INFORMATION THEORY
Master of Logic, University of Amsterdam, 2016
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Practice problem set 6

This week's exercises deal with channel capacities. You do not have to hand in these exercises, they are for practicing only. Problems marked with a ★ are generally a bit harder. If you have questions about any of the exercises, please post them in the discussion forum on Moodle, and try to help each other. We will also keep an eye on the forum.

Problem 1: Channel with memory

[CT 7.36] Consider the discrete memoryless channel with input alphabet $X_i \in \{1, -1\}$ and output $Y_i = Z_i X_i$.

(a) What is the capacity of this channel when $\{Z_i\}$ is i.i.d. with $P[Z_i = 1] = P[Z_i = -1] = \frac{1}{2}$?

(b) Now consider the channel with memory. Before transmission begins, Z is randomly chosen and fixed for all time. Thus, $Y_i = ZX_i$. What is the capacity of this channel when $P[Z = 1] = P[Z = -1] = \frac{1}{2}$?

Problem 2: Source and channel

[CT 7.31] We wich to encode a Bernoulli(α) process $V_1, V_2, ...V_n$ for transmission over a binary symmetric channel with crossover probability p, using the channel n times.

$$V^n \longrightarrow X^n(V^n) \longrightarrow \text{BSC}(p) \longrightarrow Y^n \longrightarrow \hat{V}^n$$

Find conditions on α and p so that the probability of error $P(\hat{V}^n \neq V^n)$ can be made to go to zero as $n \to \infty$.

Problem 3: Encoder and decoder as part of the channel

Consider a binary symmetric channel (BSC) $(\mathcal{X}, P_{Y|X}, \mathcal{Y})$ with crossover probability 0.1. A possible coding scheme for this channel with two codewords of length 3 is to encode message w_1 as 000 and w_2 as 111. The decoder uses majority vote. With this coding scheme, we can consider the combination of encoder, channel, and decoder as forming a new BSC $(\mathcal{X}', Q_{Y'|X'}, \mathcal{Y}')$, with two inputs w_1 and w_2 and two outputs w_1 and w_2.

(a) Draw this new channel and calculate its crossover probability.

(b) What is the capacity of this channel in bits per transmission of the original channel $P_{Y|X}$?

(c) What is the capacity of the original BSC $P_{Y|X}$ with crossover probability 0.1? Compare the two capacities.

(d) Prove the following general result: for any channel, considering the encoder, channel and decoder together (as a new channel from message W to estimated messages $\hat{W}$) will not increase the capacity in bits per transmission of the original channel.

Problem 4: Practice!

If you are done with all of the above problems, take the rest of the exercise session to practice older exercises from previous practice and/or homework problem sets.