INFORMATION THEORY
Master of Logic, University of Amsterdam, 2016
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Practice problem set 5

This week's exercises deal with random walks, noisy channels and error-correcting codes. You do not have to hand in these exercises, they are for practicing only. Problems marked with a ★ are generally a bit harder. If you have questions about any of the exercises, please post them in the discussion forum on Moodle, and try to help each other. We will also keep an eye on the forum.

Problem 1: Random walk on a chessboard

Consider a 4x4 chessboard. We let a knight (who can move 2 spaces horizontally and 1 vertically or 1 space horizontally and 2 vertically) perform a random walk on this chessboard, choosing his move uniformly random every time. What is the entropy rate of this process?

Problem 2: Repetition code

Consider the repetition code R_9. One way of viewing this code is as a *concatenation* of R_3 with itself. We first encode the source stream with R_3, then encode the resulting output with R_3 again. We could call this code R_3^2. This idea motivates an alternative decoding algorithm, in which we decode the bits three at a time using the decoder for R_3, and then decode the decoded bits from that first decoder using the decoder for R_3.

Evaluate the probability of error for this decoder and compare it with the probability of error for the optimal decoder for R_9.

Can you think of reasons to use R_3^2 (instead of R_9) in practice?

Problem 3: Another linear code

Consider the following linear code C given by the generator matrix

$$G^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(a) Find the parity check matrix H.

(b) How many bits can C encode? How long are its codewords? How many different codewords are there?

(c) What is the minimal distance?

(d) Encode the strings 101, 111 according to C.

(e) Decode 1011010, 1110110, 1111110, and 1111111.