nm=1 n2A2n(1 « e«ny)2 < 2" |
| 0<y<Ñ, Ñ . . | |  |