Computer Vision 1 - Cheat Sheet

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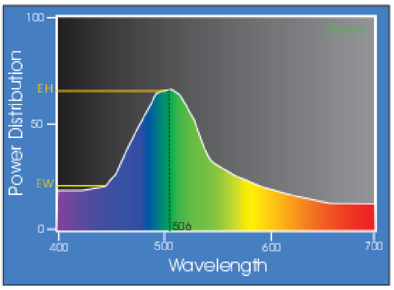
1 Color Models

Color is part of the electromagnetic spectrum with energy in the range from 380 to 780 nm wavelength. Most of the colors we perceive are a mixture of wavelengths where the amount of energy at each wavelength is given by the spectral energy distribution (SED). When white light shines upon an object, some wavelengths are absorbed and other are reflected (a green object will reflect light with wavelength around 500nm, other wavelengths will be absorbed). The **hue** corresponds with the dominant wavelength of the SED; **saturation** is defined as the proportion of pure light with respect to white light needed to produce the color; **lightness** is the intensity of the reflected light meanwhile **brightness** is the intensity of the light source. Let EH be the dominant wavelength in the SED and EW the wavelength contributing to the white light, then the *hue* is equal to EH, the *saturation* equals to the difference $EH - EW$ and the *lightness* equals to the area underlined by the SED.



Experiments have been conducted in which a human observer was asked to adjust three knobs which control the intensity of a three primary colors so to match the (perceived) color of the test light. The three primary lights were additively mixed in and the knobs' values were recorded yielding the so called color matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$. The problem with these was that a negative amount of at least one of the primaries was necessary to produce the full spectra. So the CIE proposed a mathematical transformation, the **XYZ** model, which uses another set of color matching functions: $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$.

The observer perceives color in terms of three color signals based on the trichromacy theory and can be modeled as:

$$C = \int_\lambda E(\lambda)S(\lambda)f_C(\lambda)d\lambda \tag{1}$$

where $C \in \{R, G, B\}$, E is the SPD, S is the light reflected by objects and f_C is the color matching function. If we use $\bar{x}$, $\bar{y}$ and $\bar{z}$ then we have the XYZ color space. To better represent graphically this color space we can compute the xyz values as follows:

$$x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z} \quad z = \frac{Z}{X+Y+Z} \tag{2}$$

where we factor out the intensity. Since the chromaticity values sum to unity, two elements are sufficient to represent a color. When the x and y values are represented on a plane the chromaticity diagram is obtained. We can also infer the hue from the chromaticity diagram: first we need to select a reference white light, the hue is then the wavelength at the spectral curve that intersects the line from reference light through the color point to the spectral curve (this point is G_2). If $||G_1||$ is the distance from the color to the white light and $||G_2||$ is the distance from G_2 to the white source, then the saturation is given by $\frac{||G_1||}{||G_2||}$.

RGB values can be obtained using equation 1 and the $\bar{r}$, $\bar{g}$, and $\bar{b}$ color matching functions. The projection of RGB points on the rgb chromaticity triangle is defined by:

$$r = \frac{R}{R+G+B} \quad g = \frac{G}{R+G+B} \quad b = \frac{B}{R+G+B} \tag{3}$$

In the HSI chromaticity diagram, we compute hue and saturation in the following way: by assuming white light we define a reference point of $r = g = b = 1/3$, the saturation can than be computed as:

$$S_{rgb}(r, g, b) = \sqrt{(r-1/3)^2 + (g-1/3)^2 + (b-1/3)^2} \tag{4}$$

or

$$S(R, G, B) = 1 - \frac{min(R, G, B)}{R+G+B} \tag{5}$$

while the hue is given by:

$$H_{rgb}(r, g, b) = arctan(\frac{r-1/3}{g-1/3}) \tag{6}$$

or

$$H(R, G, B) = arctan(\frac{\sqrt{3}(G-B)}{(R-G)+(R-B)}) \tag{7}$$

A color invariant system contains color invariant models that are more or less insensitive to the varying image conditions. For matte surfaces RGB is sensitive to orientation while rgb is (assuming constant white light) insensitive to orientation, illumination direction and intensity (similarly are S and H). For shiny surfaces H is color invariant.

2 Surface Reflection

The Bidirectional Reflectance Distribution Function (BRDF) is the most general model of light scattering. Describes how much light arriving at an incident direction $\mathbf{v_i}$ is emitted in a reflected direction $\mathbf{v_r}$. It can be written as a function of the angles of incident and reflected light like so:

$$f_{BRDF}(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{radiance\_of(\theta_r, \phi_r)}{irradiance\_at(\theta_i, \phi_i)} \tag{8}$$

Typically BRDF can be split into *diffuse* and *specular* components. The **diffuse** component (*Lambertian* or *matte* reflection) scatters light uniformly in all directions and is associated with the phenomena of *shading*. Light is scattered uniformly across all directions (the BRDF is constant):

$$f_d(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\rho_d}{\pi} \tag{9}$$

but the amount of light depends on the angle between the incident direction and the surface normal θ_i (is independent of the viewing direction $\mathbf{v}$):

$$radiance\_of = \frac{\rho_d}{\pi} I cos(\theta_i) \tag{10}$$

where ρ_d is the surface albedo and I is the source intensity. This is because the surface area exposed to a given amount of light becomes larger at oblique angles. The **specular** (glossy or highlight) BRDF reflection depends strongly on the outgoing light direction. All the incident light energy is reflected in a *single* direction (only when $v_i = v_r$). So the mirror BRDF is a delta function:

$$f_d(\theta_i, \phi_i, \theta_v, \phi_v) = \rho\delta(\theta_i - \theta_v)\delta(\phi_i + \pi - \phi_v) \tag{11}$$

$$radiance\_of = I f_d(\theta_i, \phi_i, \theta_v, \phi_v) \tag{12}$$

Another reflectance model is the **Phong** model which uses an *ambient illumination* component besides diffuse and specular.

3 Image Processing